ARCHIVES OF ELECTRICAL ENGINEERING VOL. 70(1), pp. 89–102 (2021)

DOI 10.24425/aee.2021.136054

Modified Voronin models of electric arc with disturbed geometric dimensions and increased energy dissipation

ANTONI SAWICKI

Association of Polish Electrical Engineers (NOT-SEP), Czestochowa Division
Poland

e-mail: sawicki.a7@gmail.com

(Received: 22.05.2020, revised: 09.08.2020)

Abstract: Mathematical models of electric an arc with disturbed geometric sizes were created based on initial assumptions adopted from the Mayr and Cassie models. Two cases of approximation of arc characteristics were considered separately. The Mayr–Voronin model was created in the low-current range with an exponential dependence of conductance on plasma enthalpy. However, the Cassie–Voronin model created is valid in the high-current range with a linear dependence of conductance on plasma enthalpy. In addition, the effect of two different assumptions about the method of energy dissipation, proportional to the lateral surface of the column or proportional to the volume of the column, on the parameters of both mathematical models was compared. It has been shown that under constant geometrical parameter values, created models can be reduced to classic Mayr and Cassie models. Then, these models were modified by taking into account the additional increase in heat dissipation as the current increases. Increasing voltage and current characteristics correspond to such an arc. Using the computer simulations, the effectiveness of using developed mathematical models in mapping the dynamic characteristics of the electric arc has been shown.

Key words: electric arc, Mayr model, Cassie model, Voronin model

1. Introduction

In most electrotechnical devices, special measures are taken to stabilize arcing. These include maintaining a constant distance between the electrodes and undisturbed external factors of the column cross-sectional area [1, 2]. But even in these devices, the processes of starting and stopping plasma-controlled parameters are associated with distortions introduced into the geometric dimensions of the arc. These occur most intensively in special devices (e.g. gliding arc plasma generators, Cold Metal Transfer CMT welding machines) and in electric devices [3–7]. To adapt the known simple fixed length mathematical models to map such a disturbed arc column, various modifications have been introduced [8–10]. Sometimes they violate the input assumptions of the created model, e.g. regarding the energy balance equation. In addition to assuming a cylindrical shape of the column, the Mayr model assumes a constant cross-sectional area and the Cassie model assumes a constant plasma temperature. In addition, various methods of electricity dissipation are adopted. According to some authors, the dissipated energy is proportional to the lateral surface area of the cylindrical column S_s [10, 11], and according to others it is proportional to the volume of such a column V [1, 12]. However, there is an unambiguous relationship between these assumptions ($V = 0.5 r S_s$), which affects only the form of auxiliary functions, but not that of differential or integral equations. It does not matter in the classic Mayr and Cassie models, because there these functions are taken as fixed quantities. The mathematical models of extension arcs developed by P.A. Kulakov [10], O.Y. Novikov – M. Schellhase [8], S. Berger [13], A.A. Voronin use various additional simplifications and therefore have limited use. The mathematical model of I.V. Pentegov – V.N. Sidorec [14] and its modifications [15] have a much greater universality in considering the effects of external disturbances on the arc column. However, it requires complicated analytical solving of derivatives and integrals of analytical expressions.

The structure of the electric arc column is heterogeneous [16]. In addition to very thin electrode layers with non-equilibrium plasma, the long arc consists of a conical part located at the cathode and a quasi-cylindrical part towards the anode. In the short arc, where the conical part is significantly longer than the cylindrical part, there is a significant share of convection power, and in a long arc, where the cylindrical part is significantly longer than the conical part, there is an advantage of radiation power. The arc column length is always greater than the distance between the electrodes. In simplified mathematical models, a cylindrical shape of the arc column is assumed. It follows that the plasma in such an arc is stationary with reduced convective heat dissipation. In fact, depending on the relative lengths of the arc components, there is a different proportion of individual components of dissipated energy [16]. Simplified AC arc models assume an energetically equivalent cylindrical shape of the column and a simplified structure of the energy dissipation channels [2, 11, 17]. The introduction of an additional component in the form of additional dissipated power at the initial stage extends the possibilities of approximating the characteristics of high-current arcs (e.g. in high-pressure gas, between sharpened electrodes) [18].

2. The Mayr–Voronin arc column model with variable geometric dimensions

The following simplifying assumptions are adopted in the Voronin model [10]: the arc column has a cylindrical shape; the plasma is homogeneous with respect to the cross-sectional area and arc axis; heat dissipation occurs only from the side surface of the arc; the length of the arc column may change over time.

As the basis for creating a mathematical model of the arc, a simplified equation of the plasma column heat balance is introduced:

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = P_{el} - P_{dis} = u_{col} i - P_{dis} , \tag{1}$$

where: Q is the plasma enthalpy, J; P_{el} is the electric power supplied to the column, W; P_{dis} is the thermal power dissipated from an arc column, W; u_{col} is the voltage on the column, V; i is the arc current, A. In addition, the following markings are introduced:

$$Q = q_V V = q_V lS, \tag{2}$$

$$g = \frac{\sigma S}{l}, \tag{3}$$

$$P_{dis} = P_{ds}\left(S_s\right) = P_{ds}(l, S) = p_S S_s = p_S l\sqrt{4\pi S}, \tag{4}$$

where: q_V is the enthalpy volume density, J/m^3; g is the conductance, S; σ is the specific conductivity of the arc column, S/m; l is the arc column length, m; p_S is the power density dissipated by the side surface of the column, W/m^2; P_{ds} is the power dissipated by the side surface of the column, W; S is the cross-sectional area of the arc, m^2; S_s is the lateral surface area of the arc, m^2; $S_s = l\sqrt{4\pi S}$.

Arc conductance is a function of arc enthalpy $g = F(Q)$. If one of the assumptions of the Mayr model is adopted [10]:

$$\sigma = \sigma_{0M} \cdot \exp\left(\frac{q_V}{q_{0M}}\right), \tag{5}$$

where σ_{0M} and q_{0M} represent the approximation coefficients of the plasma conductivity function. The equation of the dynamic arc model with changing length $l(t)$, based on (1)–(5), has the form:

$$\frac{1}{g}\frac{\mathrm{d}g}{\mathrm{d}t} = \frac{1}{q_{0M}Sl}\left(u_{col}i - P_{ds}(l,S)\right) - \frac{1}{l}\frac{\mathrm{d}l}{\mathrm{d}t}\left(1 + \ln\frac{gl}{\sigma_{0M}S}\right) + \frac{1}{S}\frac{\mathrm{d}S}{\mathrm{d}t}\left(1 - \ln\frac{gl}{\sigma_{0M}S}\right). \tag{6}$$

After further transformations, we get the equation:

$$\frac{1}{g}\frac{\mathrm{d}g}{\mathrm{d}t} = \frac{p_S}{q_{0M}}\sqrt{\frac{4\pi}{S}}\left(\frac{u_{col}i}{p_S l\sqrt{4\pi S}} - 1\right) - \frac{1}{l}\frac{\mathrm{d}l}{\mathrm{d}t}\left(1 + \ln\frac{gl}{\sigma_{0M}S}\right) + \frac{1}{S}\frac{\mathrm{d}S}{\mathrm{d}t}\left(1 - \ln\frac{gl}{\sigma_{0M}S}\right). \tag{7}$$

The final form of the Mayr–Voronin equation is as follows:

$$\frac{1}{g}\frac{\mathrm{d}g}{\mathrm{d}t} = \frac{1}{\theta_{Ms}(S)}\left(\frac{u_{col}i}{P_{Ms}(l,S)} - 1\right) - \frac{1}{l}\frac{\mathrm{d}l}{\mathrm{d}t}\left(1 + \ln\frac{gl}{\sigma_{0M}S}\right) + \frac{1}{S}\frac{\mathrm{d}S}{\mathrm{d}t}\left(1 - \ln\frac{gl}{\sigma_{0M}S}\right), \tag{8}$$

where:

– damping function in S:

$$\theta_{Ms}(S) = \frac{q_{0M}}{p_S}\sqrt{\frac{S}{4\pi}}, \tag{9}$$

– power dissipation function in W:

$$P_{Ms}(l,S) = p_S l\sqrt{4\pi S}. \tag{10}$$

Due to the adopted assumption, (5), this model approximates well the characteristics of low-current arcs.

In the special case of parameter stability ($\mathrm{d}l/\mathrm{d}t = 0$, $\mathrm{d}S/\mathrm{d}t = 0$) Equation (8) boils down to the Mayr model ($P'_{Ms} = P_{Ms}\left(l_0, S_0\right) = \text{const}$, $\theta'_{Ms} = \theta_{Ms}\left(S_0\right) = \text{const}$). This means that both the length and diameter of the column do not change temporarily. Then the relationships between the Mayr model parameters and the Mayr–Voronin (8) model coefficients are as follows:

$$p_S = \frac{P'_{Ms}}{l_0\sqrt{4\pi S_0}}, \tag{11}$$

$$q_{0M} = \frac{\theta'_{Ms}P'_{Ms}}{l_0 S_0}. \tag{12}$$

If instead of condition (4) we assume that the dissipated power is proportional to the volume of the column:

$$P_{dis} = P_{dv}(V) = P_{dv}(l, S) = p_V V = p_V l S, \tag{13}$$

we will get other dependencies on auxiliary functions:

– damping constant in S:

$$\theta_{Mv} = \frac{q_{0M}}{p_V} = \text{const}, \tag{14}$$

– power dissipation function in W:

$$P_{Mv}(l, S) = p_V l S, \tag{15}$$

where p_V is the dissipated power volume density, W/m^3. In Equation (8) only the auxiliary functions are changed. If we assume constancy of size ($\mathrm{d}l/\mathrm{d}t = 0$, $\mathrm{d}S/\mathrm{d}t = 0$), also in this case Equation (8) boils down to the Mayr model ($P''_{Mv} = P_{Mv}(l_0, S_0) = \text{const}$, $\theta''_{Mv} = \theta_{Mv} = \text{const}$). Then the relations between the Mayr model parameters and the Mayr–Voronin model coefficients are as follows:

$$p_V = \frac{P''_{Mv}}{l_0 S_0}, \tag{16}$$

$$q_{0M} = \frac{\theta''_{Mv}P''_{Mv}}{l_0 S_0}. \tag{17}$$

Regardless of the adopted method of energy dissipation from the arc (proportional to the lateral surface or proportional to the volume of the plasma column) there is a relationship between the functions of the Mayr–Voronin model:

$$\theta_{Ms}(S) \cdot P_{Ms}(l, S) = \theta_{Mv} \cdot P_{Mv}(l, S) = q_{0M} l S = q_{0M} V. \tag{18}$$

In contrast, there are relationships between the auxiliary functions of the model with various heat dissipation assumptions:

$$\frac{\theta_{Ms}(S)}{\theta_{Mv}} = \frac{p_v}{p_s}\sqrt{\frac{S}{4\pi}}, \tag{19}$$

$$\frac{P_{Ms}(l, S)}{P_{Mv}(l, S)} = \frac{p_s}{p_v}\sqrt{\frac{4\pi}{S}}. \tag{20}$$

Hence the inverse proportionality between the auxiliary functions of these models, equivalent to the relationship (18):

$$\frac{\theta_{Ms}(S)}{\theta_{Mv}} = \frac{P_{Mv}(l,S)}{P_{Ms}(l,S)}. \tag{21}$$

Integral forms of mathematical models are used in modeling arcs of selected electrotechnological devices (e.g. gliding-arc plasma generators). In the general case, the equivalent integer form of the equation in relation to the equation of model (8) is as follows:

$$g = g_{0M} \exp\left\{\int_0^t \left[\frac{1}{\theta_{Ms}(S)}\left(\frac{u_{col}i}{P_{Ms}(l,S)} - 1\right) - \frac{1}{l}\frac{\mathrm{d}l}{\mathrm{d}t}\left(1 + \ln\frac{gl}{\sigma_{0M}S}\right) + \frac{1}{S}\frac{\mathrm{d}S}{\mathrm{d}t}\left(1 - \ln\frac{gl}{\sigma_{0M}S}\right)\right]\mathrm{d}\tau\right\}. \tag{22}$$

3. A modified Mayr–Voronin arc column model with variable geometric dimensions and with additional energy dissipation

A simplified equation for the arc column heat balance is introduced as the basis for the new model:

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = P_{el} - \left(P_{dis} + P_{dM}\left(i_\theta\right)\right), \tag{23}$$

where: P_{dM} is the additional thermal power dissipated from the arc column, W, i_θ is the state current related to plasma temperature, A [14]. In order to simplify the analysis, further considerations omitted determinations related to heat dissipation methods. After assuming (2)–(5), a modified form of Formula (8) is obtained:

$$\frac{1}{g}\frac{\mathrm{d}g}{\mathrm{d}t} = \frac{1}{\theta_M(S)}\left(\frac{u_{col}i}{P_M(l,S)} - 1 - \frac{P_{dM}\left(i_\theta\right)}{P_M(l,S)}\right) - \\ - \frac{1}{l}\frac{\mathrm{d}l}{\mathrm{d}t}\left(1 + \ln\frac{gl}{\sigma_{0M}S}\right) + \frac{1}{S}\frac{\mathrm{d}S}{\mathrm{d}t}\left(1 - \ln\frac{gl}{\sigma_{0M}S}\right). \tag{24}$$

Let's consider the steady state ($i_\theta = I = \text{const}$) arc with assumptions ($\mathrm{d}g/\mathrm{d}t = 0$, $\mathrm{d}l/\mathrm{d}t = 0$, $\mathrm{d}S/\mathrm{d}t = 0$). Then we will get the dependence:

$$U_{col}I = P_M(l,S) + P_{dM}(I), \tag{25}$$

where U_{col} is the voltage on the arc column. If we assume that the additional dissipated power is determined by the formula:

$$P_{dM}(I) = U_{dM}(I)\cdot I, \tag{26}$$

with voltage–current static characteristics $U_{dM}(I)$, we will get a generalized dependence on static characteristics in the form:

$$U(l,S,I) = \frac{P_M(l,S) + P_{dM}(I)}{I} = \frac{P_M(l,S)}{I} + U_{dM}(I). \tag{27}$$

If approximation of additional voltage drop was used with a polynomial:

$$U_{dM}(I) = a_{M1}I + a_{M2}I^2, \tag{28}$$

(where constant coefficients are expressed in units: a_{M_1} in Ω, a_{M_2} in VA^{-2}), then the characteristics of additional dissipated power will be described by the dependence

$$P_{dM}(I) = I\left(a_{M1}I + a_{M2}I^2\right). \tag{29}$$

After substituting (29) to (24), the final form of the new Mayr–Voronin model with additional energy dissipation is as follows:

$$\frac{1}{g}\frac{\mathrm{d}g}{\mathrm{d}t} = \frac{1}{\theta_M(S)}\left(\frac{u_{col}i}{P_M(l,S)} - 1 - \frac{U_{dM}(i_\theta)\,i_\theta}{P_M(l,S)}\right) - \\ - \frac{1}{l}\frac{\mathrm{d}l}{\mathrm{d}t}\left(1 + \ln\frac{gl}{\sigma_{0M}S}\right) + \frac{1}{S}\frac{\mathrm{d}S}{\mathrm{d}t}\left(1 - \ln\frac{gl}{\sigma_{0M}S}\right). \tag{30}$$

Due to the assumption adopted, (5), this model approximates well the characteristics of low-current arcs. In this case, the equivalent integer to model (30) will be as follows:

$$g = g_{0M}\exp\left\{\int_0^t\left[\begin{array}{l}\frac{1}{\theta_M(S)}\left(\frac{u_{col}i}{P_M(l,S)} - 1 - \frac{U_M(i_\theta)\,i_\theta}{P_M(l,S)}\right) + \\ -\frac{1}{l}\frac{\mathrm{d}l}{\mathrm{d}t}\left(1 + \ln\frac{gl}{\sigma_{0M}S}\right) + \frac{1}{S}\frac{\mathrm{d}S}{\mathrm{d}t}\left(1 - \ln\frac{gl}{\sigma_{0M}S}\right)\end{array}\right]\mathrm{d}\tau\right\}. \tag{31}$$

4. A modified Cassie–Voronin arc column model with variable geometric dimensions

In order to create this model, simplifying assumptions should be made, as in the Voronin model [10]. Here, as the basis of the model, a simplified equation of the energy balance of the arc column (1) is introduced, followed by the following conditions marked as (2)–(4). Dependence of arc conductance on plasma enthalpy $g = F(Q)$ is now in line with the assumption of the Cassie model [11]:

$$\sigma = \sigma_{0C}\cdot\frac{q_V}{q_{0C}}, \tag{32}$$

where σ_{0C} and q_{0C} are the approximation coefficients of the plasma conductivity function. Equation of dynamic arc model with changing geometric sizes $S(t)$ and $l(t)$ has the form:

$$\frac{1}{g}\frac{\mathrm{d}g}{\mathrm{d}t} = \frac{\sigma_{0C}}{q_{0C}gl^2}\left(u_{col}i - P_{dis}(l,S)\right) - \frac{2}{l}\frac{\mathrm{d}l}{\mathrm{d}t}. \tag{33}$$

After taking into account Formula (4) and transformations, it follows that:

$$\frac{1}{g}\frac{\mathrm{d}g}{\mathrm{d}t} = \frac{\sigma_{0C}p_S\sqrt{4\pi S}}{q_{0C}gl}\left(\frac{u_{col}i}{p_S l\sqrt{4\pi S}} - 1\right) - \frac{2}{l}\frac{\mathrm{d}l}{\mathrm{d}t}, \tag{34}$$

where $P_{dis} = P_{ds}(l,S) = p_S l\sqrt{4\pi S}$. After transformations, the final form of the new Cassie–Voronin model is as follows:

$$\frac{1}{g}\frac{\mathrm{d}g}{\mathrm{d}t} = \frac{1}{\theta_{Cs}(S)}\left(\frac{u_{col}^2}{U_{Cs}^2(l,S)} - 1\right) - \frac{2}{l}\frac{\mathrm{d}l}{\mathrm{d}t}, \tag{35}$$

where on the basis of (32) the relationship can be used as:

$$g = \frac{\sigma_{0C}}{q_{0C}} \frac{q_V S}{l}, \tag{36}$$

and yields:

– damping function in s:

$$\theta_{Cs}(S) = \frac{q_{0C}\, g l}{\sigma_{0C}\, p_S \sqrt{4\pi S}} = \frac{q_V}{p_S}\sqrt{\frac{S}{4\pi}}, \tag{37}$$

– voltage function on the arc column in V^2:

$$U^2_{Cs}(l,S) = \frac{p_S l \sqrt{4\pi S}}{g} = \frac{q_{0C}\, p_S}{\sigma_{0C}\, q_V}\sqrt{\frac{4\pi}{S}}\, l^2 = \frac{q_{0C}\, p_S}{\sigma_{0C}\, q_V}\frac{l^2}{\theta_{Cs}(S)} = E^2_{Cs}(S)\cdot l^2, \tag{38}$$

where E_{Cs} is the electric field strength in the arc column, V/m. Due to the adopted assumption (32), this model well approximates the characteristics of high current arcs. This model is very similar to the Kulakov model [10].

In special cases, the stability of the geometric parameter $(\mathrm{d}l/\mathrm{d}t = 0)$ Equation (35) can be reduced to the Cassie model ($U'_{Cs} = U^2_{Cs}(l_0, S_0) = \text{const}$, $\theta'_{Cs} = \theta_{Cs}(S_0) = \text{const}$). Then the relationships between the Cassie model parameters and the Cassie–Voronin model coefficients are as follows:

$$\frac{p_S}{q_V} = \frac{1}{\theta'_{Cs}}\sqrt{\frac{S_0}{4\pi}}, \tag{39}$$

$$\frac{q_{0C}}{\sigma_{0C}} = U'^2_{Cs}\frac{q_V}{p_s}\sqrt{\frac{S}{4\pi}}\frac{1}{l_0^2} = U'^2_{Cs}\theta'_{Cs}\frac{1}{l_0^2} = E'^2_{Cs}, \tag{40}$$

where E'_{Cs} is the electric field strength in the arc column, V/m.

If instead of condition (4) we accept condition (13) that the dissipated power is proportional to the volume of the column $P_{dis} = P_{dv}(V) = P_{dv}(l,S) = p_V l S$, instead of (37) we get the S damping function:

$$\theta_{Cv} = \frac{q_{0C}\, g l}{\sigma_{0C}\, p_V S} = \frac{q_V}{p_V} = \text{const}. \tag{41}$$

However, the auxiliary function of voltage on the arc column takes the form:

$$U^2_{Cv}(l) = \frac{P_{dv}(l,S)}{g} = \frac{p_V l S}{g} = \frac{q_{0C}}{\sigma_{0C}}\frac{p_V}{q_V} l^2 = \frac{q_{0C}}{\sigma_{0C}}\frac{l^2}{\theta_{Cv}} = E^2_{Cv} l^2, \tag{42}$$

where E_{Cv} is the electric field strength in the arc column, V/m.

After adopting condition (13) (instead of condition (4)) in Equation (35), the functions θ_{Cs} and U_{Cs} should be replaced by the functions θ_{Cv} and U_{Cv}. If we assume parameter stability $(\mathrm{d}l/\mathrm{d}t = 0,\ \mathrm{d}S/\mathrm{d}t = 0)$, then also in this case Equation (35) boils down to the Cassie model with parameters ($U''_{Cv} = U^2_{Cv}(l_0) = \text{const}$, $\theta''_{Cv} = \theta_{Cv} = \text{const}$). Then the relations between the Cassie model parameters and the Cassie–Voronin model coefficients are as follows:

$$\frac{q_V}{p_V} = \theta''_{Cv}, \tag{43}$$

$$\frac{q_{0C}}{\sigma_{0C}} = \frac{U''^2_{Cv}}{l_0^2}\frac{q_V}{p_V} = \frac{U''^2_{Cv}\theta''^2_{cv}}{l_0^2} = E''^2_{Cv}\theta''_{cv}, \tag{44}$$

where E''_{Cv} is the electric field strength in the arc column, V/m.

Regardless of the adopted method of energy dissipation from the arc (proportional to the lateral surface or proportional to the volume of the cylindrical plasma column) there is a relationship between the auxiliary functions of the Cassie–Voronin model:

$$\theta_{Cs}(S) \cdot U^2_{Cs}(l,S) = \theta_{Cv} \cdot U^2_{Cv}(l) = \frac{q_{0C}}{\sigma_{0C}} l^2. \tag{45}$$

In contrast, there are relationships between the auxiliary functions of models with different assumptions about heat dissipation:

$$\frac{\theta_{Cs}(S)}{\theta_{Cv}} = \frac{p_v}{p_s}\sqrt{\frac{S}{4\pi}}, \tag{46}$$

$$\frac{U^2_{Cs}(l,S)}{U^2_{Cv}(l)} = \frac{p_s}{p_v}\sqrt{\frac{4\pi}{S}}. \tag{47}$$

Hence the inverse proportionality between the auxiliary functions of these models equivalent to relationship (45):

$$\frac{\theta_{Cs}(S)}{\theta_{Cv}} = \frac{U^2_{Cv}(l)}{U^2_{Cs}(l,S)}. \tag{48}$$

In the general case, the equivalent overall form, (35), of the model will be as follows:

$$g = g_{0M} \exp\left\{\int_0^t \left[\frac{1}{\theta_C(S)}\left(\frac{u^2_{col}}{U^2_C(l,S)} - 1\right) - \frac{2}{l}\frac{\mathrm{d}l}{\mathrm{d}t}\right]\mathrm{d}\tau\right\}. \tag{49}$$

5. A modified Cassie–Voronin arc column model with variable geometric dimensions and with additional energy dissipation

A simplified equation for the arc column heat balance is introduced as the basis for the new model:

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = P_{el} - \left(P_{dis} + P_{dC}\left(i_\theta\right)\right), \tag{50}$$

where: P_{dC} is the additional thermal power dissipated from the arc column, W, i_θ is the state current related to plasma temperature, A [14]. In order to simplify the analysis, further considerations omitted the markings associated with heat dissipation methods.

After taking into account assumptions (2)–(4) and (32) and after transforming Formula (50) we get:

$$\frac{1}{g}\frac{\mathrm{d}g}{\mathrm{d}t} = \frac{1}{\theta_{Cs}(S)}\left(\frac{u^2_{col}}{U^2_{Cs}(l,S)} - 1 - \frac{P_{dC}\left(i_\theta\right)}{P_{dis}(l,S)}\right) - \frac{2}{l}\frac{\mathrm{d}l}{\mathrm{d}t}. \tag{51}$$

Let us now consider the steady state ($i_\theta = I = \text{const}$) arc with assumptions ($\mathrm{d}g/\mathrm{d}t = 0$, $\mathrm{d}l/\mathrm{d}t = 0$). Then from (51) we will obtain:

$$U_{col} I = P_{dis}(l, S) + P_{dC}(I). \tag{52}$$

The additional dissipated power is determined by the formula:

$$P_{dC}(I) = \frac{I^2 - G^2 U_C^2(l, S)}{G}, \tag{53}$$

with static conductivity characteristics saved in a given form:

$$G = \frac{I}{U_{col}} = \frac{I}{U_C(l, S) + U_{dC}(I)}. \tag{54}$$

In this characteristic, the U_{col} component related to the P_{dC} power was separated from the U_{col} voltage. After substituting (54) to (53), we obtain the expression for additional dissipated power:

$$P_{dC}(I) = \frac{2U_C(l, s) + U_{dC}(I)}{U_C(l, s) + U_{dC}(I)} U_{dC}(I) I. \tag{55}$$

Here, the additional voltage drop can be approximated by a polynomial:

$$U_{dC}(I) = a_{C1} I + a_{C2} I^2, \tag{56}$$

where constant coefficients are expressed in units: a_{C_1} in Ω, a_{C_2} in VA^{-2}. The modified form of the new Cassie–Voronin model is as follows:

$$\frac{1}{g}\frac{\mathrm{d}g}{\mathrm{d}t} = \frac{1}{\theta_C(S)}\left(\frac{u_{col}^2}{U_C^2(l, S)} - 1 - \frac{U_{dC}(i_\theta)\, i_\theta}{P_{dis}(l, S)} \cdot \frac{2U_C(l, S) + U_{dC}(i_\theta)}{U_C(l, S) + U_{dC}(i_\theta)}\right) - \frac{2}{l}\frac{\mathrm{d}l}{\mathrm{d}t}, \tag{57}$$

where i_θ is the state current [14]. Due to the adopted assumption (32), this model approximates well the characteristics of high current arcs. In this case, the equivalent overall form, (57), of the mathematical model is as follows:

$$g = g_{0C} \exp\left\{\int_0^t \left[\frac{1}{\theta_C(S)}\left(\frac{u_{col}^2}{U_C^2(l, S)} - 1 - \frac{U_{dC}(i_\theta)\, i_\theta}{P_{dis}(l, S)} \cdot \frac{2U_C(l, s) + U_{dC}(i_\theta)}{U_C(l, s) + U_{dC}(i_\theta)}\right) - \frac{2}{l}\frac{\mathrm{d}l}{\mathrm{d}t}\right] \mathrm{d}\tau\right\}. \tag{58}$$

6. Results of simulation tests of dynamic states in the circuit with models of an electric arc of disturbed geometric dimensions

The cross-sectional area of the arc column depends on many factors: current, gas composition, gas pressure, gas temperature, column cooling intensity (gas flow, walls of the constructor or plasma generator nozzle, flux), etc. The literature [5–7] presents the findings of theoretical considerations and experimental studies of direct current arcs. Depending on the arc conditions, the current density varies from units to hundreds of amps per mm^2. The radius of the arc is primarily a function of current $r_c = f\left(I^{2/3}\right)$. This function is very close to the empirical formula [6]:

$$r_c \cong kI^n, \text{ cm}, \tag{59}$$

(where: $n = 0.6 \div 0.7$) obtained in the case of an arc longitudinally washed by a gas stream. If the arc is in the air, then $k = 0.135$ cm·A^{-n}.

In the case of an AC arc, the formulas presented in the literature usually do not take into account the physical processes that occur when the current passes through zero. It has been experimentally shown [19] that a decrease in the initially high current is accompanied by a mild decrease in the damping function $\theta \propto i$. For the specific construction of an electrode system and an arc burning in a nitrogen atmosphere, the minimum was reached at the point ($I = 20$ A, $\theta = 10 \cdot 10^{-6}$ s). Further reduction of the current to zero was accompanied by a rapid and significant increase in the value of the damping function $\theta \propto 1/i$.

In the cases of auxiliary functions (9) and (37), as the current decreases to zero, the column diameter also decreases to zero, and then the damping functions also decrease instead of increasing. The simulation effect of such a model results in excessively high arc ignition voltage and thus in instability of the programs. In contrast, versions of models with the value of dissipated power proportional to the volume of the arc column (Formulas (14) and (41)) maintain a constant value of the damping function, which only partially meets the conditions roughly approximating data from physical experiments. Therefore, it is proposed to approximate the column radius by means of the dependence:

$$r_{col} = r_0 \varepsilon_{ri}(i) + r_c(i) \cdot (1 - \varepsilon_{ri}(i)), \tag{60}$$

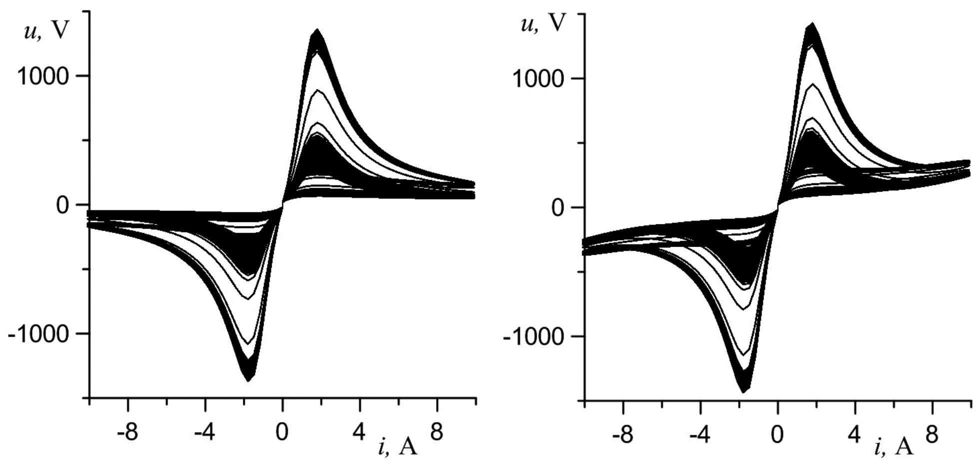
where the weight function can have the form:

$$\varepsilon_{ri}(i) = \exp\left(\ln(k_r)\,\frac{i^2}{I_{r_0}^2}\right), \tag{61}$$

where: r_0 is the tasks fixed radius value ($r_0 > r_c(I_{r_0})$), k_r is the coefficient of participation of individual components r_0 and r_c as a function of radius at the point with the abscissa I_{r_0}, ($0 < k_r < 1$), I_{r_0} is the current value corresponding to the minimum value of the damping function determined experimentally, e.g. 20 A [19]. The use of these dependencies avoids problems with denominators zeroing in expressions on mathematical models of arcs and instability of simulation programs.

The approximation possibilities of the proposed modified mathematical models of the electric arc were examined in a simulation manner. To this end, macromodels of the arc were created in equivalent differential and integral versions. The calculations took into account the set sum of electrode voltage drops $U_{AC} = 18$ V. A current source generating a 50 Hz sine wave was used as forcing in the arc circuit. In the cases of the modified Mayr–Voronin model (without P_{dM} and with P_{dM}) a current of 10 A amplitude was used, and in the cases of the modified Cassie–Voronin model (without P_{dC} and with P_{dC}) a current of 200 A was used. In contrast to static characteristics, to present dynamic characteristics, currents with variable flow direction are most often used. If in the formulas for mathematical models of the arc we use alternating current (and not the state current i_θ [14]), then its properties should be taken into account in the functions determining the additional dissipated power ($U_{dM}(|i|)|i|$, $U_{dC}(|i|)\,|i|$).

Figure 1 shows the dynamic characteristics of the Mayr–Voronin arc model with dissipation power proportional to the side surface of the column. The change in the length of the column followed a given relationship $l(t) = 10^{-2} \cdot \left(1.4 + t - \exp\left(-(t/0.5)^{40}\right)\right)$. So, during a linear increase in the length of the column after 0.5 s there was a quasi-jump increase. The total stretching time reached 0.8 s. The column cross-sectional area was a function of current according to Formulas (59)–(61). The coefficient values were as follows: $n = 0.6$, $k = 0.135$ cm·A^{-n}, $r_0 = 4 \cdot 10^{-3}$ m, $k_r = 0.9$, $I_{r_0} = 8$ A.



(a) (b)

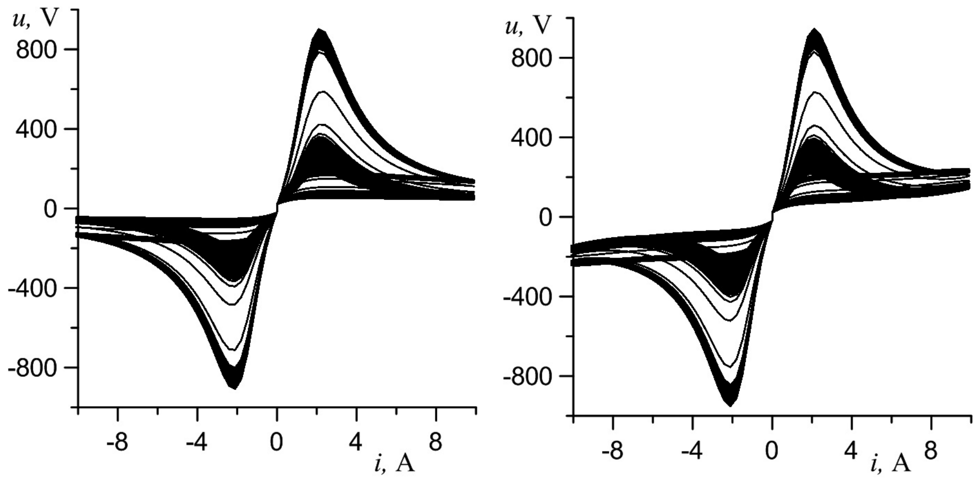
Fig. 1. Dynamic characteristics of the extended arc described by the Mayr–Voronin model ($p_S = 2.5 \cdot 10^6$ W/m^2, $q_{0M} = 0.5 \cdot 10^6$ J/m^3, $\sigma_{0M} = 800$ S/m): a) without additional dissipated power in the high-current range, Formulas (8)–(10); b) with additional dissipated power in the high-current range, Formulas (9), (10), (30) ($a_{M1} = 0.1\ \Omega$, $a_{M2} = 2$ VA^{-2})

Figure 2 shows the dynamic characteristics of the Mayr–Voronin arc model with dissipation power proportional to the column volume. Changes in the column length were carried out according to the same dependence as in Figure 1. As a result of simulation of processes in the electric circuit with the Mayr–Voronin mathematical model, different shapes of the dynamic voltage-current characteristics of the arc were obtained. They demonstrate great approximation potential for low-current arcs, stretched without and with increased dissipated power. In the cases of long arcs, no influence of assumptions about the method of heat dissipation from the column on the formation of clear differences in the model selection options was found.

Figure 3 shows the dynamic characteristics of the Cassie–Voronin arc model with dissipation power proportional to the side surface of the column. The column length was changed according to a similar relationship $l(t) = 10^{-3} \cdot \left(6 + 20t - 10\exp\left(-(t/0.5)^{40}\right)\right)$. The column cross-sectional area was a function of current according to Formulas (59)–(61). The coefficient values were as follows: $n = 0.6$, $k = 0.135$ cm·A^{-n}, $r_0 = 6 \cdot 10^{-2}$ m, $k_r = 0.1$, $I_{r0} = 15$ A.

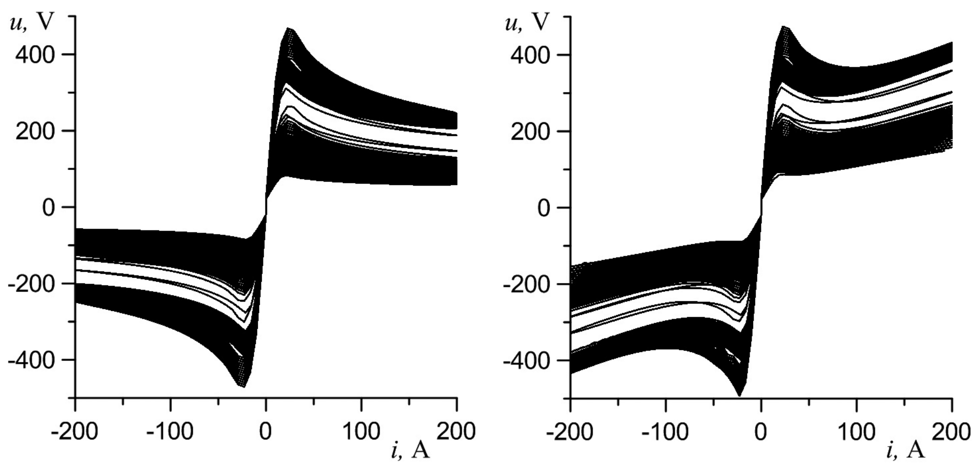
Figure 4 shows the dynamic characteristics of the Cassie–Voronin arc model with dissipation power proportional to the column volume. The change in column length was the same as in the previous case. Only in the case of Figure 4(a), the auxiliary functions, (41) and (42) of

Equation (34), do not depend on the cross-sectional area of column S. As a result of simulations of processes in the electric circuit with the Cassie–Voronin mathematical model, various shapes



(a) (b)

Fig. 2. Dynamic characteristics of the extended arc described by the Mayr–Voronin model ($p_V = 1 \cdot 10^9$ W/m^3, $q_{0M} = 0.5 \cdot 10^6$ J/m^3, $\sigma_{0M} = 800$ S/m): a) without additional dissipated power in the high-current range, Formulas (8), (14) and (15); b) with additional dissipated power in the high-current range, Formulas (30), (14) and (15) ($a_{M1} = 0.1\ \Omega$, $a_{M2} = 1$ VA^{-2})



(a) (b)

Fig. 3. Dynamic characteristics of the expanded arc described by the Cassie–Voronin model ($p_S = 1 \cdot 10^6$ W/m^2, $q_{0C} = 2 \cdot 10^5$ J/m^3, $\sigma_{0C} = 800$ S/m, $q_V = 1.7 \cdot 10^4$ J/m^3): a) without additional dissipated power in the high-current range, Formulas (35), (37) and (38); b) with additional dissipated power in the high-current range, Formulas (57), (37) and (38) ($a_{C_1} = 1 \cdot 10^{-3}\ \Omega$, $a_{C_2} = 1 \cdot 10^{-3}$ VA^{-2})

of dynamic voltage-current characteristics of the arc were obtained. They indicate to some approximation possibilities of high-current arc models stretched without and with increased dissipated power ere, too, in the cases of long arcs, there was no impact of assumptions about the method of heat dissipation from the column on the formation of clear differences in the model selection options.

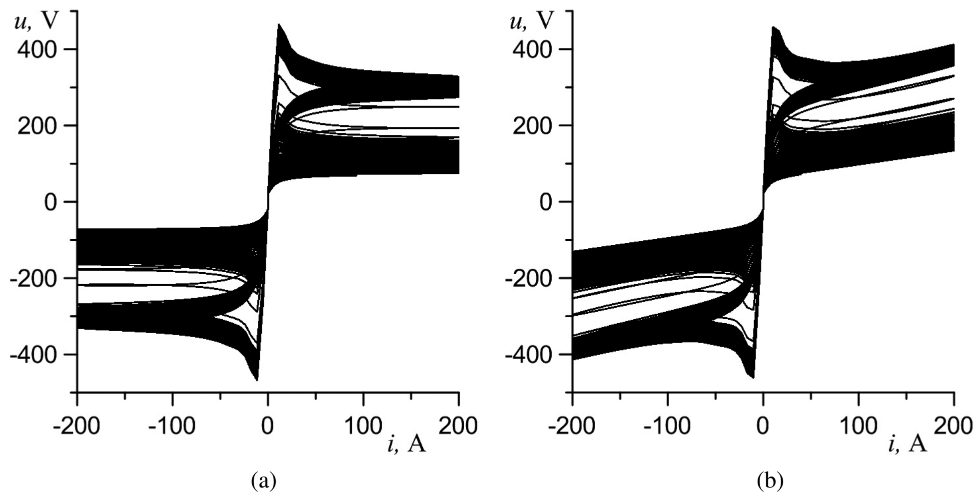


Fig. 4. Dynamic characteristics of the extended arc described by the Mayr–Voronin model ($p_V = 50 \cdot 10^6$ W/m^3, $q_{0C} = 16 \cdot 10^6$ J/m^3, $\sigma_{0C} = 800$ S/m, $q_V = 1 \cdot 10^4$ J/m^3): a) without additional dissipated power in the high-current range, Formulas (35), (41) and (42); b) with additional dissipated power in the high-current range, Formulas (57), (14) and (42) ($a_{C1} = 1 \cdot 10^{-4}$ Ω, $a_{C2} = 2 \cdot 10^{-4}$ VA^{-2})

7. Conclusion

The article specifies the assumptions of the Voronin model by considering two variants of heat dissipation from the plasma column (proportional to the lateral surface area, proportional to volume). In addition, these models were modified so that it was possible to take into account additional heat dissipation from the column in the high-current range.

Separation of the Voronin model into two cases of changes in conductivity relative to enthalpy (approximation by exponential function, approximation by linear function) enabled the development of mathematical models of expanded arc meeting the correct assumptions of low-current and high-current arcs.

Repeatedly simulated processes by the author in circuits with macromodels of an electric arc stretched at different speeds have shown the possibility of selecting model parameters to ensure the needful shapes of dynamic voltage and current characteristics.

In the presented mathematical models of the extended electric arc, the plasma column was treated as an element of the electric circuit with concentrated parameters. These models were written in two forms of differential and integral. It gives the possibility of their computer implementations by building macromodels in simulation programs using controlled voltage or current sources.

References

[1] Krouchinin A.M., Sawicki A., *Modelling of the constricted arc in plasma generators*, Monographs series, no. 109, The Publishing Office of Częstochowa University of Technology, Częstochowa (2005).

[2] Solonenko O.P., *Thermal Plasma Torches and Technologies*, Cambridge International Science Publishing, vol. 1 (2000).

[3] Jaroszyński L., Stryczewska H.D., *Computer simulation of the electric discharge in GlidArc plasma reactor*, 3rd International Conference: Electromagnetic devices and processes in environment protection ELMECO-3, pp. 31–36 (2000).

[4] Schavemaker P.H., van der Sluis L., *An Improved Mayr-Type Arc Model Based on Current-Zero Measurements*, IEEE Trans. Power Delivery, vol. 15, no. 2, pp. 580–584 (2000).

[5] Kopersak V.M., *The theory of welding processes – 1*, KPI (in Ukrainian), Kiev (2011).

[6] Zalessky A.M., *Fundamentals of the theory of electrical apparatus*, Higher School Publishing House (in Russian), Moscow (1974).

[7] Taev I.S., *Electrical contacts and arcing devices of low voltage devices*, Energy Publishing House (in Russian), Moscow (1973).

[8] Marciniak L., *Dynamic models of short-circuit arc for networks with low ground fault current*, Energy Archive (in Polish), vol. 37, pp. 357–67 (2007).

[9] Ziani A., Moulai H., *Hybrid model of electric arcs in high voltage circuit breakers*, Electric Power Systems Research, vol. 92, pp. 37–42 (2012).

[10] Voronin A.A., *Improving the efficiency of contact-extinguishing systems of high-current switching devices with an extending arc*, Abstract of thesis (in Russian), Samara (2009).

[11] Ciok Z., *Mathematical models of connecting arc*, Warsaw University of Technology (in Polish), Warsaw (1995).

[12] Sawicki A., *Models of adjustable length electric arc*, Wiadomości Elektrotechniczne (in Polish), no. 7, pp. 15–19 (2012).

[13] Berger S., *Mathematical approach to model rapidly elongated free-burning arcs in air in electric power circuits*, ICEC 2006, 6–9 June 2006, Sendai, Japan (2006).

[14] Pentegov I.V., Sydorets V.N., *Comparative analysis of models of dynamic welding arc*, The Paton Welding Journal, no. 12, pp. 45–48 (2015).

[15] Sawicki A., *The universal Mayr–Pentegov model of the electric arc*, Electrical Review, vol. 94, no. 12, pp. 208–211 (2019), DOI: 10.15199/48.2019.12.47.

[16] Krouchinin A.M., Sawicki A., *A theory of electrical arc heating*, The Publishing Office of Technical University of Częstochowa, Częstochowa (2003).

[17] Sawicki A., *Arc models for simulating processes in circuits with a SF_6 circuit breaker*, Archives of Electrical Engineering, vol. 68, no. 1, pp. 147–159 (2019), DOI: 10.24425/aee.2019.125986.

[18] Katsaounis A., *Heat flow and arc efficiency at high pressures in argon and helium tungsten arcs*, Welding Research Supplement I, pp. 447-s- 454-s (1993).

[19] Kalasek V., *Measurements of time constants on cascade d.c. arc in nitrogen*, TH-Report 71-E18, Eindhoven, pp. 1–30 (1971).