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(Inter)regional Input-Output Table Estimation: from Surveys to Spatial Econometrics

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Abstract

Researchers and practitioners who consider economic regional (sub-national) policy questions often face the problem of regional input-output (I–O) table unavailability. The literature offers a number of approaches to approximate such tables. In this paper, a survey of the leading state-of-the-art methods in the field is presented, and their pros and cons are discussed. I take into account both single-region approaches, such as the location quotient (LQ) family, as well as interregional input–output (IRIO) methods, especially their class referred to as gravity-RAS. I pay particular attention to recent developments in using spatial econometric methods (Spatial IRIO). The discussion is illustrated with simple numerical examples and selected empirical results.

Keywords: input-output, regional input–output tables, interregional input-output tables, location quotients, gravity-RAS, Spatial IRIO

JEL Classification: C31, C67, R12, R15

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1 Regional and interregional tables

National statistical offices provide economists with input-output (I–O) tables for national economies, usually at 5-year intervals. However, tables for sub-national regions, as well as flows of intermediate goods between these regions, remain beyond the scope of the standard data delivery of public statistics. At the same time, such sub-national tables remain a useful tool in regional economic studies or regional development policy design. This is why economists have been attempting to propose and apply increasingly accurate and sophisticated estimation methods. The literature proposes a number of various approaches, characterized with different pros and cons, with earliest proposition dating back to Isard (1951), Chenery (1953), Moses (1955) and Leontief and Strout (1963), i.e. times when the I–O analysis itself was relatively young.

This text, addressed predominantly to applied researchers, aims to present a spectrum of most impactful approaches to building input-output tables for a grid of (subnational) regions. The emphasis is put on rationale behind the presented concepts (why it was proposed and might be useful), their critical discussion (why it sometimes fails to replicate the scarce real-world evidence), as well as enhancing replicability and building intuition (link to code and numerical examples). I also discuss in more detail the recent attempts to apply spatial econometric methods in the field.

An important distinction needs to be made between (intra)regional and interregional (or multiregional) I–O tables, henceforth referred to as RIO and IRIO, respectively. The structure of RIO resembles the structure of national I–O datasets, but they refer to a territorial subset of the national economy (see Table 1a and left/middle pane of Figure 1). In some published RIO tables, an additional row for interregional "imports" appears, so as to distinguish it from international imports in the strict sense (cf. Koutaniemi and Louhela, 2006). IRIO, in turn, covers multiple geographies, describing intermediate flows beween each sector-region pair. As a result, the intermediate demand matrix is sized $S \times S$ in national or regional I–O tables, whereas the respective size in IRIO is $(S \cdot R) \times (S \cdot R)$ (S – number of sectors, R – number of regions; Table 1b).

One can think of an IRIO table, covering a complete set of subnational regions of a country described by a national table (see right pane of Figure 1), as an interpolation in which every scalar entry is replaced with an $R \times R$ matrix (see Table 1). Conversely, if IRIO was arranged with regions as the "slow" and sectors as the "fast" dimension (the "Chenery-Moses notation"), one might regard the intermediate demand matrices from RIO tables as diagonal blocks thereof. From the application perspective, it must be recognized that single-region tables do not account for cross-regional feedback effects (Folmer and Nijkamp, 1985; Wiedmann et al., 2011; Miller and Blair, 2009, ch. 3, pp. 76-101) and this is especially the case when the I–O simulation is to be conducted for a small region. According to Loveridge (2004), "theory would say that

the [local] multiplier should grow as the size of the region over which the impact is estimated grows".

Table 1: IO, RIO and IRIO table structure: intermediate demand

(a) national/regional I–O table

	Sec A	**Sec B**
Sec A	$z_{A,A}$	...
Sec B	⋮	⋱

(b) IRIO table

		Sec A		**Sec B**	
		Reg 1	**Reg 2**	**Reg 1**	**Reg 2**
Sec A	**Reg 1**	$z^{1,1}_{A,A}$	$z^{1,2}_{A,A}$	...	
	Reg 2	$z^{2,1}_{A,A}$	$z^{2,2}_{A,A}$		
Sec B	**Reg 1**	⋮		⋱	
	Reg 2				

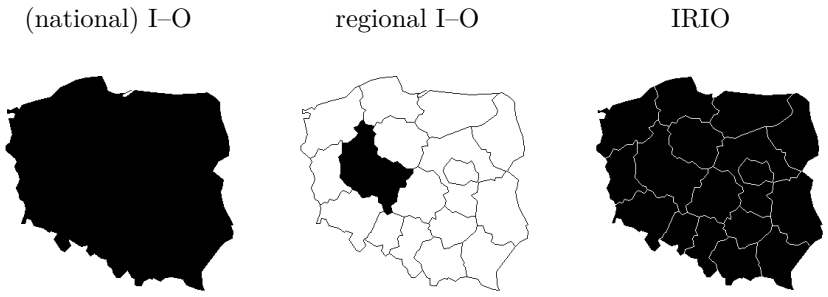
This survey starts with taking a look at scarce available data sources, including the Commodity Flow Survey study from the United States (on which multiple further studies are based), RIO tables, international I–O databases (multiple countries or multicountry blocks) and interregional I–O tables (subnational regions) in Section 2. In Section 3, leading approaches to RIO construction – mostly belonging to the Location Quotient family – are presented and applied in a simple numerical example. Section 4 proceeds to methods of IRIO construction and Section 5 focuses in particular on recent contributions that build on using spatial econometric methods. Section 7 concludes.

2 Existing data sources

2.1 Commodity Flow Survey

Instances of official RIO or IRIO publication at subnational level are scarce. For such a publication to be feasible, a survey-based assessment of trade geography would have to be made for all sectors in every region of interest. Such an effort would be overly expensive for most practical applications (Sargento et al., 2012). However, the U.S. government runs a related survey, named Commodity Flow Survey (CFS,

Figure 1: Geographic scope of I–O, RIO and IRIO



https://www.census.gov/programs-surveys/cfs.html), that provides researchers with high-quality direct (though incomplete) evidence that facilitates building tables for subnational regions of the U.S. economy. The survey is designed as shipper-side survey: enterprises provide information related to selling and sending physical goods and selected services, related to the value, weight, origin and destination (both within-country and exports, final and raw goods; see Figure 2 for an example). The way in which CFS-like data, along with other data sources, is used in building a balanced regional and interregional table can be traced in detail in Jackson et al. (2006) (see also an extensive discussion in Miller and Blair, 2009, ch. 8).

2.2 International tables

Among available IRIO-type tables, international versions are more prevalent. Table 2 summarizes their availability and sources. The pioneering but discontinued World Input-Output Database project issued its most recent edition available as of 2014. More recent data has been published by both Eurostat (Figaro project) and OECD (Inter-Country Input-Output database, OECD, 2023). The limitation of the former is focus on European countries, whereas the latter does not provide an explicit breakdown of value added into labour and capital income, which limits the scope of studying labour-related questions or excludes the possibility of simulating induced effects in the Leontief model without additional external assumptions. Another source, available as part of the Global Trade Analysis Project database that feeds data into Computable General Equilibrium models, is GTAP MRIO (Carrico et al., 2020).

The compilation of such tables is by no means free from methodological challenges. Their long list includes i.a. reframing of data generated under various statistical accounting standards (e.g. different sectorial breakdowns), resolving discrepancies of various types (e.g. reporter and trade partner side), the need for ex-post balancing or accounting for international trade margins. The recent literature pays attention to the problem of currency conversion when international tables are compiled for

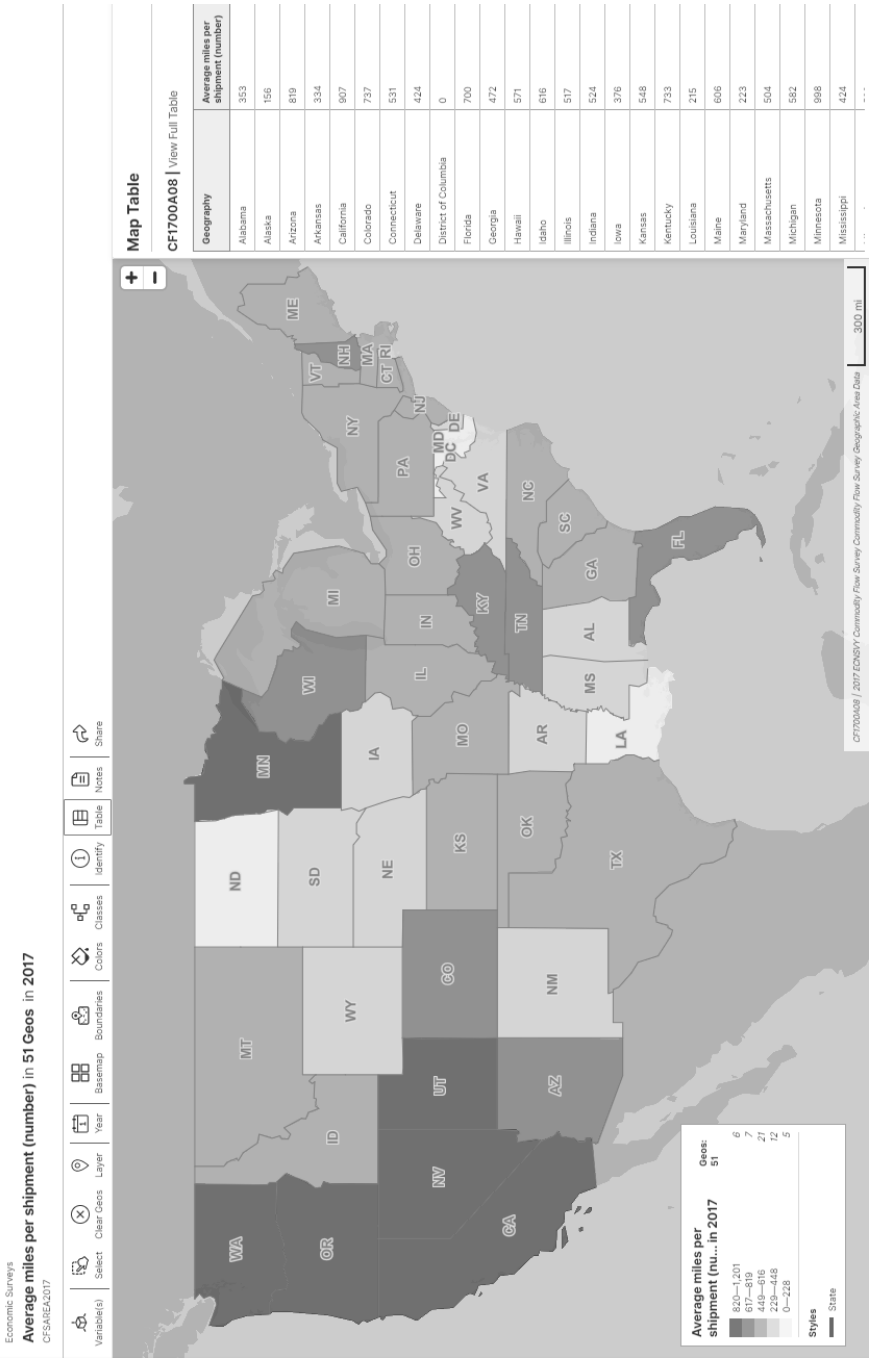


Figure 2: Example information from Commodity Flow Survey database: average miles per shipment in 2017 by U.S. state of origin

Source: https://www.census.gov.

Table 2: International IRIO tables

Source	Institution	Regions	Sectors	Additional information	Last edition (verified 09.2024)
World Input-Output Database	WIOD	43	56	extensive set of auxiliary indicators (including employment)	2016 (data as of 2014)
FIGARO	Eurostat	46	64	focused on EU economy	2024 (data as of 2022)
OECD ICIO	OECD	77	45	no breakdown of value added into capital and labour remuneration	2023 (data as of 2020)

different currency areas (Timmer et al., 2016). This is why purchasing power parity based conversion appears to be superior to nominal exchange rate conversion. Further discussion on this can be found in Reich (2018) and Lach (2020).

2.3 Subnational tables

The instances of survey-based subnational IRIO can be regarded as rare, one-off or low-frequency publications. Table 3 provides a summary, including hyperlinks to sources, information on the year to which the most recent version relates, as well as selected texts that build on a given source. The summary is possibly not exhaustive, but it contains all the sources widely considered in the literature, mostly mentioned in a similar summary provided by Davidson et al. (2022, p. 44).
Two more data sources deserve special attention. Koutaniemi and Louhela (2006) provide (partly) survey-based RIO tables for 19 Finnish NUTS-3 regions. A unique table covering multiple sub-national regions of multiple countries has been provided by the European Commission Joint Research Centre (2020) in the EUREGIO project, with 256 EU NUTS-2 regions, 17 other regions and 15 sectors.

3 Regional tables: LQ family and cross-hauling

For the purpose of numerical illustration of building RIO, let us consider a simple 3-sector, 2-region economy. A typical dataset available in practice in such a case, e.g. for the EU countries acting under the Eurostat standards, consists of the following:

Table 3: Subnational IRIO tables: published data examples

Country	year	regions	sectors	Applied in...
South Korea	2015	16	33	Flegg and Tohmo (2019), Jahn et al. (2020)
Japan	2005	9	53	Gabela (2020), Sonis et al. (2000), Polenske (1970)
China	2017	31	32	Zhang et al. (2015)
Belgium	2015	2	97	L. Avonds (2021)
Canada	2008	14	51	United Nations (2018)

Source: Davidson et al. (2022, p. 44); author.

i) The national I–O table (e.g. Table 4a), including the intermediate demand matrix $\mathbf{Z}$, as well as vectors of global demand $\mathbf{x}$, final demand $\mathbf{f}$ and value added $\mathbf{v}$. The number of sectors typically varies from 50 to 80 (e.g. 77 in Statistics Poland I–O tables). In what follows, let i index supply-side sectors (commodities), and j demand side sectors (activities).

ii) The 2-dimensional, regional and sectorial distribution of economic activity (e.g. Q_j^r in Table 4b, with r indexing regions). Depending on the level of spatial and sectorial aggregation, this can be value added or employment. Note the trade-off between the level of detail in both dimensions. For example, by European standards, one can choose between data on value added for 20 NACE sections or NUTS-2 granularity or 7 groups of sections for NUTS-3 granularity. With NUTS-4 granularity, it is necessary to use employment data in a breakdown into groups of sections. The drawback of the latter is only partial coverage of workforce (without small enterprises) and not accounting for cross-regional differences in regional productivity when translating employment proportions into activity proportions. This is particularly acute when sectors are heterogeneous, and NUTS-4 units are relatively specialized within a given sector (e.g. some of them are dominated by, say, a steel mill, whereas the employment is translated into activity proportions only with national manufacturing productivity, or at best higher level regional manufacturing productivity at NUTS-2). The data in Table 4b coincides with national totals in terms of global output, which is normally not available in regional breakdown, but can be derived from data on value added under the assumption of homogeneous profitabilities in a given sector across regions, which simplifies exposition here.

Table 4a implies a national cost structure matrix $\mathbf{A}^N = \begin{bmatrix} 0.4 & 0.1 & 0.063 \\ 0.05 & 0.3 & 0.125 \\ 0.05 & 0.1 & 0.313 \end{bmatrix}$.

Table 4: Example of 3-sector 2-region economy

(a) national I–O table

		Sector 1	Sector 2	Sector 3
Intermediate demand ($\mathbf{Z}$)	Sector 1	40	5	5
	Sector 2	5	15	10
	Sector 3	5	5	25
Global output ($\mathbf{x}^T = \begin{bmatrix} Q_1^N & Q_2^N & Q_3^N \end{bmatrix}$)		100	50	80

(b) regional activity data

Q_j^r	Sector 1	Sector 2	Sector 3	TOTAL
Region 1	70	20	10	100 [43%]
Region 2	30	30	70	130 [57%]
TOTAL	100 [43%]	50 [22%]	80 [35%]	$Q^N = 230$

The general principle of the family of LQ approaches is to use these coefficients as starting values and modify them with region-specific scaling factors $q_{i,j}^r$ to obtain the regional cost structure $\mathbf{A}^r$:

$$a_{i,j}^r = a_{i,j}^N \cdot q_{i,j}^r. \tag{1}$$

The (Simple) Location Quotient approach (see Isard, 1951; Isard and Kuenne, 1953; Leontief and Strout, 1963; Flegg et al., 1995; Miller and Blair, 2009) defines the scaling factors as follows:

$$q_{i,j}^r = \min\left(SLQ_i^r; 1\right), \tag{2}$$

$$SLQ_i^r = \frac{Q_i^r/Q^r}{Q_i^N/Q^N}. \tag{3}$$

For a given sector-region pair, SLQ compares the regional and the national share of a given sector in the whole economy (3). If the former is higher, the region is considered as specialized, SLQ exceeds unity and no scaling is applied (2). In the opposite case, the regional cost structure coefficient is downscaled, so as to reflect the fact that a need for cross-regional imports arises and the degree of local sourcing is lower than nationwide. In principle, local sourcing is always treated as a first choice by the SLQ approach, and the method is widely considered to produce upward-biased $\mathbf{A}^r$ entries. The literature regards SLQ as a fine-tuning device of the regional cost structures to reflect cross-regional exchange rather than region-specific adjustment of technologies. Using this interpretation, one can compute implicit cross-regional imports of indirect goods from sector i as $\sum_j \left(a_{i,j}^N - a_{i,j}^r\right) \cdot x_j^r$ by region r. Note that $q_{i,j}^r$ only depends on i, but not j: common scaling factors are applied to individual rows of the cost structure matrix (see Table 5b). Due to this row-wise scaling scheme, each of the two

Table 5: Simple example, cont'd: (Simple) Location Quotients

(a) Calculation of SLQ

Region	$i=1$	$i=2$	$i=3$
		SLQ_i^r	
$r=1$	1,610	0,920	0,288
$r=2$	0,531	1,062	1,548
Interregional imports: $\sum_j \left(a_{i.j}^N - a_{i,j}^r\right) \cdot x_j^r$			
$r=1$	0,000	**0,860**	**6,145**
$r=2$	**9,091**	0,000	0,000

(b) Regional cost structure matrices

Region 1 ($\mathbf{A}^1$)			Region 2 ($\mathbf{A}^2$)		
0.400	0.100	0.063	**0.212**	**0.053**	**0.033**
0.046	**0.276**	**0.115**	0.050	0.300	0.125
0.014	**0.029**	**0.090**	0.050	0.100	0.313

Note: Figures in bold are corrected as compared to the national coefficients $\mathbf{A}^N = \begin{bmatrix} 0.4 & 0.1 & 0.063 \\ 0.05 & 0.3 & 0.125 \\ 0.05 & 0.1 & 0.313 \end{bmatrix}$.

regions can either be an importer or an exporter of a given good to other regions, but it cannot be both at once (see Table 5a). Simultaneous exports and imports of the same commodity is highly prevalent in real-world data and referred to as cross-hauling, and it is assumed away under SLQ. In the considered example, region 1 is highly specialized in producing commodity 1, and imports other commodities from region 2, especially commodity 3 as a regional specialization of region 2.

One proposed extension of SLQ is to exclude from the computation of Q^r and Q^N in (3) the sectors that do not purchase commodity i. This approach is known as Purchases-Only Location Quotient (PLQ), and the modified location quotients are interpreted in terms of comparing regional and national ability to deliver supply of i for the sectors that acutally demand this commodity.

A more widespread method, named Cross-Industry Location Quotients (CILQ), modifies the off-diagonal elements of the correction factor matrix $\left[q_{i,j}^r\right]$ as follows:

$$q_{i,j}^r = \begin{cases} \min\left(SLQ_i^r; 1\right) & i = j \\ \min\left(CILQ_{i,j}^r; 1\right) & i \neq j \end{cases}, \tag{4}$$

$$CILQ_{s,v}^r = \frac{SLQ_i^r}{SLQ_j^r}. \tag{5}$$

Table 6: Simple example, cont'd: Cross Industry Location Quotients

(a) Calculation of CILQ

Region	Sectors	$j=1$	$j=2$	$j=3$	Interregional imports:
		$CILQ^r_{i,j}$			$\sum_j \left(a^N_{i.j} - a^r_{i,j}\right) \cdot x^r_j$
	$i=1$		1.750	5.600	0.000
$r=1$	$i=2$	0.571		3.200	**1.980**
	$i=3$	0.179	0.313		6.477
	$i=1$		0.500	0.343	10.006
$r=2$	$i=2$	2.000		0.686	**2.750**
	$i=3$	2.917	1.458		0.000

Note: two positive figures in bold indicate cross-hauling in commodity 2.

(b) Regional cost structure matrices

Region 1 ($\mathbf{A}^1$)			Region 2 ($\mathbf{A}^2$)		
0.400	0.100	0.063	**0.212**	**0.050**	**0.021**
0.029	**0.276**	*0.125*	0.050	0.300	***0.086***
0.009	**0.031**	**0.090**	0.050	0.100	0.313

Note: figures in bold are corrected as compared to the national coefficients $\mathbf{A}^N = \begin{bmatrix} 0.4 & 0.1 & 0.063 \\ 0.05 & 0.3 & 0.125 \\ 0.05 & 0.1 & 0.313 \end{bmatrix}$. In region 2, element (2,3) is corrected downwards, unlike the rest of row 2 (italics).

Again, the scaling is only possible downwards, but individual entries in each row of $\mathbf{A}^N$ now face different scaling factors, depending on both i and j. The scaling is nuanced depending on the relative size of the recipient sector in the region r. The less a region is specialized in producing a given commodity against a nationwide benchmark, the more it is importing, and this differentiation is overproportional. As a result, cross-hauling can emerge. In the numerical example (see Table 6a), commodity 2 is traded between the two regions in both directions, since the input of its local production into activity 3 has dropped below the countrywide average to 0.086. This is because region 2 is less specialized in producing commodity 2 than the country average, but more specialized in sector 3 that necessitates commodity 2 as an input. Hence, the case for imports arises.

Despite CILQ does not preclude cross-hauling, its size is limited (Flegg and Tohmo, 2013a) and, consequently, local cost structures and multiplier are still likely to be overestimated (Robinson and Miller, 1988, 1991).

As noted by Round (1978), both SLQ and CILQ miss one piece of information each: SLQ does not take into account the size of regional buying sector, whereas the

Table 7: Simple example, cont'd: Round's Location Quotients

(a) Calculation of RLQ

Region	Sectors	$j=1$	$j=2$	$j=3$	Interregional imports:
		$RLQ^r_{i,j}$			$\sum_j \left(a^N_{i.j} - a^r_{i,j}\right) \cdot x^r_j$
	$i=1$		1.711	4.416	0.000
$r=1$	$i=2$	0.665		2.524	1.653
	$i=3$	0.208	0.305		6.389
	$i=1$		0.509	0.393	9.759
$r=2$	$i=2$	1.728		0.787	1.867
	$i=3$	2.520	1.483		0.000

(b) Regional cost structure matrices

Region 1 ($\mathbf{A}^1$)			Region 2 ($\mathbf{A}^2$)		
0.400	0.100	0.063	**0.212**	**0.051**	**0.025**
0.033	**0.276**	0.125	0.050	0.300	**0.098**
0.010	**0.031**	**0.090**	0.050	0.100	0.313

Note: figures in bold are corrected as compared to the national coefficients $\mathbf{A}^N = \begin{bmatrix} 0.4 & 0.1 & 0.063 \\ 0.05 & 0.3 & 0.125 \\ 0.05 & 0.1 & 0.313 \end{bmatrix}$.

computation of CILQ cancels out the relative size of the region's economy, Q^r/Q^N. As a reconciliation, he proposed Round's Location Quotients, based on the following formula to be applied in eq. (4) instead of $CILQ^r_{i,j}$:

$$RLQ^r_{i,j} = \frac{SLQ^r_i}{\log_2\left(1 + SLQ^r_j\right)}. \tag{6}$$

The application of RLQ limits the role of extreme SLQ values on the purchasing sector side, taking into account the region size. In the numerical example under consideration, it leads to a slight decrease in interregional trade as compared to using CILQ, although the general picture remains roughly unchanged (see Table 7).

Flegg's Location Quotients (FLQ), probably the most widespread technique in single-region applications (Flegg et al., 1995; Bonfiglio, 2009; Flegg and Tohmo, 2014; Flegg et al., 2021), extends CILQ as follows:

$$FLQ^r_{i,j} = \begin{cases} SLQ^r_i \cdot \lambda^r & i = j \\ CILQ^r_{i,j} \cdot \lambda^r & i \neq j \end{cases}, \tag{7}$$

$$\lambda^r = \left[\log_2\left(1 + Q^r/Q^N\right)\right]^\delta, \tag{8}$$

Table 8: FLQ correction (λ) depending on convexity (δ) and region size (Q^r/Q^N)

λ^r	convexity (δ)										
region share	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	1.00	0.65	0.43	0.28	0.18	0.12	0.08	0.05	0.03	0.02	0.01
0.01	1.00	0.77	0.59	0.45	0.35	0.27	0.20	0.16	0.12	0.09	0.07
0.05	1.00	0.82	0.67	0.55	0.45	0.37	0.30	0.25	0.20	0.17	0.14
0.1	1.00	0.87	0.77	0.67	0.59	0.51	0.45	0.39	0.34	0.30	0.26
0.2	1.00	0.91	0.82	0.75	0.68	0.62	0.56	0.51	0.46	0.42	0.38
0.3	1.00	0.93	0.87	0.81	0.75	0.70	0.65	0.60	0.56	0.52	0.49
0.5	1.00	0.95	0.90	0.85	0.81	0.76	0.72	0.69	0.65	0.62	0.58

using then $FLQ^r_{i,j}$ instead of $SLQ^r_{i,j}$ in eq. (2). The convexity parameter δ ($0 \leq \delta < 1$) determines the degree of interregional trade and cross-hauling. For $\delta = 0$, the formulae (7)-(8) collapse to (5). As δ grows, the interregional trade intensifies, especially for small regions (see Table 8).

Although FLQ ensures a more realistic treatment of the openness of the regional economy and more considerable role of cross-hauling, the results are conditional upon the choice of δ. Flegg and Tohmo (2013b), Flegg et al. (2021) and Azorin et al. (2022), among others, discuss potential values and analyse the degree of empirical fit based on available resouces. Flegg and Tohmo (2019, eq. (22)) propose a helpful formula that links an optimum level of δ (derived as minimizing mean percentage error for Finnish regional tables) to other factors desribing a regional economy: (i) region's size (% of national output), (ii) proportion of region's gross output imported from other regions, averaged over sectors, (iii) region's average use of intermediate inputs. A more nuanced proposal has been put forward by Kowalewski (2015) who suggests using sector-specific values of δ_s. Zhao and Choi (2015) analyse this proposal in detail and conclude that this version of FLQ formula is debatable and still needs major improvement.

In the numerical example, using FLQ with $\delta = 0.75$ generally intensifies the cross-regional trade as compared to CILQ or SLQ (cf. Tables 9b and 6b). It also extends the degree of cross-hauling: the bi-directional trade in commodity 2 intensifies, and affects now commodity 1 as well. This is specifically because the correction factor λ^1 forces $FLQ^1_{1,1}$ below unity and hence enforces intermediate imports.

All abovementioned LQ techniques presume building $\mathbf{A}^r$ either by reducing $\mathbf{A}^N$ elements into or by leaving them unchanged. This precludes the third possibility, that of increasing national cost structure coefficients, which could be justified on the grounds of economic geography. As argued by McCann and Dewhurst (1998), regional specializations can lead to creation of regional clusters; these, in turn, increase

Table 9: Simple example, cont'd: Flegg's Location Quotients

(a) Calculation of FLQ with $\delta = 0.75$

Region	Sectors	$j=1$	$j=2$	$j=3$	Interregional imports:
		$FLQ^r_{i,j}$			$\sum_j \left(a^N_{i.j} - a^r_{i,j}\right) \cdot x^r_j$
	$i=1$	0.987	1.073	3.433	**0.362**
$r=1$	$i=2$	0.350	0.564	1.962	**4.890**
	$i=3$	0.109	0.192	0.176	7.308
	$i=1$	0.383	0.360	0.247	**12.621**
$r=2$	$i=2$	1.442	0.765	0.494	**6.538**
	$i=3$	2.103	1.051	1.116	0.000

Note: figures in bold emphasize commodities for which cross-hauling arises.

(b) Regional cost structure matrices

Region 1 ($\mathbf{A}^1$)			Region 2 ($\mathbf{A}^2$)		
0.395	0.100	0.063	**0.153**	**0.036**	**0.015**
0.018	**0.169**	0.125	0.050	**0.230**	**0.062**
0.005	**0.019**	**0.055**	0.050	0.100	0.313

Note: figures in bold are corrected as compared to the national coefficients $\mathbf{A}^N = \begin{bmatrix} 0.4 & 0.1 & 0.063 \\ 0.05 & 0.3 & 0.125 \\ 0.05 & 0.1 & 0.313 \end{bmatrix}$.

Table 10: Simple example, cont'd: Augmented Flegg's Location Quotients

(a) Calculation of AFLQ with $\delta = 0.75$

Region	Sectors	$j=1$	$j=2$	$j=3$
		$AFLQ^r_{i,j}$		
$r=1$	$i=1$	1.366	1.073	3.433
	$i=2$	0.485	0.564	1.962
	$i=3$	0.152	0.192	0.176
$r=2$	$i=1$	0.383	0.376	0.334
	$i=2$	1.442	0.799	0.667
	$i=3$	2.103	1.097	1.506

(b) Regional cost structure matrices

Region 1 ($\mathbf{A}^1$)			Region 2 ($\mathbf{A}^2$)		
0.546	0.100	0.063	0.153	0.038	0.021
0.024	0.169	0.125	0.050	0.240	0.083
0.008	0.019	0.055	0.050	**0.110**	**0.471**

Note: figures in bold are higher than national coefficients $\mathbf{A}^N = \begin{bmatrix} 0.4 & 0.1 & 0.063 \\ 0.05 & 0.3 & 0.125 \\ 0.05 & 0.1 & 0.313 \end{bmatrix}$.

the probability of intraregional purchase. For this reason, Augmented FLQ (AFLQ) method has been presented by Flegg and Webber (2000) as:

$$AFLQ^r_{i,j} = \log_2\left(1 + \max\left\{SLQ^r_j; 1\right\}\right) \cdot FLQ^r_{i,j}, \tag{9}$$

$$q^r_{i,j} = \begin{cases} AFLQ^r_{i,j} & SLQ^r_j > 1 \\ \min\left(FLQ^r_{i,j}; 1\right) & SLQ^r_j \leq 1 \end{cases}. \tag{10}$$

Note that, contrary to all other formulae presented throughout this section, equations (9)-(10) do not imply that $q^r_{i,j} \leq 1$. In spite of the sound justification of this extension, the applicability of AFLQ faces some constraints. First, it limits cross-hauling and interregional trade in general, which is against the scarce empirical evidence. Table 10 demonstrates this effect with the numerical example. Second, the resulting regional tables are not fully consistent with national tables in some respects (see Flegg and Webber, 2000; Miller and Blair, 2009 for further discussions).

Another method RIO construction, put forward by Kronenberg (2009) and further extended by Többen and Kronenberg (2015), is Cross-Hauling Adjusted Regionalization Method (CHARM). It differs from the LQ approach in two major

aspects. First, while LQ remains focused on the cost coefficients, CHARM traces a complete and balanced regional accounting framework. Second, it is therefore assumed that the initial dataset additionally consists of the local intermediate demand matrix $\mathbf{Z}$, constructed with some other method or known, as well as the final demand $\mathbf{f}$, value added $\mathbf{v}$ and output $\mathbf{x}$ vectors for the region of interest, which is a more challenging data endowment than the regional activity structure alone (cf. Table 11 in Section 4). For this reason, I do not continue the numerical example as more input assumptions would have to be made. Given the additional data, CHARM exploits multiple regional accounting identities, as well as further assumptions, to arrive at two further key vectors of size S: regional imports $\mathbf{m}$ and regional exports $\mathbf{e}$, leading to ultimate total supply and total use figures for each sector in the region. The key assumption is that nationwide cross-hauling shares for each supply-side sector i, defined as $(e_i + m_i - |e_i - m_i|)/(x_i + \sum_j z_{i,j} + f_i)$ extend to regional economies.
Kronenberg (2009) refers to this commodity-specific quantity (subscripted with i) as heterogeneity, emphasizing that the cross-hauling is predominantly due to product heterogeneity within a given sector. He argues further that heterogeneity is a property of (commodity-side) sector, not region, and hence it can be extrapolated from the national to sub-national levels given any sector. Court and Jackson (2015) "...disagree with the assertion (...) that equality of national and regional heterogeneity is a reasonable assumption". This is because the within-sector product mix can vary from region to region. Consider an example of the chemical industry (NACE C.20) comprising the manufacture of fertilisers and agrochemical products (20.1+20.2), soap and detergents (20.4) and other subsections. While the C.20 branch of national economy may be regarded as a mix, there are sub-national regions that specialize in producing fertilisers or soap, and those will exhibit a higher degree of cross-hauling in C.20. Flegg et al. (2015) emphasises that the cross-hauling – heterogeneity nexus is important, but the difficulty of obtaining satisfactory estimates of regional heterogeneity (especially for small regions) remains a dominant obstacle in developing further methods.
Többen and Kronenberg (2015) enumerate a number of limitations of the CHARM approach, including i.a. better applicability to type A (or E) "technical" tables (aggregating the use of domestic and imported indirect input of a given commodity) rather than type B "domestic" tables (focusing on the domestic input, and shifting the imported input of various commodities to another row). They also present a multi-region version of the CHARM approach (see Piskin and Hannum, 2017, for a numerical application thereof). Evidence on the empirical performance of CHARM, as compared to e.g. FLQ, remains mixed (Flegg et al., 2015).

4 Interregional tables: gravity-RAS and IRIOLQ

Conventional approaches to building IRIO tables, such as ones depicted in Table 1b or Table 11 (extended to a wider accounting framework), use the preliminary input from the RIO tables in respective positions of the intermediate demand matrix. This defines the intraregional blocks. Conditionally upon that, the rest of the **Z** matrix remains to be defined as interregional blocks. In this text, we focus on presenting IRIOLQ method proposed by Jahn (2017), as an interesting and tractable example belonging to a wider class of so-called gravity-RAS approaches. The gravity-RAS methods, also referred to as doubly constrained gravity (cf. Cai, 2020, 2023), encompass three major steps:

1. Build intraregional blocks with RIO method of choice.
2. Build interregional blocks by using gravity modelling, filling the missing elements for $r \neq p$ in line with an estimated gravity formula (see i.a. Leontief and Strout 1963; Polenske 1970; Gordon 1976; Lindall et al. 2006):

$$\ln z^{r,p} = \beta_0 + \beta_1 \ln d^{r,p} + \beta_2 \ln Q^r + \beta_3 \ln Q^p + ... + \eta^{r,p}. \tag{11}$$

   Note that additional regressors can be used, and that – depending on the size of dataset for which the regression is fit – the equation can either describe total flows between regional economies, or be sector-specific (usually by supply-side sector).
3. Perform balancing to enforce consistency with the existing regional data or internal consistency of the obtained results.

The major practical problem with eq. (11) obviously consists in the lack of data on the dependent variable: if the matrix **Z** was observable, IRIO would not need to be estimated. Gabela (2020) suggests three potential solutions:

1. use data for a different period;
2. use data for a different geography;
3. calibrate or adapt coefficients from the literature (e.g. in Sargento et al., 2012).

Option 3 mostly boils down to 1 and 2 in some form, and option 1 is not available for most geographies, so the most frequent solution in practice is option 2. For example, Jahn (2017) estimates the following equation on the set of data on trade between EU-28 countries in 2010 (a similar specification with the same dataset is also considered by Cai, 2023):

$$\ln z^{r,p} = \beta_0 + \beta_1 \ln d^{r,p} + \beta_2 \ln Q^r + \beta_3 \ln Q^p + \beta_4 border^{r,p} + \eta^{r,p}. \tag{12}$$

Table 11: IRIO structure: full accounting framework

Shaded area = **Z** ↘			$j=1$			...	$j=S$			**f** (incl. exports*)	$\mathbf{x}^T$
			$p=1$	$p=2$	$p=R$		$p=1$	$p=2$	$p=R$		
$i=1$	$r=1$									f_1^1 (incl. e_1^1)	x_1^1
	$r=2$										
	$r=R$									f_1^R (incl. e_1^R)	x_1^R
...											
$i=S$	$r=1$									f_S^1 (incl. e_S^1)	x_S^1
	$r=2$										
	$r=R$									f_S^R (incl. e_S^R)	x_S^R
Imports* (**m**)			m_1^1	m_1^2	m_1^R		m_S^1	m_S^2	m_S^R		
Value added (**v**)			v_1^1	v_1^2	v_1^R		v_S^1	v_S^2	v_S^R		
Output (**x**)			x_1^1	x_1^2	x_1^R		x_S^1	x_S^2	x_S^R		

Note: * – from/to other countries.
Source: Author (based on Torój, 2021).

The parameters estimates are usually used to compute fitted values for the spatial grid that shall be considered in IRIO construction. Those, in turn, allow one to capture proportions between the value of deliveries to other regions. Specifically, Jahn (2017) proceeds as follows. After using FLQ to build intraregional blocks as described in Section (3), he looks at the implied value of total intermediate demand of sector j for commodity i that is not used in the regions that produced this commodity, i.e. that is traded between regions. He refers to this value as FLQ residual:

$$\epsilon_{i,j}^{FLQ} = z_{i,j} - \sum_{r} z_{i,j}^{r,r}. \tag{13}$$

Subsequently, he uses the proportions between the predicted values of $\mathbf{Z}$, defined as follows:

$$h_{i,j}^{r,p} = \begin{cases} \hat{z}^{r,p} & r \neq p \\ 0 & r = p \end{cases} \quad ; \quad g_{i,j}^{r,p} = \frac{h_{i,j}^{r,p}}{\sum\limits_{i,j} h_{i,j'}^{r,p}}$$

to interpolate the FLQ residual into components for each regional pair r, p ($r \neq p$):

$$z_{i,j}^{r,p} = g_{i,j}^{r,p} \cdot \epsilon_{i,j}^{FLQ}. \tag{14}$$

In the empirical illustration, Jahn (2017) applies this approach to approximate IRIO for the German federal states. In a later work, Jahn et al. (2020) approximate the South Korean IRIO for 16 regions and find a decent degree of fit (though varying depending on some specification details, including δ in eq. (8) at the first step). Nakano and Nishimura (2013) and Yamada (2015) use their own gravity estimates to break down the Japanese IRIO tables into a higher number of regions than presented in Table 3. Nakano and Nishimura (2013), Cai (2023) and Torój (2024, Appendix D in online supplemental material) present their own estimates of gravity models that differ from specification (11), as they are fit for each supply-side sector individually. The results from Torój (2024), provided here in Table 12, exhibit some common features within this strand:

i) negative (highly significant) estimates of β_1: spatial decay in supplies;

ii) estimates of β_2 and β_3 generally within the interval $(0.7; 1)$, i.e. slightly sub-proportional dependency on supply in the region of origin and demand in the region of purchaser;

iii) with sector-specific estimates, stronger dependency on distance for commodities from sections F (construction) or O-U (public, education, health services etc.), and weaker depencency for section C (manufacturing);

iv) significant deviation from the exponential spatial decay profile for the adjacent region pairs (both positive and negative).

Table 12: Estimates of the gravity models for Japanese regions, EU states and entire OECD ICIO country set

Sample	Japan	Japan	OECD ICIO (EU-27)
Constant	-11.534*** (0.002)	-14.012*** (0.002)	-9.022*** (0.013)
Output at source (log)	0.747*** (0.0001)	0.892*** (0.0001)	0.767*** (0.0005)
Output at destination (log)	0.724*** (0.0001)	0.74*** (0.0001)	0.78*** (0.0005)
Distance (log)	-0.496*** (0.0002)		-0.831*** (0.002)
Distance (log, source A)		-0.402*** (0.0002)	
Distance (log, source B-E ex. C)		-0.724*** (0.0003)	
Distance (log, source C)		-0.392*** (0.0002)	
Distance (log, source F)		-1.122*** (0.001)	
Distance (log, source G-J)		-0.434*** (0.0002)	
Distance (log, source K-N)		-0.644*** (0.0002)	
Distance (log, source O-U)		-0.789*** (0.0002)	
Adjacency	0.165*** (0.0002)		-0.008*** (0.002)
Adjacency (source A)		0 (0.0013)	
Adjacency (source B-E ex. C)		1.177*** (0.002)	
Adjacency (source C)		0.138*** (0.0003)	
Adjacency (source F)		-0.885*** (0.006)	
Adjacency (source G-J)		0.098*** (0.0004)	
Adjacency (source K-N)		-0.083*** (0.001)	
Adjacency (source O-U)		-0.232*** (0.001)	

Source: Torój (2024, online Appendix D).

Table 12: Estimates of the gravity models for Japanese regions, EU states and entire OECD ICIO country set, cont.

Sample	OECD ICIO (EU-27)	OECD ICIO (all)	OECD ICIO (all)
Constant	-9.553*** (0.013)	-11.334*** (0.003)	-11.498*** (0.003)
Output at source (log)	0.855*** (0.001)	0.813*** (0.0002)	0.87*** (0.0002)
Output at destination (log)	0.79*** (0.0005)	0.85*** (0.0002)	0.856*** (0.0002)
Distance (log)		-0.696*** (0.0004)	
Distance (log, source A)	-0.926*** (0.002)		-0.784*** (0.0004)
Distance (log, source B-E ex. C)	-1.125*** (0.002)		-0.72*** (0.0004)
Distance (log, source C)	-0.836*** (0.002)		-0.707*** (0.0004)
Distance (log, source F)	-1.353*** (0.003)		-1.249*** (0.001)
Distance (log, source G-J)	-0.846*** (0.002)		-0.748*** (0.0004)
Distance (log, source K-N)	-0.937*** (0.002)		-0.794*** (0.0004)
Distance (log, source O-U)	-1.396*** (0.003)		-1.22*** (0.001)
Adjacency		0.238*** (0.001)	
Adjacency (source A)	0.334*** (0.009)		0.252*** (0.004)
Adjacency (source B-E ex. C)	0.451*** (0.011)		0.126*** (0.003)
Adjacency (source C)	0.072*** (0.003)		0.296*** (0.001)
Adjacency (source F)	-0.554*** (0.026)		-0.471*** (0.019)
Adjacency (source G-J)	-0.219*** (0.003)		0.224*** (0.002)
Adjacency (source K-N)	-0.318*** (0.004)		-0.307*** (0.002)
Adjacency (source O-U)	-0.811*** (0.021)		-0.583*** (0.012)

The third step of the gravity-RAS approaches, the one related to the RAS part, presumes that individial elements of **Z**, defined with equations like (14), are just an initial version of the final estimate, but need to be further modified to ensure consistency with regional data. The most popular balancing algorithm is RAS, which is not an acronym but has been named after the matrix notation used by Stone (1961). It is applicable when the row and column sums of a square matrix are known, just as well as the initial estimate of this matrix that does not necessarily fulfil these conditions. The balancing algorithm iterates as follows:

1. Premultiply the initial version (**A**) with a diagonal matrix of compatible size (**R**) containing ratios of desired row sums to current actual row sums located on the diagonal.

2. Postmultiply the matrix obtained in step 1 (**RA**) with a diagonal matrix of compatible size (**S**) containing ratios of desired column sums to current actual column sums located on the diagonal.

3. Replace the initial version with the version obtained in steps 1-2 (**RAS**) and redo steps 1-2. Iterate until convergence.

To ensure consistency with the national table, Torój (2021) introduced an additional step, related to blockwise (rather than column or row) sums. This is to ensure that block sums, computed over a given (i, j) block in Table 11, are equal to the respective $z_{i,j}$ elements from the national table. The procedure, though considerably slower than original RAS, is also convergent.
However, the balancing constraints are not always easily expressed as row, column or block sums, depending on what type of national or regional data is, or is not, available. For example, a number of authors, including Többen and Kronenberg (2015), Jahn (2017) and Torój (2021; 2024), assume that final demand vector is known for commodities from each region, which leads to row sum constraints. In fact, it is unknown (at least for the EU countries at e.g. NUTS-3 level) and multiple authors propose approximations based on value added or population. (Patryk Czechowski demonstrated that these approximations are highly accurate, at least for the case of Japan and South Korea, and more sophisticated approaches like regression-based imputation do not outperform them. Results are available upon request.) Instead of that, Jahn (2017), as well as Canning and Wang (2005) among others, perform the final step not in the form of RAS, but in a more general setup, minimizing an objective function of the form:

$$f(\mathbf{Z}) = \sum_c \frac{(g_c(\mathbf{Z}) - G_c)^2}{G_c}$$

with respect to **Z**, where c indexes constraints based on the right-hand-side constraint values G_c known from national or regional accounts data and $g_c(\mathbf{Z})$ is the respective

left-hand side of constraint (usually sum of a subset of **Z** elements). Inequality constraints are also considered, and their conventional use is to ensure that a given column of **Z** does not exceed the respective global output x_j^r.

5 Interregional tables: Spatial IRIO

Starting with Rey (2000) and Loveridge (2004), the literature started to notice the applicability of spatial econometric methods in IRIO estimation, within the so-called integrated econometric–I–O frameworks. Early attempts of using these methods included i.a. the work of Liu et al. (2015) that applied a Geographically Weighted Regression model in exploring trade links between Chinese regions. In the recent years, Torój (2016; 2021; 2024) developed a framework building on the multi-equation version of the Spatial Durbin Model (SDM), labelled Spatial IRIO in the most recent contribution. The features of this approach, including the positioning against the previous literature, the pros and cons, will be discussed in this section.

The starting point for model specification is the commonly applied assumption of symmetric sectorial technologies between regions. This allows one to compute the column sums of each (i, j) block of **Z** matrix in Table 11 in proportion to value added:

$$x_i^r = \frac{v_i^r}{\sum_{r=1}^{R} v_i^r} \cdot x_i.$$

Two-dimensional table v_i^r is available for i.a. EU countries for various levels of spatial and sectorial aggregation and serves roughly the same role as Q_i^r in Section 3. The column sums are further interpolated into individual column entries using proportions $\mathbf{w_i^P} = \begin{bmatrix} w_i^{1,p} & w_i^{2,p} & \dots & w_i^{R,p} \end{bmatrix}^T$, allocating between the regions of origin the deliveries of commodity i to region p. For all $r = 1, ..., R$, these vertical vectors can be collected into a single, commodity-specific matrix $\mathbf{W_i} \equiv \begin{bmatrix} \mathbf{w_i^1} & \mathbf{w_i^2} & \dots & \mathbf{w_i^R} \end{bmatrix}$ that can be found through the econometric analysis of the following set of equations for each $i = 1, ..., S$:

$$\begin{bmatrix} v_i^1 \\ \vdots \\ v_i^R \end{bmatrix} = \mathbf{W_i} \begin{bmatrix} v_1^1 & \dots & v_S^1 & f_i^1 \\ \vdots & \ddots & \vdots & \vdots \\ v_1^R & \dots & v_S^R & f_i^R \end{bmatrix} \begin{bmatrix} \beta_{i,1} \\ \vdots \\ \beta_{i,S} \\ \beta_{i,f} \end{bmatrix} + \begin{bmatrix} \varepsilon_i^1 \\ \vdots \\ \varepsilon_i^R \end{bmatrix}. \tag{15}$$

This equation originates from the national distribution identity for i-th commodity, $x_i = \sum_j z_{i,j} + f_i$, written for the regional level with a lead matrix $\mathbf{W_i}$ that accommodates both intra- and interregional trade. Torój (2016) presents an analytic

proof that if column sums of matrices $\mathbf{W_i}$ sum to unity (which is a just-identifying assumption for proportion indicators $\mathbf{w_i^r}$), regional distribution equations hold and the resulting $\mathbf{Z}$ matrix in the IRIO model yields identical simulation results as the national-level I–O model. Technically, equation (15) can be regarded as a specification that reads as SDM since $\mathbf{v_i}$ is both a dependent variable and one of the regressors in a spatially lagged form, as long as $\mathbf{W_i}$ is interpreted as a spatial weight matrix. The fundamental difference is that the coefficient vector $\boldsymbol{\beta}$ is known (from national I–O ratios), and $\mathbf{W_i}$ – estimated. As matrices of size $R \times R$ obviously cannot be estimated freely in S versions, each entry $w_i^{r,p}$ is parametrically formulated as a function of properties of the pair (r, p), just as in eq. (11). The initial version of $\mathbf{Z}$ is subject to balancing as described in Section 4.

The Spatial IRIO approach exhibits three major advantages:

1. Flexibility with small regions. Although multiple functional forms linking distance $d^{r,p}$ and $\mathbf{W_i}$ entries can be considered, the previous applications demonstrated that the following, based on the cumulated distribution function P_Γ, accommodates a number of useful cases with only two commodity-specific parameters: shape and scale. First, local suppliers can be strongly preferred (case 1 in Fig. 3a). Second, the supply intensity can decrease mildly with distance (case 2 in Fig. 3a). Third, a threshold of tolerance to distance exists (case 3 in Fig. 3a). While cases 1 and 2 can easily be modelled with a single decay parameter as in gravity models like (11), case 3 and many further ones cannot. When relatively large regions are considered, the specification (12) with an additional binary variable indicating adjacent region pairs is a reasonable solution. This is not any more the case when the regions are small enough because the threshold distance considered in case 3 can substantially exceed a typical perimeter of a region's circumcircle.

The posterior means for the shape and scale parameters in the sector-specific spatial decay profiles suggest that agricultural commodities and advanced services are supplied to the most distant locations, while the simple services and construction activities – to the least distant ones, on average. These results are roughly comparable to sector-specific gravity estimates from Table 12.

The related literature provides a few similar studies, both with the Spatial IRIO framework (Mogiła et al., 2024) and non-spatial gravity models (Nakano and Nishimura, 2013; Cai, 2023). Although the sectoral breakdowns vary, nonmanufacturing industries (mining, construction) generally tend to exhibit a relatively local profile both in Mogiła et al. (2024) and in Cai (second-highest distance elasticity estimate of 1.69 in absolute terms). For a more detailed commodity breakdown, Nakano and Nishimura find below-unity elasticities for a number of non-manufacturing industry subsections. All three studies conclude that services are supplied to higher distances, on average. The second-lowest Cai's distance elasticity is for "other services" (excluding trade, hospitality, transport and communications), whereas Nakano and Nishimura's profile for some advanced services prefers high over low distance with an inversely signed estimate. This last possibility is precluded by

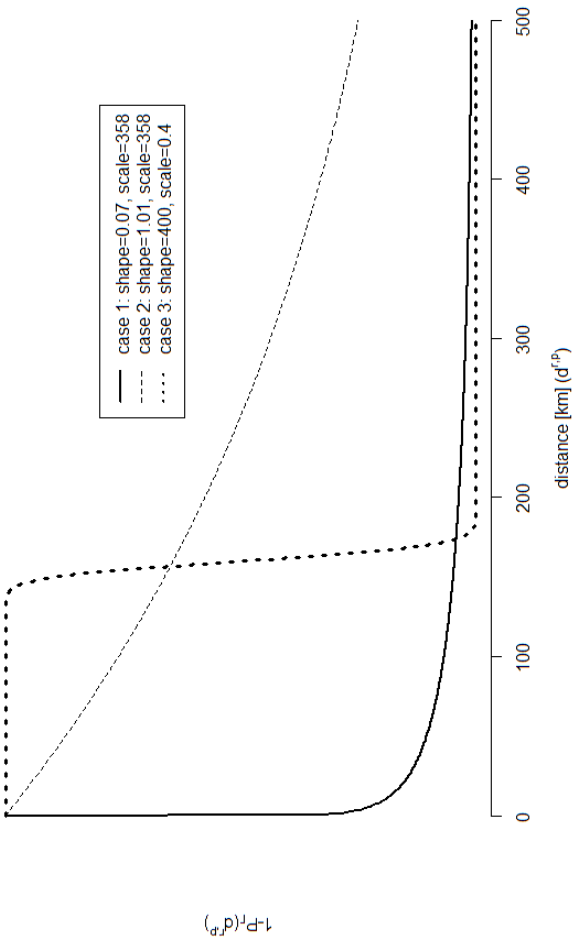


Figure 3: Spatial decay profiles obtained by Torój (2024) with Spatial IRIO approach
(a) three illustrative scenarios for $w_i^{r,p}$ as a function of $d^{r,p}$

Source: Torój (2024).

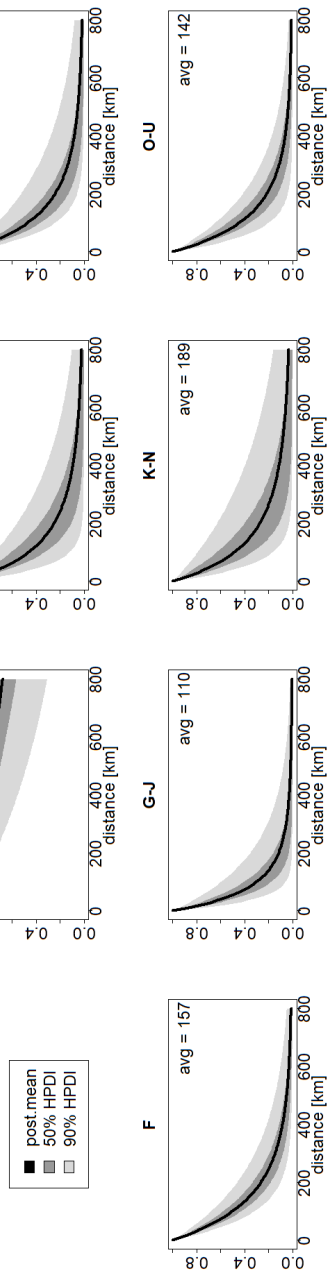


Figure 3: Spatial decay profiles obtained by Torój (2024) with Spatial IRIO approach, cont.

(b) estimated spatial decay profiles for NACE groups of sections, Poland

Spatial IRIO.

2. Using data for the spatial area and granularity of interest. Estimating gravity models with a different geography than a geography of interst for IRIO building, and recycling their parameters for IRIO construction as suggested in Section 4, poses a risk of mismatch between the specific properties of both economies, but also between spatial granularities. It is customary to use estimates obtained with greater, more data-abundant regional grids (i.e. countries) for sub-national regions, but also survey-based IRIO data for greater countries and regions to build IRIO estimates for smaller ones. In a benchmarking exercise with the use of South Korean data (cf. Table 3), Torój (2024) compares the elements of $\mathbf{Z}$ matrix estimated with (i) Spatial IRIO method (using Korean spatio-sectoral distribution of value added), (ii) IRIOLQ method with limited information. In the latter experiment, IRIOLQ does not use real-world Korean data on $\mathbf{Z}$ for estimating gravity equation, but replaces it with data for other geographies: Japan and OECD ICIO (EU-27 or entire set; cf. Table 2). In this setup, the original survey-based data is only used for the *ex post* accuracy assessment. This comparison replicates the choice that researchers usually face when generating IRIO for a new spatial grid, with typical limited regional data endowment. The RMSE under IRIOLQ turned out to be greater by 4.9-9.1%, consistently across various specifications. A potential reason for this can be the usage of Korean (rather than Japanese or multicountry) regional and national data for estimation.
However, in an environment of scarce data, the idea of combining information from various data sources shall not be rejected on the basis of this limited evidence. In a version of Spatial IRIO presented by Torój (2024), this possibility is accommodated through the use of Bayesian methods. The posterior distributions of shape and scale parameters that constitute the elements of $\mathbf{W_i}$ in eq. (15) are a synthesis of (i) information extracted from the data on $\mathbf{v}$ and $\mathbf{f}$ for the respective country and regional granularity (Poland, NUTS-3), (ii) the prior distribution that has been derived with the data for NUTS-3 regions in Finland (see Section 2). In particular, the survey-based set of Finnish RIO tables implies the prior expected values of scale, conditionally on shape, so as to match the fraction of intermediate supply of commodity i delivered to the home region. One can imagine similar prior elicitation based on the Japanese, Korean or any other existing dataset.

3. Local and cross-regional cost coefficients determined in a joint procedure (with endogenous cross-hauling). Note that the intraregional cost coefficients arise naturally from the estimates of shape and scale (as well as the balancing procedure), along with the interregional ones. This is contrary to the IRIOLQ procedure from Section 4, in which the RIO tables for individual values, determined with the FLQ method conditionally some choice of δ, imply FLQ residuals (eq. (13)) as sums of interregional blocks. Note that, in such sequential procedures, the interregional multipliers are independent on the gravity modelling and the degree of cross-hauling is largely determined by δ calibration at step 1. In the Spatial IRIO

procedure, cross-hauling arises and its intensity is endogenous; in particular it is not related to the value of any calibration.

Boero et al. (2018) noted that *the methods for the regionalization of IO tables and the ones for estimating trade flows are independent and separated, with the possibility to obtain inconsistent estimates*. At the same time, Cai (2020, p. 91) emphasizes that *there is nothing about intra-regional trade that would warrant special treatment*. Consistently with this line of argument, Spatial IRIO determines both types of coefficents jointly, and in doing so it is similar to the procedure proposed by Gabela (2020) under the name *extended approach* (although these authors themselves assess the application of this approach as *quite unsatisfactory in practice*).

The use of Bayesian methods for estimating shapes and scales offers a convenient possibility of drawing from the posterior distribution to obtain multiple versions of **Z** and hence different values of local I–O multipliers. This, in turn, allows to quantify the statistical uncertainty around various quantities relevant in regional studies. One of them is the share of indirect effects in the domestic region in total indirect effects for a specific I–O simulation. In a numerical example considered by Torój (2024), this share amounts to 86.9% at point estimates, and the 90-percent highest posterior density interval ranges from 81.8% to 89.7%. Similar statements can be made with regard to individual **Z** or **A** entries, multipliers, etc.

As any approach, Spatial IRIO faces some limitations. Firstly, it is more analytically complicated than the previously proposed methods (although replication codes are available online, `https://github.com/AndrzejToroj/SpatialIRIO`; see Torój, 2024). Secondly, eq. (15) involves $\mathbf{f_i}$ (distribution of final demand for each commodity i between regions of origin) as an observable variable, while in fact it is only approximated. Even if the quality of approximation is high, as available sources might suggest, this is another source of uncertainty, unaccounted for by the model. Third, the benchmarking study by Torój (2024) suggests that a relatively good performance of this method is predominantly due to (much) higher fit of intraregional blocks, whereas the interregional blocks are (somewhat) less accurately replicated than with IRIOLQ in different variations.

Like many methodological approaches, Spatial IRIO builds on the assumption that regional technologies are symmetric (i.e. cost coefficients of a given branch do not vary between regions for a given activity sector). On the one hand, there is some empirical evidence that this assumption is decently (though not perfectly) valid (Torój, 2021). On the other hand, if the regions under consideration are small enough to be labelled as metropolitan or peripheral, which is the case for Polish NUTS-3 regions, then the arguments from New Economic Geography can be used to argue against symmetric technologies, as in the case of AFLQ presented in Section 3. Starting with the core-periphery model of Krugman (1991), a number of contributions justify how agglomeration effects reinforce regional productivity differences which are potentially impactful for regional technological input-output coefficients (see i.a. Fujita et al., 1999; Miller and Blair, 2009; Bartelme and Gorodnichenko, 2015). This results from

knowledge spillovers, labor market pooling and skill accumulation, lower transaction and transport costs and a number of externality types occuring in the metropolitan areas, shaping both cost levels and structures of the existing firms and encouraging new firms to co-locate if they have potential upstream or downstream connections,
Fourthly, as noted by Mogiła et al. (2024), when the number of regions is low, shape and scale are not well identified as individual parameters. One can imagine this as the lines in Figure 3a being fitted to a lower number of data points. This has no direct impact on the estimate of **Z** in IRIO tables. However, different combinations of shape and scale can yield a similar data fit and hence the MCMC simulation fails to converge. An additional inspection of such cases by the researcher is required. Usually, when the simulated MCMC chains of shape and scale for a given commodity are correlated with Pearson coefficient close to -1, one can assume that the desired distance profile has been found even if the potential scale reduction factor fails to converge to one. This is inconvenient, however, as it requires additional discretion from the researcher and may have a detrimental impact on interval assessment of the kind discussed in point 3 above.

6 Empirical performance of RIO and IRIO methods: evidence from South Korea

The scarce cases of survey-based tables, enumerated in Table 3, shed some light on the empirical performance of RIO and IRIO construction methods. To illustrate the discussion in Sections 3-5, I attempt to reconstruct the intermediate demand matrix **Z** from the South Korean IRIO tables. I consider three regions: Sejong (a municipal province located in country interior), Jeollabuk-do (a relatively large province in the East) and Busan (a maritime logistics megahub in the West of the country) – see Figure 4. For each of these, I consider RIO table estimates obtained with SLQ, CILQ or FLQ, as well as intraregional partition of IRIO obtained with IRIOLQ and Spatial IRIO methods. The results are summarized in Table 13.
In line with most of the empirical literature, FLQ exhibits the highest degree of accuracy in the class of RIO methods. Although FLQ is the first step in the IRIOLQ procedure and fully determines the initial version of **Z**, the subsequent balancing step makes the RMSE for both A and Z under FLQ deviate from the analogous values for intraregional blocks under IRIOLQ, with a slight drop in accuracy. Spatial IRIO outperforms IRIOLQ in accuracy for some provinces (Jeollabuk-do, Sejong), while it underperforms in others (Busan). It can be concluded that the generally promising properties of Spatial IRIO, discovered so far for a cross-section of Korean regions, cover some degree of heterogeneity that requires further research.
These results are extended with an illustrative I–O simulation. A representative enterprise from the 'Machinery and Equipment' industry is considered, with a total production value of 200m won and a sectorial cost structure matching the branch.

Table 13: Performance of RIO and IRIO construction methods for selected South Korea provinces

(a) Sejong

Method	ME (A)	RMSE (A)	ME (Z)	RMSE (Z)	indirect home reg. (dX)	indirect other reg. (dX)
Stat. KO (RIO)	–	–	–	–	128.8658	–
SLQ	0.0055	0.0181	3.4383	16.9795	162.3481	–
CILQ	0.0063	0.0199	3.3820	15.7498	180.5257	–
FLQ	0.0022	0.0159	1.1413	10.7446	151.8649	–
Stat. KO (IRIO)	–	–	–	–	129.0908	99.3200
IRIOLQ	0.0019	0.0360	0.6519	14.9713	142.2828	135.7459
SpatialIRIO	0.0008	0.0123	0.7459	13.9312	134.8641	91.2483

(b) Jeollabuk-do

Method	ME (A)	RMSE (A)	ME (Z)	RMSE (Z)	indirect home reg. (dX)	indirect other reg. (dX)
Stat. KO (RIO)	–	–	–	–	143.2297	–
SLQ	0.0049	0.0159	12.1851	81.1959	188.8277	–
CILQ	0.0044	0.0159	10.1281	82.1895	192.8414	–
FLQ	0.0001	0.0149	-2.9414	89.6928	155.5470	–
Stat. KO (IRIO)	–	–	–	–	144.0464	84.3990
IRIOLQ	0.0006	0.0242	-1.0916	108.9345	157.2591	95.3329
SpatialIRIO	0.0044	0.0195	10.4878	85.6058	182.2438	47.3087

(c) Busan

Method	ME (A)	RMSE (A)	ME (Z)	RMSE (Z)	indirect home reg. (dX)	indirect other reg. (dX)
Stat. KO (RIO)	–	–	–	–	149.7326	–
SLQ	0.0043	0.0136	22.8836	81.5593	197.9233	–
CILQ	0.0043	0.0142	21.5576	82.8990	191.3172	–
FLQ	0.0006	0.0120	0.7335	75.9180	157.8265	–
Stat. KO (IRIO)	–	–	–	–	151.1832	74.3972
IRIOLQ	0.0007	0.0202	1.2744	92.9558	199.6407	144.4143
SpatialIRIO	0.0043	0.0164	22.7974	96.8304	195.4857	33.2813

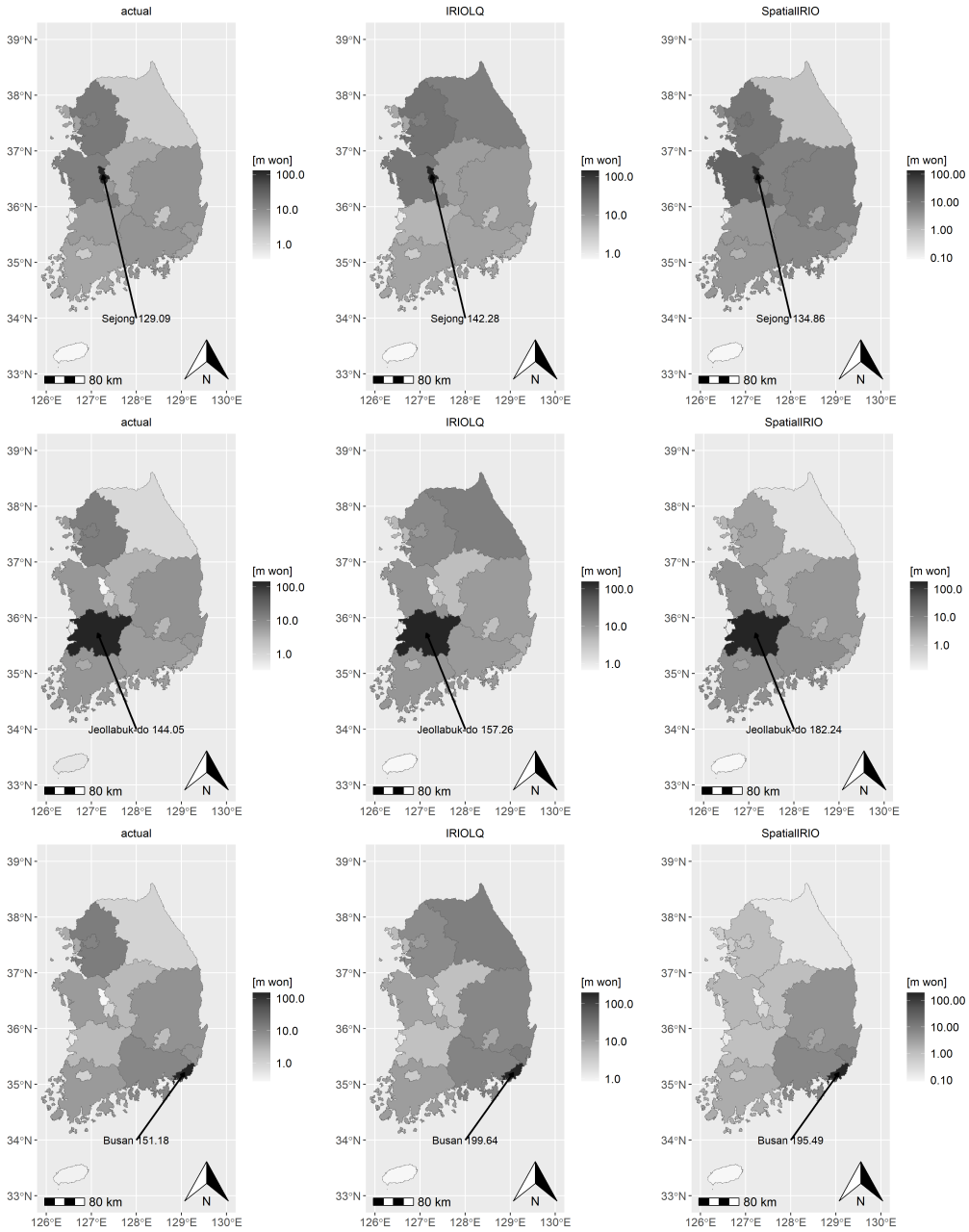
Note: ME = Mean Error. RMSE = Root of Mean Square Error. **A** = cost coefficient matrix (as in text). **Z** = intermediate demand matrix [bn won] (as in text).

All purchases are assumed to be made in the home province. Table 13 presents total indirect effects related to the purchases made by this enterprise, in terms of global output. In the case of RIO approaches, the results refer to the home region (second last column). For IRIO approaches, the results are broken down into home province and the sum of other South Korean provinces (two last columns).
For all regions under consideration, SLQ- and CILQ-based RIO tables yield the highest indirect effects for the home region within the class of single region approaches. Under FLQ, the indirect effects for the home region are considerably and consistently smaller, although they still deviate on the upside from the amount that can be simulated with the original table. When the original IRIO tables are used, and the cross-regional feedback effects materialize, the indirect effects for the home region are just marginally higher. However, the indirect effects for the other regions can be traced in this case, and their sum turns out to be generally smaller than for the home region, both when the original table is considered, as well as its approximations.
The simulation results obtained with the original table are matched to a varying extent by both IRIO construction approaches, depending on the province. For Sejong, effects computed with Spatial IRIO are reasonably close, and clearly outperform IRIOLQ which overshoots the original result. The opposite is the case for Jeollabuk-do, where Spatial IRIO overestimates the indirect effect for the home province and underestimates it for the other regions. In the case of Busan, both methods perform just as badly, overscoring the indirect effects in the home region. At the same time, IRIOLQ overestimates the effects for other regions, while the Spatial IRIO underestimates them, both with considerable errors.

7 Conclusions

Regional input–output (RIO) tables and their interregional extensions (IRIO) for sub-national granularities remain a useful tool for regional policy analysis. A number of regional development questions – starting with the impact of a greenfield investment on a local economy, up to the socio-economic impact of an existing company on the local community – can be answered with input–output (I–O) simulations powered by these tables. At the same time, the instances of such tables based on direct empirical, survey-based evidence remain limited, predominantly due to a prohibitively high cost of compilation. The rare exceptions include the datasets derived from the Commodity Flow Survey in the United States, and infrequent estimates for a grid of 9-18 regions in i.a. Finland, Japan or South Korea.
It is therefore not surprising that the demand for estimation methods of RIO and IRIO tables persists and new proposals are formulated in the literature. In this article, I review a few most popular methods, including also some most recent developments, and discuss their advantages and disadvantages. To this aim, I use a consistent notation embedded within an interregional accounting framework, as well as simple numerical examples and empirical results from the literature.

Figure 4: Simulation results with RIO and IRIO tables: indirect effects on global output of an illustrative 'Machinery and Equipment' company located in selected South Korean regions



The workhorse approaches for RIO construction belong to the Location Quotient family. Simple Location Quotients (SLQ) preclude cross-hauling (simultaneous imports and exports of the same commodity) and assume that the local demand shall be fully satisfied with local supply, as long as the latter is sufficient, which results in a tendency to overestimate the local cost share and local multipliers. The issue is not fully solved by Cross-Industry Location Quotients (CILQ) or closely related Round's Location Quotients (RLQ), although they generate some cross-hauling. The dominant approach, Flegg's Location Quotient (FLQ), substantially increases the value of interregional trade and cross-hauling by using a calibrated convexity parameter. Augmented FLQ (AFLQ) additionally allows the local cost coefficient to grow above the national one due to local clustering, but this generally leads to a lower degree of fit. A different approach, Cross-Hauling Adjusted Regionalization Method (CHARM), keeps track of a wider regional accounting perspective and applies the national cross-hauling intensities to regions.

A combination of demand, supply and distance considerations is taken into account when building interregional I–O (IRIO) tables. The widespread family of gravity-RAS approaches either uses RIO tables as intraregional blocks and proceeds to apply some gravity model to consistently build interregional blocks (as e.g. IRIOLQ combines FLQ with gravity modelling), or determines both in a joint procedure. A relatively recent, promising development in the latter group builds on spatial econometric analysis involving regional activity data and national cost structures (Spatial IRIO). The extant evidence, though scarce, demonstrates that it can be regarded as a promising approach in small-area studies.

The age of big data will likely modify this landscape. An increase in the use of direct observations can be expected, but with sensors and GPS devices rather than surveys. Early attempts in the literature involved refining the measures of distance for gravity or spatial modelling, e.g. by using journey time between regional capitals rather than physical distance (Jahn et al., 2020; Torój, 2024), with only marginal effect on the final results. Data from mobile devices has been used to differentiate between regions where households earn and spend their income due to commuting (Torój, 2024). On the other hand, the big repositories of tax data created for tax enforcement purposes (such as Standard Audit File for Tax) can be used to extract locations of both selling and purchasing side, and potentially merged with sectors of both. This, in turn, might provide an imprecise spatial perspective, since invoice data might not coincide with the actual production site. The real breakthrough would come with the possibility to merge GPS-based shipment data with commodity or activity codes.

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