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Identifying core nodes in interaction graphs for critical component analysis in cascading failures of power grids

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Abstract: This paper introduces a novel approach to identifying critical lines in power systems during cascading failures, addressing significant limitations in previous methodologies such as computational inefficiency and limited effectiveness. The proposed methodology is inspired by social network analysis techniques for evaluating the importance of nodes and determining core roles within a network. By adopting these techniques, multiple centrality metrics – degree, ego-betweenness centrality, and eigenvector centrality – are applied to assess the importance and role of lines in interaction graphs of cascading failures. The methodology can be applied to any type of interaction graph of cascading failures and involves selecting centrality metrics with strong correlations to outline the particularity of critical lines. The importance of each line is evaluated as the normalized sum of its three metrics. Critical lines are identified based on their deviation from the statistical correlation of the overall interaction graph, analogous to the role determination process in social networks. The effectiveness of the proposed approach is demonstrated through extensive testing on IEEE 39-bus and 118-bus systems. The identified critical lines are validated with a method from the literature and by analysing the effects of upgrading the critical lines, demonstrating the accuracy and reliability of the proposed methodology. By leveraging interaction graphs and simulation data, our approach provides a robust framework for mitigating cascading failures. The results indicate that this methodology not only improves computational efficiency but also enhances the precision of critical line identification, making it highly suitable for real-time applications in power system stability and reliability.

Key words: cascading, failures, graphs, interaction, power

1. Introduction

Significant blackouts, which have substantial impacts on society and the economy, often result from cascading failures within power grid transmission networks [1]. The complexity and vast number of components in power grids that interact intricately make preventing these cascading failures challenging. Research efforts have focused on understanding these complex interactions to predict failure propagation and identify critical or vulnerable components within power grids.

Notable blackouts, such as those in India in July 2012 [2], the northeaster US and Canada on August 14, 2003 [3], and the widespread outage affecting Italy and other EU nations on September 28, 2003 [4, 5], have been extensively studied using both complex networks theory (CNT) and electrical engineering methodologies [6]. CNT offers a robust framework for analysing and managing power grids by conceptualizing generators and loads as interconnected nodes through power cables or communication links [7]. This approach, rooted in graph theory, focuses on network structure, measurements, and component influence [8, 9].

In the context of the application of CNT, many studies have used centrality concepts to model and analyse cascading failures with the physical topology of power grids. Centrality identifies nodes and edges with maximal influence on network performance [10, 11], and research shows that targeted removal of specific nodes causes faster network disintegration than does random removal [12]. This highlights the importance of identifying central components for both network resilience and potential attack strategies. However, empirical evidence suggests that physical topology alone is insufficient. Historical data show that the failure of a critical transmission line can overload distant lines, up to 100 miles away, despite no direct physical connection. This phenomenon is due to the principles of electricity governing power flow dynamics and the functional and cyber dependencies managed by automated systems and operators. Consequently, recent research has focused on using interaction graphs, instead of the physical topology of power grids, to represent influences among outaged components, simplifying the study of failure propagation and component roles during cascading events.

An interaction graph is a graph G(C,L), where vertices C represent system components and edges L represent interactions or influences among components. These interactions can be directed, undirected, weighted, or unweighted, depending on the analysis requirements. Interaction graph construction methods fall into two categories: data-driven and electric distance-based methods. Data-driven methods use system data (historical or simulation data) to infer interactions during cascading processes, while electric distance-based approaches rely on Kirchhoff's laws to define interactions based on electrical distances.

Interaction graphs have proven to be effective tools for identifying critical components in cascading failures. For instance, methods such as [13, 14] sum the weights of the interaction links originating from a node to find critical transmission lines whose failure could trigger large cascades. Other studies employ methods like cascade probability vectors to quantify the failure likelihood of transmission lines during cascading stages, providing a probabilistic view of critical line identification [15–17]. Meanwhile, approaches based on electric distance, such as effective resistance-based interaction graphs [18] and impedance-based graphs [19, 20], compute metrics like effective graph resistance and net-ability to evaluate grid performance before and after the removal of specific components.

While these methods provide valuable insights, they often require extensive simulations, detailed system models, or rely heavily on historical failure data, which may limit their applicability to unseen scenarios. To address these challenges, other works have applied CNT to interaction graphs to identify critical components. For example, studies such as [21, 22] have identified fault chain-based interaction graphs as scale-free, implying that a small number of nodes hold a disproportionate degree centrality, leading the authors to conclude that nodes with high degrees are the critical components. Similar conclusions were reached in impedance-based interaction graphs [23, 24]. However, the reliance on degree centrality (DC) in these studies, while computationally efficient, limits the approach to local information [25]. This lack of global context can lead to an overemphasis on high-degree nodes, potentially missing other critical components that do not have high connectivity but are still essential to the network's function [26]. Eigenvector centrality (EC) used in other studies [27] addresses this by considering not only the number of connections but the influence of connected nodes [28]. Yet, EC-based approaches often concentrate influence on a few dominant nodes, which can obscure the importance of other critical nodes [29].

Several other works have incorporated global centrality measures to overcome the limitations of local metrics. Closeness centrality (CC) [30] and betweenness centrality (BC) [31], for instance, are frequently employed to identify nodes that hold critical positions within the overall network structure. BC identifies nodes that act as bridges between different parts of the network, while CC focuses on nodes with short paths to all other nodes. Despite their effectiveness in identifying key components, using these methods as single indicators is not sufficient to analyse the multiple and complex characteristics of a node in an interaction graph. Similarly, the K-shell decomposition method [32], which identifies nodes based on their position within the network hierarchy, provides a spatially informative analysis but often lacks the granularity to distinguish between nodes within dense core regions, hindering precise identification of critical components [33].

Finally, several studies [34, 35] have also applied community detection and community-centrality measures to identify critical lines. These methods go beyond traditional centrality measures by considering the role of nodes within network substructures. However, the effectiveness of community detection algorithms can vary based on network structure; disconnected components or isolated nodes can significantly affect their performance. Additionally, these techniques often require substantial computational resources, particularly in large and complex networks, and their outcomes are sensitive to the choice of algorithm, making results difficult to generalize across different grid models.

Given the limitations of single-metric approaches and community detection algorithms in identifying critical components, this paper introduces a multi-metric methodology aimed at assessing component importance in cascading failures of power systems. Drawing on concepts from social network analysis [36], the proposed methodology combines degree centrality, ego-betweenness centrality, and eigenvector centrality to evaluate node importance and identify core nodes within interaction graphs. By calculating the normalized sum of these metrics and validating node roles through statistical correlations, this approach captures both local and global network properties, offering a more nuanced understanding of component influence.

The proposed methodology's effectiveness is demonstrated through application to data-driven interaction graphs constructed using the method from [13], with cascading failure data generated by a simulator based on [37]. Detailed descriptions of both the interaction graph construction and cascading failure simulator are provided.

The main contribution of this paper is to demonstrate that the proposed multi-metric approach provides an effective and competitive alternative within the scope of current interaction-graph-based methods, offering high computational efficiency and accuracy in identifying critical components. This is exemplified through comparative analysis with the method in [13], which is widely recognized as a representative state-of-the-art approach in interaction-graph-based analysis and has been cited extensively in review articles and recent studies. The case studies on the IEEE 39-bus and IEEE 118-bus systems further underscore the potential of the proposed methodology to improve cascading failure analysis in power systems.

The paper is organized as follows: Section 2 explains the cascading failure simulator, which generates the failure data used to build the interaction graphs. Section 3 describes the methodology for constructing the interaction graphs based on the failure data from cascading failure simulations. Section 4 introduces the proposed methodology to be applied to the interaction graphs. Section 5 provides a detailed analysis to identify critical transmission lines using the IEEE 39-bus and IEEE 118-bus systems, which serve as representative models of real power systems, and Section 6 summarizes the findings and draws conclusions.

2. Cascading failure simulator

Various methods for constructing interaction graphs are typically based on cascading failure simulations and statistical analysis of utility outage data. A variety of tools for simulating cascading failures in power systems have been developed, as reviewed in [38]. There are two principal approaches to cascading failure simulation: dynamic transient models [39–41] and quasi-steady-state (QSS) models [37, 42, 43], each with distinct advantages and limitations.

Dynamic models incorporate the detailed behaviour of system components such as rotating machines, exciters, and governors, typically described by differential equations. These models also account for the protective elements and their dynamic responses, providing a more granular representation of system behaviour. However, this level of detail significantly increases computational complexity, especially when applied to large-scale systems, limiting the feasibility of performing numerous Monte Carlo (MC) simulations needed to evaluate blackout risk across multiple planning scenarios. Additionally, dynamic simulations can suffer from numerical issues related to solving differential equations, and they often rely on various assumptions that can affect accuracy. The study by Dai *et al*. [41] presents an example of enhancing dynamic modelling in cascading failure studies using Python scripting for DIgSILENT PowerFactory, which addresses some of these computational challenges.

In contrast, QSS models, which assume a steady-state condition for the system, have been widely adopted in the literature for cascading failure analysis due to their relative simplicity and efficiency. These models calculate network re-dispatch flows using power flow analysis, offering a practical trade-off between computational burden and the ability to assess large-scale blackouts.

For the purposes of this work, a QSS model is sufficient to generate the cascading failure data needed. The study focuses on the failure of transmission lines and transformers, where a cascade refers to successive component disconnections following an initial contingency. The sequence of failures in each simulated cascade is recorded to form the dataset used for analysis.

The cascading failure model developed in this work was based on that presented in [37], which uses AC power flow to account for reactive power and voltage. The cascading failure simulation is implemented through the following steps:

1. **Select and disconnect overloaded components** after a contingency occurs.
2. **Update the system** (generators, lines, and loads).
3. If the system becomes divided into islands, **balance the generation and load** in each island by shedding the load or adjusting the generator outputs.
4. **Compute AC power flow** in each island.
5. If there are components violating their capacity limits, return to Step 1; otherwise, proceed to Step 6.
6. **Calculate load losses** or other cascade size metrics and stop the simulation.

In Step 3, if the total supply of active power within an island surpasses the total demand, the active power outputs of the generators are curtailed. Conversely, if the total supply of active power falls short of satisfying the total demand, demand reduction is employed to balance supply and demand within the island. This involves scaling down the energy supply or demand across all nodes by a common factor. In the event that an island lacks both supply and demand, all supply and demand nodes within the island are deactivated.

This cascading failure model has limitations because it does not encompass the full complexity of cascading failures and specifically overlooks transients and phase angle dynamics. Nonetheless, it enables the identification of the impact of component overloads, which are consistently present in any cascading failure and blackout. Additionally, this model can produce consistent data distributions, such as the well-observed power-law distribution of demand loss [44].

3. Estimating the interactions between component failures in a cascade process

To extract the interactions among the components of the power grid during the cascade process, a set of cascading failures are simulated using the cascade model described in the previous section. Given that the cascade model is based on transmission line overloading, the interactions among the transmission lines of the power grid are obtained. The specific technique outlined in [13] is adopted in this work for interaction extraction and subsequent interaction graph construction. Additionally, the technique presented in [13] for the identification of the critical components based on the obtained interactions is considered in this work for validating the proposed methodology. This section offers a concise overview of these techniques.

3.1. Graph of interactions

Within cascading failures, a group of failures can trigger subsequent failures in other components. These groups, defined as the generation of failures, encapsulate failures occurring in close temporal proximity. In this work, the sequence of cascade failures is partitioned into these generations, where cause-and-effect relationships between successive generations are explored. Specifically, it is assumed that outages in generation $k + 1$ are caused by outages in generation k.

Thus, M cascades can be expressed as

	Generation 0	**Generation 1**	**Generation 2**	$\cdots$
Cascade 1	$F_0^{(1)}$	$F_1^{(1)}$	$F_2^{(1)}$	$\cdots$
Cascade 2	$F_0^{(2)}$	$F_1^{(2)}$	$F_2^{(2)}$	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Cascade M	$F_0^{(M)}$	$F_1^{(M)}$	$F_2^{(M)}$	$\cdots$

where $F_k^{(m)}$ is the set of failed components produced in generation k of cascade m.

For a system with n components, a matrix $\boldsymbol{A} \in Z^{n\times n}$ can be constructed using the M simulated cascades. The entry a_{ij} of matrix $\boldsymbol{A}$ is the number of times that component i fails in one generation before the failure of component j among all cascades. The assumption based on which $\boldsymbol{A}$ is obtained consists of considering that all components in generation k have interactions with all components in generation $k+1$. However, all line outages in one generation may not be the cause of a line outage in the next generation. Therefore, in the work presented in [13] it is considered that the cause of failure of a line j in generation $k+1$ is due to the failure of a line in generation k with the maximum influence value on line j. The maximum influence value for component i in generation k is defined as the number of times that component i has failed in generation k before the failure of line j in the successive generation $k+1$ in the simulated cascades. This can be expressed mathematically as

$$\left\{ i_c | i_c \in F_k^{(m)} \quad \text{and} \quad a_{i_c j} = a_{ij} \right\}. \tag{1}$$

Once matrix $\boldsymbol{A}$ is obtained, a corrected $\boldsymbol{A}' \in Z^{n\times n}$ is determined, where entry a'_{ij} is the number of times that the failure of component i causes the failure of component j.

To determine how components interact with each other in a cascade process, the interaction matrix $\boldsymbol{B} \in R^{n\times n}$ is calculated from A'. Its entry b_{ij} is given by

$$b_{ij} = \frac{a'_{ij}}{N_i}, \tag{2}$$

where b_{ij} is the empirical probability that the failure of component i causes the failure of component j and N_i is the number of failures of component i.

Finally, an interaction graph $\mathrm{G(C, L)}$ can be obtained from matrix $\boldsymbol{B}$, where vertices C are components and nonzero element b_{ij} corresponds to a link $l \in \mathrm{L} : i \longrightarrow j$ that starts from component i and ends with component j; this also indicates that a failure of the source vertex component causes the failure of the destination vertex component with a probability greater than 0.

3.2. Identifying critical components

To identify critical components, an index I_l is defined for each link $l : i \longrightarrow j$ to be the expected value of the number of failures that are propagated through link l. The failures propagated through link l can be directly triggered by the failure of component i or can be triggered by the failures of other components that eventually cause component i to fail.

Therefore, to calculate I_i, a directed acyclic subgraph $\mathrm{G}_j\left(\mathrm{C}_j, \mathrm{L}_j\right)$ starting with component j from interaction network G is obtained. For each link, there is a unique directed acyclic subgraph that can be extracted from the whole interaction graph and is composed of all the components influenced by this link. Then, from the subgraph G_j, the index I_l is obtained, which indicates the contribution of a link to the propagation of cascading failures. The greater the index is, the more important the link is for cascading failure propagation. Thus, the links with large I_i can be defined as key links.

By taking I_i as the weights of the links, a directed-weighted interaction network G^W is obtained. Then, the vertex out-degree and in-degree of the interaction network can be calculated. Components with large out-degree can cause great consequences and thus are critical for the propagation of cascading failures.

4. The proposed methodology

The proposed methodology for identifying critical components within an interaction graph of cascading failures in power systems builds upon the work presented in [36]. The methodology focuses on finding core nodes within interaction graphs. A core node is defined as one that binds a group of nodes together and significantly influences other nodes, typically occupying a central position within the group. To identify these core nodes, multiple network metrics are utilized. Based on the experiments and findings detailed in [36], the methodology employs three specific metrics: degree, ego betweenness centrality, and eigenvector centrality.

Degree is the most used metric and is crucial for identifying important nodes, as nodes with a high degree are more likely to serve as core nodes. Ego betweenness centrality is chosen to assess a node's role in facilitating information exchange. It has been demonstrated across various applications that nodes with high ego betweenness are critical for mediating communication and collaboration between non-adjacent nodes. Eigenvector centrality measures a node's connectivity to other important nodes, reinforcing a node's status as a core if it has a high eigenvector centrality.

These metrics exhibit strong positive correlations, which help highlight the topological features of the nodes, particularly when the correlation of their metrics conflicts with statistical relationships. In this methodology, the importance of nodes in influencing others is evaluated using the normalized sum of these three indicators. The following subsections describe the methodology and its components in detail.

4.1. Evaluation of node importance

In the proposed methodology, the importance of a node is equal to the normalized sum of its three metrics. For a given interaction graph G(C, L), where C is the set of vertices and L is the set of links, if C_{RDi}, C_{EBi}, and C_{Ei} denote the normalized value of the degree, the ego-betweenness centrality and the eigenvector centrality of node $i \in \mathrm{C}$, respectively, then the importance IM_i of node i is thus:

$$IM_i = C_{RDi} + C_{EBi} + C_{Ei}, \tag{3}$$

where the three metrics are defined as follows:

Degree centrality

The degree of a node is the number of edges incident to node i, in other words, it is the number of nearest neighbours that i has. The most characteristic feature of degree centrality is that it accounts only for the influence of nearest neighbours to a given node [45]. The relative degree of node i is $RD_i = D_i/(\mathrm{C} - 1)$, where D_i is the number of edges incident to node i.

Ego betweenness centrality

An ego network consists of a focal node ("ego") and the nodes to which the ego is directly connected (these are called "alters") plus the ties, if any, among the alters. The betweenness of node i is:

$$B_i = \sum_{s,t \neq i \in \mathrm{C}} \frac{g_{st}(i)}{g_{st}}, \tag{4}$$

where $g_{st}(i)$ is the number of shortest paths between s and t passing through i and g_{st} is the total number of shortest paths between s and t. Betweenness centrality measures the usefulness of the node in the transmission of information within the network. The node plays a central role if many of the shortest paths between two nodes have to go through this node. With these definitions of ego network and betweenness, the ego betweenness EB_i of node i is the betweenness of i between its immediate neighbours.

Eigenvector centrality

The eigenvector centrality E_i of node i is defined to be proportional to the sum of the eigenvector centralities of i's neighbours, so that

$$E_i = q^{-1} \sum\nolimits_{\substack{\text{nodes } j \text{ that are} \\ \text{neighbors of } i}} E_j, \tag{5}$$

where q must be equal to the largest eigenvalue of the adjacency matrix of the interaction graph. With this definition, a node can achieve high centrality either by having many neighbours with modest centrality or by having a few neighbours with high centrality (or both) [46].

4.2. Identification of critical components by identifying the core nodes in the interaction graph

In the proposed methodology, the identification of critical components in cascading failures for power systems is achieved by identifying the core nodes in the corresponding interaction graph. The procedure for identifying core nodes is described in this section.

First, all the nodes of the interaction graph are sorted in descending order by degree, ego-betweenness centrality, and eigenvector centrality. The rank of node i is denoted as R_{Di}, R_{EBi}, and R_{Ei} respectively. Second, the following rank differences are obtained: $Diff_{EBi} = R_{Di} - R_{EBi}$ and $Diff_{Ei} = R_{Di} - R_{Ei}$ for node i. Third, the general correlation of the overall interaction graph is computed as follows:

$$\overline{Diff_{EB}} = \frac{1}{C} \sum_i Diff_{EBi}, \tag{6}$$

$$\overline{Diff_E} = \frac{1}{C} \sum_i Diff_{Ei}. \tag{7}$$

Finally, the core nodes are identified according to the following rule: if $1 \le R_{Di} \le K$, $Diff_{EBi} \ge \overline{Diff_{EB}}$, and $Diff_{Ei} \ge \overline{Diff_{E}}$ node i is a core node, where K is the greatest integer less than or equal to the average degree of the interaction graph.

If $Diff_{EBi} \ge \overline{Diff_{EB}}$, the rank of the ego-betweenness centrality of node i is greater than the rank of the degree; thus, the node not only connects with many other nodes but also shows a more significant feature of information exchange. If $Diff_{Ei} \ge \overline{Diff_{E}}$, the rank of the eigenvector centrality of node i is higher than the rank of the degree; thus, the node shows a more significant feature of connecting with other important nodes.

Overall, the methodology for identifying critical components in cascading failures in power systems can be presented with the pseudocode shown in Table 1.

Table 1. The pseudocode of the proposed methodology

Pseudocode	Description
1: Input: power system data;	
2: Output: arrays critical components[];	
Begin	
3: Simulate cascading failures;	
4: Build the matrix $\boldsymbol{A}$;	
5: Obtain the corrected matrix $\boldsymbol{A'}$;	
6: Calculate the interaction matrix $\boldsymbol{B}$;	
7: Build an interaction graph G(C, L);	It is obtained from matrix $\boldsymbol{B}$ (See Section 3.1)
8: Set *integer* K = the greatest integer less than or equal to 2L/C;	
9: Set *double* av_dif_eb = 0 and av_dif_e = 0;	$\overline{Diff_{EB}}$ and $\overline{Diff_{E}}$
10: Set *integer* #core = 0;	Number of detected cores
11: Set *array* metric[*i*][3], rank[*i*][3] and IM[*i*];	The value and the rank of the three metrics, and the importance of all nodes of the interaction graph
12: Set *array* temporary[*C*];	
13: metric = *calculate_metric* (G(C,L));	Computing the metrics' value of all the nodes
14: rank = *getRank* (metric);	Ranking all the nodes based on the metrics
for each node *i* *do*	
15: Calculate IM[*i*];	Computing the importance of node *i*
16: temporary[*i*] = rank[*i*][2];	Recording the rank of ego-betweenness of node *i*
17: rank[*i*][2] = rank[*i*][1] – rank[*i*][2];	Computing $Diff_{EBi}$
18: rank[*i*][3] = rank[*i*][1] – rank[*i*][3];	Computing $Diff_{Ei}$
19: av_dif_eb = av_dif_eb + rank[*i*][2];	Summing up the rank difference $Diff_{EB}$
20: av_dif_e = av_dif_e + rank[*i*][3];	Summing up the rank difference $Diff_{E}$
end	

Continued on next page

Table 1 – *Continued from previous page*

Pseudocode	**Description**
21: av_dif_eb = av_dif_eb/C;	Computing $\overline{Diff_{EB}}$
22: av_dif_e = av_dif_e/C;	Computing $\overline{Diff_{E}}$
for each node i *do*	
if rank[i][1] ≥ 1 *and* rank[i][1] ≤ K	Selecting nodes with great degree as core candidates
and rank[i][2] ≥ av_dif_eb	Detecting cores from the candidates
and rank[i][3] ≥ av_dif_e;	
23: core_nodes[#core] = i;	Recording the detected core nodes
24: #core = #core + ;	
end	
end	
25: Rank = *getRank*(core_nodes[], IM[]);	Ranking the core nodes based on their importances
26: critical_components[] = core_nodes[]	The critical components are the core nodes ranked from highest to lowest importance IM
end	

The process of the proposed methodology is as follows: first, outage datasets are obtained by simulating cascading failures using the model presented in Section 2. The sequence of cascade failures of each simulation is partitioned into generations (see Section 3.1). Second, with the outage datasets, the matrices $\boldsymbol{A}, \boldsymbol{A}'$ and $\boldsymbol{B}$ are calculated according to the procedure explained in Section 3. The entry b_{ij} of matrix $\boldsymbol{B}$ is the empirical probability that the failure of component i causes the failure of component j. Third, the interaction graph is obtained from matrix $\boldsymbol{B}$. Fourth, the values of multiple metrics of each node are computed; see step 13 in Table 1. Fifth, the importance and rank difference of each node and the average difference of the interaction graph are calculated in steps 15 to 22. Sixth, the core nodes are identified according to the identification rule in step 23. Seventh, in steps 25 and 26, the core nodes are ranked based on their importance and are the critical components in cascading failures of the power system under analysis.

5. Experiments and analysis

5.1. The case study with the IEEE 39-bus system

The first test system used in this work is the IEEE 39-bus system, which includes 10 generators and 46 lines. This system is a representative model of the 345 KV New England power system (see Fig. 1). The basic data for this system can be found in [47] and are not duplicated here. In this study, 10 000 cascading failures were simulated using the cascading model described in Section 2. To introduce diversity in the cascading outages, the system loads at the start of each run were varied randomly around their mean values by multiplying by a factor uniformly distributed in $[2 - \gamma, \gamma]$, where $\gamma = 1.15$. The initial failures comprised two lines selected randomly, with each line assumed to fail independently with a probability $p = 0.01$. These values of γ and p are typical in the literature.

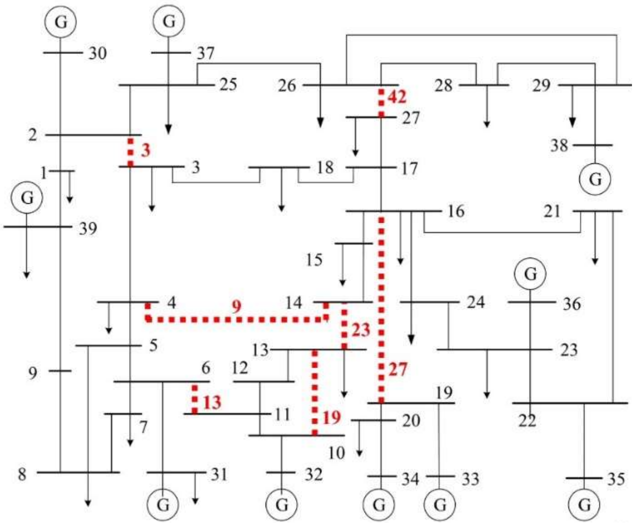

Fig. 1. One-line diagram of the IEEE 39-bus system. The critical lines found by the proposed methodology are highlighted in dashed red

By applying the method from [13], matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ were obtained. Due to space limitations, these matrices are not displayed here. Figure 2 shows the interaction graph derived from matrix $\boldsymbol{B}$. The nodes represent lines in the IEEE 39-bus system, and the links denote the interactions between lines during the cascading process. The weights of the links are omitted for clarity. This graph is distinct from the one-line diagram in Fig. 1, where the vertices are buses and the undirected links between vertices are transmission lines. Table 2 lists the seven most critical components identified using the method from [13].

Table 2. Critical components (transmission lines) in cascading failures of the IEEE 39-bus system

Rank	Method of [13]	Proposed methodology
1	3	19
2	13	23
3	19	27
4	42	9
5	23	3
6	9	13
7	38	42

To identify critical components using the proposed methodology, we calculated the metric degree, ego-betweenness, and eigenvector for each node in the interaction graph of Fig. 2. Using these metrics, the importance of each node was determined with (3). The rank difference of

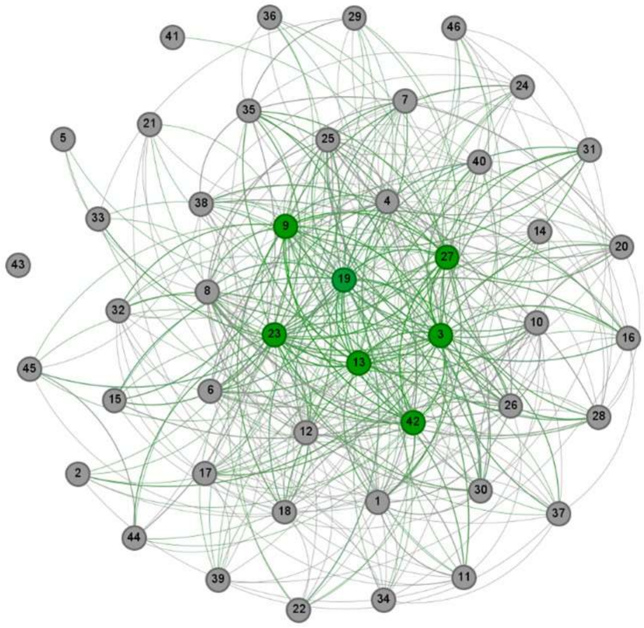

Fig. 2. Interaction graph of the IEEE 39-bus system. The critical lines found by the proposed methodology are highlighted in green

each node and the average difference of the interaction graph were obtained using (6) and (7): $\overline{Diff_{EB}} = -6.3696$ and $\overline{Diff_E} = -6.6957$. To find the core nodes in the interaction graph, $K = 17$ is set to apply the identifier rule. The core nodes identified were ranked from highest to lowest in importance, indicating the most critical components of the power system in descending order of importance.

Table 2 also shows the seven most critical components identified by the proposed methodology. Comparing the critical components identified by both methods, it can be observed that only one component differs between the two methods, indicating that the proposed methodology is competitive with the method from [13]. In the one-line diagram of Fig. 1, the critical lines identified by the proposed methodology are highlighted in red.

To validate the critical lines, we compared the mitigation effects of upgrading the critical lines versus upgrading seven random lines. Figures 3 and 4 display the complementary cumulative distributions of the total number of line outages and the total load shed, respectively. As shown in Figs. 3 and 4, upgrading the critical lines identified by the proposed methodology more effectively mitigates the cascading process than does upgrading random lines.

Computation time analysis

To provide a comprehensive evaluation, we also compared the computation times required by both methods to identify the critical components. The experiments were conducted on a laptop with an Intel(R) Core(TM) i5-11400H 11th Gen processor operating at a speed of 2.70 GHz and equipped with 24 GB of RAM. MATLAB R2022b was used for the computations. The m-files used for this research included custom scripts developed specifically for calculating the

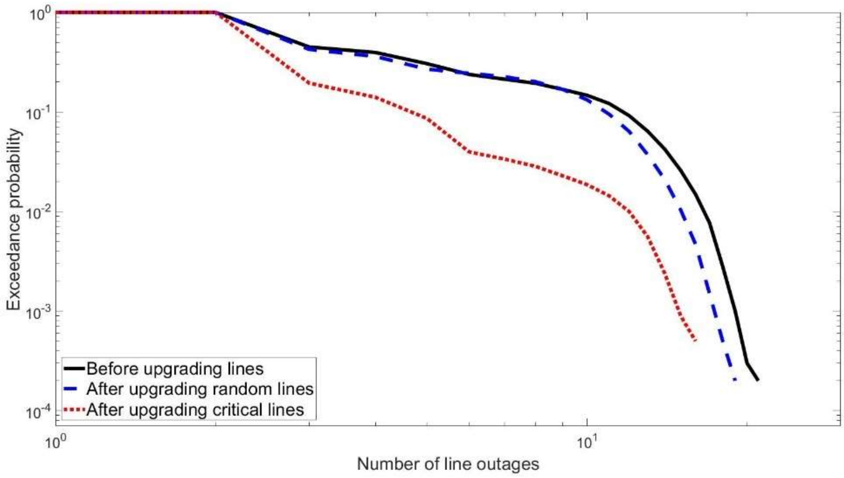

Fig. 3. The complementary cumulative distribution functions of line outages from 10 000 cascading failure simulations for the IEEE 39-bus system in three scenarios: before upgrading lines, after upgrading random lines, and after upgrading the critical lines identified by the proposed methodology

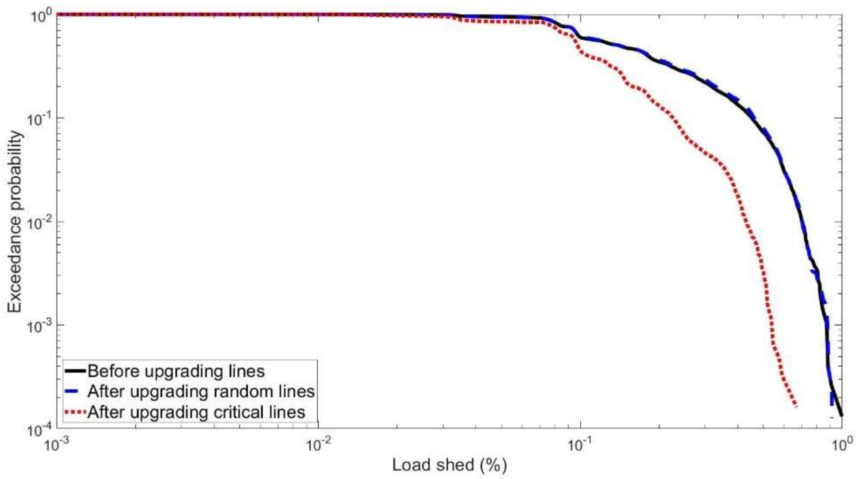

Fig. 4. The complementary cumulative distribution functions of load shed from 10 000 cascading failure simulations for the IEEE 39-bus system in three scenarios: before upgrading lines, after upgrading random lines, and after upgrading the critical lines identified by the proposed methodology

centrality metrics – degree centrality, ego-betweenness centrality, and eigenvector centrality – and for building and analysing the interaction graphs.

For the IEEE 39-bus system, the method from [13] took 6.878 seconds, while the proposed methodology took only 0.954 seconds. This significant reduction in computation time highlights the efficiency of the proposed methodology.

5.2. The case study with the IEEE 118-bus system

The second test system used in this work is the IEEE 118-bus system (see Fig. 5). This system approximates the American Electric Power System in the Midwestern US as it was in 1962. It contains 19 generators, 35 synchronous condensers, 177 lines, 9 transformers, and 91 loads. The data for this system are standard, except that the line limits are $c_l = (1+\alpha)\max\left\{|f_l|, \overline{f}\right\}$, where α is the line tolerance and $\overline{f}$ is the mean initial line flow. Basic data for this system are provided in [48].

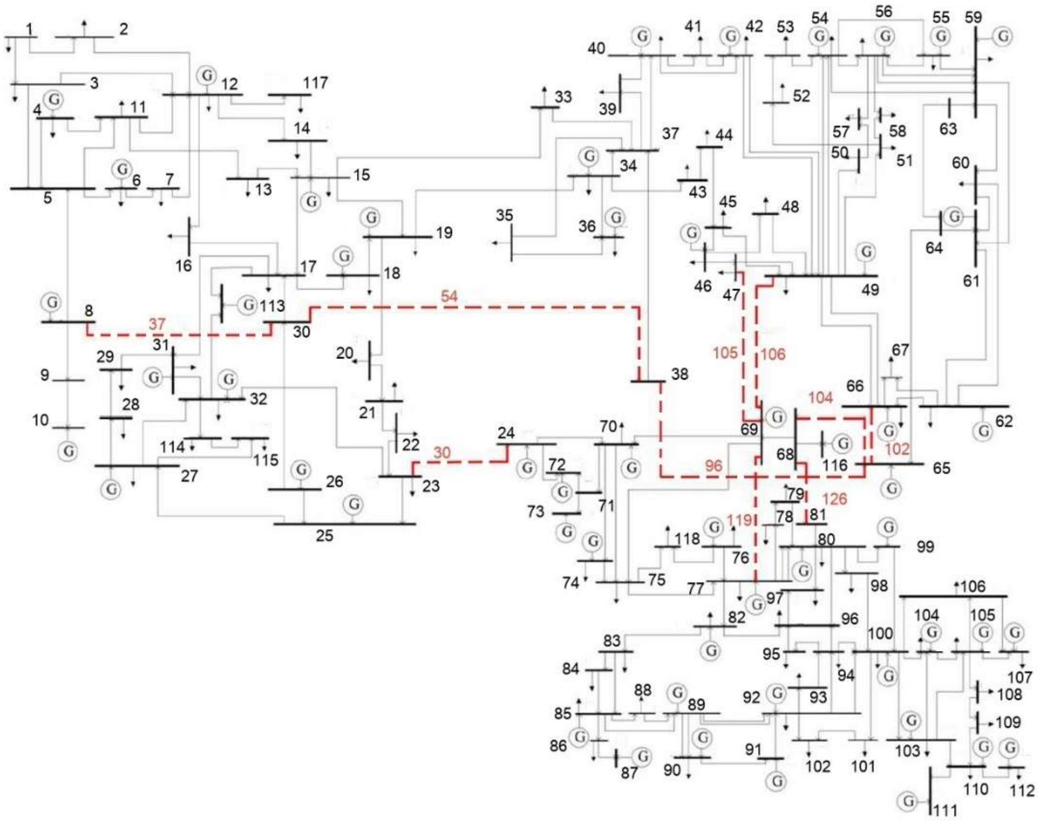

Fig. 5. One-line diagram of the IEEE 118-bus system. The critical lines found by the proposed methodology are highlighted in dashed red

In this case, 30 000 cascading failures were also simulated, using $\gamma = 1.15$ and $p = 0.01$ to randomly vary system loads and select initial failures. For line capacity limits, $\alpha = 1.0$ was used.

By applying the method from [13], the resulting interaction graph is shown in Fig. 6, and the ten most critical components identified by this method are listed in Table 3. Using the proposed methodology, we also identified the ten most critical components, as shown in Table 3. As in the previous case, the two methods differ by only one component, demonstrating that the proposed method is competitive with the method from [13].

Table 3. Critical components (transmission lines) in cascading failures of the IEEE 118-bus system

Rank	**Method of [13]**	**Proposed methodology**
1	104	119
2	106	104
3	30	126
4	126	106
5	102	105
6	119	54
7	37	30
8	96	96
9	105	102
10	32	37

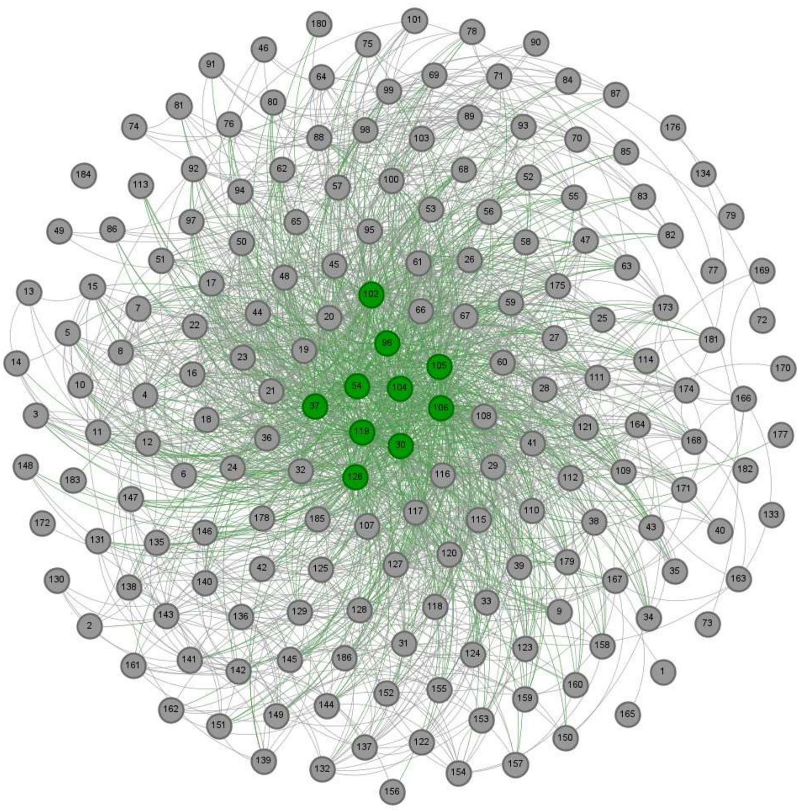

Fig. 6. Interaction graph of the IEEE 118-bus system. The critical lines found by the proposed methodology are highlighted in green

As in the previous case, we compared the mitigation effects of upgrading the critical lines versus upgrading ten random lines. Figures 7 and 8 display the complementary cumulative distributions of the total number of line outages and the total load shed, respectively. Once again, upgrading the ten critical lines identified by the proposed methodology more effectively mitigates the cascading process than does upgrading random lines.

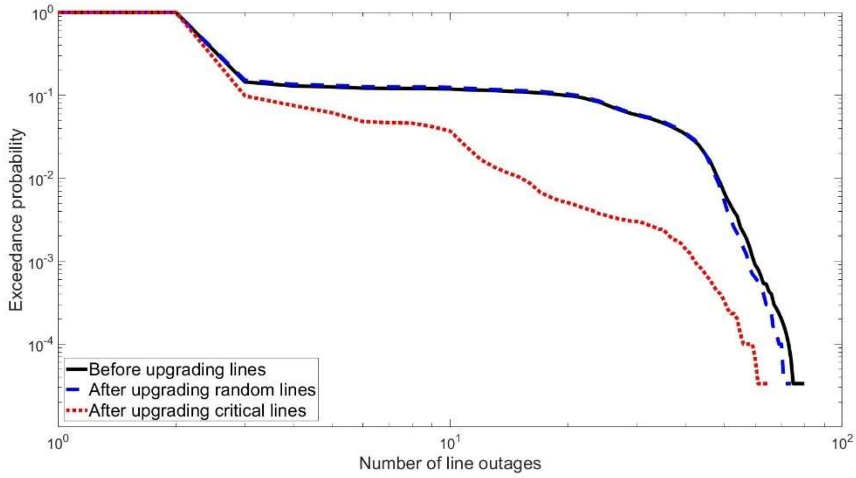

Fig. 7. The complementary cumulative distribution functions of line outages from 30 000 cascading failure simulations for the IEEE 118-bus system in three scenarios: before upgrading lines, after upgrading random lines, and after upgrading the critical lines identified by the proposed methodology

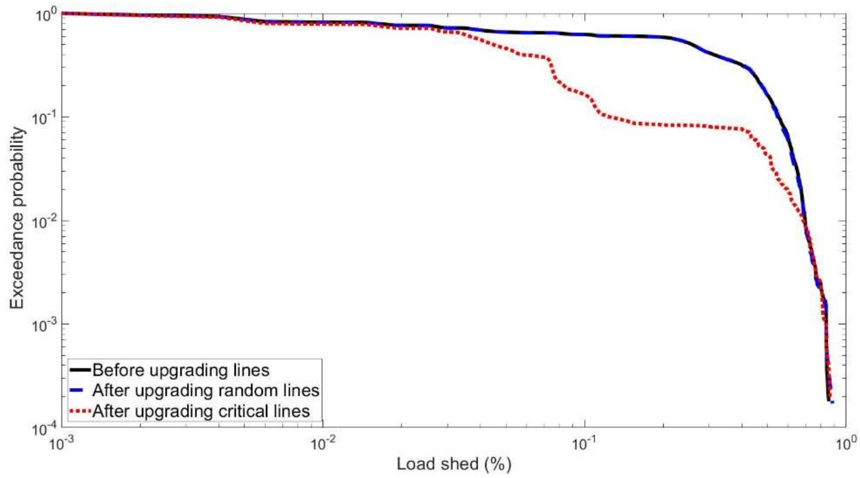

Fig. 8. The complementary cumulative distribution functions of load shed from 30 000 cascading failure simulations for the IEEE 118-bus system in three scenarios: before upgrading lines, after upgrading random lines, and after upgrading the critical lines identified by the proposed methodology

Computation time analysis

We also measured the computation times for the IEEE 118-bus system using the same hardware and software setup. The method from [13] took 1730 seconds, while the proposed methodology took only 8.63 seconds. This drastic reduction in computation time further demonstrates the efficiency and practical applicability of the proposed methodology.

The significant reduction in computation time, as demonstrated in the case studies, not only demonstrates the efficiency of the method but also indicates its potential for use in real-time monitoring and control systems. By quickly identifying critical components in the interaction graph, the methodology could be integrated into supervisory control and data acquisition (SCADA) systems or energy management systems (EMS) to prevent cascading failures before they escalate.

6. Conclusions

This research introduces a multi-metric methodology for identifying critical transmission lines in power systems during cascading failures, providing novel contributions that address several limitations in current methods. Traditional approaches often rely on single-metric measures, such as degree centrality, which lack the global context necessary for accurately pinpointing critical components. Other methods, such as community detection algorithms, are sensitive to network structure and can be affected by disconnected components or isolated nodes. The proposed approach addresses these challenges by adapting principles from social network analysis, creating a more comprehensive framework that simultaneously evaluates multiple centrality metrics – degree, ego-betweenness, and eigenvector centrality – to capture both local and global network properties.

Novel contributions to the methodology

The proposed methodology uniquely integrates multiple centrality metrics into a composite score, allowing for a balanced assessment of node importance that surpasses the limitations of single-metric and even dual-metric methods. This approach introduces a new level of accuracy to power system interaction graphs by combining the strengths of local (degree) and influence-based (eigenvector and ego-betweenness) metrics. Furthermore, the inclusion of rank differences to validate core nodes introduces an innovative validation layer that distinguishes truly influential nodes by analysing inconsistencies across metrics, an aspect not addressed in previous methods.

Impact of the novel methodology on results

Theoretical benefits: By leveraging multiple metrics, the proposed methodology achieves a nuanced and precise identification of critical components. The multi-metric approach ensures that nodes with high composite scores are genuinely influential, as demonstrated by the method's effective performance in reducing cascading failures.

Quantitative comparison: The IEEE 39-bus and IEEE 118-bus case studies illustrate the method's competitive performance. For the IEEE 39-bus system, the proposed methodology identified six out of seven critical components that matched the results from the method in [13], with only one differing component. Notably, the proposed methodology achieved these results with significantly lower computation times – 0.954 seconds compared to 6.878 seconds for the method in [13]. This represents an improvement of approximately 86.1% in computation time.

Similarly, for the IEEE 118-bus system the proposed methodology identified nine out of ten critical components matching the method in [13], with only one differing component. This result further confirms the reliability of the approach across larger power systems. Also, for this larger system, the proposed methodology completed the analysis in 8.63 seconds, whereas the method in [13] required 1730 seconds. This corresponds to a 99.5% reduction in computation time, demonstrating the scalability and computational efficiency of the proposed approach.

Empirical validation

The critical lines identified by the proposed methodology were further validated through mitigation testing. As demonstrated in Figs. 3, 4, 7, and 8, upgrading the critical lines identified by the proposed methodology resulted in a more effective reduction of cascading failures than upgrading random lines, underscoring the accuracy and reliability of the proposed approach in identifying genuinely critical components. Additionally, the significant reduction in computation time across both case studies illustrates the scalability of the methodology, supporting its potential for real-time applications in large-scale power systems. The combination of high accuracy and reduced computational demands positions the methodology as an effective and competitive alternative for rapid decision-making in real-time grid operations.

It is important to note that these results were obtained through comparative analysis with the method in [13], a well-recognized representative of interaction-graph-based methods, and validated on the IEEE 39-bus and IEEE 118-bus systems. While the conclusions drawn in this paper are specific to these test systems and the comparative context provided, the demonstrated advantages in accuracy and computational efficiency suggest that the proposed methodology holds potential for broader applicability in cascading failure analysis across diverse power systems. Further studies are encouraged to explore its performance under varying conditions and in comparison with other state-of-the-art methods to assess its generalizability.

In summary, this research presents a highly accurate and efficient tool for identifying critical components in power systems, with results validated through both quantitative comparison and mitigation testing. By addressing the limitations of previous single-metric and interaction-graph methods, the proposed multi-metric methodology offers substantial improvements in accuracy, computational efficiency, and scalability. These advancements represent a valuable contribution to power systems research, with promising implications for enhancing the stability and resilience of modern power grids. Future research could explore the integration of machine learning techniques to further adapt and refine the methodology in real-time grid environments, enabling more robust solutions for cascading failure prevention and response.

Nomenclature

G(C, G)	Interaction graph
C	Set of vertices of an interaction graph
L	Set of edges of an interaction graph
$F_k^{(m)}$	Set of failed components produced in generation k of cascade m
M	Set of simulated cascades
n	Number of components of a power system

a_{ij}	Entry of matrix $\boldsymbol{A}$, indicating the number of times component i fails in one generation before the failure of component j among all cascades
a'_{ij}	Entry of matrix $\mathbf{A}''$, representing the number of times the failure of component i causes the failure of component j
b_{ij}	Entry of matrix $\boldsymbol{B}$, the empirical probability that the failure of component i causes the failure of component j
N_i	Number of failures of component i
l	A link corresponding to a nonzero element b_{ij}
I_l	Expected value of the number of failures propagated through link l
$\mathrm{G}_j\ (\mathrm{C}_j, \mathrm{L}_j)$	Directed acyclic subgraph starting with component j from interaction network G
G^W	Directed-weighted interaction network
C_{RDi}, C_{EBi}, C_{Ei}	Normalized values of degree, the ego-betweenness centrality and eigenvector centrality of node $i \in \mathrm{C}$, respectively
IM_i	Importance index of node i
RD_i	Relative degree of node i
B_i	Betweenness of node i
$g_{st}(i)$	Number of shortest paths between s and t passing through i
g_{st}	Total number of shortest paths between s and t
EB_i	Ego betweenness of node i
E_i	Eigenvector centrality of node i
q	Largest eigenvalue of the adjacency matrix of the interaction graph
R_{Di}, R_{Ebi}, R_{Ei}	Rank of node i sorted in descending order by degree, ego-betweenness centrality, and eigenvector centrality, respectively
$Diff_{EBi}, \quad iff_{Ei}$	Rank differences
$\overline{Diff_{EB}}, \overline{Diff_E}$	General correlation of the overall interaction graph
K	Greatest integer less than or equal to the average degree of the interaction graph

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