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THEORETICAL AND LABORATORY INVESTIGATIONS ON OIL DISPLACEMENT FROM THE ROCK MATRIX UNDER CAPILLARY FORCES

BADANIA TEORETYCZNE I LABORATORYJNE WYPIERANIA ROPY Z MATRYCY SKALNEJ POD WPŁYWEM SIŁ KAPILARNYCH

The paper contains two parts. The theoretical basis for the process of oil displacement through the water from the fractured porous medium, due to capillary forces, has been discussed in the part No. 1. The functional equation describing the relative water content increase in the fractured rock matrix, surrounded by the water layer, has also been introduced.

This may reflect the actual reservoir conditions, if a fractured oil reservoir is flooded with water, at the moment when the rock matrix porous block has already been surrounded by the water forced in as a result if shifting of watering front in the fractures system. The impact of particular parameters of this process on its performance has also been determined. This equation is non-linear, and may be solved using numerical methods, in general case. The analytical solution may be arrived at when the constant value of the mass exchange coefficients has been assumed, as well as simple geometry, such as spherical geometry and process symmetry. Having assumed as above, the analytical solution has been arrived at and compared with a numerical solution. Taking into consideration the assumptions, the error committed has been slight, in order of few percent. The results of the theoretical contemplations have been compared with laboratory experiments' results, in the second part of the paper. The detailed description of performed research and its methodology have been given. The experiments have confirmed theoretical results.

Key words: oil displacement, capillary forces, double porosity, imbibition, mass exchange.

Zjawisko wnikania wody do przestrzeni porowych wypełnionych ropą pod wpływem sił kapilarnych jest istotnym czynnikiem wpływającym na efektywność nawadniania złóż ropy, zwłaszcza w przypadku, gdy skałą zbiornikową jest skała porowato-szczelinowata. Z punktu widzenia hydromechaniki złoże takie składa się z systemu spękań i szczelin oraz bloków ośrodka porowatego przepuszczalności wielokrotnie mniejszej od systemu szczelin. Bloki te noszą nazwę matrycy skalnej. Front wypierania ropy przez wodę przemieszcza się przede wszystkim (szybciej) w systemie szczelin, następnie zaś następuje proces wymiany masy

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pomiędzy szczelinami a porami matrycy skalnej. Według modeli znanych z literatury, np. G r e e n k o r n (1983), M a r l e (1981), wypieranie ropy przez wodę następuje pod wpływem sił kapilarnych. W niedawno opublikowanej pracy Z h a n g X. i M o r r o w N. R. (1996) wykazali na podstawie prób laboratoryjnych, że przebieg procesu kapilarnego wypierania ropy przez wodę zależny jest od kształtu próbki, warunków brzegowych w czasie eksperymentu, stosunku lepkości cieczy, napięcia powierzchniowego na granicy ropa-woda oraz przepuszczalności względnych i przepuszczalności absolutnej. Interpretacja wybików laboratoryjnych jest jednak utrudniona, gdyż model matematyczny procesu jest silnie nieliniowy.

W niniejszej pracy rozważono pojedynczy blok matrycy ośrodka porowatego o podwójnej porowatości i przepuszczalności. Blok ten nasycony jest cieczą węglowodorową (ropą lub naftą), zaś jego powierzchnia boczna znajduje się w kontakcie z wodą, która jest cieczą wypierającą. Przy założeniu, że ciśnienie kapilarne jest jedynym czynnikiem powodującym wypieranie ropy otrzymano równanie (22) opisujące względny przyrost nasycenia wodą bloku matrycy w czasie trwania procesu wypierania. W równaniu tym występuje współczynnik dyfuzji D określony wzorem (14).

Określono wpływ poszczególnych parametrów tego procesu na jego przebieg. Równanie (22) jest nieliniowe i w ogólnym przypadku może być rozwiązane metodami numerycznymi. Rozwiązanie analityczne może być znalezione przy przyjęciu stałego współczynnika dyfuzji i prostych geometrii i symetrii procesu. Po przyjęciu takich założeń otrzymano rozwiązanie analityczne i porównano je z numerycznym. Jak widać z wykresu na rys. 3 różnica otrzymanych wyników jest niewielka. Wynik otrzymany na drodze rozważań teoretycznych porównano następnie z wynikami badań laboratoryjnych wymienionego zagadnienia. Podano szczegółową metodykę przeprowadzonych badań. Ich wyniki potwierdziły słuszność rozważań teoretycznych. Wyniki pomiarów oraz dopasowanie krzywych teoretycznych do danych pomiarowych pokazano na rys. 4 i 5.

Słowa kluczowe: wypieranie ropy naftowej, siły kapilarne, podwójna porowatość, transport masy.

NOMENCLATURE

D — diffusivity, m^2/s
g — relative water content
k — permeability, m^2,
k_i — relative permeability for i-th phase, $i = o, w$
M — mobility ratio
p — pressure, Pa
p_c — capillary pressure, Pa
R — effective radius of the porous grain, m
S_i — saturation with i-th phase, $i = o, w$.
t — timee, sek
u_i — Darcy velocity for i-th phase, $i = w, g$, m/s
W — volume averaged numerical solution
w — numerical solution
r — distance, m
ϱ_i — density, kg/m^3
τ — tortuosity, 1/m
μ_i — viscosity, Pa s
ϕ — porosity
Ω — porous grain volume

1. Introduction

The phenomenon of water penetrating the pores filled with oil as a result of capillary forces constitutes the essential factor, decisive in the efficiency of flooding crude oil reservoirs with water, particularly if the reservoir rock is characteristic of double porosity or permeability. The double porosity and permeability system contains of slots and fractures system, as well as the so-called rock matrix, constructed from the porous body block, with permability much less than that of the fractured system, with regard to hydraulic properties. The flow in the slots is usually accompanied by the mass exchange with the matrix. The mechanism of these processes has not yet been fully known, although the phenomenon model known in the literature as inhibition, is very well-known and has been described by G r e e n k o r n and M a r l e. The self penetration of water into the rock matrix occurs as a result of capillary forces, according to the said model.

The most recent results obtained in this discipline were analysed in the report developed by D i x i t et al. (1996), where the results of numerical simulation with the application of random pore network models, as well as interfacial contact angle, were showed. The authors also recognized a few categories of flows with different quantitative characteristics. In the Z h a n g and M o r r o w works (1996), a series of laboratory research results was presented, performed on cylindrical cores, as a result of which the capillary displacement process performance was found dependent on the sample shape, input conditions (the side surface part was insulated or not), liquid viscosity, surface tension, permeability, relative permeability and porosity. The research into capillary imbibition phenomenon is difficult due to the fact that the mathematical model of the process is heavily nonlinear (Marle 1981), which in turn hinders and restricts the interpretation of the laboratory research results (Chen J. et al. 1995). This problem will be subject to analysis in the further sections of this paper.

The analysis of the phenomenon of crude oil displacement through water from the single block of the rock matrix due to capillary forces constitutes the subject of this article.

The results of laboratory research have been presented, as well as the theory describing this phenomenon has been proposed. Also, the problem of linearization of the equation describing the process has been discussed and mass transport average coefficients have been arrived at.

2. The mathematical model of oil displacement by the water from the rock matrix due to capillary forces

A single block of porous body with double permeability and porosity has been considered in this report, the diagram of which is presented in figure 1. This block is assumed to be initially saturated with the crude oil. From the moment $t = 0$, the block side service remains in contact with the wetting liquid, e. g. water, as a result of

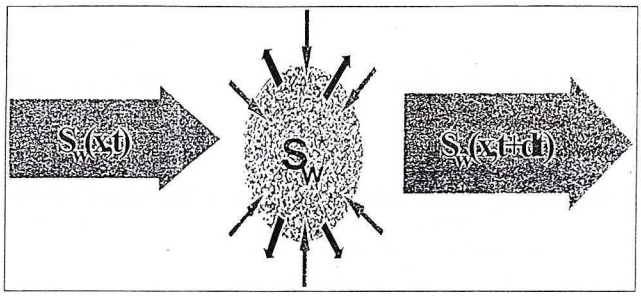

Fig. 1. Flow diagram of oil displacement from rock matrix through water penetrating into due to capillary forces

which the water penetrates into the matrix block and causes the crude oil to be displaced. This process occurs due to capillary and surface forces.

The water saturation S_w and oil saturation S_0 constitute the reflections of water and oil content in the matrix:

$$S_w = \frac{V_w}{V_w + V_0} = \frac{V_w}{V_p} \qquad S_0 = I - S_w,$$

where:

V_w — pore volume occupied by water,
V_0 — pore volume occupied by oil,
S_w — pore saturation with water,
S_0 — pore saturation with oil.

The following equations may be applied to the local coordinate system, related to the single matrix block, neglecting gravitational effects and assuming that the capillary pressure is the only factor causing water to penetrate into the matrix:

$$u_i = -\frac{k\,k_i}{\mu_i} \operatorname{grad} p_i \qquad i = w, o, \tag{1}$$

$$\phi \frac{\partial (S_i \varrho_i)}{\partial t} + \operatorname{div}(u_i \varrho_i) = 0 \qquad i = w, o, \tag{2}$$

$$u_w = -u_o, \tag{3}$$

$$p_0 - p_w = p_c(S_w), \tag{4}$$

$$S_0 + S_g = 1, \tag{5}$$

where:

k — absolute permeability ratio,
k_i — relative permeability ratio for i-th phase,
p_i — pressure in t-th phase,
μ_i — dynamic viscosity for i-th phase,
S_i — saturation with i-th phase,

u_i — Darcy velocity for i-th phase,
p_c — capillary pressure.

The index "w" refers to water, while the index "o" refers to oil.

The equations (1) are the motion equations (Darcy Law), while the equations (2) constitute the continuity equations for water and oild. The equation (3) means that the water volume penetrating into the block is equal to all the volume displaced by this water. The relation (4) describes the capillary pressure on the oil-water interface, while the formula (5) balances the saturation in the pore system. Having transformed the equations (1), (3), (4), the following formula may be arrived at:

$$u_w = k\left(\frac{\mu_w}{k_w} + \frac{\mu_0}{k_0}\right)^{-1} \frac{dp_c}{dS_w} \operatorname{grad} S_w . \tag{6}$$

Combining the equation (6) and (2), and assuming that the fluids' densities are approximately constant, the following partial differential equation may be arrived at:

$$\frac{\partial S_w}{\partial t} = \operatorname{div}(D \operatorname{grad} S_w), \tag{7}$$

where:

$$D = \frac{k}{\left(\frac{\mu_w}{k_w} + \frac{\mu_g}{k_0}\right)\Phi} \frac{dp_c}{dS_w} . \tag{8}$$

As the water has been assumed to be wetting phase, $\left(\frac{dp_c}{dS_w}\right)$ is lower than zero, therefore $D < 0$, which means that the equation (7) is the diffusion type equation. The initial and boundary conditions may be formulated as follows:

$$S_w = S_0 \quad t = 0, \tag{9}$$

$$S_w = S_1 \text{ on the block surface, } t > 0. \tag{10}$$

It should be stressed that the equation (7), with conditions (9) and (10), is subject to solving in the local coordinate system, related to the single block. Having incorporated the approximated formulas of Correy and Brooks (11, 12, 13), provided in the work of Greenkorn R (1983), for calculating phase permeabilities and capillary pressure, (γ is the surface tension, while τ is the porous body tortuosity), the equation (8) may be presented in the following form (14):

$$k_w = (S_w)^4, \tag{11}$$

$$k_g = (1 - S_w)^2\left[1 - (S_w)^2\right], \tag{12}$$

$$p_c = \frac{\gamma}{\tau}\left(\frac{\phi}{5k}\right)^{0.5} (S_w)^{-0.5}, \tag{13}$$

$$D = D' \frac{(S_w)^{\frac{5}{2}}(1-S_w)^2[1-(S_w)^2]}{(1-S_w)^2[1-(S_w)^2]+\frac{\mu_0}{\mu_w}(S_w)^4}, \tag{14}$$

where $D' = \text{const}$.

The graphs for the function $(F(S_w M) = D(S_w)/D')$ for different values of the $(M = \mu_0/\mu_w)$ parameter have been shown in figure 2.

The above function in its physical sense provides the characteristics of the capillary process intensity of hydracarbon liquid displacement (oil displacement) through water due to capillary pressure. As can be seen, from the theoretical viewpoint, the higher the intensity of the process, the lower is the viscosity of the hydrocarbon liquid with relation to the water viscosity (assuming that other process parameters remain constant).

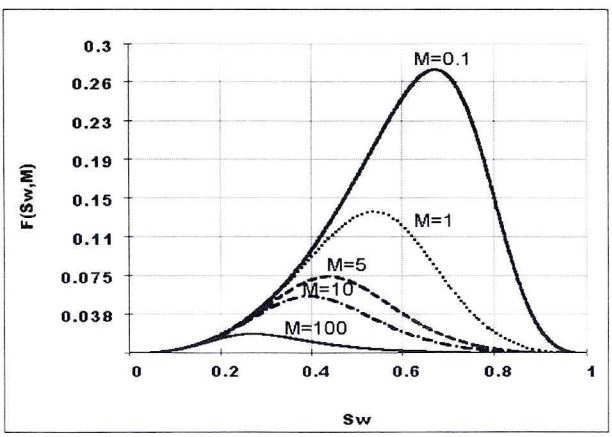

Fig. 2. $F(S_w, M)$ function graphs, for different values of M parameter

The equation (7) is non-linear and may be solved by means of numerical methods in general case [5]. The analytical solution may be arrived at, if D coefficient is constant and geometry is simple. Specifically, in spherical geometry, for $D = \text{const}$ and assuming the process symmetry, the equation (7) can be written in the following form (15).

$$\frac{\partial S_w}{\partial t} = D\left(\frac{\partial^2 S_w}{\partial r^2} + \frac{2}{r}\frac{\partial S_w}{\partial r}\right). \tag{15}$$

The conditions (9) and (10) in this case are (16), (17):

$$S_w = S_0 \quad t = 0 \quad \text{and} \quad 0 < r < R, \tag{16}$$

$$S_w = S_1(t) \quad r = R \quad \text{and} \quad t > 0, \tag{17}$$

where R constitutes the average radius of the block.

The value-boundary problem (15), (16), (17) has its analytical solution, which may be expressed with the formula [1]:

$$\vartheta(r,t)=S_w-S_0=\left\{\frac{2\pi D}{Rr}\sum_{k=1}^{\infty}(-1)^{k+1}k\sin\left(\frac{r\pi k}{R}\right)\exp\left(-\frac{k^2\pi^2 D}{R^2}t\right)\right\}\otimes\{S_1(t)-S_0\}, \quad (18)$$

where $\otimes$ means convolution operation:

$$(f\otimes g)(t)=\int_0^{+\infty} f(u)g(t-u)\,du \qquad f,g\in L^2[0,+\infty),\quad f(t)=g(t)=0 \text{ for } t<0. \quad (19)$$

In order to arrive at the average change of water content in the block, the solution (18) may be averaged by calculating the integral over the porous grain Ω:

$$\left\langle \vartheta \right\rangle_{\Omega}(t)=\left\langle S_w-S_0\right\rangle_{\Omega}=\frac{1}{|\Omega|}\iiint_{\Omega}\vartheta(r,t)\,d\Omega. \quad (20)$$

For spherical grains, with R radius, the following equation may be presented, having calculated the latter integral:

$$V_R(t)=\left\langle S_w-S_0\right\rangle_R=\frac{6D}{R^2}\int_0^t\left[S_1(t)-S_0\right]\cdot\sum_{k=1}^{\infty}\exp\left(-\frac{k^2\pi^2 D}{R^2}(t-\tau)\right)d\tau. \quad (21)$$

If $S_i(t)=S_1=\text{const}$, which corresponds to the immediate surrounding of the entire matrix block by the displacing liquid, then the equation (21) becomes simplified, and is given in the following form (22):

$$g(t)\frac{\left\langle S_w-S_0\right\rangle_{\Omega}}{S_1-S_0}=1-\frac{6}{\pi^2}\sum_{k=1}^{\infty}\frac{1}{x^2}\exp\left\{-\frac{k^2\pi^2 Dt}{R^2}\right\}. \quad (22)$$

The formula (21) provides the increase of water content in the matrix block during imbibtion. The formula (22) describes the relative increase of water content in the blocks in time, at the immediate surrounding of the blocks by water. This may correspond to the immersing of a sample in the vessel with water, which in turn corresponds to reservoir conditions of the porous block surrounding (matrix) by the forced water as a result of shifting the wetting front in the fractures system.

The problem of equation (7) linearization by assuming the average value of the D coefficient, i.e. assuming that $D=\text{const}$, remains open. In order to estimate the linearization error, the equation (7) has been solved with coefficient D that is described by the equation (14), with boundary conditions (16), (17), for the spherical

geometry case, with the application of the finite difference method and explicite method of the following form:

$$w_{i+1,j} = w_{i,j} + \frac{dt}{(r_{j+1} \cdot dr)^2}\left[(r_{j+1})^2 \cdot D\left(w_{i,j+1}\right) \cdot \left(w_{i,j-1} - w_{i,j}\right) - \right.$$

$$\left. - (r_j)^2 \cdot D(w_{i,j-1}) \cdot (w_{i,j} - w_{i,j-1})\right], \quad (23)$$

where index i corresponds to the time, while j index corresponds to the space. The average value has been arrived at by means of formula (20), which in its finite difference version for spherical geometry assumes the following form:

$$W_i = \frac{3}{R^3}\sum_j w_{i,j+1} \cdot (r_{j+1} - r_j) \cdot (r_{j+1})^2 . \quad (24)$$

The analytical solution (22), by means of optimization of D coefficient, has been matched with a numerical solution, obtained using (23) and (24). The following has been assumed in the calculations: $D' = 10^{-10}$ m^2/s, (μ_w = 1mPa·s, μ_0 = 5mPa·s, $0.2 < S_w < 0.5$, $R = 0.013$ m). As a result of

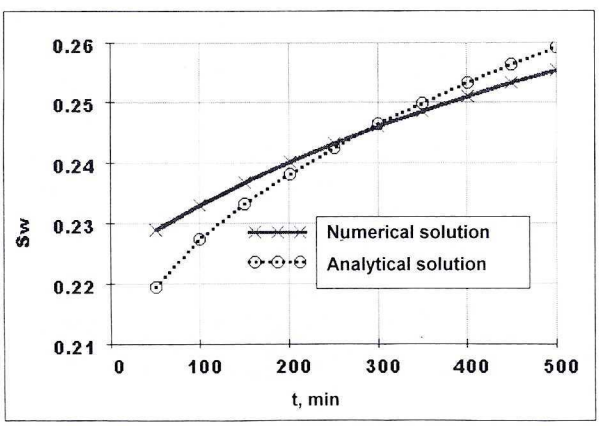

Fig. 3. Comparison of the numerical solution of the non-linear diffusion equation with linearized analytical solution

matching, the average value of D coefficient equal to $D_{av} = 1.983 * 10^{-11}$ has been arrived at. The comparison of analytical and numerical solutions has been summarized in figure No. 3. Table 1 presents the analysis of error committed as a result of the said linearization. It should be stressed that the presented results refer only to the average solution against the matrix block volume. The errors arising from linearization are much higher, even up to few hundred percent, if saturation's distributions in the matrix are compared with one another.

TABLE 1

Diffusion equation linearization error analysis. Comparison of linearized equation solution value with numerical value of non-linear equation solution

Time, min	Analytical solution	Numerical solution	Error %
50	0.219	0.229	4.126
100	0.227	0.233	2.484
150	0.233	0.237	1.535
200	0.238	0.240	0.859
250	0.243	0.243	0.318
300	0.246	0.246	−0.137
350	0.250	0.249	−0.533
400	0.253	0.251	−0.886
450	0.256	0.253	−1.204
500	0.259	0.255	−1.495

3. Laboratory research

Two series of experiments of research into kinetics of water penetration into the matrix and displacing the oil from the matrix have been performed. The verification of obtained models and theoretical solutions, as well as the determination of the numerical values of D coefficients, constituted the prime objective of the research in question.

The sandstone from Radków surface mine was used as a model of porous body block (rock matrix). This selection has been justified by the willingness of using the porous body with high porosity and permeability. The porosity of the sample used, arrived at by means of weight-volume method, was equal to $\phi = 0.139$, while the absolute permeability equalled $k = 1.59 \times 10^{-12}\,\mathrm{m}^2$, the air being used as operating medium.

Two spherical samples were taken from the sandstone, which were then saturated with oil from the Grobla reservoir (experiment A), and with kerosene (experiment B), following the removal of air from their pores. The initial ratio of pore saturation with hydrocarbon liquid for both samples may be assumed to be equal to $S_0 = 1.0$, while with water to be equal to $S_w = 0.0$. The viscosity coefficient was equal to 2.9 mPas for oil and 1.07 mPas for kerosene. The surface tension on the boundary with water was equal to $40.57 \times 10^{-3}\ \mathrm{Nm}^{-1}$ for oil, and $34.83 \times 10^{-3}\ \mathrm{Nm}^{-1}$ for kerosene, respectively.

Following this, each sample was hung on the laboratory, scales and then immersed in water. Its weight at the initial moment was measured, as well as after subsequent time periods, showing the increase in its weight. This was due to the fact that with time the water content in sample pores also increased as a result of capillary oil displacement through water, while the pore saturation with oil decreased accordingly.

Having performed the mass balance, the following relation between the average sample saturation with water and timely scales indication, may be arrived at:

$$\left\langle S_w(t)\right\rangle_{\Omega} - S_0 = \frac{G(t) - G_0}{\Omega\phi(\varrho_w - \varrho_0)},$$

where G means the scales indication.

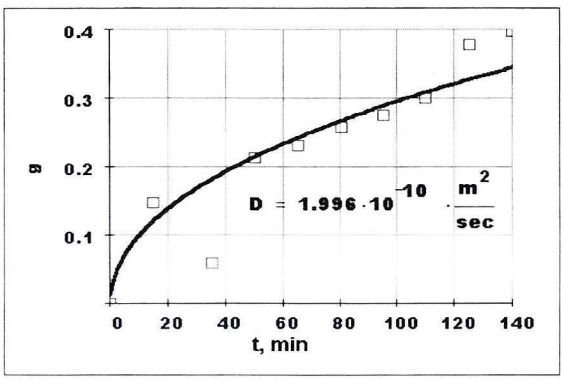

Fig. 4. Matching theoretical curve with measurement data, experiment no. 1

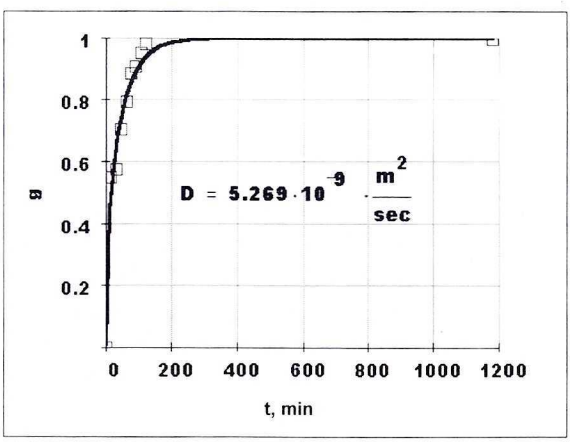

Fig. 5. Matching theoretical curve with measurement data, experiment no. 2

This allows using the formula (22) for determining the optimal value of the D coefficient, by means of the least squares method. The measurements' results and matching theoretical curves with measurement data are summarized in figure No 4 and 5. Having determined D/R^2 coefficients on the above basis, the "diffusion" coefficient values have been arrived at, assuming the average values of the effective radius of the rock matrix sample used (R), determined upon the basis of sample volume value. The results are summarized in Table 2.

TABLE 2

Measurement results

Experiment	Ω cm^3	R cm	ϕ	ϱ_w kg/m^3	ϱ_o kg/m^3	D $[m^2/sek]$
A	6.258	1.431	0.139	1000	800	$1.996 \cdot 10^{-10}$
B	8.410	1.262	0.139	1000	780	$5.269 \cdot 10^{-9}$

The significant difference of diffusion ratio obtained in the analysis performed should be taken into consideration. This is the result of the differences of viscosity, density and surface tension of the hydrocarbon liquids used on the boundary of water (oil in the case A, kerosene in the case B).

4. Conclusions

To sum up, the aforesaid considerations allow assuming the existence of the capillary process of hydrocarbon liquid displacement (oil. etc.) saturating the original pores of the rock matrix element through water occurring in the fractures system. The process efficiency may be characterized by means of the "diffusion" coefficient, which is dependent on the rock matrix physical properties, as well as water and hydrocarbon liquid. The laboratory attempts performed have confirmed the theoretical considerations.

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