

Designing the optical system with a real time lighting effect control

K. Kubiak 

Electrical Power Engineering Institute, Warsaw University of Technology,
75 Koszykowa St., 00-662 Warsaw, Poland

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Abstract

In the paper, an effective way to design asymmetrical optics for a uniform vertical surface illumination was presented. Assessment of the obtained distribution of luminance (illuminance) on the illuminated surface is done almost at the same time as designing the optical system elements. Advantage of the final application of the presented method in 3D will be independence from the implementation of time-consuming simulations in order to verify the already designed optics. Understanding the method and its application is simple and intuitive. Observing the luminance distribution, created on the illuminated surface almost at the same time as its design, allows to see the effect of adding the next elements of the optical system on this distribution.

1. Introduction

Electric lighting has a significant impact on human beings. Suitable lighting gives visual performance that is crucial for correct, efficient, and safe working [1]. In addition, it was recently found out that lighting affects our health because of the circadian rhythm [2-4].

As part of suitable lighting, it is desirable, among others, to ensure its uniformity on the work plane [1]. Uniform lighting is needed because its aim is to allow the work on the illuminated plane regardless of its part. For example, a shop assistant cannot be forced to go from one end to the other of a shop counter to read the invoice data when dealing with a complaint. Uniform lighting also ensures that the observation of individual parts of the illuminated plane takes place under the same lighting conditions as, e.g., a painting in an art gallery.

A common situation is when the work plane is horizontal, e.g., a table surface or a floor surface. Obtaining uniform lighting parameters is not difficult in this case. Choosing a luminaire with a suitable distribution (e.g., symmetrical with a certain beam angle) is what is just needed. Next, placing one or more of these luminaires in

array above the work plane is also needed. Various types of luminaires are available on the market: spot, linear and even surface ones. Obtaining good lighting results is possible, due to the fact that the luminaire or luminaires can be located directly above the illuminated surface in any arrangement in space.

A different situation is illustrated in Fig. 1. When the work plane is positioned vertically, e.g., a painting or a wall, it is difficult to illuminate it uniformly. The luminaires cannot be placed in front of such a surface (as it was in the case of horizontal surfaces) because the observer will appear between the luminaire and the illuminated surface [Fig. 1(a)]. This will create shadows on the surface which we want to illuminate uniformly. A good solution in such situations may be the use of recently fashionable self-luminous surfaces, but they are useful only when we do not work on such a surface [5,6]. In this case, the object of the work, e.g., an image, will be in a shadow which will prevent it from working. Such self-luminous surfaces can be used successfully on ceilings and walls, but only when there are no objects on them (because they will be in the shadow). Otherwise, it is necessary to arrange the luminaires asymmetrically to the illuminated surface [Fig. 1(b)]. In addition to the asymmetrical arrangement of luminaires, it is also necessary to use asymmetrical optics in luminaires to make such a lighting solution efficient.

Corresponding author at: e-mail: kamil.kubiak@ien.pw.edu.pl

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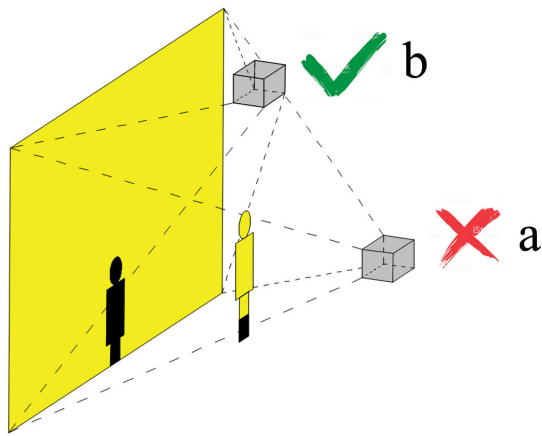


Fig. 1. Symmetrical a) and asymmetrical b) lighting conditions.

Asymmetrical lighting is associated with an asymmetrical assembly of luminaires to the illuminated surface and asymmetrical luminaires optics (photometry). Asymmetry of luminaires assembly is easy to achieve, however, luminaires photometric asymmetry requires a more complicated optical system of the luminaires. Figure 2 illustrates common problems connected with asymmetrical lighting caused by insufficient optics in luminaires (in this case, optics with rotational symmetry). These problems include excessive lighting of the surface closest to the luminaire (points "a" in Fig. 2) in relation to the rest of the surface (i.e., non-uniform lighting). At the same time, around the luminaire there is a sharp border of light and shadow in the shape of a parabola or an inverted obtuse trapezoid (point "b" in Fig. 2). The next problem is that the significant part of the luminous flux is directed away from the illuminated object causing light pollution because the greatest luminous intensities are directed vertically upwards. The mentioned above problems are the consequence of the insufficient optical system design.

Thus, making sure that the optics of the luminaire is properly selected is an important issue in asymmetrical lighting ensuring uniform illumination of the surface. The

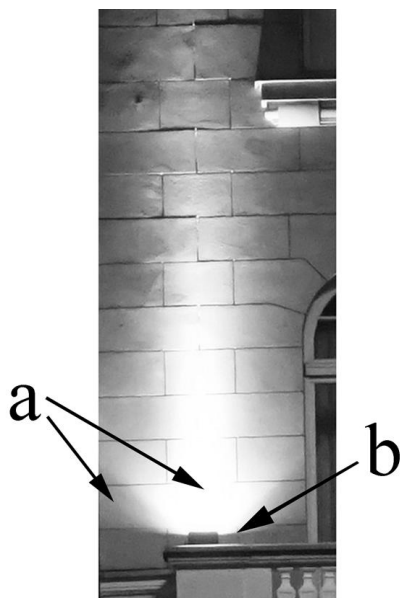


Fig. 2. Common problems with asymmetrical lighting.

optics design process should be efficient because of the current level of technology development in the field of simulations, as well as techniques for production and processing of optical elements. The entire process of creating lighting equipment cannot be slowed down by designing an optical system.

Obtaining a suitable (uniform) asymmetrical lighting should be focused on the resulting distribution of luminance or illuminance on the object's surface. The shape of such a distribution should be constantly controlled during the optical system design process. It is not enough to perform many time-consuming analytical and geometrical operations and, then, control the obtained distribution using computer simulations [7-9]. This leads to a danger of creating unacceptable designs and starting work from the beginning. Therefore, there is a need to control the achieved lighting effect (luminance or illuminance distribution on the illuminated surface) during ongoing operations of optics design and not only after many operations and verification simulations. In his research, the author has found the way to design asymmetrical optics efficiently, so as to evaluate the obtained luminance (illuminance) distribution on the illuminated surface during subsequent operations in the field of designing elements of the optical system.

2. Method

2.1 Contemporary design methods

Designing optical systems is still time-consuming because it is done "manually" by an optical designer performing iterative analytical and geometrical operations in order to meet the set of requirements [7-11]. Some simulation tools, of course, in addition to performing simulations of the "manually" obtained optical system also allow a designer to automate some of these time-consuming design activities [12]. However, applicability of these automations is rather limited and suitable for simple constructions only.

Most of the optical design methods are based on a specific (reference) distribution of illuminance on the illuminated surface we are willing to achieve. Next, distribution of the luminous flux incident on discrete elements of the illuminated surface is determined which, then, will create a reference distribution of illuminance. The determined luminous flux distribution is next compared with the luminous flux distribution emitted from the light source. Any inequality in this comparison is corrected by introducing reflective and/or refractive surfaces in optical system [8-11].

In the above approach, we use the luminous flux (power) expressed as the product of luminous intensity and solid angle. Thus, in the design process, we use luminous intensity in a proposed direction and it does not directly provide information on the obtained lighting effect. Only the introduction of a distance between the light source and the illuminated object and their mutual angular position gives information on the obtained illuminance which distribution is the most important for us. Therefore, disadvantage of this approach is that during so many operations we know nothing about the achieved lighting effect (distribution of illuminance or luminance on the

illuminated surface). We can perform simulations to know the obtained distribution of illuminance or luminance on the illuminated surface only after completing a series of actions and when we have the shape of the entire surface of the optical system. At this point, differences occur while comparing the reference distribution with the obtained distribution as the result of the simulations. They result, among others, from commonly accepted assumptions about a point light source [13-15]. Of course, the larger the optical system relative to the light source used, the smaller these differences are. However, they should not be ignored. The mentioned above approach is not efficient enough. It would be advisable to have a design method verifying these distributions generated on the illuminated surface during the optical system designing process. According to the author's knowledge there is no such a method in scientific literature. Thanks to the author's new approach, distributions can be obtained using fast calculations without time-consuming simulations. This would be a significant simplification and, thus, facilitation of designing process.

2.2 New designing method

To solve the problem, we can be inspired by the process of floodlighting designing [16]. The purpose of floodlighting is to obtain a specific distribution of luminance on the illuminated facade, understood as brightening of certain parts of the facade. In order to obtain such an effect, we select luminaires with an appropriate power and an appropriate luminous intensity distribution, place them in the right places and aim accordingly. Observation of the effect allows us to accept a given solution or to introduce some changes (no matter whether we are dealing with field trial or simulations) [16]. We can do the same when designing individual components of the optical system. We can imagine the illuminated surface and the light source placed in its vicinity. It illuminates this surface in a certain way, usually unacceptable from the point of view of our design assumptions. Therefore, we are introducing an optical system that will change/adjust the resulting lighting effect. Each next optical element (e.g., a rectangular reflective surface with dimensions similar to

the dimensions of the light source) placed near the light source and properly aimed will illuminate the surface by changing the existing luminance distribution on the object's surface. By adjusting dimensions, position and aiming of such a reflective surface, we can improve the existing distribution of luminance on the object's surface. By introducing subsequent reflective and/or refracting elements, we obtain the surface illumination which is convergent with the reference surface illumination.

The mentioned above comparison of an optic design and floodlighting one was used to create a block diagram using the author's designing method. The block diagram is presented in Fig. 3. The results comparison (obtained luminance distribution or illuminance distribution on the illuminated surface) with the reference (expected) distribution is the most important to us. Luminance and illuminance distributions are linear-dependent on each other because we assume diffuse reflection properties of the illuminated surface. At the beginning, however, there is a specific light source with its specific dimensions, luminance distributions on its surface, luminous intensity distributions, and its position. The light source, the illuminated surface and the reference distribution of luminance or illuminance are assumed in advance, and we do not change them during the designing process. Then, the iterative process of material selection, dimensions, positioning relative to the light source and aiming of optical elements takes place. Preview of the resulting distribution of illuminance or luminance on the illuminated surface and its comparison with the reference distribution is an inseparable element of these iterations. It allows us to assess previously performed actions and perform the next ones. In addition, the preview of the optical system resulting geometry is available. This is the general idea of the design (calculation) method. However, such a block diagram is commonly used. It concerns the entire optical system, not its individual elements like in Fig. 3. In the case of the entire optical system, the "optical element" block should be replaced with the "optical system" block. Thus, we have a global analysis instead of a local one. However, all changes can only be made at the local level and this paper focuses on this issue.

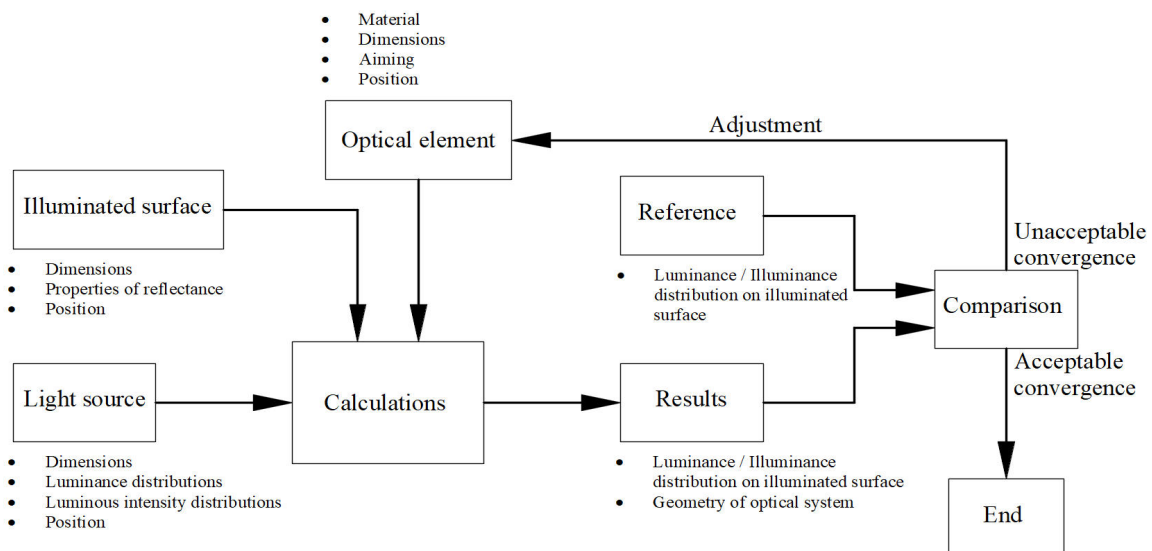


Fig. 3. Block diagram of the author's idea of designing method.

In order to implement this method, a certain assumption has been made namely that calculations are performed on a plane (2D). Therefore, the light source solid is changed for its cross-section, the flat illuminated surface for its cross-section (line), and the reflector's surface for its profile (line). When switching to 3D, this 2D approach leads to the formation of a trough-shape optical system which is the result of extruding a profile along the vector perpendicular to the plane with the profile. Such a trough-shape optical system can be manufactured. However, in terms of the designing method in 2D, only the luminance distribution along one (characteristic) line on the surface of the illuminated object is known, i.e., the vertical axis of symmetry of the resulting illuminated area. Such a limitation of the calculated luminance distribution on the object's surface to one line does not disqualify the work at the current stage, when the analysis of a uniform illumination of the object's surface is limited to the analysis of variability of the created luminance only to one (characteristic) line. It can be assumed that the primary goal is that along this line the luminance distribution is uniform and illuminates the object as high as possible. Distribution of luminance on the entire object can be obtained by means of time-consuming simulation, but this is done as part of the final verification, and not as part of the subsequent designing activities of the optical system.

Quasi-point LED light source Lumileds LXS9-PW30-0017 was chosen [17] because implementation of the designing method is in 2D. Its model used in the calculations of the described designing method consists of three parts: one geometric part and two luminance parts. The geometric part is a cross-section plane geometry of the light source solid. It gives a line segment that lies on the surface of the light source and its length is equal to the length of the light source. Of course, we can rotate the light source by about 90° and in this case the length of the obtained line segment is equal to the light source width. The luminance parts contain two sets of luminance distributions. The first set is luminance distributions of the LED luminous surface registered with the matrix luminance meter during meter rotation around the LED in one plane with a properly selected step. The second set is luminance distributions also registered with the matrix luminance meter but on the plane of a measuring screen. This screen is only illuminated by the light source. Each measurement has the same location of the light source to the measuring screen but different rotation of the light source to its own axis. Luminance distributions can be obtained from the mentioned above laboratory measurements or simulations but with the guarantee that the results obtained are sufficiently accurate.

With this light source model, the following transformations can be made. If a flat reflective optical element with unlimited dimensions located near the light source is considered, the luminance distribution created on the illuminated surface can be obtained by simply transforming one of the luminance distributions on the measuring screen which is the part of the light source model. The measuring screen that is in the same direction relative to the light source as the reflective element that we analyze needs to be selected. Transformation consists of scaling each of discrete luminance values of the measuring screen by a factor depending on a distance and a mutual

orientation change between the light source and the object. The object here is first, the measuring screen and, then, the illuminated surface. As a result, the luminance distribution is obtained, formed on the illuminated surface after applying an optical system consisting of a light source and an unlimited reflective surface. Of course, the obtained results should be supplemented with a luminance distribution derived from surface illumination by the light source itself. This distribution is also one of the luminance distributions registered on the measuring screen.

The next step is to make this situation more realistic, i.e., to limit dimensions of the reflective surface used. These dimensions can be freely limited. It should be remembered, however, that from the moment when dimensions of the reflective surface no longer guarantee the reflection of the light source entire image, the luminous intensity of the reflected beam from this reflective element will begin to decrease. This decrease is relative to the initial value which is equal to the light source luminous intensity towards this reflective element times reflectance. When limiting the surface dimensions, we should scale each of discrete luminance values on the surface of the illuminated object obtained from the use of a given reflective element. The scaling factor will depend on the extent to which the image of the light source visible on the reflective surface from this direction has been limited. The limitation of this image directly translates into a decrease in luminous intensity and, thus, generated luminance on the illuminated surface. The above described transformations are devoid of many details due to the limited length of this publication. However, these details are available in Ref. 18.

We use this method for the remaining introduced flat reflective elements. We introduce one reflective element right after another, so they touch each other. Then, described transformations are performed to obtain the luminance distribution on the illuminated surface originating from the light source, previously added reflective elements and currently considered ones. The best solution is to automate such transformations.

3. Results and Discussion

The author's designing method aims to enable the assessment of the obtained luminance distribution on the illuminated surface almost at the same time as designing next elements of the optical system.

3.1 Implementation of the new designing method

All the above described transformations have been implemented in 2D into an Excel spreadsheet, constituting the trial of the designing method. The reasons for using Excel were the availability of such a program, the use of its functions and the small volume of data processed. The screenshot of the spreadsheet is presented in Fig. 4. In addition to the mathematical functions layer, the spreadsheet shows the obtained luminance distribution on the illuminated surface along one (characteristic) line and allows us to enter a series of input data. The input data include dimensions of the light source and two sets of luminance distributions associated with the light source. These data are entered at the beginning of the design process and do not change later unless we decide to change

reflector height [mm]	20
reflector aiming [deg]	141.4
reflector start point x [cm]	-4,00
reflector start point y [cm]	-3,00
direction of increase +x=1, -x=0	0
housing start point x [cm]	5
housing start point y [cm]	9,4
distance to object [cm]	100
arc tube radius [mm]	4,191
distance to screen [cm]	100
reflection index	0,63
L=1, E=0	1
constant level L / E	12

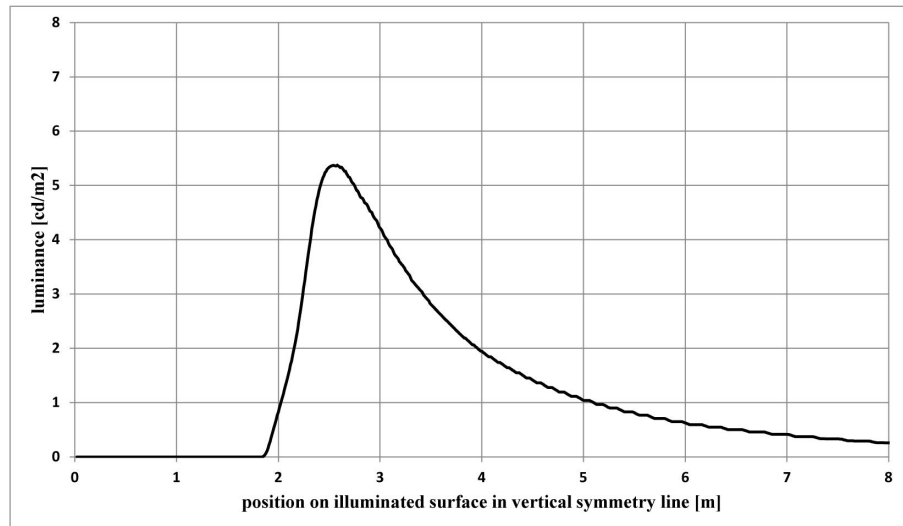


Fig. 4. The screenshot of the author's design method spreadsheet.

the light source. These data come from laboratory measurements or simulations. On the other hand, location, dimensions and aiming of the reflector's analyzed element are subject to current changes and are also input data.

3.2 Application of the new method in designing a trough-shape reflector

The following paragraph presents a method in 2D of designing a 3D trough-shape reflector working with the LED light source using the described spreadsheet. After choosing the right light source for the application and taking into account its parameters we conduct its laboratory measurements or simulations. The purpose of these measurements or simulations is to collect a set of luminance distributions of the luminous surface of the light source and a set of luminance distributions of the measuring screen, as well as basic geometric dimensions. Then, we organize the obtained luminance distributions, e.g., assigning individual luminance values to their actual position relative to the coordinate system associated with the luminous surface of the light source or with the measuring screen surface, respectively. Having such luminance distributions prepared, we put them in the appropriate sheets. Then, we enter the coordinates of the ends of the segment constituting the cross-section of the light source luminous surface with a perpendicular plane. In addition, we determine the height of the object's flat illuminated surface and its distance from the center of the light source (1 m in this case), and, most importantly, the reference luminance distribution on the illuminated surface that we want to obtain. Having the parameters defined in this way, the spreadsheet determines the shape of the luminance distribution along one (characteristic) line formed on the illuminated surface as the result of using only the light source. This distribution is presented in Fig. 5. This is the base point for further design activities. At this point, it is known to what extent the reference luminance distribution on the object's surface is obtained by the light source itself. This is valuable information because we know in what direction we need to strengthen the light to weaken it, and to eliminate it completely. Then, we move on to designing individual elements of the optical system. We

choose whether they will be specular reflection elements or those scattering the light beam reflected. The spreadsheet allows calculations for both of these reflection characteristics [19].

Suppose we choose a specular reflection because we want to strengthen the luminous intensity. We define the geometry of the reflector's element according to the designations in Fig. 6. Assuming that the light source will be located at the bottom of the asymmetrical trough-shape reflector, we determine the first point of the reflector (x; y) close enough to the light source to ensure appropriate thermal conditions. Then, we pre-determine the height of this reflector's element (h) and its aiming (α). When we enter these data, we can immediately see where and what is the shape of the luminance distribution on the illuminated surface along one (characteristic) line which is effect of this

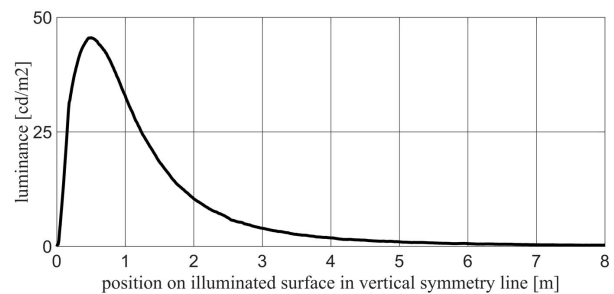


Fig. 5. The luminance distribution along one (characteristic) line formed on the illuminated surface as a result of using only the light source.

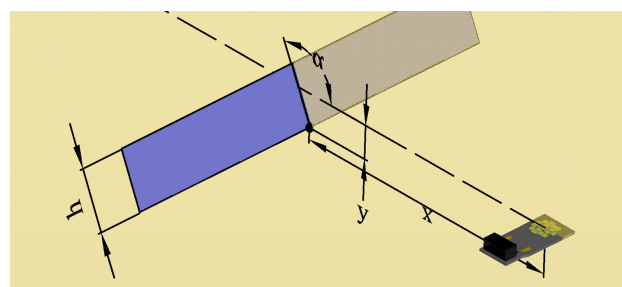


Fig. 6. Definition of the reflector element geometry.

optical element (Fig. 7). By changing the aiming, we change this luminance distribution to complement the existing one to meet the reference luminance distribution. Next, we choose the height (h) of the reflector to obtain satisfactory results. We repeat this procedure for the next reflector's element. When we reach the assumed dimensions of the optical system or the introduction of next elements of the reflector, it does not introduce significant changes in the luminance distribution obtained on the illuminated surface, so we finish this part of the reflector and begin to shape the opposite side of the reflector. Performing subsequent steps and observing their effects at the time of designing process in a form of luminance distribution along one (characteristic) line, we come to the final form of the optical system. Ultimately, we obtain the reflector's profile and the resulting luminance distribution on the illuminated surface along one (characteristic) line. Figure 8 presents the resulting luminance distribution along one (characteristic) line on the illuminated surface from the designing method and its verification in Photopia™ simulation tool [20]. Comparison of the curves shows their convergence. Figure 9 shows the obtained geometry of the optical system after extruding the profile and including additional elliptical end caps. The results obtained are the effect of making subsequent design decisions based on the comparison of the obtained luminescence distribution with the reference distribution.

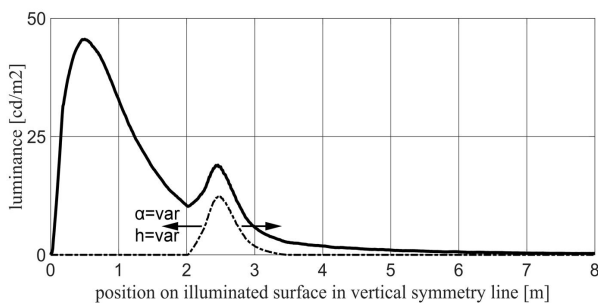


Fig. 7. The luminance distribution along one (characteristic) line on the illuminated surface from the optical element.

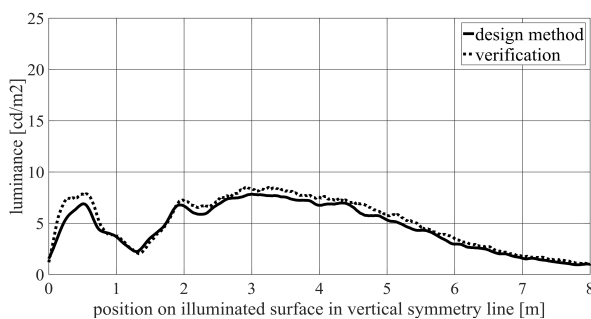


Fig. 8. The comparison of resulting luminance distribution along one (characteristic) line on the illuminated surface.

3.3 Advantages of the new designing method

The main advantage of the author's method is an opportunity to observe the luminance distribution formation at the time of designing process despite the fact that the method is implemented in 3D or 2D as a prototype.

Advantage of this method in 3D is also that the designing process is independent from time-consuming

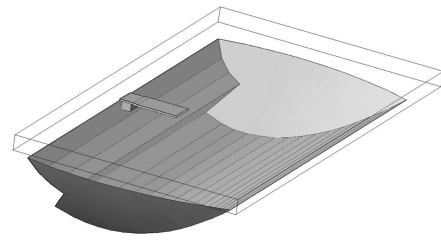


Fig. 9. The obtained geometry of the optical system.

computer simulations in order to verify already designed optics. Performing a simulation always requires time and having the optical system geometry in the appropriate graphic format. We load such a geometry into the simulation application, ensuring its correct positioning and orientation relative to the coordinate system in the application. Then, we insert a properly located and space-oriented light source from the program library. Finally, we define simulation parameters. In the case of a random optical system analysis, for example in the Monte Carlo method, we perform a simulation which duration is directly proportional to the accuracy of the results obtained. If we do not have much time, we will only get approximate results [21].

By eliminating this time-consuming and laborious stage of performed activities verification of designing the optical system in the form of simulations, we accelerate the process of making subsequent design decisions. We do not need to perform simulations to see how close we have been to the reference distribution of illuminance or luminance on the illuminated surface. We acknowledge this during introducing changes to the optical system. In the era of needing customized luminaires, designing speed has become important. We use 3D printers more and more often and they print elements of the optical system quickly [22]. This directly cuts time and costs of implementing new luminaires.

Understanding the method and its application in the form of the presented spreadsheet is simple and intuitive. It resembles the play of a child who, holding a mirror in his hand, sets it so that the reflected sunlight reaches the face of his parent. That is why, the designer (the method's user) does not need to be a professional designer. This method facilitates and disseminates the optics designing process among lighting professionals who do not have advanced knowledge in the field of analytics and optics geometry. It can become a solution to a difficult situation for lighting companies willing to produce their own luminaires but not having an experienced optical designer.

The applied light source model in the form of geometry and two sets of luminance distributions is easy to implement and does not cause computational difficulties when limited to 2D. If we develop the method in 3D, the main challenge will be a significant increase in the amount of data to be captured and processed, which may not be a problem in the future. The model's accuracy is high because it uses accurate geometric dimensions and high-resolution luminance distributions which results from the use of a matrix luminance meter. The luminance distribution of the LED light source luminous surface used in the described case has 291 points for one exemplary viewing

direction. To contrast, the luminance distribution of the light source luminous surface used in the Photopia™ optical design software has only 6 points of which only two have a non-zero luminance value [20]. One should be aware that in some simulated optical systems this may significantly affect the results of the simulation and such optical systems cannot always be recognized easily [23].

While working on this designing tool, attempts have been made to move away from flat reflective surfaces. Concave reflective surfaces forming a part of the side surface of the cylinder have been taken into account [24]. Specific modifications with their verification have been performed. However, this was performed until it appeared that such a shape of optical system elements does not facilitate obtaining uniform illumination of the illuminated surface along one (characteristic) line. Obtaining uniform illumination along such a line of the illuminated surface was the primary goal of the author's research. Thus, concave optical elements can be included in the presented tool calculations after making appropriate changes.

Observation of the luminance distribution arising on the illuminated surface almost at the same time of designing the optical system makes it possible to see what effect the addition of subsequent elements of the optical system has on this distribution. We can see what shape of a luminance distribution they produce and how this distribution changes with the change in aiming, i.e., distribution location on the illuminated surface. In addition, we can see the impact of slopes of such a distribution at the very moment when we want to combine it with the already existing distribution. These slopes are often a critical element affecting the extent to which we can achieve a uniform distribution of luminance on the illuminated surface.

4. Conclusions

Asymmetrical lighting is obtained mainly by means of an asymmetrical optical system. This kind of lighting system is necessary in situations when we illuminate vertical planes. By using such a system, we avoid the risk of an observer's shadow appearing on the illuminated surface. In this article, the author presents his idea for an effective design of asymmetrical optical systems of luminaires. Thanks to this method, the designing process becomes more transparent, straight forward and faster. The created trial of the calculation tool allows a designer to use the proposed designing method in 2D. The described example of a trough-shaped reflector's design shows the ease of this process and no need of deep professional knowledge which can limit the group of people able to design it. This work is the basis for a development of this method in the 3D environment and its implementation in a suitably selected programming environment. The use of various automation methods in such an application is recommended. Using these automation methods can help assessing the compliance degree of current results with reference distributions, selecting sizes and aiming at optical system elements. This can contribute to minimizing discrepancies with the reference distribution. Such an application can be a great improvement both in terms of working time and design costs of asymmetrical luminaires.

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