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THE TENSILE TEST CURVE OF WIRE AFTER ITS CURVATURE CHANGING

KRZYWA ROZCIĄGANIA DRUTU PO ZMIANIE JEGO KRZYWIZNY

Stress and strain state during the curvature change are characterized by great non-uniformity. There are circumferential tensile stresses in the external elongated zone and compressive ones in the internal zone. Both zones are separated by the neutral surface where circumferential stresses equal zero.

Circumferential strains during curvature change can be determined from:

$$\varepsilon_\theta = \frac{x}{\rho}.$$

In the neighbourhood of the neutral surface the material is in the elastic state and the circumferential stresses can be calculated on the basis on equation:

$$\sigma_\theta = \frac{x}{\rho} \cdot \mathrm{E}.$$

Range of the elastic zone can be determined from the condition that the circumferential stresses reach the value of yield stress.

$$\frac{x_0}{\rho} \cdot \mathrm{E} = \sigma_p.$$

As a result of superimposing of the unloading stresses on the stresses appearing during the active phase of the process some stresses called residual stresses will remain after the process is finished.

During tension of a wire having the residual stresses first elastic deformation is observed in the wire, and the tensile force changes linearly depending on the deformation. When sum of the stresses resulting from the external loading and residual stresses reaches the yield stress value, the material is in the plastic condition. With the increase of the stress resulting from the external loading the plastic zone is gradually expanding. In the elastic zone stresses grow proportionally to the strain while in the plastic zone rise in stresses will occur in the range from residual stresses to the yield stress value.

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Borders between the zones can be determined from the boundary conditions saying that sum of stresses from external loading and residual stresses reaches value of yield stress.

On the basis of given equations first distribution of the residual stresses on the cross-section of the wire and then values of forces for the elastic and plastic zones were determined. Considering the peculiarity of the change of stresses in the elastic and plastic zones equations describing tensile force for elastoplastic stage of tension were derived and compared with forces observed in the experiment.

Podczas zmiany krzywizny stan naprężenia i odkształcenia charakteryzuje się dużą nierównomiernością. W zewnętrznej wydłużonej strefie naprężenia obwodowe są rozciągające, natomiast w wewnętrznej są ściskające. Obie te strefy oddziela od siebie powierzchnia obojętna, w której naprężenia obwodowe są równe zeru.

Odkształcenia obwodowe podczas zmiany krzywizny można wyznaczyć ze wzoru:

$$\varepsilon_\theta = \frac{x}{\rho}.$$

W pobliżu powierzchni obojętnej materiał znajduje się w stanie sprężystym, a naprężenia obwodowe można wyznaczyć ze wzoru:

$$\sigma_\theta = \frac{x}{\rho} \cdot \mathrm{E}.$$

Zasięg strefy sprężystej może być wyznaczony z warunku, że naprężenia obwodowe osiągają wartość naprężenia uplastyczniającego:

$$\frac{x_0}{\rho} \cdot \mathrm{E} = \sigma_p.$$

W wyniku nałożenia się naprężeń odciążających na naprężenia występujące podczas czynnej fazy procesu pozostaną po jego zakończeniu pewne naprężenia zwane naprężeniami własnymi.

Podczas rozciągania drutu, w którym występują naprężenia własne drut początkowo odkształca się sprężyście, a siła rozciągająca zmienia się liniowo w zależności od odkształcenia. Uplastycznienie nastąpi wówczas, gdy suma naprężeń od obciążenia zewnętrznego oraz naprężeń własnych osiągnie wartość naprężenia uplastyczniającego. W miarę przykładania coraz to większych naprężeń zewnętrznych rozszerza się zasięg uplastycznionej strefy.

W strefie sprężystej naprężenia rosną proporcjonalnie do zachodzących odkształceń, natomiast wzrost naprężeń w strefie uplastycznionej będzie zachodził w zakresie od wartości naprężeń własnych do wartości naprężenia uplastyczniającego.

Granice stref wyznacza się z warunków brzegowych, zgodnie z którymi suma naprężeń własnych oraz naprężeń od obciążenia zewnętrznego osiąga wartość równą naprężeniu uplastyczniającemu.

W oparciu o podane wzory wyznaczono rozkład naprężeń własnych na przekroju poprzecznym drutu a następnie wartość siły dla strefy sprężystej i plastycznej. Uwzględniając specyfikę zmian naprężeń w strefie sprężystej i plastycznej wyprowadzono wzory do określenia siły rozciągania dla sprężysto-plastycznego etapu rozciągania oraz porównano je z siłami wyznaczonymi doświadczalnie.

Symbols specifications:

r_0 – wire radius

x – distance of the given point

x_0 – range of elastic zone

x_1, x_2, x_3 – coordinates of zones
E – Young's modulus
M_g – bending moment
P_p – resultant force for plastic zone
P_t – tensile force
P_e – resultant force for elastic zone
W_g – bending strength index
ε_θ – circumferential strain
ρ – radius of neutral surface curvature
σ_{un} – unloading stress
σ_p – yield stress
σ_{re} – residual stress
σ_{ex} – external stress
σ_θ – circumferential stress

1. Introduction

The main strain observed during manufacturing of products made of wire such as lines, springs, strings, steel cord, chains etc. is the circumferential strain and external symptom of strain occurrence is change of wire curvature.

After the technological process of manufacturing of the product is over and after unloading of the deformed material residual stresses, reaching sometimes significant values, remain. Residual stresses cause initial effort of material even before applying external load.

Residual stresses must fulfill the condition of stresses equilibrium. If there is a tensile stress in a given zone, there must exist another zone having compressive stresses. When applying a load on products made of wire stresses from external loading superimpose onto residual stresses. In zones where residual stresses and stresses from external loading possess the same sign, resulting stresses equal to their sum. Thus, resultant material effort is higher than material effort caused only by external loading. In that case, the residual stresses influence is harmful. However, when residual stresses and the stresses from external loading have the opposite sign we deal with the situation where resultant material effort is lower than the material effort caused by the external loading. Residual stress influence is favorable now.

Residual stresses are derived based on the theoretical analysis of stress state during loading and unloading. Empirical residual stresses are obtained with aid of layer removal method and the X-ray methods. Magnetic method offers great opportunities, too.

Tensile stresses are dominant during external loading of the products made of wire and thus, the most suitable test to estimate wire properties is the tensile test. Residual stresses remaining in the material after technological process change material state and will affect shape of the stress-strain curve in tension. Yielding of the tensed specimens with residual stresses does not occur on the whole wire cross-section but starts from the place of biggest effort and gradually expands with the increase of external loading, reaching consequently the yield stress value in whole cross-section. In the present paper analysis of the influence of residual stresses on the shape of stress-strain curve in tension is presented.

2. Stress and strain state during change of curvature

Stress and strain state during changing of curvature is characterised by great non-uniformity. There are circumferential tensile stresses in the external longitudinal zone and compressive ones in the internal zone. Both zones are separated with the neutral surface where circumferential stresses equal zero.

Circumferential strain during change of curvature can be obtain from the equation:

$$\varepsilon_\theta = \frac{x}{\rho}. \tag{1}$$

Near the neutral surface material is in the elastic state and the circumferential stresses can be calculated from:

$$\sigma_\theta = \frac{x}{\rho} \cdot \mathrm{E}. \tag{2}$$

It follows from equation (2) that circumferential stresses grow linearly with the increase of the distance of given point from neutral surface. In some distance from neutral surface circumferential stresses will reach value of yield stress and material will pass into plastic state. The range of the elastic zone can be determined from the condition that circumferential stresses reach the yield stress value:

$$\frac{x_0}{\rho} \cdot \mathrm{E} = \sigma_p \tag{3}$$

and:

$$x_0 = \frac{\sigma_p \cdot \rho}{\mathrm{E}}. \tag{4}$$

It comes from equation (4) that range of the elastic zone is bigger when yield stress value and radius of curvature are bigger and the Young's modulus is smaller.

In the plastic zone, as a result of plastic deformation, a strengthening of material is observed. Considering the fact that plastic strains are relatively small it can be assumed that circumferential stresses are constant and equal to yield stress.

3. Residual stresses in unloaded wire

During unloading longitudinal stresses change linearly according to:

$$\sigma_{unx} = \frac{x}{r_0} \cdot \sigma_{un}. \tag{5}$$

As the result of superimposing of unloading stresses onto the stresses occurring during active phase of the process some stresses called residual stresses will remain after the process is finished. Residual stress distribution on the cross-section of the wire is described by the following relationships:

elastic zone (I):

$$\sigma_{re} = \frac{x}{x_0}\sigma_p - \sigma_{un} \cdot \frac{x}{r_0}, \tag{6}$$

tensed plastic zone (II)

$$\sigma_{re} = \sigma_p - \sigma_{un} \cdot \frac{x}{r_0} \tag{7}$$

compressed plastic zone (III),

$$\sigma_{re} = -\sigma_p - \sigma_{un} \cdot \frac{x}{r_0}. \tag{8}$$

The value of the unloading stress should be taken from:

$$\sigma_{un} = \frac{M_g}{W_g}, \tag{9}$$

where

$$M_g = 4 \cdot \left\{ \frac{\sigma_p}{x_0} \cdot \left[\frac{r_0^4}{8} arc\sin \frac{x_0}{r_0} + \frac{r_0^2 \cdot x_0}{8} \sqrt{(r_0^2 - x_0^2)} - \frac{x_0}{4} \cdot \sqrt{(r_0^2 - x_0^2)^3} \right] + \frac{\sigma_p}{3} \cdot \sqrt{(r_0^2 - x_0^2)^3} \right\}. \tag{10}$$

4. Shape of stress-strain curve of the tensed wire with the residual stresses

During tensing a wire with residual stresses first, elastic deformation is observed (first stage), and the tensile force changes linearly depending on relative strain:

$$P_t = \Pi \cdot r_0^2 \cdot \frac{l - l_0}{l_0} \cdot \mathrm{E}. \tag{11}$$

Yielding will ensue when sum of the stresses resulting from the external loading and residual stresses reaches the yield stress value:

$$\sigma_p - \sigma_{un} \cdot \frac{x_0}{r_0} + \sigma_{ex} = \sigma_p \tag{12}$$

thus:

$$\sigma_{ex} = \sigma_{un} \cdot \frac{x_0}{r_0} \tag{13}$$

Value of the tensile force at the end of the first phase can be obtained from:

$$P_t = \Pi \cdot \sigma_{un} \cdot x_0 \cdot r_0. \tag{14}$$

In the second stage of tension, part of the specimen is in the plastic state. Yielding of the material will start from the point, where the residual stresses are biggest. Depending on the wire curvature radius during the active phase zone with the highest residual stresses can be situated either near the centre of the wire (between zone II and I) or on the outer part (zone III). The earlier case of highest residual stresses concentrated in the neighbourhood of the centre of the wire is discussed below.

As higher and higher external stresses are applied the range of plastic zone spreads out (yielding in zone I broadens towards the centre of wire, while in zone II towards external layers). Total tensile force at the second stage of tension will equal to the sum of forces in plastic and elastic zone. Inside the elastic zone stresses increase uniformly, and the force consumed for that increase can be derived from:

$$P_e = 2 \cdot \int_{-r_0}^{x_1} \varepsilon \cdot \mathrm{E}\sqrt{(r_0^2 - x^2)}\, dx + 2 \cdot \int_{x_2}^{r_0} \varepsilon \cdot \mathrm{E}\sqrt{(r_0^2 - x^2)}\, dx. \tag{15}$$

However the increase of stresses inside the plastic zone which is connected with the increase of stresses from a value of residual stresses to the yield stress can be obtained from:

$$P_p = 2 \cdot \int_{x_1}^{x_0} \left[\sigma_p - x \cdot \left(\frac{\sigma_p}{x_0} - \frac{\sigma_{un}}{r_0}\right)\right] \sqrt{(r_0^2 - x^2)}\, dx + 2 \cdot \int_{x_0}^{x_2} \left[\sigma_p - \left(\sigma_p - \frac{\sigma_{un} \cdot x}{r_0}\right)\right] \sqrt{(r_0^2 - x^2)}\, dx. \tag{16}$$

After integrating and substituting the limits of integration the total tensile force is described by:

$$\begin{aligned} P_t = {} & \frac{2}{3} \cdot \left(\frac{\sigma_p}{x_0} - \frac{\sigma_{un}}{r_0}\right) \cdot \left(\sqrt{(r_0^2 - x_0^2)^3} - \sqrt{(r_0^2 - x_1^2)^3}\right) + \\ & + (\sigma_p) \cdot \left(x_0 \cdot \sqrt{(r_0^2 - x_0^2)} + r_0^2 \cdot arc\sin\frac{x_0}{r_0} - x_1 \cdot \sqrt{(r_0^2 - x_1^2)} - r_0^2 \cdot arc\sin\frac{x_1}{r_0}\right) - \\ & + (\varepsilon \cdot E) \cdot \left(x_1 \cdot \sqrt{(r_0^2 - x_1^2)} + r_0^2 \cdot arc\sin\frac{x_1}{r_0} - x_2 \cdot \sqrt{(r_0^2 - x_2^2)} - r_0^2 \cdot arc\sin\frac{x_2}{r_0}\right) + (\varepsilon \cdot \mathrm{E}) \cdot \left(\frac{\Pi \cdot r_0^2}{2}\right). \end{aligned} \tag{17}$$

In the third stage of tension next zone becomes plastic (zone III). Yielding develops towards zone I. Value of the tensile force for elastic zone is described by:

$$P_{\mathrm{e}} = 2 \cdot \int_{x_3}^{x_1} \varepsilon \cdot \mathrm{E}\sqrt{(r_0^2 - x^2)}\, dx + 2 \cdot \int_{x_2}^{r_0} \varepsilon \cdot \mathrm{E}\sqrt{(r_0^2 - x^2)}\, dx. \tag{18}$$

However value of the force for plastic zone can be derived from:

$$\begin{aligned} P_p = {} & 2 \cdot \int_{x_1}^{x_0} \left[\sigma_p - x \cdot \left(\frac{\sigma_p}{x_0} - \frac{\sigma_{un}}{r_0}\right)\right] \sqrt{(r_0^2 - x^2)}\, dx + 2 \cdot \int_{x_0}^{x_2} \left[\sigma_p - \left(\sigma_p - \frac{\sigma_{un} \cdot x}{r_0}\right)\right] \sqrt{(r_0^2 - x^2)}\, dx + \\ & + 2 \cdot \int_{-r_0}^{x_3} \left[\sigma_p - \left(-\sigma_p - \frac{\sigma_{un} \cdot x}{r_0}\right)\right] \sqrt{(r_0^2 - x^2)}\, dx. \end{aligned} \tag{19}$$

After integrating and substituting the limits of integrations the following relationship for the total tensile force is received:

$$P_t = \frac{2}{3}\cdot\left(\frac{\sigma_p}{x_0}-\frac{\sigma_{un}}{r_0}\right)\cdot\left(\sqrt{(r_0^2-x_0^2)^3}-\sqrt{(r_0^2-x_1^2)^3}\right)+$$

$$+(\sigma_p)\cdot\left(x_0\cdot\sqrt{(r_0^2-x_0^2)}+r_0^2\cdot arc\sin\frac{x_0}{r_0}-x_1\cdot\sqrt{(r_0^2-x_1^2)}-r_0^2\cdot arc\sin\frac{x_1}{r_0}\right)+$$

$$-\left(\frac{2}{3}\cdot\frac{\sigma_{un}}{r_0}\right)\cdot\left[\sqrt{(r_0^2-x_2^2)^3}-\sqrt{(r_0^2-x_0^2)^3}\right]-\frac{2}{3}\cdot\left(\frac{\sigma_{un}}{r_0}\right)\cdot\left(\sqrt{(r_0^2-x_3^2)^3}\right)+$$

$$+2\cdot\sigma_p\cdot\left(\frac{\Pi\cdot r_0^2}{2}+x_3\cdot\sqrt{(r_0^2-x_3^2)}-r_0^2\cdot arc\sin\frac{x_3}{r_0}\right)+$$

$$+(\varepsilon\cdot E)\cdot\left(x_1\cdot\sqrt{(r_0^2-x_1^2)}+r_0^2\cdot arc\sin\frac{x_1}{r_0}-x_3\cdot\sqrt{(r_0^2-x_3^2)}-r_0^2\cdot arc\sin\frac{x_3}{r_0}\right)+$$

$$(\varepsilon\cdot \mathrm{E})\cdot\left(\frac{\Pi\cdot r_0^2}{2}-x_2\cdot\sqrt{(r_0^2-x_2^2)}-r_0^2\cdot arc\sin\frac{x_2}{r_0}\right). \tag{20}$$

The third stage of tension ends when tensed wire is plastic in the whole cross-section. After reaching plastic state only small deformation occur in the wire and than a neck forms and the specimen cracks.

Limits of the regions x_1, x_2, x_3 (plastic zones range) can be determined from the boundary conditions stating that the sum of residual stresses and the ones from external loading equal to the yield stress:

zone I

$$x_1\cdot\left(\frac{\sigma_p}{x_0}-\frac{\sigma_{un}}{r_0}\right)+\varepsilon\cdot E=\sigma_p \tag{21}$$

and

$$x_1=\frac{\sigma_p-\varepsilon\cdot \mathrm{E}}{\left(\frac{\sigma_p}{x_0}-\frac{\sigma_{un}}{r_0}\right)}, \tag{22}$$

zone II

$$\left(\sigma_p-\frac{x_2}{r_0}\cdot\sigma_{un}\right)+\varepsilon\cdot \mathrm{E}=\sigma_p \tag{23}$$

and

$$x_2=\frac{\varepsilon\cdot \mathrm{E}\cdot r_0}{\sigma_{un}}, \tag{24}$$

zone III

$$\left(-\sigma_p - \frac{x_3}{r_0} \cdot \sigma_{un}\right) + \varepsilon \cdot \mathrm{E} = \sigma_p \tag{25}$$

and

$$x_3 = -\frac{(2 \cdot \sigma_p - \varepsilon \cdot \mathrm{E}) \cdot r_0}{\sigma_{un}}. \tag{26}$$

5. Experimental tests

The experimental tests were performed with the specimens of drawn wire made of D70 steel – diameter 2.6 mm, curvature radius 34.5 mm and the yield point 1224 MPa. Wires were annealed and then straightened.

Afterwards the wires were put to the tensile test and both the tensile force and elongation were registered using mechanical sensor of scale interval 0.01 mm, gauge length of a test piece was 100 mm.

Figure 1 shows that the range of elastic zone I is not big (x_0 = 0.211 mm). The highest value of residual stresses occurs at the contact between zone I and II (elastic zone and plastic tensed zone during active process of changing the curvature), where σ_{re} = 890 MPa. Value of residual stresses on the periphery of zone III is slightly lower and equals σ_{re} = 827 MPa.

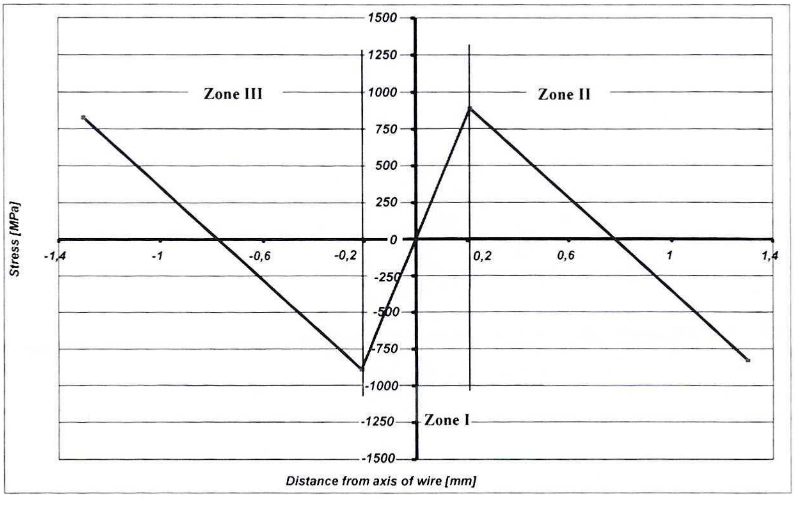

Fig. 1. Residual stress distribution after changing curvature and unloading

During the tensile test of a wire, when value of external stresses and residual ones reach the value of yield stress, yielding takes place. The beginning of yielding can be observed on the border between zone II and I. Figure 2 shows the stress distribution at the moment of yielding appearance and the directions of plastic zones broadening.

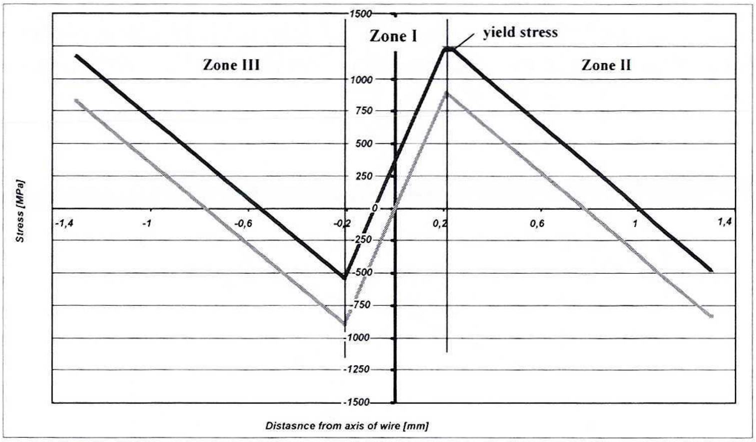

Fig. 2. Second stage of tension (beginning of yielding of zone I and II)

In the next stage of tension external layer of the zone III yields as it was shown in Figure 3. In this third stage of tension yielding develops towards zone I.

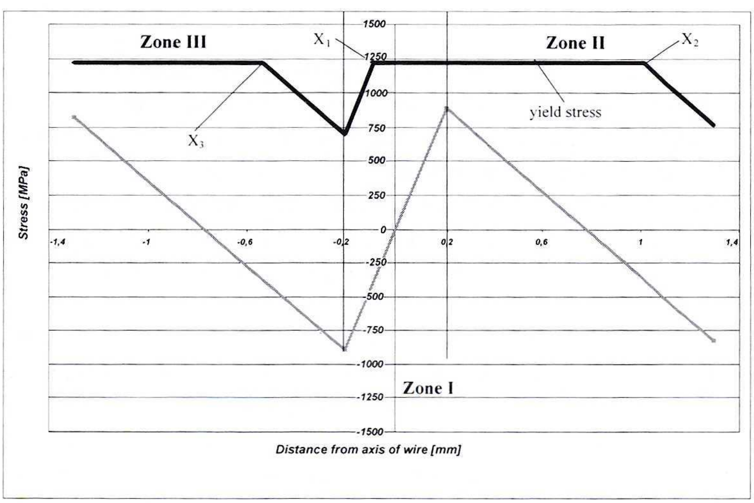

Fig. 3. Third stage of tensing (yielding occurs in zone III)

At the moment of yielding a point at the contact of zone III and I (residual stresses after change in curvature of a wire had had the smallest value of σ_{re} = −890 MPa) whole cross-section of the wire is in the plastic state. Further increase of tensile force causes forming of neck and breaking the specimen.

Figures stress-strain curve for specimen with and without residual stresses were shown in Figure 4 to illustrate the effect of change of wire curvature on the stress-strain curve in tension. Introducing residual stresses into operation of changing wire curvature caused significantly earlier occurrence of elastoplastic zone and thus earlier occurrence of plastic deformation of the wire what is caused by initial effort of the material.

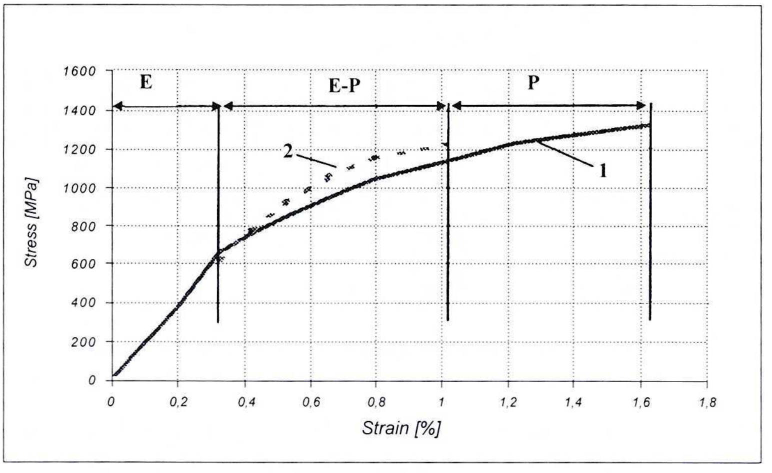

Fig. 4. Stress-strain curve in tension 1 – experiment, 2 – calculation

Experimental results of wire tension and results of calculation of the tensile force (Fig. 4) show satisfactory agreement and amounted to less than 10%. Theoretical values were generally slightly higher than experimental ones. It my show that real residual stresses were slightly lower than theoretically determined stresses.

The experimental tests revealed that uniform deformation predominated not only during uniform tension of the specimen in elastic and elastoplastic range but also in the third stage, i.e. plastic tension. Moreover strain hardening further proceeds during uniform deformation.

In figures 5 and 6 stress-strain curves in tension for a wire of 2.6 mm diameter after stress relief annealing and for a wire with residual stresses connected with the change of curvature (wire was straightened) tensed with different strain rates have been presented. It results from the tests that the residual stresses in a wire influence significantly shape of stress-strain curve in tension. Rectilinear segment of elastic deformation is much longer for a wire without residual stresses than for the one with the residual stresses present. It is connected with the fact that in the zone where there are tensile residual

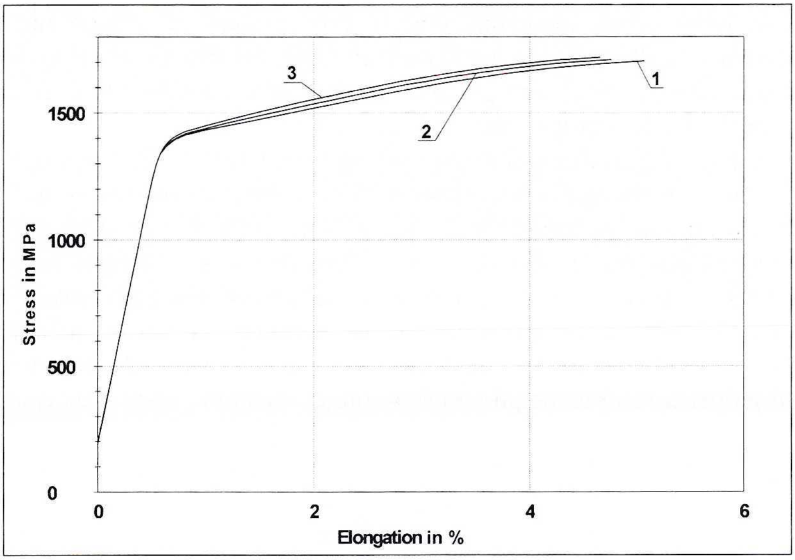

Fig. 5. Stress-strain curve in tension for wire without residual stresses Strain rate: 1 – 0.025; 2 – 0.0025; 3 – 0.00025 [s^{-1}]

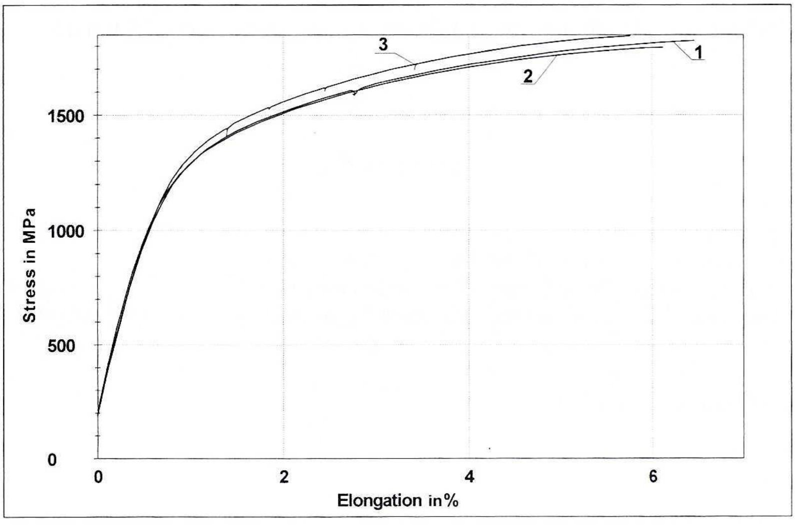

Fig. 6. Stress-strain curve in tension for wire with residual stresses Strain rate: 1 – 0.025; 2 – 0.0025; 3 – 0.00025 [s^{-1}]

stresses, resultant stresses equal to the sum of residual stresses and the ones from external loading. Consequently resultant stresses will reach a value of yield stress earlier and material will reach plastic state in the place of greatest effort. Yielding of material starts from the place of greatest effort and the range of plastic zone gradually broadens while tensile force grows monotonically. In this stadium of tension the specimens deform uniformly finally reaching the maximum force. Further tension is connected with strain concentration in a neck and gradual decrease of the tensile force. After reaching the end of ability to deform the specimen cracks.

The tests have shown that strain rate ranging from 0.00025 sek^{-1} to 0.025 sek^{-1} does not affect a run of stress-strain curve either with or without residual stresses. The highest values of tensile stresses occurred for tension with the strain rate 0.00025 sek^{-1}. Increase in strain rate usually causes the increase in tensile stresses though, in some cases, especially for high strain rates, the decrease of yield stress has been stated. In the tests the highest strain rate was ten times bigger than nominal one but it was not very high. On the other hand plastically deformed steel wires usually showed a non-uniformity of mechanical properties and it should be taken into consideration when analysing the tests results.

Conclusions

1. The change of curvature of a wire causes formation of residual stresses in a wire. In the surface layer which is lengthened during the change of curvature there are compressive residual stresses while in shortened layer there are tensile residual stresses.
2. Residual stresses change the shape of stress-strain curve in tension.

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