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Research paper

Identification of Physical Parameters of a Porous Material Located in a Duct by Inverse Methods

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Lined ducts with porous materials are found in many industrial applications. To understand and simulate the acoustic behaviour of these kinds of materials, their intrinsic physical parameters must be identified. Recent studies have shown the reliability of the inverse approach for the determination of these parameters. Therefore, in the present paper, two inverse techniques are proposed: the first is the multilevel identification method based on the simplex optimisation algorithm and the second one is based on the genetic algorithm. These methods are used of the physical parameters of a simulated case of a porous material located in a duct by the computation of its acoustic transfer, scattering, and power attenuation. The results obtained by these methods are compared and discussed to choose the more efficient one.

Keywords: porous materials; inverse methods; scattering matrix; acoustic power attenuation.

1. Introduction

The application of porous materials in duct systems is widely used in many industrial applications such as aircraft engines, compressors, and ventilation systems to reduce noise. Many works are presented to model the acoustic behaviour of lined ducts with porous materials using the acoustic multimodal scattering matrix due its efficiency. In fact, this matrix presents an intrinsic characterisation of the studied duct element allowing the computation of its transmission and reflection coefficients as well as its acoustic power attenuation as presented in (Benjedidia *et al.*, 2014) and in (Othmani *et al.*, 2015) who studied respectively the influence of the temperature and porous parameters on the acoustic behaviour and the acoustic power attenuation of a cylindrical duct lined with a porous material using this matrix. Masmoudi *et al.* (2017) and Dhief *et al.* (2020) used the same matrix to study the acoustic behaviour of ducts lined with porous materials with geometric and impedance discontinuities. The scattering matrix can be used also to model the acoustic behaviour of duct elements containing a porous material as presented in (Kani *et al.*, 2019).

The porous materials are characterised by physical parameters such as air flow resistivity, porosity, tortuosity and viscous and thermal characteristic lengths. These physical parameters are used to predict and model the acoustic behavior of this kind of liner as presented in (Delany, Bazley, 1970; Attenborough, 1983; 1987; Johnson *et al.*, 1987; Hess *et al.*, 1987; Mikki, 1990; Allard, Champux, 1992; Hamet, Bérengier, 1993; Bérengier *et al.*, 1997; Lafarge *et al.*, 1997; Allard, Attala, 2009) allowing the computation of the characteristic impedance, the characteristic wave number and the acoustic absorption coefficient of a porous material.

These parameters can be measured directly, but this needs a set of experimental dispositive to measure each parameter separately and requires a lot of time.

To overcome this problem, inverse methods present an alternative technique. Therefore, many authors have proposed different inverse techniques for estimating of acoustical parameters of porous materials. Sellen *et al.* (2002) used the active control method to identify the parameters of porous materials. The principle of this method is based on varying the boundary conditions at the face of the material by an active control system. The modification of boundary conditions aims at bringing the theoretical results given by the model of Lafarge *et al.* (1997) closer to the experimental ones. A very good agreement is observed between estimated values and experimental values for different cases. Attalla and Panneton (2005) developed an inverse method for the characterisation of porous materials based on the measurement of the surface acoustic impedance of the porous material and using the model of Allard and Champoux (1992). The proposed method showed its efficiency even in the case of multilayer porous materials. Garoum and Simon (2005) and Garoum and Tajayouti (2007) developed an inverse procedure based on the genetic algorithm and the Levenberg-Marquardt method in order to estimate the physical parameters of two sustainable materials. They minimise a cost function based on the difference between the experimental and calculated acoustic absorption coefficients. Two models of porous materials are used: model of Johnson *et al.* (1987) and the model of Attenborough *et al.* (1983). The agreement between parameters estimations given by models and experimental data is shown. Shravege *et al.* (2008) proposed an inverse method based on the genetic algorithm to minimise the difference between the measured and theoretical surface acoustic impedance of the material modelled on the basis of Allard and Champoux (1992). Mareze and Lenzi (2011) developed an inverse method using models of Allard and Champoux (1992) and Johnson *et al.* (1992). It is based on the minimisation of the difference between the measured and computed surface acoustic impedances of the porous material. The minimisation is assured by genetic and gradient methods. The obtained results present a good agreement with direct measurements of each parameter. They showed that the gradient method is the fastest one to estimate the porous material parameters. Alba *et al.* (2011) developed an inverse technique to estimate the porosity by using measurements of the normal incidence sound absorption coefficient. They demonstrate that the numerical results agree well with the experimental ones. Zielinski (2012; 2014) developed an inverse identification method based on the model of Allard and Champoux (1992) and a minimisation of the difference between theoretical and experimental acoustic absorption coefficients. Also a microscopic estimation of the acoustic parameters of the porous materials is presented. Chazot *et al.* (2012) presented an inverse characterisation method to get poroelastic intrinsic parameters of porous materials. This method is based on a Bayesian approach getting probabilistic data of each parameter allowing the determination of the confidence interval of each parameter. Hentati *et al.* (2016) developed a three level identification method for the identification of the physical parameters of porous materials based on the minimisation between the experimental and computed absorption coefficients of a porous material using the Nelder-Mead simplex algorithm. In each level one or two parameters are deduced. They used the models of Delany-Bazley (1970), Hamet and Bérengier (1993) and Allard and Champoux (1992) respectively of each level.

In this paper, the inverse identification method developed by Hentati *et al.* (2016) is applied to a simulated case of a duct element containing a porous material to estimate its physical parameters. In this case, the acoustic power attenuation is used as a cost function to be minimised. This power is computed from the scattering matrix of the duct element containing the porous material. This matrix is deduced from the transfer matrix of the duct element. The proposed method is compared with results obtained by the genetic algorithm. The outline of this paper is as follows: in Sec. 2, the transfer matrix method (TMM) is presented as well as the models to predict the acoustic behavior of porous materials according their physical parameters. Sections 3 and 4 present respectively details of the computation of scattering and the acoustic power attenuation of a duct containing a porous material. Then, the validation of this method is obtained by a comparison with the wave finite element method (WFE) as presented in Sec. 5. Section 6 presents in detail the used two inverse methods. Finally, the results of these inverse methods are presented and discussed in Sec. 7.

2. The transfer matrix method (TMM)

The transfer matrix method (TMM) is a method allowing the modelling of the propagation of sound along a duct element. In the present study, only the acoustic plane wave is assumed to propagate in and out of the element and the sound field can be characterised by two state variables: the acoustic pressure and the particle velocity on each side of the studied duct element presented in Fig. 1.

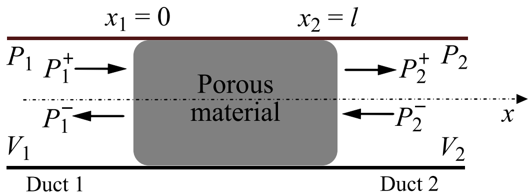

Fig. 1. Studied duct element containing a porous material.

A porous material with a length equal to l located in a duct element is considered as presented in Fig. 1. The transfer matrix $[T]$ relates the sound pressure and the particle velocity in part 1 (P_1 and V_1) with the sound pressure and the particle velocity in part 2 (P_2 and V_2). The transfer matrix is expressed as follows (Ying, 2010; Kani *et al.*, 2019):

$$\begin{bmatrix} P_1 \\ V_1 \end{bmatrix} = [T] \begin{bmatrix} P_2 \\ V_2 \end{bmatrix},$$

$$[T] = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} \cos K_C(\omega)l & jZ_C \sin K_C(\omega)l \\ \dfrac{j}{Z_S} \sin K_C(\omega)l & \cos K_C(\omega)l \end{bmatrix}, \tag{1}$$

where $Z_C(\omega)$ and $K_C(\omega)$ are respectively the normalised complex acoustic impedance and the complex wave number of the porous material, and $\omega = 2\pi f$ is the pulsation, where f is the frequency. These two quantities are estimated by different models using the physical parameters of the porous material as presented in Sec. 1. Further on, only used in this paper models are discussed in detail:

1) The first model, the simplest one, is an empirical model proposed by Delany and Bazley (1970) which allows estimation of the normalised complex acoustic impedance and the wave number of the porous material in function of the frequency and the flow resistivity σ by:

$$Z_C(\omega) = Z_0 \left(1 + 9.08 \left(\frac{f}{\sigma} \right)^{-0.754} - 11.9j \left(\frac{f}{\sigma} \right)^{-0.732} \right), \tag{2}$$

$$K_C(\omega) = \frac{\omega}{c_0} \left(1 + 10.8 \left(\frac{f}{\sigma} \right)^{-0.700} - 10.3j \left(\frac{f}{\sigma} \right)^{-0.595} \right), \tag{3}$$

where $Z_0 = \rho_0 c_0$ is the characteristic impedance of the air; ρ_0 and c_0 are respectively the air density and the sound celerity in the air.

2) The second model is one by Hamet and Bérengier (1993) and it expresses the same quantities as follows:

$$Z_C(\omega) = Z_0 \sqrt{\frac{\alpha_\infty}{\gamma}} \frac{1}{\phi} \sqrt{1 - j\frac{f_\mu}{f}} \left[1 - \left(1 - \frac{1}{\gamma} \right) \frac{1}{1 - j\frac{f_\theta}{f}} \right]^{-1/2}, \tag{4}$$

$$K_C(\omega) = k_a \sqrt{\alpha_\infty \gamma} \sqrt{1 - j\frac{f_\mu}{f}} \sqrt{1 - \left(1 - \frac{1}{\gamma} \right) \frac{1}{1 - j\frac{f_\theta}{f}}}, \tag{5}$$

where

$$f_\theta = \frac{\sigma}{2\pi\rho_0 N_{\mathrm{Pr}}} \qquad \text{and} \qquad f_\mu = \frac{\sigma\phi}{2\pi\rho_0\alpha_\infty}$$

describe respectively the thermal and viscous dependences of the porous material, ϕ and α_∞ are respectively the material porosity and tortuosity, $\gamma = C_p/C_v$ is a specific heat ratio and N_{Pr} is the Prandtl number.

3) The third model is authored by Lafarge *et al.* (1997) and expresses $Z_C(\omega)$ and $K_C(\omega)$ using the five porous materials parameters (σ, ϕ, α_∞, Λ, and Λ') as follows:

$$Z_C(\omega) = \frac{1}{\phi} \sqrt{\rho(\omega) K(\omega)}, \tag{6}$$

$$K_C(\omega) = \omega \sqrt{\frac{\rho(\omega)}{K(\omega)}}, \tag{7}$$

with

$$\rho(\omega) = \alpha_\infty \rho_0 \left[1 - j \frac{\sigma\phi}{\rho_0 \alpha_\infty \omega} \sqrt{1 + \frac{4j\rho_0 \alpha_\infty^2 \omega\eta}{\sigma^2 \phi^2 \Lambda^2}} \right], \tag{8}$$

$$K(\omega) = \gamma P_0 \left[\gamma - \frac{\gamma - 1}{1 + \frac{\phi\eta}{j\omega\rho_0 N_{\mathrm{Pr}} k_0'} \sqrt{1 + \frac{4j\omega\rho_0 N_{\mathrm{Pr}} k_0'^2}{\eta\phi^2 \Lambda'^2}}} \right]^{-1}, \tag{9}$$

where $k_0' = \frac{\phi\Lambda'^2}{8}$ is the thermal permeability, P_0 is the atmospheric pressure, and η is the dynamic viscosity.

3. Computation of the scattering matrix

When the linear theory is valid, the acoustic behaviour of the studied duct element can be completely described by its scattering matrix $[S]$, which gives a linear relationship between the incoming wave pressures vector $\begin{Bmatrix} P_1^+ \\ P_2^- \end{Bmatrix}$ and the outgoing pressure wave vector $\begin{Bmatrix} P_1^- \\ P_2^+ \end{Bmatrix}$ (Fig. 1) as follows:

$$\begin{Bmatrix} P_1^- \\ P_2^+ \end{Bmatrix} = [S] \begin{Bmatrix} P_1^+ \\ P_2^- \end{Bmatrix} \quad \text{with} \quad [S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}. \tag{10}$$

The relationship between the transfer and scattering matrices is given by (Ying, 2010) and (Kani *et al.*, 2019) as follows:

$$\begin{aligned} S_{11} &= \frac{X^+ - W^+}{X^+ + W^+}, \\ S_{12} &= \frac{X^+W^- - W^+X^-}{X^+ + W^+}, \\ S_{21} &= \frac{2}{X^+ - W^+}, \\ S_{22} &= \frac{X^- + W^-}{X^+ - W^+}, \end{aligned} \tag{11}$$

with

$$\begin{aligned} X^\pm &= T_{11} \pm T_{12}, \\ W^\pm &= \frac{T_{12}}{Y} \pm T_{22}, \end{aligned} \tag{12}$$

where Y is the porous material complex acoustic admittance $\left(Y = \frac{1}{Z_C}\right)$.

In the studied case, it is supposed that only the acoustic plane wave propagates in the duct, and the sample is symmetrical. Then, the scattering matrix contains only two unknown coefficients instead of four defined by:

- $S_{11} = S_{22} = R_{00,00}$: the reflection coefficient of the plane mode;
- $S_{12} = S_{21} = T_{00,00}$: the transmission coefficient of the plane mode.

4. Computation of the acoustic power attenuation

The acoustic power attenuation W_{att} of the duct element containing a porous material is defined as the difference between the input acoustic power W_{in} and the output acoustic power as defined in (Taktak *et al.*, 2010) and (Kani *et al.*, 2019):

$$W_{\text{att}} = W_{\text{in}} - W_{\text{out}}. \tag{13}$$

As described in (Taktak *et al.*, 2010), these acoustic power attenuations are defined by:

$$W_{\text{in}} = \Phi^2 \left(|P_1^+|^2 + |P_2^-|^2 \right), \tag{14}$$

$$W_{\text{out}} = \Phi^2 \left(|P_1^-|^2 + |P_2^+|^2 \right), \tag{15}$$

with

$$\Phi = \sqrt{\frac{1}{2\rho_0 c_0}}. \tag{16}$$

W_{in} and W_{out} can be rewritten as in (Taktak *et al.*, 2010):

$$W_{\text{in}} = \{\Pi\}_{\text{in}}^H \times \{\Pi\}_{\text{in}}, \tag{17}$$

$$W_{\text{out}} = \{\Pi\}_{\text{out}}^H \times \{\Pi\}_{\text{out}}, \tag{18}$$

with

$$\{\Pi\}_{\text{in}} = \left\{ \begin{matrix} \Phi P_1^- \\ \Phi P_2^+ \end{matrix} \right\} \quad \text{and} \quad \{\Pi\}_{\text{out}} = \left\{ \begin{matrix} \Phi P_1^+ \\ \Phi P_2^- \end{matrix} \right\}, \tag{19}$$

where superscript H is the conjugate transpose.

Substituting Eqs (17) and (18) into Eq. (13), we get:

$$W_{\text{att}} = \{\Pi\}_{\text{in}}^H \times \{\Pi\}_{\text{in}} - \{\Pi\}_{\text{out}}^H \times \{\Pi\}_{\text{out}}. \tag{20}$$

According to the definition of the scattering matrix, we have

$$\{\Pi\}_{\text{out}} = [S] \cdot \{\Pi\}_{\text{in}}. \tag{21}$$

Then

$$W_{\text{att}} = \{\Pi\}_{\text{in}}^H \left([I] - [H] \right) \cdot \{\Pi\}_{\text{in}}. \tag{22}$$

Here, $[H] = [S]^H \cdot [S]$; $[I]$ is a unit matrix.

$[H]$ is a positive definite matrix and it can be expressed as follows:

$$[H] = U \cdot \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \cdot [U]^H. \tag{23}$$

Here, λ_1, λ_2 are the eigenvalues of the matrix $[H]$; $[U]$ is the eigenvector of the matrix $[H]$, and $[I] = [U] \cdot [U]^H$.

Finally, the acoustic power attenuation is expressed:

$$W_{\text{att}} = \{\Pi\}_{\text{in}}^H [U] \begin{bmatrix} 1-\lambda_1 & 0 \\ 0 & 1-\lambda_2 \end{bmatrix} [U]^H \{\Pi\}_{\text{in}} \tag{24}$$

with

$$\{d\} = [U]^H \{\Pi\}_{\text{in}}. \tag{25}$$

So,

$$W_{\text{att}} = \{d\}^H \left[\text{diag}\left(1 - \lambda_j\right) \right] d, \quad j = 1, 2. \tag{26}$$

Equation (26) can be rewritten as

$$W_{\text{att}} = \sum_{j=1}^{2} \xi \left| d_j \right|^2, \quad \xi = 1 - \lambda_j. \tag{27}$$

The acoustic power attenuation is defined in decibel [dB] in (Taktak *et al.*, 2010):

$$\begin{aligned} W_{\text{att}}\,[\text{dB}] &= 10 \log_{10} \left(\frac{W_{\text{in}}}{W_{\text{out}}} \right) \\ &= 10 \log_{10} \left(\frac{|d_1|^2 + |d_2|^2}{\lambda_1 |d_1|^2 + \lambda_2 |d_2|^2} \right). \end{aligned} \tag{28}$$

5. Numerical validation

To validate the proposed modelling method presented in Sec. 2, a comparison with the results obtained with the WFE (Wave Finite Element) method developed by Kessentini *et al.* (2016) and Ben Souf *et al.* (2017) is presented in this section. The parameters of the studied example are presented in Table 1. Figure 2 shows the variation of transmission and reflection coefficients as well as the acoustic power attenuation of the studied duct in function of frequency of the studied case computed by the two modelling methods.

The comparison between the TMM and the WFE methods for calculating the transmission and reflection coefficients as well as the acoustic attenuation for a duct containing porous material showed a good agreement between the two results (Fig. 2). This validation allows us to use the TMM method for determining the parameters of the porous material located in a duct, which is presented in the following section.

Table 1. Summary of the parameters of the studied duct element duct containing a porous material.

Parameters	Value
Air	
Celerity of sound c_0	340 m/s
Density ρ_0	1 kg/m^3
Duct	
Radius r	0.015 m
Length l	0.25 m
Porous material	
Flow resistivity σ	10 000 N/(s·m^4)
Porosity ϕ	0.88
Tortuosity α_∞	1
Viscous characteristic length Λ	$129 \cdot 10^{-6}$ m
Thermal characteristic length Λ'	$198 \cdot 10^{-6}$ m
Length	0.015 m

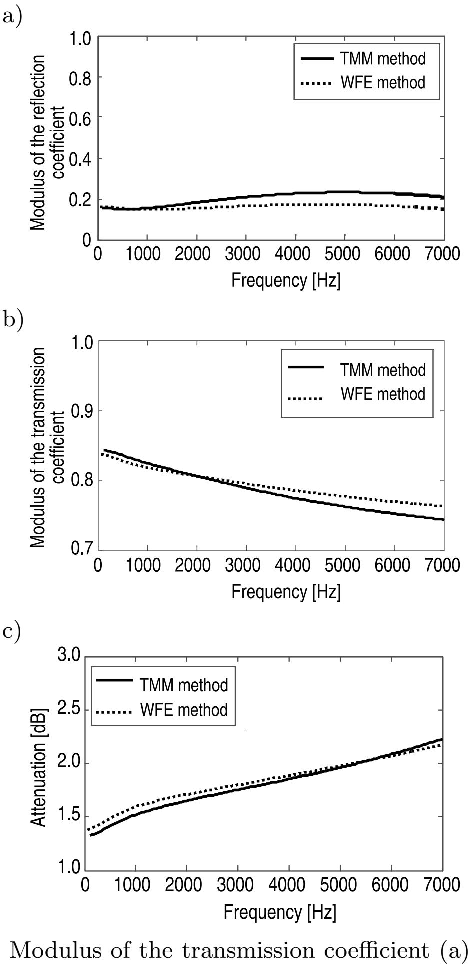

Fig. 2. Modulus of the transmission coefficient (a), the reflection coefficient (b), acoustic power attenuation (c) of the studied duct element.

6. Inverse identification techniques

In this section, the proposed inverse identification technique is presented. It has the same principle as the previous work (Hentati *et al.*, 2016) based on the Nelder-Mead's algorithm (Lagarias *et al.*, 1998) but instead of using the acoustic absorption coefficient in the cost function, the acoustic power attenuation of the duct element containing the studied porous material is used. For this, we use reference values of each physical parameter to compute the acoustic power attenuation of the duct configuration. Then, by applying the optimisation algorithm, the physical parameters are estimated and compared to the reference one to evaluate the efficiency of the proposed method. The results obtained by this technique are compared with the results obtained with the help of the genetic algorithm (Goldberg, 1989). In the following sections, both used methods are presented in detail.

6.1. The multilevel optimisation method

In this method, the identification is carried out at three levels as presented in (Hentati *et al.*, 2016). This estimation is based on the minimisation of the cost function based on the Nelder-Mead method. This function is defined by the square of the modulus of the difference between the computed and the reference acoustic power attenuation defined as:

$$F_{\min} = \left(\left|\frac{W_{c_i} - W_{\text{ref}}}{W_{\text{ref}}}\right|\right)^2, \qquad i = 1, 2, 3, \tag{29}$$

where i is the level number and W_{c_i} is the computed acoustic power attenuation in level i and W_{ref} is the reference acoustic power attenuation.

The estimation process can be described in detail similar to the flow chart presented in Fig. 3:

- At the first level, the flow resistivity σ_{est} of the studied material is estimated. For this, the Delany-Bazely model is used to compute the acoustic power attenuation W_{c_1} by injecting the initial flow resistivity σ_i. The real flow resistivity will be deducted by varying the initial flow resistivity in order to minimise the cost function. The value of the flow resistivity giving the minimum of this cost function is chosen as the estimated flow resistivity σ_{est} of the studied material.
- The estimated porosity ϕ_{est} and the estimated tortuosity $\alpha_{\infty\text{est}}$ are determined at the second level. Using the Hamet-Bérengier model, the computed acoustic power attenuation W_{c_2} is calculated using the estimated flow resistivity σ_{est} (determined in the first level), the initial porosity ϕ_i, and the initial tortuosity α_i. These last two parameters are then varied to minimise the difference between the reference acoustic power attenuation W_{ref} and the computed acoustic power attenuation W_{c_2} to get finally the estimated porosity ϕ_{est} and the estimated tortuosity $\alpha_{\infty\,\text{est}}$.
- At the third level, the estimated viscous length Λ_{est} and the estimated thermal length Λ'_{est} are

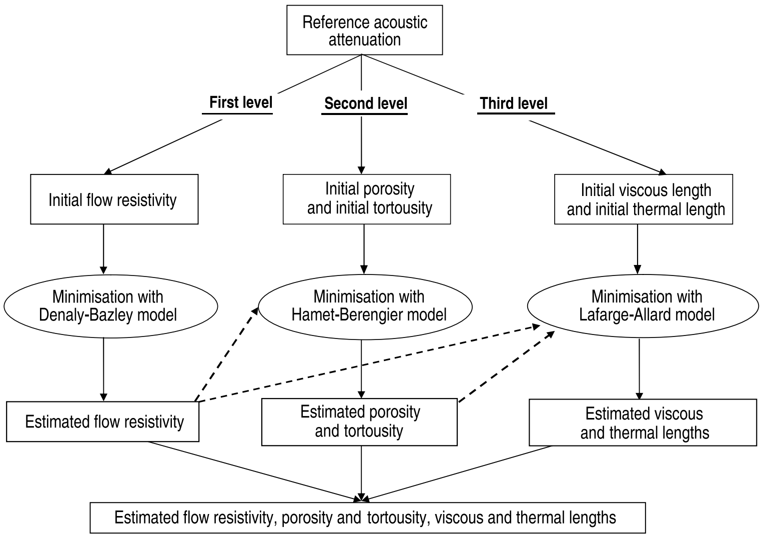

Fig. 3. Flow chart of the steps of the simplex algorithm for calculating the parameters of the porous material located in a duct.

determined using the Lafarge-Allard model. Using the estimated flow resistivity σ_{est} (computed at the first level) and estimated porosity ϕ_{est} and tortuosity $\alpha_{\infty\,\text{est}}$ (computed at the second level), the computed acoustic power attenuation W_{c_3} is calculated. The values of viscous and thermal lengths Λ_i and Λ_i' are varied to minimise the difference between the reference and the computed acoustic power attenuation W_{ref} and W_{c_3} to get finally the estimated viscous and thermal lengths Λ_{est} and Λ'_{est}.

Finally we obtain five estimated values which are the flow resistivity σ_{est}, the porosity ϕ_{est}, the tortuosity $\alpha_{\infty\,\text{est}}$, and the two characteristic lengths: viscous Λ_{est} and thermal Λ'_{est} ones, by an average of obtained values over the frequency band.The flow chart of the developed multilevel identification is described in Fig. 3.

6.2. Genetic algorithm

The genetic algorithm is based on Darwin's theory of evolution. It is used to solve the optimisation problem with constraints and limitations on the solution. It repeatedly modifies a population of individual points by using modelled rules on gene combinations in biological reproduction.

At each stage, the genetic algorithm selects random individuals from the current population to be "parents" and uses them to produce "children" for the next generation. In successive generations, the genetic algorithm improves the chances of finding a global solution. In the final analysis, acoustic power attenuation is used as a cost function. The cost minimisation function is defined as:

$$F_{\min} = \left(\left| \frac{W_c - W_{\text{ref}}}{W_{\text{ref}}} \right| \right)^2, \tag{30}$$

where W_c is the computed acoustic power attenuation and W_{ref} is the reference acoustic power attenuation.

The classical architecture synthesising the major steps implemented in a genetic algorithm is presented in Fig. 4 (Goldberg, 1989).

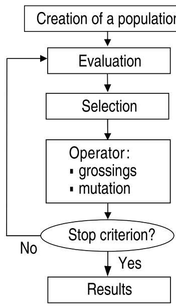

Fig. 4. Schematic diagram of the principle of the genetic algorithm.

7. Results and discussion

Two identification methods are applied to the same case presented in Sec. 3. The used reference values are presented in Table 2. The initial values are obtained using formulas presented in (Leclaire *et al.*, 1996) and (Panneton, Olny, 2006).

Table 2. Used reference and initial physical parameters.

Parameters	σ	φ	α_∞	Λ	Λ'
Reference	10000	0.88	1	129	198
Initial	9500	0.85	0.95	127	195

For the genetic algorithm, Table 3 presents the bounds of each physical parameter used in the optimisation process.

Table 3. Bounds on physical parameters used in the genetic algorithm.

Bounds	σ	φ	α_∞	Λ	Λ'
Lower bounds	1000	0.1	0.1	10	10
Upper bounds	20000	1	5	1000	1000

Table 4 presents the estimated values of the physical parameters of the studied porous material given by the proposed and the genetic methods and compared with the reference ones. The value of each parameter is obtained as an average over the frequency domain.

Table 4. Reference and estimated values porous material parameters.

Parameters	Reference	Estimated multilevel method	Parameters genetic method
Flow resistivity [N/(s·m^4)]	10 000	9478	12 175.65
Porosity	0.88	0.85	0.235
Tortuosity	1	0.98	0.903
Viscous characteristic length [μm]	129	92.07	95.96
Thermal characteristic length [μm]	198	141.3	121.605

The relative error for all physical parameters is calculated by the relation (31) and the relative error for all materials in the duct is given in Table 5

$$\text{Error [\%]} = \left| \frac{\text{estimated value} - \text{reference value}}{\text{reference value}} \right|. \qquad (31)$$

Table 5. Average relative error for all material samples.

Error [%]	σ	φ	α_∞	Λ	Λ'
Multilevel method	5.22	3.4	2	28.63	28.63
Genetic method	21.75	73.29	9.7	25.61	38.58

Finally, Fig. 5 shows a comparison between the variation of acoustic power attenuation calculated using the reference parameters and those calculated using the estimated parameters in the frequency range from 0 at 7000 Hz using the Lafarge-Allard model.

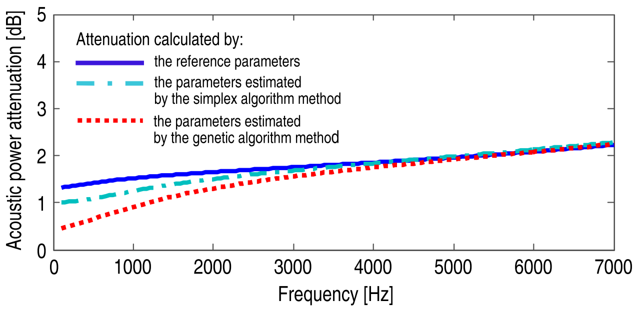

Fig. 5. Comparison of sound attenuation for a duct containing a porous material.

By analysing these results we derive the following:

- The acoustic attenuation achieved by the parameters estimated by the genetic algorithm is close to the attenuation calculated by the reference parameters of the studied porous material in the high frequencies range 4000–7000 Hz. However, the obtained values of this method are far from the reference one as presented in Table 3 (error exceeds 50% as presented in Table 4). The fact that these differences do not touch the computation of the acoustic power attenuation is explained by the fact that the computation of this later uses all the estimated parameters and the errors observed in Tables 4 and 5, which influences the final result.
- The multilevel method gives results close to the reference values presented in Table 3. (The relative errors obtained by the presented method are 5.22% for the air flow resistivity, 3.4% for the porosity, 2% for the tortuosity, and 28.62% for characteristic lengths), and the acoustic attenuation obtained using these values is close to the reference one except for the frequency range of 0–2000 Hz, with small and acceptable differences. This difference is also due to the fact that the acoustic power attenuation is obtained by combining the estimated parameters. On the other hand, the use of three different models to estimate the parameters of the porous material minimises considerably the errors in the estimated results. This approach estimates the physical parameters with the minimum of error as presented in Table 3 and gives an acoustic attenuation close to the reference one as presented in Fig. 5. We can conclude that the developed multilevel identification method using the simplex algorithm is more effective than the genetic algorithm for estimating the acoustic parameters of porous materials located in a duct.

8. Conclusion

The multilevel identification method of the acoustic power attenuation of a duct element containing a porous material was developed to estimate the physical parameters of the porous material. It is based on the simplex optimisation algorithm. This method is based on the computation of the scattering matrix. The comparison between the results of the proposed method with those of the genetic method shows clearly that the parameters estimated by the multilevel identification method are closer to the reference parameters than the parameters estimated by the genetic algorithm alone. The obtained errors are acceptable. In addition, the acoustic attenuation calculation using the parameters estimated by this method allowed to reach the acoustic attenuation calculated by the reference parameters.

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