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DETERMINATION OF STRESS-STRAIN RELATIONSHIP OF SHEET METAL IN UNIAXIAL AND BIAXIAL TENSION

WYZNACZANIE ZALEŻNOŚCI NAPRĘŻENIE–ODKSZTAŁCENIE W PRÓBIE JEDNOOSIOWEGO ORAZ DWUOSIOWEGO ROZCIĄGANIA BLACH

The strain hardening parameters of steel, aluminium and brass sheets were determined by uniaxial and balanced biaxial (hydraulic bulging) tensile tests. Sheet thickness gradation in different points of hemisphere formed in bulge test was analysed. The Hollomon equation was used to described uniaxial and biaxial strain hardening curves, and a comparison of strain hardening exponent was performed. Both the mean value of strain hardening exponent n (which describe the strain hardening of the whole strain range) and differential n_t-value were determined on the base of the results of uniaxial and biaxial testing. The influence of the stress-state on the strain hardening behaviour of the material, as well as strain localization process, under both deformation modes are analysed.

Parametry krzywej umocnienia blach stalowych, aluminiowych oraz mosiężnych zostały wyznaczone w próbach jednoosiowego i dwuosiowego (wybrzuszanie hydrauliczne) rozciągania. Przeprowadzona została analiza rozkładu grubości blachy w różnych punktach uformowanej czaszy. Równanie Hollomona zostało zastosowane do opisu zależności naprężenie-odkształcenie przy jedno- oraz dwuosiowym rozciąganiu, i porównowartości wykładnika krzywej umocnienia. Zarówno wartość średnia wykładnika krzywe umocnienia n (opisująca skłonność do umocnienia w całym zakresie odkształcenia), jak i jego wartość chwilowa n_t, zostały wyznaczone na podstawie wyników prób jedno- oraz dwuosiowego rozciągania. Przeanalizowano wpływ stanu naprężenia na skłonność do umocnienia materiałów, jak również na proces lokalizacji odkształcenia, dla obydwu rodzajów testów.

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1. Introduction

The stress-strain relation and hardening behaviour of a material are very important in determining its resistance to plastic instability. In sheet forming operation biaxial as well as uniaxial stress state exists. Thus, one must know and understand material hardening behaviour as a function of stress state.

Satisfactory modeling of sheet forming is dependent on availability of accurate data for plastic behaviour to the high strain level in such operations. Routine forecasts of formability could also benefit from this information. However, for some reasons, standard uniaxial tension tests cannot provide this data [1]:

- the range of stable uniform strain is restricted to less than half that sustainable under biaxial stress,
- observable stress-strain relationships are, generally, imprecisely ascertained,
- variation of strain hardening behaviour is difficult to discern, but would obviously affect the probable extrapolation,
- biaxial strain deformation is sensitive to plastic anisotropy.

It is obviously desirable to generate the required data directly from biaxial strain test.

The hydraulic bulge test is widely used in determining the strain hardening properties of sheet materials in biaxial tension. In the bulge test, stress and strain can be determined up to failure of the specimen, while in the conventional uniaxial test only the uniform strain range can be utilized. Since the strains in press forming are normally larger then the uniform strain, the bulge test can better describe the plastic properties of a sheet metal at large strains [2]. This is especially important in determining the stress-strain behaviour of sheets, which are in cold-rolled condition.

Hydraulic bulging has long been known as a convenient method for judging the ductility of sheet metal and is an appropriate method for ascertaining biaxial stress-strain relationships because, provided that the die aperture is in order of a hundred times the sheet thickness, the only insurmountable drawback is some very slight bending; whereas other methods, employing cruciform of tubular specimen, induce local stress concentrations or necessitate prior deformation.

When the object of hydraulic bulging is to evaluate plastic properties of the sheet material, the strain distribution may not be ascertained by any method that requires presupposition of those properties. Joint resolution of both bulging strains and material properties together is feasible, but would require complex instrumentation to provide enough information for the computation. Therefore detailed geometric analysis of the measured central dome appears to be most practicable method for calculating the local strain gradients of curvature.

The aim of the present work was to compare plastic behaviour of different sheet material under uniaxial and biaxial tension.

2. Material and mechanical testing

The tests were carried out on the 1.0 mm thick half hard 63–37 brass sheet (M63), 0.8 mm thick DDQ (deep drawing quality) steel sheet and 0.8 mm thick AW1050 aluminium sheet in annealed state. The tensile specimens of 50 mm gauge length and 12.5 mm width were prepared from strips cut at 0^0, 45^0 and 90^0 according to the rolling direction of the sheet. The experiments were carried out on the Schoenk UTS tensile testing machine, using a special device, which recorded simultaneously the tensile load, the current length and width of specimen, using a microcomputer.

In order to determine the flow properties of a material in biaxial stretching, the bulge test was carried out, using hydraulic bulge apparatus (Fig. 1) with a circular die aperture of 80 mm diameter. The bulging pressure and the curvature of the pole were measured and recorded continuously up to specimen failure.

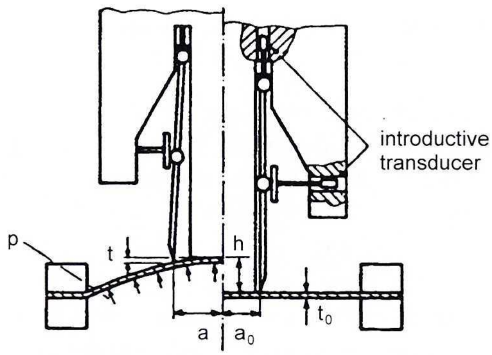


Fig. 1. Hydraulic bulge test apparatus

Both, the uniaxial and biaxial testing were curried out at the room temperature, with initial strain rate of $2.2 \cdot 10^{-1}$ s^{-1} and $2.6 \cdot 10^{-1}$ s^{-1} respectively.

3. Thickness distribution

In bulging sheet metal through a die aperture by lateral fluid pressure, the expansion of surface area is only modest but the meridional strain gradient, from very little at the periphery to quite large at the pole, is severe. Let us consider free forming of a circular membrane. The current half arc length of any meridian passing trough the dome apex is equal to Rα – where R is the dome radius and α is half of the angle subtended by the dome surface at the center of curvature (Fig. 2). Since the initial half arc length of the meridian under consideration equals to the radius R_0, it is stretched $R\alpha/R_0 = \alpha/\sin\alpha$ times. Proceeding from symmetry, it follows that the principal positive strains are equal to each other and thickness at the dome apex equals

$$t_d = t_0(\sin\alpha/\alpha)^2. \tag{1}$$

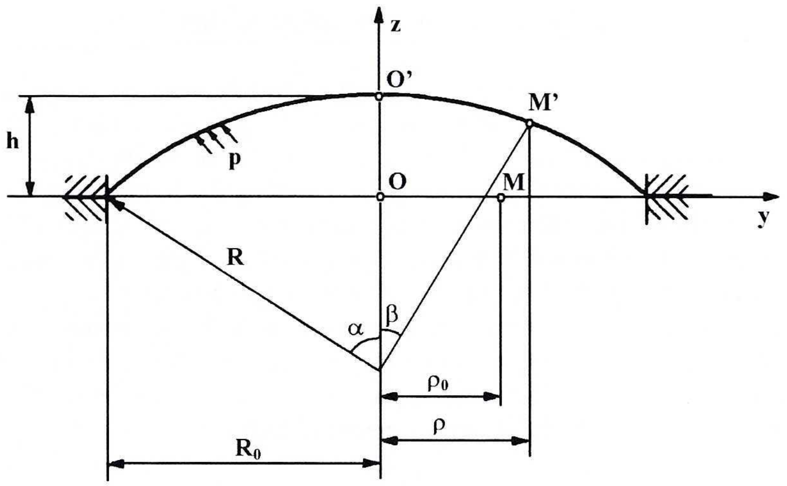


Fig. 2. Schematic of deformation modeling

Since the clamp does not deform during forming, the circumferential deformation along the periphery is negligible. On the other hand, any meridian approaching the periphery is stretched by $\alpha/\sin\alpha$ times, and from this it follows that dome thickness at the periphery equals to

$$t_p = t_0(\sin\alpha/\alpha). \tag{2}$$

At some moment of deformation the point *M* transfer to point *M*', and point *O* to *O*' (Fig. 2). Let β be the angle between the symmetry axis and the dome radius to the point *M*' under consideration. The latitude passing the point *M*' is stretched by ρ/ρ_0 times and the dome thickness at the point *M*' may be found as follow

$$t = t_0(\rho_0/\rho)(\sin\alpha/\alpha). \tag{3}$$

Taking into account that $= R\sin\beta$, $\rho_0 = v\, R_0$ and $\beta = v\alpha$ the dome thickness at any point could be calculated from the following equation [3]:

$$t(\alpha, \beta) = t_0(\sin\alpha/\alpha)^2\beta/\sin\beta. \tag{4}$$

Measurements of sheet thickness in different points of brass and aluminium hemisphere formed in bulging test (at the moment close to material failure) were compared with calculations using eq. (4). From this presentation (Fig. 3) it is visible that thickness variation along the dome wall obtained in experiment is larger than determined theoretically. Due to a specimen clamping the sheet thickness at the hemisphere periphery is the largest. The smallest sheet thickness at the dome pole could be a result of strain localization. Also it is noteworthy, that visibly variation of the hemisphere formed radius was observed. Because of this deviation, measurement of the sheet thickness at the pole, e.g. with an ultrasonic probe, was suggested [4].

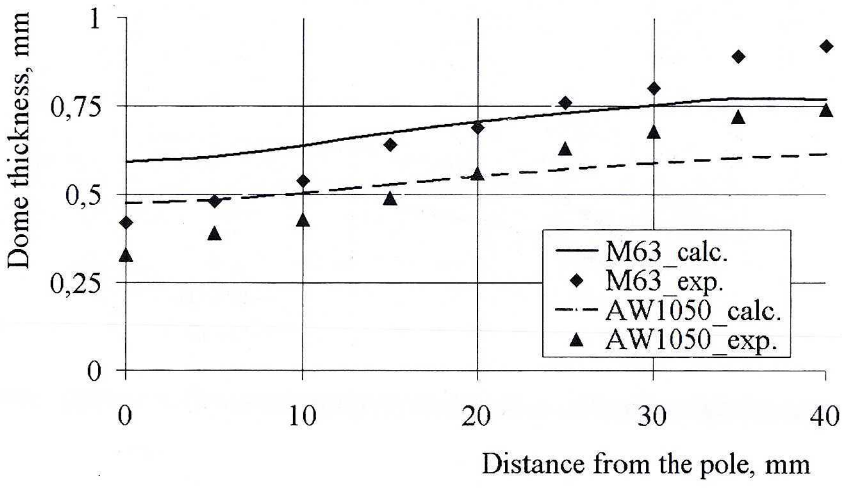


Fig. 3. Dependence of dome thickness on the distance from the dome apex of brass and aluminium sheet at the end of bulge test

4. Stress-strain relationship

In the bulge test a circular diaphragm, rigidly clamped at the periphery, is stretched by uniform lateral pressure. The sheet bends during deformation due to clamping. Provided that the sheet thickness/bulge diameter ratio is small, the effects of bending can be neglected in calculating membrane stresses. The average value of effective stress can be calculated on the basis of the force equilibrium of a small circular element at the center of a membrane from:

$$\sigma = \frac{pR}{2t}, \tag{5}$$

where p is the bulging pressure, and R and t are the radius of curvature and the thickness of the element, respectively. The radius of curvature could be obtained from:

$$R = \frac{a^2 + h^2}{2h}, \tag{6}$$

where a is width and h is height of the central part of membrane (see Fig. 1).

On the base of measured width and height of the central part of membrane the effective strain (equals to the thickness strain) could be calculated as [5],

$$\varepsilon = 2\ln\left[1 + \left(\frac{h}{a}\right)^2\right]. \tag{7}$$

Comparison of stress-strain relationships obtained in uniaxial tensile test and equibiaxial stretching (bulge test) have shown visibly differences (Fig. 4) – larger region

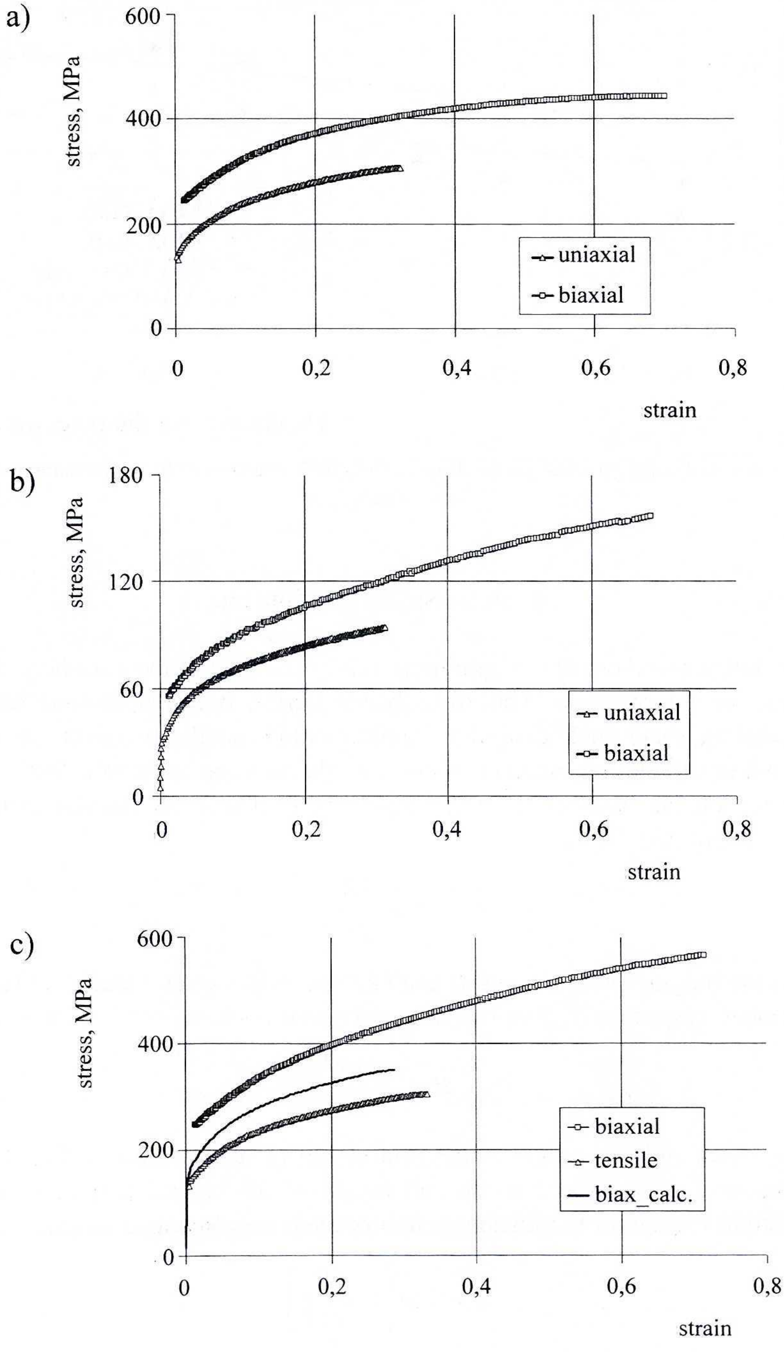


Fig. 4. Uniaxial tensile test and biaxial stress-strain curves of brass (a), aluminium (b) and steel (c) sheet

of straining and higher stress value. The latest could be a result of different textural changes accompanying plastic deformation in these two tests [6].

According to some works [7], uniaxial and equibiaxial stress-strain curves could be related using the following relationships:

$$\sigma = \left(\frac{1+r_{av}}{23}\right)^{1/2} \sigma_{av}, \tag{8a}$$

$$\varepsilon = \left(\frac{2}{1+r_{av}}\right)^{1/2} \varepsilon_{av}, \tag{8b}$$

where r is anisotropy ratio, and material parameters are averaged as $x_{av} =$ $= (x_0 + 2x_{45} + x_{90})/4$.

The plastic anisotropy factor r_{av} has been determined on the base of the relationship between the width and thickness strain in the whole range of straining using the method proposed by Welch *et al.* [8], and amount the value of 0.855, 1.167 and 1.638 for brass, aluminium and steel sheet respectively.

However, in present work, calculations of equibiaxial stress-strain curve on the base of uniaxial stress-strain curve, have shown poor agreement with experimental curve obtained from the bulge test, even in the case of the DDQ steel sheet (Fig. 4c) characterized by the highest r-value.

For many years different strain hardening laws has been used to describe the plastic behaviour of polycrystalline metals and alloys. The Hollomon law in the form of:

$$\sigma = K\varepsilon^n \tag{9}$$

has been used the most frequently. The parameters involved in these laws, particularly n-value, have been correlated to changes in the microstructure of a material and in some way represents processes, which occur during deformation. They have also been used extensively to characterize the formability of sheet material. The value of strain hardening exponent n is usually determined from the double logarithmic plot of the true stress and true strain by linear regression.

The value of n-exponent is strain state dependent [6]. In the case of all the material tested the value of biaxial strain hardening exponent was larger than that of uniaxial one (Fig. 5).

5. Instantaneous strain hardening

The n-value is strain dependent what resulted from the changes in the crystallographic texture [6,8]. Because of this the mean n-value (which describe the strain hardening of the whole strain range) and differential n_t-value were determined on the base of the results of uniaxial and biaxial testing. Taking the derivative from equation (9) yields

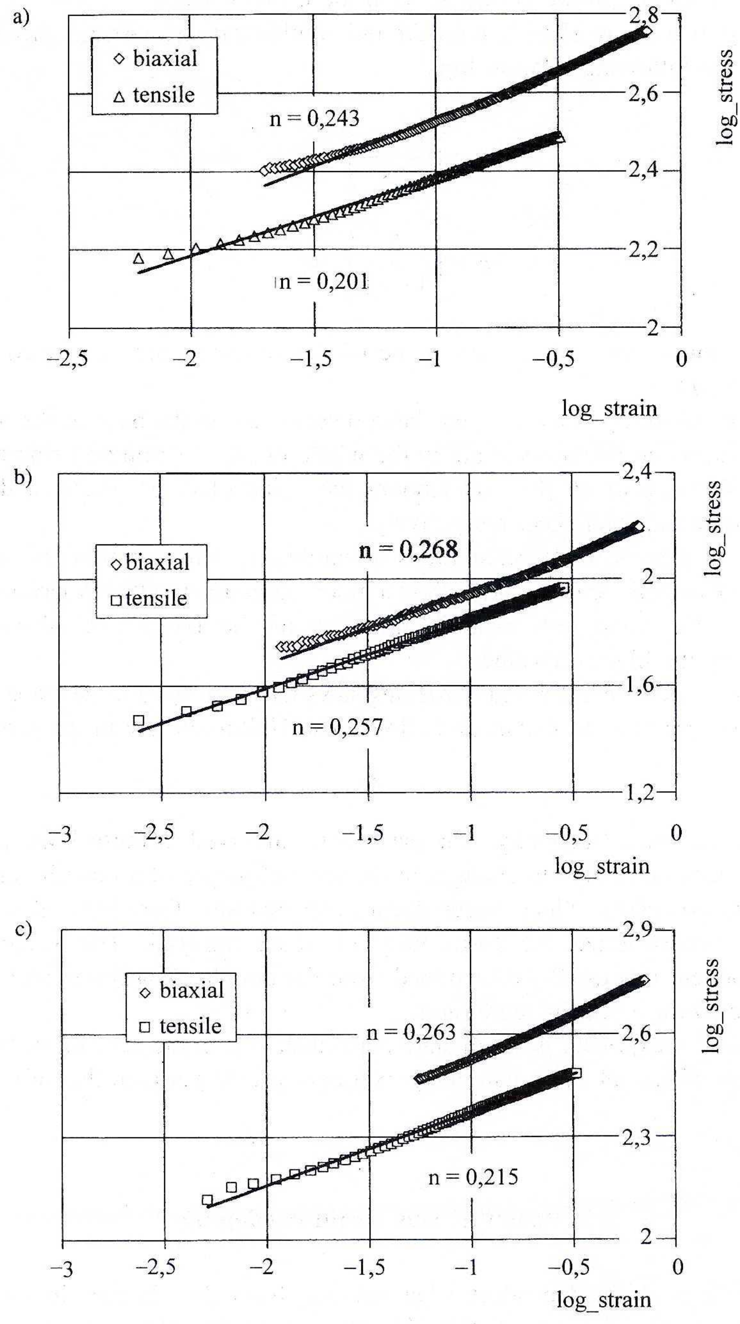


Fig. 5. Logarithmic stress-strain curves of uniaxial tensile test and biaxial of brass (a), aluminium (b) and steel sheet (c)

$$\frac{d\sigma}{d\varepsilon} = Kn\varepsilon^{n-1} = \frac{\sigma}{\varepsilon}n \tag{10}$$

which results in

$$n_t = \frac{d\sigma}{d\varepsilon}\frac{\varepsilon}{\sigma}. \tag{11}$$

The results presented in Fig. 6 show clearly that there is no unique constant n-value, which may characterize hardening process in both uniaxial and biaxial deformation of brass sheets.

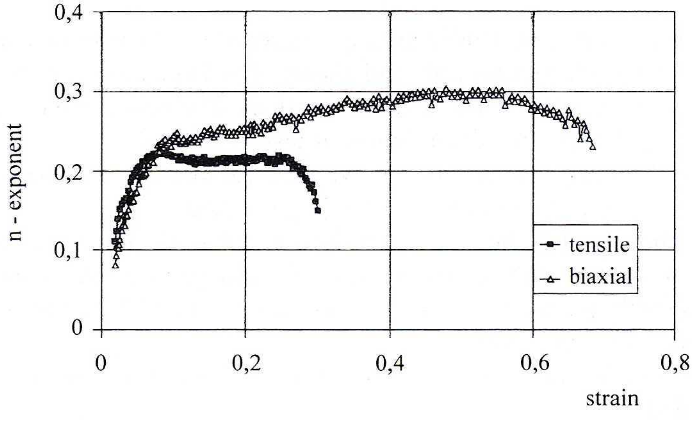
6. Strain localization

In strain rate independent materials the strain localization begins when increase of hardening cannot balance the decrease of the cross section of deformed sample:

$$\sigma = d\sigma/d\varepsilon. \tag{12}$$

The strain at the intersection of the σ and the $d\sigma/d\varepsilon$ curves is the local instability strain [10]. Meanwhile, in the rest of the sheet, practically useful "quasistable" flow succeeds the initially stable flow. The quasistable flow increment increases as the degree of biaxiality increases [11]. The strain at the intersection of the $\sigma/2$ and the $d\sigma/d\varepsilon$ curves is the diffuse instability strain. The results presented in Fig. 7 indicate that in the case of biaxial stretching test (Fig. 7a) the quasistable flow was twice larger than that in the case of uniaxial tensile test (Fig. 7b), as it could be expected on the base of the results presented in Fig. 6.

a)



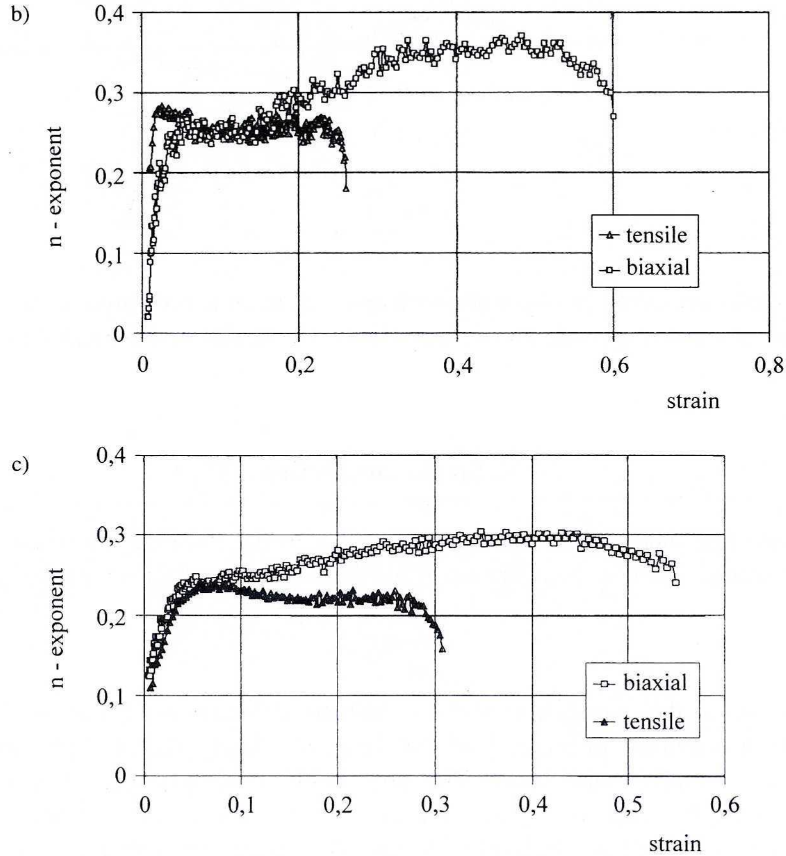


Fig. 6. Differential strain hardening exponent
of uniaxial tensile test and biaxial of brass (a), aluminium (b) and steel sheet (c)

7. Conclusions

Uniaxial and equibiaxial tensile tests were carried out to determine strain-hardening parameters of brass, aluminium and steel sheets. The Hollomon equation was found to describe stress-strain curves well, and the n-values determined from biaxial test were larger, than those of determined from uniaxial test.

Equi-biaxial bulging was found to be very useful method for determining the strain hardening behaviour of the material at very large strains.

For both the uniaxial and biaxial tensile the value of differential strain hardening exponent was strain dependent, and demonstrate the change in the strain hardening process.

The quasistable plastic flow range in the case of biaxial testing was twice larger than those under uniaxial tensile.

Calculation of biaxial stress-strain curve on the base of the results of uniaxial test was not satisfied.

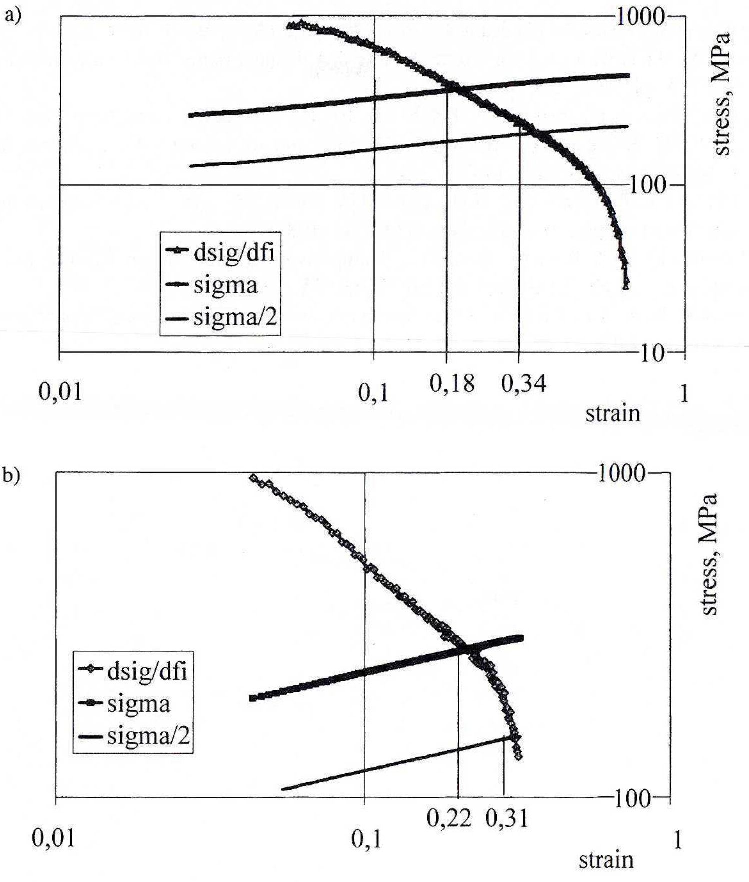


Fig. 7. Demonstrating the instability strain determination of brass sheet using biaxial (upper) and uniaxial (lower) stress-strain curve

Because of visible difference in plastic flow under bulge test and uniaxial tensile, both of these two tests should be perform due to obtain material parameters needed for satisfactory modeling of sheet forming processes.

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