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Role of the Structural and Thermal Peclet Numbers in the Brass Continuous Casting

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Abstract

The Structural *Peclet* Number has been estimated experimentally by analyzing the morphology of the continuously cast brass ingots. It allowed to adapt a proper development of the *Ivantsov's* series in order to formulate the Growth Law for the columnar structure formation in the brass ingots solidified in stationary condition. Simultaneously, the Thermal *Peclet* Number together with the *Biot*, *Stefan,* and *Fourier* Numbers is used in the model describing the heat transfer connected with the so-called contact layer (air gap between an ingot and crystallizer). It lead to define the shape and position of the s/l interface in the brass ingot subjected to the vertical continuous displacement within the crystallizer (in gravity). Particularly, a comparison of the shape of the simulated s/l interface at the axis of the continuously cast brass ingot with the real shape revealed at the ingot axis is delivered. Structural zones in the continuously cast brass ingot are revealed: FC – fine columnar grains, C – columnar grains, E – equiaxed grains, SC – single crystal situated axially.

Keywords: Fundamentals of foundry processes, Growth Law, Continuously cast brass ingot, Shape of the s/l interface, Structural zones

1. Introduction

A new tendency in the interpretation / description of structure formation has appeared recently. It has suggested a mathematical predictions of the static ingot's structure appearance, [1]. It is based on the analysis of some particular points of the obtained functions which result from the temperature field calculation. In particular, some points of inflection are used to define the structural transformations which have been, at first, described by Prof. J.D. Hunt in his fundamental paper associated with the solid / liquid interface undercooling, [2].

The mentioned mathematical predictions of the structure formation and especially predictions of the structural transitions in the static ingot are subjected to the application to the continuous casting of the brass ingots.

The fundamental role of the so-called contact layer (air gap between brass ingot and graphite which covers uniformly the crystallizer) is considered.

Therefore, a theoretical / analytical model is developed to describe / estimate the temperature of the contact layer. It allows to formulate the Numbers such as *Biot*, *Fourier*, *Stefan* and *Peclet* Number which decide on the analyzed proceeding of the brass ingot solidification and applicable to this technology

So, the role of boundary condition connected with the contact layer is considered, analogously, as in the model for the mushy zone formation, [3].

Finally, the application of the mentioned Numbers allows to define the position of the s/l interface (and simultaneously the position of the solid - liquid zone), in the simplified manner, in the imposed co-ordinate system, and for different velocities of the brass ingot displacement in a given crystallizer.

2. Structural parameters of an ingot

The recently developed method for the ingot's structure forecast is based on the revealing of the points of inflection if they are presented by the calculated function. The analyzed function is the *liquidus* isotherm rate plotted versus ingot solidification time.

2.1. Forecasts of the static ingot structural zones

The numerical simulation of the temperature filed formation during static ingot solidification is simplified in such a way that the ingot shape is transformed into a roll shape, [4]. The competition between the ν - rate of the *liquidus* isotherm movement and the ν_S - s/l interface rate of the displacement is shown in Fig.1 from the mathematical viewpoint.

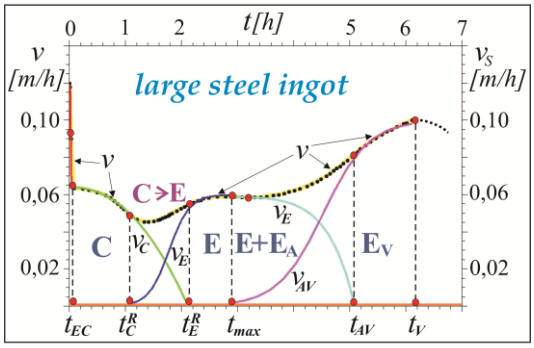

Fig. 1. Correlation between the ν - *liquidus* isotherm movement and
ν_S - s/l interface displacement; $\nu_S = \nu_C$; ν_E; ν_{AV} for columnar, equiaxed, and "A" , "V" – segregates zone formation; t_{EC} - time of the chilled equiaxed into columnar structure formation $E \rightarrow C$, t_C^R, - time of the equiaxed structure birth, t_E^R - time of the columnar structure death; t_{max} - time of the "A" segregates appearance; t_{AV} - time of the so-called "switching point", [1], t_V - time of the beginning of the "V" segregates vanishing; $C \rightarrow E$ - the CE transitions (CET), [1]

Not only points of inflection are useful in the mathematical interpretation of the static ingot's structure appearance but the extreme points (minima / maxima) of the analyzed ν - function as well, Fig. 1.

2.2. Structural *Peclet* Number role in Growth Law for columnar structure formation

A Structural *Peclet* Number can be determined experimentally by means of the columnar grains size estimation, Fig. 2. The exact measurements are to be focused on the estimation / determination of the R_{exp} - tip radius of the columnar dendrite or columnar cells revealed in the longitudinal and transversal sections of an ingot.

The proper definition of the Structural *Peclet* Number (for the Zn-solute diffusion in brass columnar structure) is as follows, [5]:

$$Pe = \frac{v_C \, R_{\exp}}{2\,D} \tag{1}$$

The longitudinal and transversal sections of the continuously cast ingots are used to estimate R_{exp} - tip radius required by Eq. (1), Fig. 2.

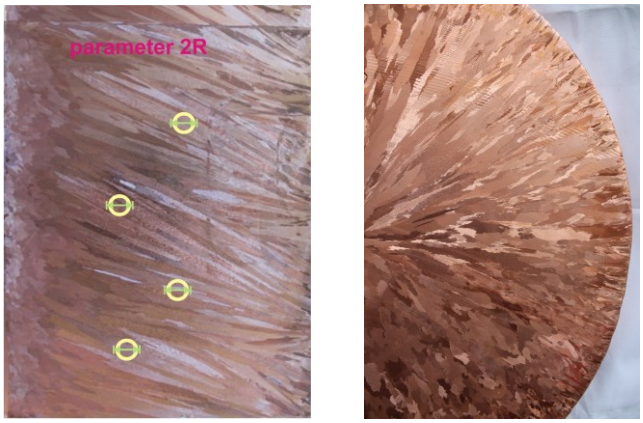

Fig. 2. Columnar / equiaxed structure; a/ principle of the R_{exp} - tip radius estimation (longitudinal-); b/ structural zones (transversal section)

According to the R_{exp} - radius measurements the *Pe* - Structural *Peclet* Number is usually greater than unity but sometimes can be less. In the last-mentioned case it is sufficient to use the so-called second term of the *Ivantsov's* function (development), [6]. The function is equal to the Ω - supersaturation governing the tip radius formation, Fig. 3:

$$I_2 = 2Pe/(2Pe+1) = \Omega \tag{2}$$

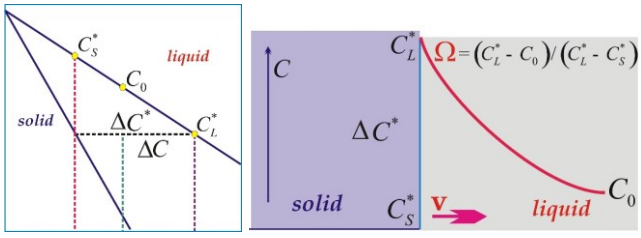

Fig. 3. Definition of the Ω - supersaturation applied to Eq. (2); C_0 is the nominal Zn-solute concentration in a given brass; C^*_L, C^*_S are the Zn-solute concentrations in the liquid and the solid, respectively, as observed at the s/l interface moving with the ν≡ν_S - velocity, [7]

The dendrite / cell tip radius is growing at the limit of stability, [8], that is, at the marginal stability, [9]. Therefore, it can be written that:

$$\lambda_i = R_{th} = 2\pi\,\sigma_E^{0.5}\left[\Delta S\left(m G_C - G\right)\right]^{-0.5} \tag{3}$$

where λ_i is the perturbation wavelength for the marginal stability, σ_E - specific surface free energy, ΔS entropy of fusion, m - slope of the liquidus line, G_C - solute concentration gradient at a columnar dendrite / cell tip, G - thermal gradient at a columnar dendrite / cell tip.

Additionally, it is justified to admit that theoretically predicted tip radius and experimentally estimated tip radius are equal to each other, $R_{th} \equiv R_{\exp} \equiv \hat{R}$. Eq. (3) yields:

$$\Omega = \frac{C_L^* - C_0}{C_L^* - C_S^*} \rightarrow C_L^* = C_0 / \left(1 + \Omega(k-1)\right) \tag{4}$$

with $C_S^* = kC_L^*$, at the s/l interface, Fig. 3. Thus, Eq. (2) gives:

$$I_2 = 2Pe/(2Pe+1) = \left(C_L^* - C_0\right)\left[(1-k)C_L^*\right]^{0.5} \tag{5}$$

Then, Eq. (3) transforms into:

$$\lambda_i = 2\pi \left[\frac{\sigma_E}{\Delta S \left\{ \frac{v_C}{D} \left[(k-1) \frac{C_0}{1-(1-k)I_2} \right] m - G \right\}} \right]^{0.5} \tag{6}$$

where, $G_C = dC/dz = -v_C(1-k)C_L^*/D$. Let admit, (Eq. (3)) that:

$$A = \left(R_{th}\right)^2 \left[m G_C - G \right] = 4\pi^2 \sigma_E \left(\Delta S\right)^{-1} \tag{7}$$

Considering the l.h.s. of Eq. (7), only, it can be written:

$$A = \left(R_{th}\right)^2 \left[m \frac{v_C}{D}(k-1) C_L^* - G \right] \tag{7a}$$

$$A = \left(R_{th}\right)^2 \left[m \frac{v_C}{D}(k-1) \frac{C_0}{1-(1-k)I_2} - G \right] \tag{7b}$$

with $I_2 = 2Pe/(2Pe+1) = \dfrac{2v_C R_{th}}{2(v_C R_{th} + D)} = \dfrac{R_{th}}{R_{th} v_C + D}$. Then,

$$A = \left(R_{th}\right)^2 \left[m \frac{v_C}{D}(k-1) \frac{C_0}{1-(1-k)\dfrac{R_{th}}{R_{th} v_C + D}} - G \right] \tag{7c}$$

$$A = \left(\hat{R}\right)^{-} \left[m \frac{v_C (k-1) C_0}{D\left[\dfrac{\hat{R} v_C + D - (1-k)\hat{R} v_C}{\hat{R} v_C + D} \right]} \right], \quad R_{th} \equiv R_{\exp} \equiv \hat{R} \tag{7d}$$

$$A = \hat{R}^{-} \left[m \frac{(k-1) C_0 \left(\hat{R} v_C + D\right)}{D\left(D + k \hat{R} v_C\right)} - G \right] \tag{7e}$$

Coming back to Eq. (7) it can be transformed into:

$$\begin{aligned} &\hat{R}^{-} \left[m v_C (k-1) C_0 \left(\hat{R} v_C + D\right) - G D \left(D + k \hat{R} v_C\right) \right] \\ &= \frac{4\pi^2 \sigma_E}{\Delta S} D \left(D + k \hat{R} v_C\right) \end{aligned} \tag{8}$$

After some rearrangements, Eq. (8) becomes:

$$\begin{aligned} &\hat{R}^{-} \left[\Delta S v_C \left(m(k-1) C_0 v_C - G D k \right) \right] \\ &+ \hat{R}^{-} \left[\Delta S D \left(m v_C (k-1) C_0 - G D \right) \right] \\ &- \hat{R} \left[4\pi^2 \sigma_E D k v_C \right] - 4\pi^2 \sigma_E D^2 = 0 \end{aligned} \tag{9}$$

with D - diffusion coefficient, k - partition ratio.

Eq. (9) presents the simplified Growth Law for the $\hat{R}$ - tip radius formation at a given solidification rate, v_C (stationary growth of columnar structure). It forecasts the $\hat{R}$ - tip radius of columnar grain when a given solidification rate, $v_S = v_C$, and thermal gradient, G, are imposed upon the system and C_0 - nominal solute content is known.

The developed Growth Law, (Eq. (9), predicts the $\hat{R}$ - tip radius not only for the columnar cells growing in the state of the marginal stability but the tip radius for columnar dendrites as well, Fig. 4.

Eq. (9) means that at a given vertical rate of the ingot displacement along the crystallizer, the resulting growth rate of columnar structure, $v_S = v_C$, and a given thermal gradient at the s/l interface, G, with the imposed nominal Zn - solute concentration in the brass, C_0, the one and only one resultant tip radius (also size of the grain) can be expected in the ingot morphology.

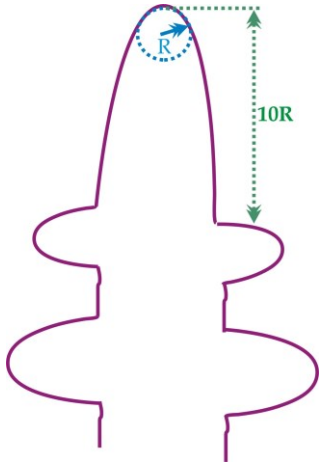

Fig. 4. Definition of the tip radius for columnar dendrites growth (R is the radius of the circle inscribed into the dendrite /cell tip

2.3. Role of the Thermal *Peclet* Number in a continuous casting of the brass ingot

The Thermal *Peclet* Number is useful in modeling of heat transfer in the system for continuous casting, [10-12]. The suggested definition of this Number adapted to the analyzed technology, Fig. 5, is as follows:

$$Pe_T = \frac{u \cdot R_{cr}}{a} \tag{10}$$

where, α is the coefficient of heat diffusion, u - rate of the ingot displacement in crystallizer, R_{cr} - inner radius of crystallizer.

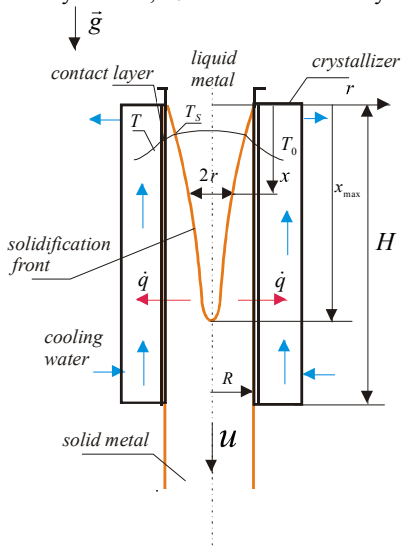

Fig. 5. Model of the system for the continuous casting of brass ingots as shown in the *x, r* - coordinates; T_S, T_L- *solidus, liquidus* isotherm temperature, respectively; T_0- crystallizer average temperature; $\vec{g}$ - gravity; $\vec{q}$ - heat flow

The energy balance for the above system, Fig. 5, is already well described in the literature, [13]. It equates the instantaneous latent heat output and the heat released from the liquid flowing to the currently solidified solid shell across the contact layer, Fig. 5.

Its modified version, adapted to the scheme shown in Fig. 5, is as follows:

$$\begin{aligned} &2\pi \cdot r \cdot \alpha \cdot (T_L - T_S) - 2\pi \cdot \rho \cdot L \cdot (1 - \mathrm{K}) \cdot r \cdot \frac{dr}{dt} = \\ &2\pi \cdot \lambda \cdot \frac{T_S - \overline{T}}{\ln\left(\frac{r}{R_{in}}\right)} = 2\pi \cdot R_{in} \cdot \alpha_{con} \cdot (T_{in} - T_W) = \\ &2\pi \cdot R_{in} \cdot \alpha_0 \cdot (T_W - T_0) \end{aligned} \tag{11}$$

with, α - heat transfer coefficient between liquid metal and solid shell; ρ - density of the liquid; L - latent heat; K - parameter responsible for the heat flux reduction caused by the mushy zone; R_{in}- total radius of the ingot; *r, x* - coordinate system; T_W- crystallizer surface temperature; T_{in}- ingot surface temperature.

The mentioned energy balance allows to reformulate adequate definitions of other Numbers adaptable to the continuous casting of the brass ingot which govern the solidification. There are:

a) *Stefan* Number which defines the ratio of sensible heat to latent heat,

$$Ste = \frac{c\,(T_F - T_0)}{L} \tag{12}$$

b) *Fourier* Number which defines the ratio of diffusive transport rate to storage rate,

$$Fo = \frac{a\,t}{R_{cr}} \tag{13}$$

c) *Biot* Number which defines heat transfer resistances inside an ingot and contact layer to heat transfer resistances on a contact layer surface,

$$\frac{1}{Bi_0} = \frac{\lambda}{\alpha_0\, R_{cr}} \tag{14a}$$

d) *Biot* Number which defines heat transfer resistance inside a contact layer to heat transfer resistance on the contact layer surface,

$$\frac{1}{Bi_{con\ lay}} = \frac{\lambda}{\alpha_{con}\, R_{cr}} \tag{14b}$$

with, c - specific heat of the liquid brass; t - time, λ - coefficient of thermal conductivity; $T_F \equiv T_L$ - in first approximation; α_0 - coefficient of the heat transfer between crystallizer and cooling water; α_{con} - coefficient of the heat transfer across the contact layer's air , Fig. 5.

The application of the above Numbers to the simulation of the continuous casting of the brass ingots allowed to predict the shape of the s/l interface in the *x, r* - coordinate system, Fig. 6.

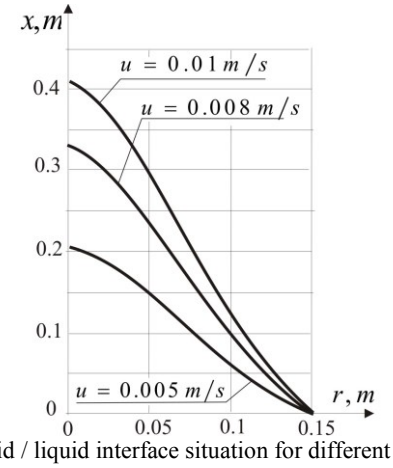

Fig. 6. Solid / liquid interface situation for different rates of the vertical displacement of a brass ingot in the crystallizer

The Pe_T =7.53 for u=0.005 [m/s], Pe_T =12.05 for u=0.008 [m/s], and Pe_T =15.06 for u=0.010 [m/s], Eq. (10).

Some different structural zones observed in the steel static ingots, [1], [14], can also be revealed in the continuously cast ingots, [15]. Particularly, the shape of columnar crystals is significant, [16], [17], since this structure is dominant in the ingot's morphology, Fig. 2.

However, the current method of solidification simulation for the continuous casting of ingots is not able to forecast precisely structural zones' in the continuously brass ingot, Fig. 6.

3. Concluding remarks

The description of a static steel ingot solidification illustrated by the behavior of the velocity of *liquidus* isotherm movement, Fig. 1, proves that the CET transformation occurs during the $t_E^R - t_C^R$ - period of time:.

Some structural observations suggest that the CET is a sharp transformation when analyzing the continuous casting of ingots, Fig. 7.

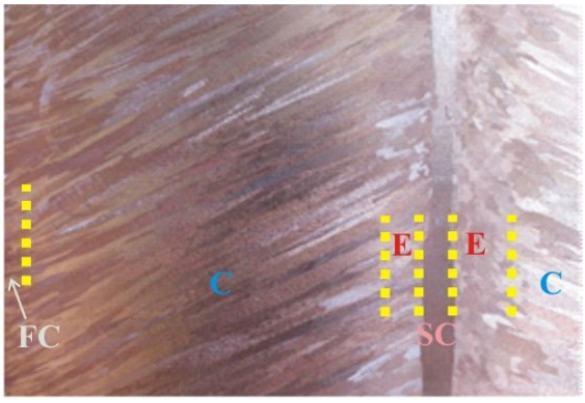

Fig. 7. Structural zones in the continuously cast brass ingot: FC – fine columnar, C – columnar, E – equiaxed structure; SC – single crystal (core) situated axially along the ingot

Some estimations of the R_{exp} - tip radius of columnar structure (half the width of columnar grains) allowed for calculating the Structural *Peclet* Number, Eq. (1) when the velocity of the ingot movement along the crystallizer is known (supported by the estimation of the $v_C(u)$).

The Growth Law for the dominant columnar structure formation has been developed with the application of the marginal stability criterion and for the Structural *Peclet* Numbers less than unity, Eq. (9).

Moreover, the analogous Growth Law can also be obtained for the Structural *Peclet* Numbers greater than unity using the presented method of the Growth Law formulation. In this case, a proper *Ivantsov's* function developed until the third term is to be applied .

The model for heat transfer during continuous casting of the brass ingots has been formulated, Fig. 5, with the use of the Thermal *Peclet* Number, Eq. (10). The contact layer (air gap between ingot and crystallizer) plays essential role in the model.

The values of the Thermal *Peclet* Number in function of the velocity of the ingot displacement along the crystallizer $Pe_T(u)$ can be estimated due to the model application.

The model did not allow to calculate the velocity of the *liquidus* isotherm movement in function of time (on analogy of Fig. 1).

However, the position of the s/l interface in the moving brass ingot can be presented, Fig. 6.

Moreover, it can be concluded, in the first approximation, that the points of inflection of the presented s/l interface shapes are responsible for the CET transformation, Fig. 8.

The points of inflection, visible in Fig. 8, are connected by the dotted line which intersects the calculated shapes of the s/l interface.

The intersections of this line with the calculated shapes allow to emphasize, intuitively, that the CET transformation appears at the greater ingot radius (later in time) when the u - velocity of ingot displacement (and resultant rate of s/l interface movement) increases.

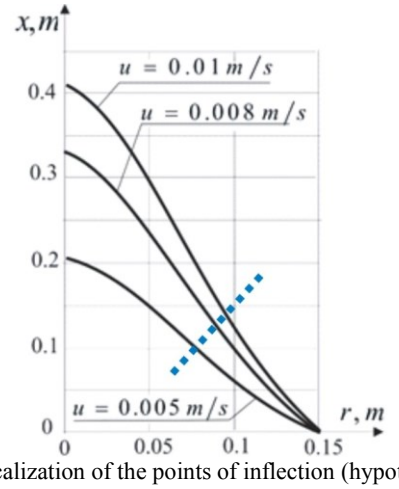

Fig. 8. Localization of the points of inflection (hypothetical localization of the sharp CET) *r=0* defines the ingot surface, *r=0.15* is connected with the ingot axis; *r* - distance from the ingot surface (directed to the ingot axis of symmetry)

The shape of the s/l interface is ending sharply at the ingot axis, Fig 8. It means that the current, simplified model, Fig. 5, Eq. (11) is not able to predict the formation of the SC - single crystal (core) situated axially as that revealed on the longitudinal section of the brass ingot, Fig. 7. In this case, the interpretation of temperature field behavior in function of the v_S velocity could be employed, [18].

Therefore, the more sophisticated model, [19], could replace the current considerations, in the near future. However, the suggested model is to be modified in order to be adaptable to the continuous casting of the brass ingots. In this case, the air gap should also be taken into account in the simulation, that is, some boundary conditions connected with this air gap are to be introduced into the mathematical description of the brass ingots solidification.

Additionally, the analysis of the thermal gradient changes during solidification of the continuously cast ingot is necessary to determine the CET transformation, [20].

Acknowledgements

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