Nilpotency indices of state matrices and linear dependence on time of state variable of electrical circuits with state feedbacks

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Abstract: Electrical circuits with state-feedbacks are addressed. It is shown that by suitable choice of the gain matrices of state-feedbacks it is possible to obtain the closed-loop system matrices with nilpotency indices equal to two and their state variables are linear functions of time. The considerations are illustrated by linear electrical circuits.

Key words: electrical circuits, nilpotency indices, state-feedback

1. Introduction

It is well-known [1-4, 7-10] that linear systems and in particular case linear electrical circuits can be described by state equations. As state variables in linear electrical circuits the current in coils and voltages across the capacitors are chosen. This way of choice of the state variables has been proposed in [4, 5]. From the solution to the state equations it follows that in linear electrical circuits the state variables are exponential functions of time.

In this paper it will be shown that by suitable choice of the state-feedbacks the closed-loop system matrices may be nilpotent matrices and the state variables of the closed systems may be linear functions of time.

The structure of the paper is the following. Basic definitions of the nilpotent matrices are recalled and some new lemmas on matrices with the nilpotency index $v = 2$ are presented in Section 2. The main result of the paper is given in Section 3 where linear electrical circuits with state-feedbacks are investigated. In Section 4 an extension of the considerations for non-square B matrices is given. Concluding remarks are given in Section 5.

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2. Nilpotent state matrices and their nilpotency indices

Let $\mathfrak{R}^{n\times m}$ be the set of $n\times m$ real matrices and I_n be the identity matrix $n\times n$.

Definition 1. A real matrix $A \in \mathfrak{R}^{n\times n}$ is called nilpotent if there exists a natural number $v \leq n$ such that $A^{v-1} \neq 0$ and $A^v = 0$. The natural number v is called the nilpotency index of the matrix A.

It is well-known [6] that the strictly upper triangular matrices and the strictly lower triangular matrices $A \in \mathfrak{R}^{n\times n}$ are nilpotent matrices with nilpotency indices $v \leq n$. These matrices have the characteristic polynomials of the form

$$\det[I_n\lambda - A] = \lambda^n \tag{1}$$

and all their eigenvalues are equal to zero.

Lemma 1. Matrices of the form

$$A = \begin{bmatrix} 0 & a_{12} & \cdots & a_{1,n-1} & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & a_{n,2} & \cdots & a_{n,n-1} & 0 \end{bmatrix} \in \mathfrak{R}^{n\times n} \tag{2}$$

have the nilpotency index $v = 2$ for any values of the entries $a_{12},...,a_{1,n-1},a_{n,2},...,a_{n,n-1}$ and the characteristic polynomials of the form

$$\det[I_n\lambda - A] = \lambda^n . \tag{3}$$

Proof. Using (2) it is easy to check that $A^2 = 0$ and

$$\det[I_n\lambda - A] = \det \begin{bmatrix} \lambda & -a_{12} & \cdots & -a_{1,n-1} & 0 \\ 0 & \lambda & \cdots & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & \lambda & 0 \\ 0 & -a_{n,2} & \cdots & -a_{n,n-1} & \lambda \end{bmatrix} \in \lambda^n .$$

Lemma 2. Matrices of the form

$$A = \begin{bmatrix} 0 & A_{12} \\ 0 & 0 \end{bmatrix} \in \mathfrak{R}^{2n\times 2n} \tag{4}$$

have the nilpotency index $v = 2$ and the characteristic polynomials of the form

$$\det[I_{2n}\lambda - A] = \lambda^{2n} \tag{5}$$

for any submatrices $A_{12} \in \mathfrak{R}^{n\times n}$.

Proof. Using (4) it is easy to verify that $A^2 = 0$ and

$$\det[I_{2n}\lambda - A] = \det\begin{bmatrix} I_n\lambda & -A_{12} \\ 0 & I_n\lambda \end{bmatrix} = \lambda^{2n}.$$

From the well-known property of the transposition (denoted by upper index T) of the matrix A, $(A^k)^T = (A^T)^k$ for $k = 1,2,\dots$ we have the following remark.

Remark 1. The transpose matrix A^T has the nilpotency index ν if and only if the matrix A has the same nilpotency index ν.

Lemma 3. A diagonal matrix A with at least one nonzero entry is not the nilpotent matrix.

Proof. This follows immediately from the relation

$$A^k = \left(\mathrm{diag}[a_1,\dots,a_n]\right)^k = \mathrm{diag}[a_1{}^k,\dots,a_n{}^k] \neq 0$$

for $k = 1,2,\dots$ if at least one from the entries $a_1,\dots,a_n$ is nonzero.

Lemma 4. Nonnegative matrix $A \in \Re_+^{n\times n}$ with at least one nonzero diagonal entry is not nilpotent.

Proof. Let decompose the matrix A as the sum of the diagonal matrix D and the nonnegative matrix B witch zero diagonal matrices. Then

$$A^k = (D + B)^k = D^k + BD^{k-1} + \dots + B^k \quad \text{for } k = 1,2,\dots. \tag{6}$$

If the matrix A has at least one nonzero diagonal entry then $D \neq 0$ and by Lemma 3 $D^k \neq 0$ for $k = 1,2,\dots$. From (6) we have $A^k \neq 0$ for $k = 1,2,\dots$ since $D^k \neq 0$ and the remaining matrices are nonnegative.

3. Linear electrical circuit with state-feedbacks

It is well-known [2, 4] that any linear electrical circuit consisting of resistors, capacitors, coils and voltage (current) sources can be described by the state equation

$$\dot{x} = Ax + Be, \tag{7}$$

where $x \in \Re^n$ is the state vector, $e \in \Re^m$ is the input vector, $A \in \Re^{n\times n}$ and $B \in \Re^{n\times m}$.

As state variables $x_1, x_2,\dots,x_n$ (the components of x) usually the currents in the coils and the voltages across the capacities are chosen. The components of the input vector e are the source voltages or source currents.

First let the following assumption be satisfied.

Assumption. The number of state variables n is equal to the number of inputs m i.e. $n = m$ and $\det B \neq 0$.

Consider the electrical circuit (7) with the state-feedback

$$e = Kx, \tag{8}$$

where $K \in \Re^{n\times n}$ is a gain matrix.

Remark 2. Note that electrical circuit with state-feedback (8) is equivalent to a linear circuit with controlled voltage or current sources.

Substitution of (8) into (7) yields

$$\dot{x} = A_c x \,, \tag{9}$$

where

$$A_c = A + BK. \tag{10}$$

We are looking for a gain matrix K such that the closed-loop system matrix (10) has nilpotency index $v = 2$.

We shall show the following theorem which is one of the main results of the paper.

Theorem 1. Let Assumption be satisfied. Then the gain matrix K can be chosen so that the variables of the circuit are linear functions of time for any given initial conditions $x(0) = x_0$.

Proof. Using Lemmas 1 and 2 we choose as the matrix A_c a matrix with nilpotency index $v = 2$. In this case

$$A_c^{\ k} = 0 \ \text{ for } k = 2,3,\dots . \tag{11}$$

If (11) holds then the solution of the equation (9) has the form

$$x = e^{A_c t} x_0 = \sum_{k=0}^{\infty} \frac{(A_c t)^k}{k!} x_0 = (I_n + A_c t) x_0 \tag{12}$$

and the state variables of the close-loop system are linear functions of the time for any initial conditions x_0.

Knowing the matrices A_c, A and B we can find the gain matrix from the equation (10) since be Assumption $\det B \neq 0$.

From (10) we have

$$K = B^{-1}(A_c - A) \tag{13}$$

this complete the proof.

Example 1. Consider the electrical circuit shown on Fig. 1 with given resistances R_1, R_2, R_3 inductances L_1, L_2 and voltage sources e_1, e_2.
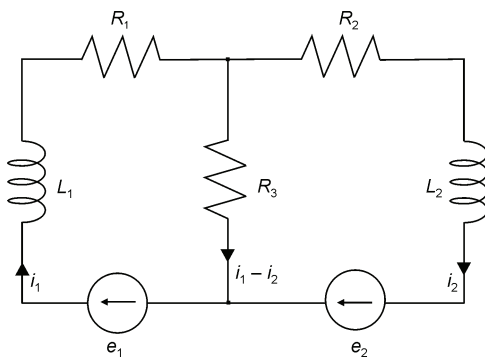

Fig. 1. Two loop circuit with RL elements

Using the Kirchhoff laws we may write the equations

$$e_1 = (R_1 + R_3)i_1 - R_3 i_2 + L_1 \frac{di_1}{dt}, \qquad (14a)$$

$$e_2 = (R_2 + R_3)i_2 - R_3 i_1 + L_2 \frac{di_2}{dt}. \qquad (14b)$$

The equations (14) can be written in the form

$$\frac{d}{dt}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = A \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + B \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}, \qquad (15a)$$

where

$$A = \begin{bmatrix} -\dfrac{R_1 + R_3}{L_1} & \dfrac{R_3}{L_1} \\[2mm] \dfrac{R_3}{L_2} & -\dfrac{R_2 + R_3}{L_2} \end{bmatrix}, \qquad B = \begin{bmatrix} \dfrac{1}{L_1} & 0 \\[2mm] 0 & \dfrac{1}{L_2} \end{bmatrix}. \qquad (15b)$$

Let

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = K \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}. \qquad (16)$$

Substitution of (16) into (15a) yields

$$\frac{d}{dt}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = A_c \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}, \qquad (17)$$

where

$$A_c = A + BK. \qquad (18)$$

We choose the matrix A_c with the nilpotency index $\nu = 2$ of the form

$$A_c = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix} \qquad (19a)$$

or

$$A_c = \begin{bmatrix} 0 & 0 \\ a & 0 \end{bmatrix}, \qquad (19b)$$

where *a* is any given parameter.

Using (13), (15b) and (19) we obtain for (19a)

$$K = \begin{bmatrix} R_1 + R_3 & aL_1 - R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \qquad (20b)$$

and for (19b)

$$K = \begin{bmatrix} R_1 + R_3 & -R_3 \\ aL_2 - R_3 & R_2 + R_3 \end{bmatrix}.$$ (20b)

From (12) we have for (19a)

$$\begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix} = \begin{bmatrix} i_{10} + ai_{20}t \\ i_{20} \end{bmatrix}$$ (21a)

and for (19b)

$$\begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix} = \begin{bmatrix} i_{10} \\ i_{20} + ai_{10}t \end{bmatrix},$$ (21b)

where $i_{10} = i_1(0)$, $i_{20} = i_2(0)$.

Remark 3. Note that the electrical circuit considered in Example 1 is positive system since the matrix A is Metzler matrix and the matrix B has nonnegative entries [7].

Example 2. Consider the electrical circuit shown on Fig. 2 with given resistances R_1, R_2, R_3. capacities C_1, C_2, and voltage sources e_1, e_2.
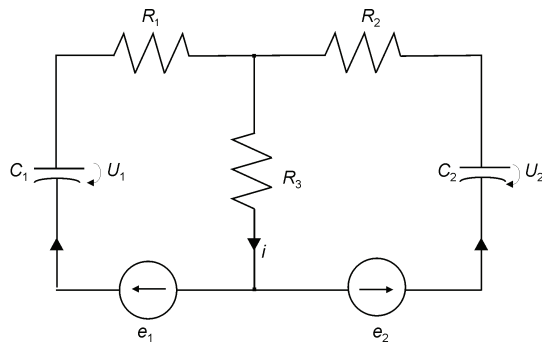

Fig. 2. Two loop circuit with RC elements

Using the Kirchhoff laws we may write the equations

$$e_1 = (R_1 + R_3)C_1 \dot{u}_1 + R_3 C_2 \dot{u}_2 + u_1,$$ (22a)

$$e_2 = (R_2 + R_3)C_2 \dot{u}_2 + R_3 C_1 \dot{u}_1 + u_2.$$ (22b)

The equations (22) can be written in the form

$$\frac{d}{dt}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = A \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + B \begin{bmatrix} e_1 \\ e_2 \end{bmatrix},$$ (23a)

where

$$A = -B = -\begin{bmatrix} C_1(R_1 + R_3) & C_2 R_3 \\ C_1 R_3 & C_2(R_2 + R_3) \end{bmatrix}^{-1}.$$ (23b)

Let

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = K \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}. \tag{24}$$

Substitution of (24) into (23a) yields

$$\frac{d}{dt} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = A_c \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \tag{25}$$

where

$$A_c = A + BK = A\,(I - K) \tag{26}$$

and

$$K = I - A^{-1} A_c . \tag{27}$$

We choose the matrix A_c of the form (19a) or (19b).

Using (27), (23b) and (19) we obtain for (19a)

$$K = \begin{bmatrix} 1 & C_1(R_1 + R_3)a \\ 0 & 1 + C_1 R_3 a \end{bmatrix} \tag{28b}$$

and for (19b)

$$K = \begin{bmatrix} 1 + C_2 R_3 a & 0 \\ C_2(R_2 + R_3)a & 1 \end{bmatrix} . \tag{28b}$$

From (12) we have for (19a)

$$\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} u_{10} + a u_{20} t \\ u_{20} \end{bmatrix} \tag{29a}$$

and for (19b)

$$\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} u_{10} \\ u_{20} + a u_{10} t \end{bmatrix}, \tag{29b}$$

where $u_{10} = u_1(0)$, $u_{20} = u_2(0)$.

Example 3. Consider the electrical circuit shown on Fig. 3 with given resistances $R_1, R_2,$ capacities $C_1, C_2,$ inductance L and voltage sources e_1, e_2, e_3.

Using the Kirchhoff laws we may write the equations

$$e_1 + e_3 = L \frac{di}{dt} + u_1 , \tag{30a}$$

$$e_2 = R_1\left(i - C_1\frac{du_1}{dt} + C_2\frac{du_2}{dt} \right) - u_1 , \tag{30b}$$

$$e_3 = R_2 C_2 \frac{du_2}{dt} + u_2 + R_1\left(i - C_1\frac{du_1}{dt} + C_2\frac{du_2}{dt} \right) . \tag{30c}$$
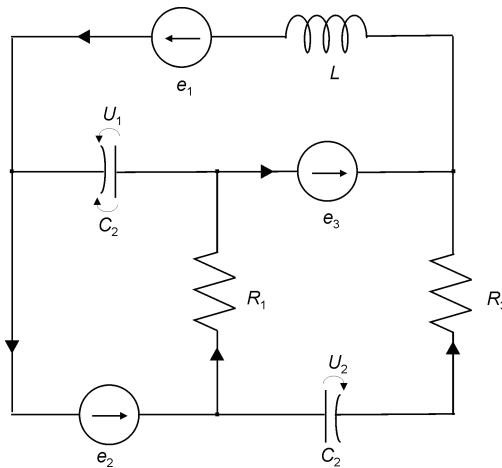

Fig. 1. Three loop circuit with RLC elements

The equations (30) can be written in the form

$$\frac{d}{dt}\begin{bmatrix} i \\ u_1 \\ u_2 \end{bmatrix} = A\begin{bmatrix} i \\ u_1 \\ u_2 \end{bmatrix} + B\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}, \tag{31a}$$

where

$$A = \begin{bmatrix} 0 & -\dfrac{1}{L} & 0 \\[2mm] \dfrac{1}{C_1} & -\dfrac{R_1+R_2}{C_1 R_1 R_2} & -\dfrac{1}{C_1 R_2} \\[2mm] 0 & -\dfrac{1}{C_2 R_2} & -\dfrac{1}{C_2 R_2} \end{bmatrix}, \qquad B = \begin{bmatrix} \dfrac{1}{L} & 0 & \dfrac{1}{L} \\[2mm] 0 & -\dfrac{R_1+R_2}{C_1 R_1 R_2} & \dfrac{1}{C_1 R_2} \\[2mm] 0 & -\dfrac{1}{C_2 R_2} & \dfrac{1}{C_2 R_2} \end{bmatrix}. \tag{31b}$$

Let

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = K\begin{bmatrix} i \\ u_1 \\ u_2 \end{bmatrix}. \tag{32}$$

Substitution of (32) into (31a) yields

$$\frac{d}{dt}\begin{bmatrix} i \\ u_1 \\ u_2 \end{bmatrix} = A_c \begin{bmatrix} i \\ u_1 \\ u_2 \end{bmatrix},$$

(33)

where

$$A_c = A + BK .$$

(34)

We choose the matrix A_c of the form

$$A_c = \begin{bmatrix} 0 & a_1 & 0 \\ 0 & 0 & 0 \\ 0 & a_2 & 0 \end{bmatrix}.$$

(35)

where a_1, a_2 are some real parameters.

Using (13), (31b) and (35) we obtain

$$K = B^{-1}(A_c - A) = \begin{bmatrix} \dfrac{1}{L} & 0 & \dfrac{1}{L} \\ 0 & -\dfrac{R_1 + R_2}{C_1 R_1 R_2} & \dfrac{1}{C_1 R_2} \\ 0 & -\dfrac{1}{C_2 R_2} & \dfrac{1}{C_2 R_2} \end{bmatrix}^{-1} \begin{bmatrix} 0 & a_1 + \dfrac{1}{L} & 0 \\ -\dfrac{1}{C_1} & \dfrac{R_1 + R_2}{C_1 R_1 R_2} & \dfrac{1}{C_1 R_2} \\ 0 & a_2 + \dfrac{1}{C_2 R_2} & \dfrac{1}{C_2 R_2} \end{bmatrix} =$$

(36)

$$= \begin{bmatrix} R_1 & 1 + a_1 L - (R_1 + R_2) C_2 a_2 & -1 \\ -R_1 & -1 + R_1 C_2 a_2 & 0 \\ R_1 & (R_1 + R_2) C_2 a_2 & 1 \end{bmatrix}.$$

From (12) we have

$$\begin{bmatrix} i(t) \\ u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} 1 & a_1 t & 0 \\ 0 & 1 & 0 \\ 0 & a_2 t & 1 \end{bmatrix} \begin{bmatrix} i_0 \\ u_{10} \\ u_{20} \end{bmatrix} = \begin{bmatrix} i_0 + a_1 u_{10} t \\ u_{10} \\ u_{20} + a_2 u_{10} t \end{bmatrix},$$

(37)

where $i_0 = i(0),\ u_{10} = u_1(0),\ u_{20} = u_2(0)$.

4. Extension of the considerations for nonsquare B matrices

If the Assumption is not satisfied but for a chosen matrix A_c the following condition is met

$$\mathrm{rank}\ B = \mathrm{rank}\ [B, A_c - A],$$

(38)

then the equation

$$BK = A_c - A \tag{39}$$

has a solution *K*.

In this case we have the following theorem.

Theorem 2. If the condition (38) is satisfied then there exist a gain matrix *K* such that the state variables of the electrical circuits are linear functions of time for any given initial conditions x_0.

Proof. If for a chosen matrix A_c the condition (38) satisfied then by Kronecker-Cappely theorem the equation (39) has a solution *K* such that the closed-loop system matrix is equal to A_c. The remaining part of the proof is similar to the proof of Theorem 1.

Example 4. Consider the electrical circuit shown on Fig. 4 with given resistance *R*, capacity *C*, inductance *L* and voltage source *e*.
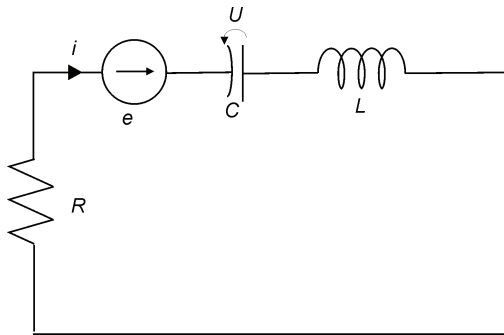

Fig. 4. One loop circuit with RLC elements

The equations

$$i = C\frac{du}{dt}, \tag{40a}$$

$$e = Ri + L\frac{di}{dt} + u \tag{40b}$$

can be rewritten in the form

$$\frac{d}{dt}\begin{bmatrix} u \\ i \end{bmatrix} = A\begin{bmatrix} u \\ i \end{bmatrix} + Be, \tag{41a}$$

where

$$A = \begin{bmatrix} 0 & \dfrac{1}{C} \\ -\dfrac{1}{L} & -\dfrac{R}{L} \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ \dfrac{1}{L} \end{bmatrix}. \tag{41b}$$

If we choose

$$A_c = \begin{bmatrix} 0 & \dfrac{1}{C} \\ 0 & 0 \end{bmatrix},$$

(42)

then the condition (38) is satisfied and the equation (39) has the form

$$\begin{bmatrix} 0 \\ \dfrac{1}{L} \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \dfrac{1}{L} & \dfrac{R}{L} \end{bmatrix}$$

(43)

and its solution is

$$K = \begin{bmatrix} k_1 & k_2 \end{bmatrix} = \begin{bmatrix} 1 & R \end{bmatrix}.$$
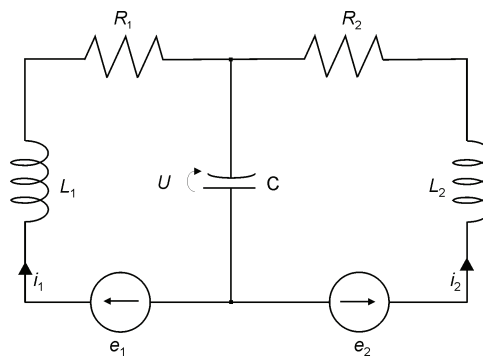
(44)

From (12) we have

$$\begin{bmatrix} u(t) \\ i(t) \end{bmatrix} = \begin{bmatrix} 1 & \dfrac{t}{C} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_0 \\ i_0 \end{bmatrix} = \begin{bmatrix} u_0 + \dfrac{i_0}{C} t \\ i_0 \end{bmatrix},$$

(45)

where $u_0 = u(0)$, $i_0 = i(0)$.

Example 5. Consider the electrical circuit shown on Fig. 5 with given resistances R_1, R_2, capacity C, inductances L_1, L_2 and voltage sources e_1, e_2.

Fig. 5. Two loop circuit with RLC elements

Using the Kirchhoff laws we may write the equations

$$e_1 = R_1 i_1 + L_1 \frac{di_1}{dt} + u \,,$$

(46a)

$$e_2 = R_2 i_2 + L_2 \frac{di_2}{dt} + u \,,$$

(46b)

$$C\frac{du}{dt} = i_1 + i_2. \tag{46c}$$

The equations (46) can be written in the form

$$\frac{d}{dt}\begin{bmatrix} i_1 \\ i_2 \\ u \end{bmatrix} = A\begin{bmatrix} i_1 \\ i_2 \\ u \end{bmatrix} + B\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}, \tag{47a}$$

where

$$A = \begin{bmatrix} -\dfrac{R_1}{L_1} & 0 & -\dfrac{1}{L_1} \\ 0 & -\dfrac{R_2}{L_2} & -\dfrac{1}{L_2} \\ -\dfrac{1}{C} & -\dfrac{1}{C} & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} \dfrac{1}{L_1} & 0 \\ 0 & \dfrac{1}{L_2} \\ 0 & 0 \end{bmatrix}. \tag{47b}$$

If we choose the matrix A_c of the form

$$A_c = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \dfrac{1}{C} & \dfrac{1}{C} & 0 \end{bmatrix}, \tag{48}$$

then the condition (38) is satisfied and the equation (39) takes the form

$$\begin{bmatrix} \dfrac{1}{L_1} & 0 \\ 0 & \dfrac{1}{L_2} \\ 0 & 0 \end{bmatrix} K = \begin{bmatrix} \dfrac{R_1}{L_1} & 0 & \dfrac{1}{L_1} \\ 0 & \dfrac{R_2}{L_2} & \dfrac{1}{L_2} \\ 0 & 0 & 0 \end{bmatrix}. \tag{49}$$

The solution of the equation (49) is

$$K = \begin{bmatrix} R_1 & 0 & 1 \\ 0 & R_2 & 1 \end{bmatrix}. \tag{50}$$

Substitution of (32) into (31a) yields

$$\frac{d}{dt}\begin{bmatrix} i \\ u_1 \\ u_2 \end{bmatrix} = A_c \begin{bmatrix} i \\ u_1 \\ u_2 \end{bmatrix}. \tag{51}$$

From (12) we have

$$\begin{bmatrix} i_1(t) \\ i_2(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \dfrac{t}{C} & \dfrac{t}{C} & 1 \end{bmatrix} \begin{bmatrix} i_{10} \\ i_{20} \\ u_0 \end{bmatrix} = \begin{bmatrix} i_{10} \\ i_{20} \\ u_0 + \dfrac{i_{10}}{C}t + \dfrac{i_{20}}{C}t \end{bmatrix}, \tag{52}$$

where $i_{10} = i_1(0)$, $i_{20} = i_2(0)$, $u_0 = u(0)$.

Remark 4. Note that the electrical circuit shown on Fig. 4 and Fig. 5 are not positive systems since the matrices A are not Metzler matrices [7].

5. Concluding remarks

Linear electrical circuits with state-feedbacks have been investigated. It has been shown that by suitable choice of the gain matrices of the state-feedbacks it is possible to obtain the closed-loop system matrices with nilpotency index $v = 2$. In such systems the state variables of the closed-loop systems are linear functions of time for any initial conditions.

If the number of state variables n is equal to the number of voltage sources m the input matrix is square but not necessary nonsingular (det $B \neq 0$). The following question arises. For which class of linear electrical circuits with $m = n$ the matrix B is nonsingular.

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