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Least squares multi-point matching for DEM with consideration of correlated neighbouring pixels and terrain height differences

One of the major tasks in digital photogrammetry is image matching technique for finding corresponding points in a stereopair. Area-based matching has been acknowledged as being more precise than feature-based matching. Least squares multi-point matching (LSMM) is one of the Global Image Matching (GIM) which was developed from the Least-squares Single point matching (LSSM) so called area-based matching. LSMM method has been more reliable than LSSM one because the relationship between the different neighbouring points is considered in simultaneous computation. LSMM is just for the simultaneous determination of the horizontal parallaxes at the node points of the regular rectangular nets for the purpose of the establishing the DEM. This paper undertakes a trial of improving the accuracy of LSMM by consideration of the correlated pixels and terrain height differences.

INTRODUCTION

From the early 1970 to mid 1980s researches related to image matching focus on digital technique. Despite more considerable effort, no general solution was found [1]. First experiment with Least squares area-based matching was proposed by Ackermann and Forstner (in Germany) in the mid 1980 [6]. The idea of this method is to minimise the grey level differences between the template and the matching window whereby the position and the shape of the matching window are parameters to be determined in the adjustment process. From mid 1980 to last years of XX century, image-matching technique has been quickly developed. Rosenholm (1987) [5] proposed the method of least squares multi-point matching in evaluating three-dimensional models. In 1992 Heipke [3] represented „A global approach for least squares image matching and surface reconstruction in object space". Integrating multi-image matching and object surface reconstruction does Heipke's method. Many other research papers related to LSMM for accuracy improvement have been published from 1990s till now. Global image matching (including LSMM) can provide much more reliable and accurate matching results than the single point matching method. For example, LSMM based on image space can reach ±0,1 ~ ±0,5 pixel accuracy and LSMM based on object space can reach ±0,15 ~ ±0.25 pixel [9]. In the LSMM there are two directions of investigating. First of them depends on the computational optimalization of the normal equation system. The method of using array relaxation technique (ART) proposed by Xiaoliang Wu [9] is about 15 times faster than the conventional LSMM. The second direction depends on the choice of weight model for observation system. Using the variable weight model for LSMM computed by ART, obtained result accuracy is about 2 times better than the conventional LSMM and is about 1.5 times better than LSMM with uniform weight model computed by ART [9].

This paper focuses on the possibility improving the accuracy of LSMM for establishing DEM by considering correlated neighbouring pixels and terrain height differences.

Least squares multi-point matching (LSMM)

Algorithms in image matching are based on the assumption that digital stereo-pair is registered in epipolar geometry. An aerial stereo-pair is not likely to be in epipolar geometry, since the attitude of the camera at the instant of exposure is different at every exposure station. The epipolar geometry is only recovered with respect to the model space by relative orientation. The real images or original photoes received at the instant of exposure have to be scanned. By this procedure we receive digital images (pixel images). There are two steps involved in the transformation of the digital images into epipolar images. First, digital images are transformed to true vertical images – the images are parallel to the *XY* plan of the object space system. Next, the true vertical images are transformed to normalised images, which must be parallel to the air base and must have the same focal length [8]. The important property of normalised images is that the epipolar lines of corresponding points are parallel to the image *x*-axis (Fig.l).

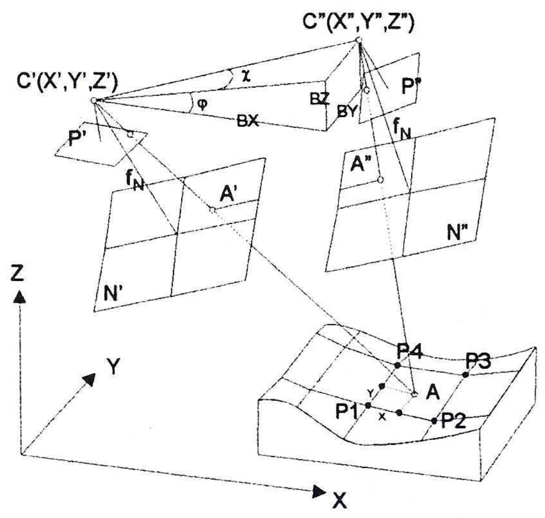

Fig. 1. The epipolar lines on the normalised images with a DEM

Assuming that a point A (Fig.l) whose horizontal parallax is to be determined and located in a rectangle formed by four corner points P_1 (i, j); P_2 $(i+1, j)$; P_4 $(i, j+1)$; P_3 $(i+1, j+1)$. By the use of the following bilinear interpolation the height Z_A of point A (X, Y) among 4 points P_1, P_2, P_4, P_3, is calculated as:

$$Z_A = a_0 + a_1X + a_2Y + a_3XY \tag{1}$$

The height Z_A of point *A* can be computed by use of bilinear interpolation method with respect to a squares in the object space whose side length is *L* as follows:

$$\begin{aligned} Z_A = Z_{P1}(1-X/L)(1-Y/L) + Z_{P2}(1-Y/L)(X/L) + \\ + Z_{P4}(1-X/L)(Y/L) + Z_{P3}(X/L)(Y/L) \end{aligned} \tag{2}$$

Similarly, in the image space, the horizontal parallax p_A of the normalised image point *A* (x, y) can be expressed by the parallaxes p_{ij}, $p_{i+1,j}$, $p_{i,j+1}$, $p_{i+1,j+1}$ of its surrounding grid points $P_1(i, j)$; $P_2(i+1, j)$; $P_4(i, j+1)$; $P_3(i+1, j+1)$ according to the following equation [10]:

$$\begin{aligned} p_A = [p_{i,j}(x_{i+1}-x)(y_{j+1}-y) + p_{i+1,j}(x-x_i)(y_{j+1}-y) + p_{i,j+1}(x_{i+1}-x)(y-y_j) + \\ + p_{i+1,j+1}(x-x_i)(y-y_i)]/(x_{i+1}-x_i)(y_{j+1}-y_j) \end{aligned} \tag{3}$$

where: $x_i \langle x \langle x_{i+1}$; $y_i \langle y \langle y_{i+1}$

If the spacing between neighbour nodes of grid is equal to 1, thus, the equation 3 should be in the form

$$pA = [(1-x)(1-y)p_{ij} + x(1-y)p_{i+1,j} + (1-x)y\,p_{i,j+1} + xy\,p_{i+1,j+1}] \tag{4}$$

If the distances from the point *A* on the normalised image to node (i, j) are marked by $d_1 = x$ and $d_2 = y$ (where $0 < d_1$; $d_2 < 1$) we obtain (4) indentycal to the formula in [9].

Assuming the parallax of left epipolar line point $A[g_1(x, y)]$ is $p°$. Thus, $g_2(x+p^0, y)$ is the corresponding point of $g_1(x, y)$ in the right epipolar line. The least squares multi – point matching is [9]:

$$\begin{aligned} \upsilon(x_A) &= n_1(x, y) - n_2(x = p^0, y) \\ &= g_2'(1-x)(1-y)dp_{ij} + g_2'x(1-y)dp_{i+1,j} + \\ &+ g_2'(1-x)y\,dp_{i,j+1} + g_2'(xy)dp_{i+1,j+1} - \Delta g \end{aligned} \tag{5}$$

where: n_1, n_2 – the noises of the left and right image, respectively;

$\Delta g = g_1(x, y) - g_2(x + p^0, y)$;

g_2' – the differential in x-direction in the right epipolar line;

$dp_{i,j}$; $dp_{i+1,j}$; $dp_{i,j+1}$; $dp_{i+1,j+i}$ – the correction values to initial parallaxes p^0 of corresponding nodes P_1, P_2, P_4, P_3.

The observation and solution of LSMM can be expressed in the matrix form:

$$\upsilon = \mathbf{A}X - L \qquad \text{with weight } \mathbf{W}$$
$$X = \left(\mathbf{A}^T \mathbf{W} \mathbf{A}\right)^{-1} \mathbf{A}^T \mathbf{W} L \tag{6}$$

where: **A** – the coefficient matrix of unknown,
X – the unknown vector of parallaxes corrections $dp_{i,j}$,
W – the weight matrix.

In the following section we should discuss about the weight matrix for computing the system (6).

The weight models

The weight model plays an important role in computing LSMM. The ideal weight model should reflect both the terrain feature and image intensity function. In practice it is difficult to choose the ideal weight model. For practical usage the variance, differential, gradient of grey level value or entropy can be considered to determine the weights. According to [6] the main factor which affects the least squares single point matching is the image texture information in the image. While estimating the variance of parallax p, there is a term σ_g^2, which indicates that it is related to the texture of the image. According to [10] the value of σ_g^2 can be determined from following Parseval equation:

$$\sigma_g^2 = \int g^2(x)dx = \int S_g(s)ds \tag{7}$$

where: $S_g(s)$ – the power spectrum with s representing the frequency expressed by number of point per 1 mm.

According to the definition of weight we can calculate uniform weight for observation system (5) as follows:

$$\mathbf{W} = t / \sigma_g^2 \tag{8}$$

where: t – constant (on dependancy of terrain texture).

Or, by array relaxation technique for solution system (6) which is transformed to following system under the first (or second) order differential constraint [9]:

$$\upsilon = \mathbf{A}X\text{-}L$$
$$\upsilon_x = c_1(p^0 + dp) \qquad \text{with weight } w_1$$
$$\upsilon_y = c_2(p^0 + dp) \qquad \text{with weight } w_2$$

where:

c_1, c_2 – the new special matrices;
w_1, w_2 – the x, y direction weights.

The w_1, w_2 may be the uniform weight model or variable one. Determining the variable weight model w_{xij} and w_{yij} for each grid node point (i, j) as follows:

$$
\begin{aligned}
w_{xij} &= c_1 + \left(\max g_x - h_{xij}\right) w_1 / \left(\max g_x - \min g_x\right) \\
w_{yij} &= c_2 + \left(\max g_y - h_{yij}\right) w_2 / \left(\max g_y - \min g_y\right)
\end{aligned}
\tag{9}
$$

where:

w_1, w_2, c_1, c_2 – experimental constants (we recommend) $w_1 = w_2 = 100$, $c_1 = c_2 = 10$;

max g_x, min g_x, max g_y, min g_y – the mean maximum and mean minimum values of differentials at x and y direction, respectively.

On the base of adjustment calculus theory, root mean squares error (RMSE) of unknowns dp depend on the free terms Δg_i (i – number of point) whereas Δg_i are the differences of grey observations g_1 (in the left image) and g_2, (in the right image). It means that Δg_i is the function of observations. Therefore Δg_i are the dependent magnitudes [7]. Next, we consider that the matching point within square P_1, P_2, P_3, P_4, of which the size of terrain squares net is equal 10 m. The size of squares in the image scale 1:10000 will be equal 1mm. Several points located in the area of 1mm^2 should be potentially correlated under the light coherence phenomenon known from physics. The correlation of neighbouring pixels in image was also confirmed in another publication [4]. For solution of observation system (6) we must substitute the weights **W** by covariance matrix **C**. The adjustment of (6) will be calculated under the condition $\upsilon^T C^{-1} \upsilon = \min$. On the power of [7] we can determine **C**, first, writing the system of differences Δg_i:

$$
\begin{aligned}
\Delta g_1 &= g_{11} - g_{21} \\
\Delta g_2 &= g_{12} - g_{22} \\
\Delta g_3 &= g_{13} - g_{23} \\
&\vdots \\
\Delta g_n &= g_{1n} - g_{2n}
\end{aligned}
\tag{10}
$$

The covariance **C**, for observation system (6) will be:

$$
\mathbf{C} = J_{\Delta g} C_g J_{\Delta g}^T \tag{11}
$$

Assuming the variances of grey values of pixels in the left and right image are equal $\sigma_{p\ln}^2 = \sigma_{p2n}^2 = \sigma_g^2$ we have

$$J_{\Delta g} = \begin{bmatrix} \frac{\partial \Delta g_1}{\partial g_{11}} & \frac{\partial \Delta g_1}{\partial g_{21}} & \frac{\partial \Delta g_1}{\partial g_{12}} & \frac{\partial \Delta g_1}{\partial g_{22}} & \cdots & \frac{\partial \Delta g_1}{\partial g_{1n}} & \frac{\partial \Delta g_1}{\partial g_{2n}} \\ \frac{\partial \Delta g_2}{\partial g_{11}} & \frac{\partial \Delta g_2}{\partial g_{21}} & \frac{\partial \Delta g_2}{\partial g_{12}} & \frac{\partial \Delta g_2}{\partial g_{22}} & \cdots & \frac{\partial \Delta g_2}{\partial g_{1n}} & \frac{\partial \Delta g_2}{\partial g_{2n}} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ \frac{\partial \Delta g_n}{\partial g_{11}} & \frac{\partial \Delta g_n}{\partial g_{21}} & \frac{\partial \Delta g_n}{\partial g_{12}} & \frac{\partial \Delta g_n}{\partial g_{22}} & \cdots & \frac{\partial \Delta g_n}{\partial g_{1n}} & \frac{\partial \Delta g_n}{\partial g_{2n}} \end{bmatrix} = \\ = \begin{bmatrix} 1 & -1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & -1 \end{bmatrix} \tag{12}$$

$$C_g = \begin{bmatrix} 1 & \rho_{11} & \rho_{12} & 0 & . & . & . & 0 \\ & -1 & 0 & \rho_{12} & . & . & . & \rho_{1n} \\ & & 1 & \rho_{22} & . & . & . & 0 \\ & & & -1 & . & . & . & \rho_{2n} \\ & & & & . & . & . & . \\ & \text{symmetr.} & \text{part} & & & . & . & . \\ & & & & & & . & . \\ & & & & & & & -1 \end{bmatrix} \sigma_g^2$$

where:

ρ_{ii} (i = 1 ...n) are the correlation coefficients of corresponding points homologous in the left and right image,

ρ_{ij} (i = 1 ...n, j = 1 ...m; $i \neq j$) are the correlation coefficients of neighbouring points in the left image.

σ_g^2 is according to (7).

Determining the correlation coefficients ρ_{ii} and ρ_{ij} can be done in the following mainer: the correlation coefficients ρ_{ii} is to be obtained on bases of:

$$\rho_{ii} = \operatorname{cov}(g_{1i}, g_{2i}) / \sigma_{g1i}\ \sigma_{g2i} \tag{13a}$$

where g_{1i}, g_{2i}, and σ_{g1i}, σ_{2i} are the measured grey values of the pixel i in the left and right image with their corresponding error, respectively.

Whereby, ρ_{ij} can be computed from the following formula:

$$\rho_{ij} = \rho_{ii}\, e^{-k.(dij)^2} \tag{13b}$$

where:

d_{ij} – the distance from point i to point j on the image;

k – experimental coefficient.

After substituting the equation (12a) and (12b) to the equation (11) we can determine the covariance matrix for solution of observation system (6) under the condition $v^T\mathbf{C}^{-1}v$ = min.

Marking $\mathbf{W} = \mathbf{C}^{-1}$, thus, the system (6) can be solved under the condition $v^T\,\mathbf{W}\,v$ = min. The problem of dependent observation adjustment transformed to the independent observation adjustment have been presented in [7].

Influence of the terrain height differences

In least squares matching the exact matching takes part only for the centre of matching window. An error due to the height differences appears for pixels near the boundaries of a window. To analyse the error of matching due to the relief, the four typical points lying at the corner of matching window are to be examined. We assume that the size of window matching is equal to size of a square of the rectangular grid. In this case we can examine the error of matching for establishing DEM. The horizontal difference between two point P_i (X_i, Y_i, Z_i,) and P_j (X_j, Y_j, Z_j,) of the corresponding epipolar line in the matching window has the form:

$$\begin{aligned} \Delta p &= p_i - p_j = (x_{1i} - x_{2i}) - (x_{1j} - x_{2j}) = \\ &= f\frac{a_{11}(X_i - X_B) + a_{21}(Y_i - Y_B) + a_{31}(Z_i - Z_B)}{a_{13}(X_i - X_B) + a_{23}(Y_i - Y_B) + a_{33}(Z_i - Z_B)} - f\frac{X_i}{Z_i} - \\ &- f\frac{a_{11}(X_j - X_B) + a_{21}(Y_j - Y_B) + a_{31}(Z_j - Z_B)}{a_{13}(X_j - X_B) + a_{23}(Y_j - Y_B) + a_{33}(Z_j - Z_B)} + f\frac{X_j}{Z_j} \end{aligned} \tag{14}$$

where:

f – principle distance of camera,

a_{ij} – the coefficients of the rotation matrix for the right image ,

$(X, Y, Z)_{i;j}$ – the model coordinates of points P_i, and P_j,

X_B, Y_B, Z_B – coordinates of exposure station of the right image to left one (see Fig .1).

In order for established DEM to be absolutely oriented and right image have to be transformed to epipolar lines, the parallax difference takes the following form for points that differ in X direction (points P_1, P_2).

$$\begin{aligned} \Delta p &= p_i - p_j = f\frac{X_i - X_B}{Z_i - Z_B} - f\frac{X_i}{Z_i} - f\frac{X_j - X_B}{Z_j - Z_B} + f\frac{X_j}{Z_j} = \\ &= f\frac{D_x Z_i - X_i D_x \mathrm{tg}\alpha}{Z_i^2 + Z_i D_x \mathrm{tg}\alpha} + f\frac{(X_i - X_B)D_x \mathrm{tg}\alpha - D_x(Z_i - Z_B)}{(Z_i - Z_B)^2 + D_x \mathrm{tg}\alpha (Z_i - Z_B)} \end{aligned} \tag{15}$$

where:

D_x – distance between points, P_1, P_2 in X direction;

α – the angle of terrain slope in X direction, between P_1 and P_2.

Assuming there is not difference between Z coordinates of projective centres of images, the expression (15) takes now the form:

$$\Delta p = f \frac{-X_B D_x \mathrm{tg}\alpha}{Z_i^2 + Z_i D_x \mathrm{tg}\alpha} \tag{16}$$

Similarly, for those points P_i, P_j that differ in Y direction (points P_1, P_4). In case $Z_B = 0$ we have:

$$\Delta p = f \frac{-X_B D_y \mathrm{tg}\beta}{Z_i^2 + Z_i D_y \mathrm{tg}\beta} \tag{17}$$

where:

D_y – distance between points P_1, P_4 in Y direction;

β – the angle of terrain slope in Y direction.

For the given image (X_B) and the terrain (α, β) formulas (16) and (17) allow to project rationally sizes of regular grid (D_x, D_y) so the values Δp in two directions (X, Y) have to be equal (the same scale for two directions). As conclusion, in case of transformation by epipolar lines, the terrain influence could be estimated by affine transformation [2]. For examining the influence of height differences in LSMM to establishing DEM the formulas (16) and (17) can be treated as the function of the height differences. For example, the equation (16) can be now written in the new form:

$$\Delta p = f \frac{-X_B \Delta Z_x}{Z_i^2 + Z_i \Delta Z_x} = -\frac{f}{Z_i} X_B \frac{\Delta Z_x}{Z_i + \Delta Z_x} = b_i \frac{-\Delta Z_x}{Z_i + \Delta Z_x}$$

$$\text{where} \quad \Delta Z_x = D_x \mathrm{tg}\alpha; \quad \text{and } b_i = \frac{f}{Z_i} X_B \tag{18a}$$

Creating ratio $\Delta Z_x/Z_i$, the parallax difference Δp will be in the following form:

$$\Delta p = -b_i \frac{\frac{\Delta Z_x}{Z_i}}{1 + \frac{\Delta Z_x}{Z_i}} = -\left(b_i \frac{\Delta Z_x}{Z_i} \right) \frac{1}{1 + \frac{\Delta Z_x}{Z_i}} \tag{18b}$$

Considering the ratio $\Delta Z_x/Z_i$, is smaller than 1. Thus, second term of (18b) could be expanded into numerical series.

$$\Delta p = -(b_i \frac{\Delta Z_x}{Z_i})(1 - \frac{\Delta Z_x}{Z_i} + \frac{\Delta Z_x^2}{Z_i^2} + ...) = b_i(\frac{-\Delta Z_x}{Z_i} + \frac{\Delta Z_x^2}{Z_i^2} - \frac{\Delta Z_x^3}{Z_i^3} + ...) \quad (18c)$$

Assuming ΔZ_x = 50 m; Z_i, = 500 m (height flight) the terms $(\Delta Z_x / Z_i)^2$; $(\Delta Z_x / Z_i)^3$ are the infinitely small number. Where from we have

$$\Delta p = p_i - p_j = -(b_i / Z_i)\Delta Z_x \quad (19)$$

The equation (18a) can be written in following form:

$$F_x = \Delta p + f \frac{-X_B \Delta Z_x}{Z_i^2 + Z_i \Delta Z_x} = (p_i - p_j) + f \frac{-X_B \Delta Z_x}{Z_i^2 + Z_i \Delta Z_x} = 0 \quad (20)$$

Linearizing the above equation, we obtain the following equation when the elements of orientation are known

$$dp_i - dp_j + \frac{\partial F_x}{\partial(\Delta Z_x)} d(\Delta Z_x) + F_x^0 + \Delta p_x^0 = 0 \quad (20a)$$

where:

F_x^0 – the initial value calculated on the base (20);

dp_i, dp_j – the correction to parallaxes p_i and p_j;

$d(\Delta Z_x)$ – correction unknown to height difference ΔZ_x between points P_i, P_j (*X*-direction)

Δp_x^0 – initial value computed from (19).

Looking at the equation (5) and Fig.1, we can change the index *i*, *j* from equation (20a) for points P_1 and P_2 lying in the *X* direction as follows:

$$dp_{i;j} - dp_{i+1,j} + \frac{\partial F_x}{\partial(\Delta Z_x)} d(\Delta Z_{i+1,j}) + F_x^0 + \Delta p_{xX}^0 = 0 \quad (21)$$

In the similar way, the equation (17) could be transformed into form (in *Y*-direction):

$$dp_{i;j} - dp_{i,j+1} + \frac{\partial F_y}{\partial(\Delta Z_y)} d(\Delta Z_{i,j+1}) + F_y^0 + \Delta p_{xY}^0 = 0 \quad (22)$$

Relating the equation (21) and (22) to equation (5) we have observation system for adjustment by indirect method with condition on the unknown:

$$\begin{aligned}
&\upsilon = g_2^{'}(1-x)(1-y)dp_{i,j} + g_2^{'}x(1-y)dp_{i+1,j} \\
&+ g_2^{'}(1-x)ydp_{i,j+1} + g_2^{'}(xy)dp_{i+1,j+1} - \Delta g; \qquad \text{given } C \\
&dp_{i;j} - dp_{i+1,j} + \frac{\partial F_x}{\partial(\Delta Z_x)} d(\Delta Z_{i+1,j}) + F_x^0 + \Delta p_{xX}^0 = 0 \\
&dp_{i;j} - dp_{i,j+1} + \frac{\partial F_y}{\partial(\Delta Z_y)} d(\Delta Z_{i,j+1}) + F_y^0 + \Delta p_{xY}^0 = 0
\end{aligned} \tag{23}$$

We can change two unknowns $d(\Delta Z_{i+1,j})$, $d(\Delta Z_{i,j+1})$ presented in the conditional equations (23) by unknowns $d\alpha$, $d\beta$ the terrain slopes in X and Y direction on the basis of (16), (17). The form of the equation system (23) will take on the form

$$\begin{aligned}
&\upsilon = g_2^{'}(1-x)(1-y)dp_{i,j} + g_2^{'}x(1-y)dp_{i+1,j} \\
&+ g_2^{'}(1-x)ydp_{i,j+1} + g_2^{'}(xy)dp_{i+1,j+1} - \Delta g; \qquad \text{given } C \\
&dp_{i;j} - dp_{i+1,j} + G_\alpha d\alpha + F_\alpha^0 + \Delta p_{x\alpha}^0 = 0 \\
&dp_{i;j} - dp_{i,j+1} + G_\beta d\beta + F_\alpha^0 + \Delta p_{x\beta}^0 = 0
\end{aligned} \tag{24}$$

where the G_α; G_β are the differentionals caculated from (16) and (17).

Systems (23) and (24) can be solved simultaneously by the method of least squares. In the solution, it would be more convenient to write two last equations at (23) and (24) into the form of error equations instead of using them as conditional equations.

The equation system (23) or (24) serves for establishing DEM with taking into account the terrain height differences (ΔZ) between the node points of regular grid.

CONCLUSION

The development of digital image process in the automatic creation of DEM. The technical basic of DEM is provided by image matching methods of witch least squares multi-points matching is of immediate and which have been realised in software program on digital image workstation. The accuracy of a DEM is simply the average vertical error of all points interpolated within the DEM grid. Choosing the rational weight model is the important task for solution of equation system (5). The grey level differences in (5) have been treated as the dependent observations. Therefore in this paper the covariance matrix proposed by author is related to the problem of correlated pixels located within square unit. The accuracy of a DEM depends also on terrain features as slope (height differences of points), breakline etc. the terrain property in the matching process. Two conditional equations on the unknown of height differences of grid nodes in the form (21) and (22) introduced by author are related to the equation (5). It makes new equation system in the form (23) for simultaneous solution of DEM. In this way the reliability of establishing DEM should be better than the conventional least square multi-point matching, especially for mountainous area.

The system (23) and (24) have also important sense to detecting and filtering objects (as buildings; trees etc) covering on flat terrain surface for accurated DEM.

The strategy of choosing the constraint weight and considering the stope of terrain (height differences) become acurrent problem in the further development of least square mult-point matching for DEM.

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*ASPRS – American Society for Photogrammetry & Remote Sensing.

**IAPRS – International Archives of Photogrammetry & Remote Sensing.

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Wielopunktowe spasowanie obrazów metodą najmniejszych kwadratów dla utworzenia DEM z uwzględnieniem korelujących pikseli sąsiadujących i różnic wysokości terenu

Streszczenie

Jednym z problemów w fotogrametrii cyfrowej jest pasowanie obrazów (Image Matching Technique) dla identyfikacji punktów homologicznych modelu utworzonego z pary stereogramów. Technika spasowania w obszarach danych rastrowych - ABM (Area-Based Matching) jest bardziej precyzyjna niż technika spasowania w przestrzeni cech danych wektorowych-FBM (Future-BasedMatching). Technika wielopunktowego spasowania obrazów metodą najmniejszych kwadratów - LSMM (Least Squares Multi-point Matching) jest jedną z technik globalnego spasowania. (obrazów) - GIM (Global Image Matching), która została zmodyfikowana na podstawie techniki pojedynczo-punktowego spasowania obrazów metodą najmniejszych kwadratów - LSSM (Least Squares Single-point Matching). Technika LSMM zapewnia wyższy stopień zaufania niż LSSM, ponieważ relacja pomiędzy różnymi sąsiadującymi punktami zostaje uwzględniona w jednoczesnym procesie obliczeniowym. Technika LSMM jest przeznaczona do wykonywania pomiaru paralaks podłużnych punktów siatki regularnej z zamiarem utworzenia DEM. W pracy przedstawiono propozycje dotyczącą uwzględniania korelacji sąsiednich pikseli i dużych różnic wysokości punktów terenu celem dokładnego automatycznego tworzenia DEM.

Луонг Чин Ке

Многоточечное совмещение изображений методом наименьших для создания цифровой модели местности с учётом соседних пикселей и разниц высоты местности

Резюме

Одной из проблем цифровой фотограмметрии является совмещение изображений (Image Matching Technique) для идентификации трансформированных центральных точек модели, созданной с стереопары. Техника совмещения в областях растровых данных - ABM (Area-Based Matching) является более точной чем техника совмещения в пространстве особенностей векторных данных - FBM (Future-Based Matching). Техника многоточечного совмещения изображений методом наименьших квадратов - LSMM (Least Square Multi-point Matching) является одной из техник глобального совмещения изображений - GIM (Global Image Matching), которая была модифицирована на основе техники одно-точечного совмещения изображений методом наименьших квадратов - LSSM (Least Square Single-point Matching). Техника LSMM обеспечивает высшую степень доверия чем LSMM, потому что связь между разными соседними точками учитывается в одновременном процессе вычислений. Техника LSMM предназначена для измерений продольных параллакс точек равномерной сетки с целью создания цифровой модели местности. В работе представлено предложение, касающееся учёта корреляции соседних и больших разниц высоты точек местности с целью точного автоматического создания модели местности.