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ELASTIC CONSTANTS OF VENEER IN BEECH PLYWOOD

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SYNOPSIS. The paper presents the results of the study concerning elastic properties of beech veneers assembled in plywood panels. There has been derived a complete set of elastic constants for veneers considered as orthotropic bodies. Young's moduli in the grain direction and in the direction perpendicular to the grain were determined by bending plywood strips with their longitudinal axis parallel and perpendicular to the grain of the face plies. The other elastic moduli of the veneers were calculated through the Young's modulus in the grain direction, with using the regression functions derived for hardwoods by Bodig and Goodman (1973). The effect of glue lines in plywood on the veneer elastic properties was evaluated.

KEY WORDS: beech wood, elastic constants, plywood, orthotropy, veneer

INTRODUCTION

Despite competition from other wood based panels, plywood is nowadays used for a wide range of purposes. It is a good quality construction material having a number of advantages compared with solid timber. As such, it is suitable for important structural elements. The mechanical properties of entire plywood are fairly well known, whereas those of veneer in plywood have rarely been the subject of studies. One should note that the properties of veneers assembled in plywood panels differ from the properties of veneers before pressing. The veneers in plywood have a greater density as a result of their compaction and an adhesive diffusion into the wood (Mansouri et al. 2006).

Curry and Hearmon (1967) investigated mechanical properties of plywood made of gaboon (*Aucoumea klaineana*) and other wood species from Commonwealth countries. They determined, among other things, the elastic moduli of veneers in three-ply plywood in the directions parallel and perpendicular to the grain, based on elastic moduli in bending of the plywood strips with the longitudinal axis parallel and perpendicular to the grain of the face plies. Okuma (1976) calculated these elastic moduli of veneer in Hoop pine (*Araucaria cunninghamii*) plywood bonded with a tannin-formaldehyde adhesive. He used the method of comparing theoretical and empirical equations for elastic moduli in bending of the plywood strips with the longitudinal axis parallel and perpendicular to the grain of the face plies. This method was very time-consuming, nine different constructions of plywood being needed to derivate the empirical equations.

WILCZYŃSKI (2007) proposed a less complex method, based, like the two above, on elastic moduli in bending of two plywood strips with different longitudinal axes, and taking into account the glue lines that bonded the veneers. The method required an assumption of anisotropy of veneer elasticity expressed as a ratio of veneer elastic modulus in the grain direction to that modulus in the direction perpendicular to the grain. The elastic moduli in two directions of veneers in beech plywood bonded by two types of adhesives (a phenol-formaldehyde and an urea--formaldehyde resin) were determined, assuming the ratio of veneer elastic moduli equalled 20. This method was also used for evaluating the effect of the thicknesses of glue lines on the elastic moduli of veneer in beech plywood (WILCZYŃSKI et AL. 2008). Moreover, the effect of the veneer anisotropy on the elastic moduli of veneers in plywood determined by this method was examined (WILCZYŃSKI and WARMBIER 2009).

Elastic properties of the veneer before making up plywood, under free conditions, were studied by LANG et AL. (2003). They determined Young's modulus for different directions in the veneer plane, using an ultrasound stress-wave method. The veneers of five hardwood species were the subject of the study, but these species did not contain beech wood.

The veneer in plywood can be regarded, like solid wood, as an orthotropic body. Therefore, besides two Young's moduli in the veneer plane, other elastic constants, Young's modulus in the direction perpendicular to the veneer, and the shear moduli and Poisson's coefficients in the three orthotropic planes are required for a full description of the veneer elasticity. The knowledge of the elastic properties of the plywood layers can enable a thorough strength analysis of structural members made of plywood. Particularly, the veneer elastic constants data can be useful for finite element analysis in the case in which orthotropic 3D elements within each layer are employed, for example when stresses and deformations at the place of the embedment of connectors are to be analysed. This data can also be useful for an application of the theory of the layered systems, for example for calculating the elastic constants of entire plywood of any given construction.

The objective of this study was to derive the set of elastic constants of the veneer in beech plywood: the three Young's moduli, three shear moduli and six Poisson's coefficients.

THEORETICAL CONSIDERATIONS

A compressed and glue-impregnated veneer in plywood (Fig. 1) can be regarded as an orthotropic body (BODIG and JAYNE 1993). The principal axes of elasticity have the following directions: x – grain direction, y – perpendicular to the grain direction, and z – perpendicular to the veneer plane. For a rotary-cut veneer the y axis agrees with the wood tangential directions, and the z axis agrees with the wood radial direction.

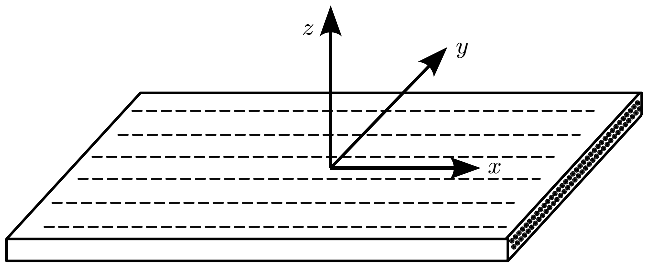

Fig. 1. Principal axes of the elasticity of the veneer

Hooke's law for veneer as an orthotropic material can be written as the generalized Hooke's law relation where strains are stated as linear functions of stresses:

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{E_x} & -\dfrac{\nu_{yx}}{E_y} & -\dfrac{\nu_{zx}}{E_z} & 0 & 0 & 0 \\ -\dfrac{\nu_{xy}}{E_x} & \dfrac{1}{E_y} & -\dfrac{\nu_{zy}}{E_z} & 0 & 0 & 0 \\ -\dfrac{\nu_{xz}}{E_x} & -\dfrac{\nu_{yz}}{E_y} & \dfrac{1}{E_z} & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{1}{G_{yz}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \dfrac{1}{G_{xz}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \dfrac{1}{G_{xy}} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{bmatrix} \tag{1}$$

where E_x, E_y and E_z are the Young's moduli, ν_{xy}, ν_{yx}, ν_{yz}, ν_{zy}, ν_{zx} and ν_{xz} are the Poisson's ratios, and G_{yz}, G_{xz} and G_{xy} are the shear moduli. The compliance matrix is symmetric. Hence, one obtains the following relationships:

$$\frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y}, \qquad \frac{\nu_{xz}}{E_x} = \frac{\nu_{zx}}{E_z}, \qquad \frac{\nu_{yz}}{E_y} = \frac{\nu_{zy}}{E_z} \tag{2}$$

Consider the plywood strips subjected to bending, in which the grain direction of the face plies runs parallel or perpendicular to the longitudinal axis of the strip. The cross-sections of these strips are presented in Figure 2. When glue lines are neglected, the flexural rigidity of the strip is the sum of the rigidity of the plies with the longitudinal axis parallel to their grain and the plies with that axis perpendicular to their grain:

$$E_1 I_2 = E_x I_{2x} + E_y I_{2y} \tag{3}$$

$$E_2 I_1 = E_y I_{1y} + E_x I_{1x} \tag{4}$$

where: E_1 and E_2 = modulus of elasticity in bending of the plywood strip with the longitudinal axis parallel and perpendicular to the grain direction of the face plies, respectively,

I_2 and I_1 = moment of inertia of the cross section of the plywood strip with the longitudinal axis parallel and perpendicular to the grain direction of the face plies about the neutral axis 2 and 1, respectively,

E_x and E_y = Young's modulus of the veneer in the grain and perpendicular to grain directions, respectively,

I_{2x} and I_{1x} = moment of inertia of all the veneer plies with the longitudinal axis parallel to the grain about the neutral axis 2 and 1, respectively,

I_{2y} and I_{1y} = moment of inertia of all the veneer plies with the longitudinal axis perpendicular to the grain about the neutral axis 2 and 1, respectively.

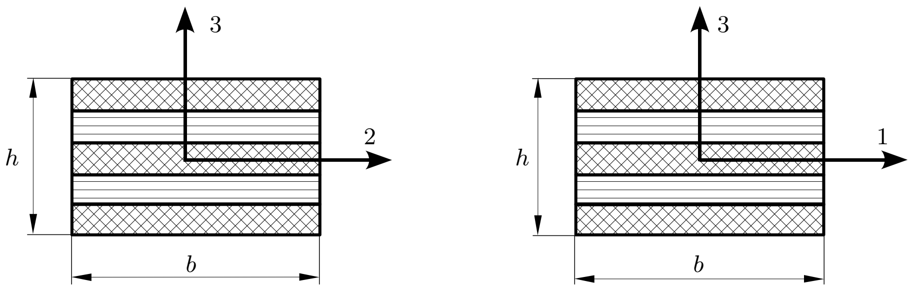

Fig. 2. Cross-sections of five-ply plywood strips with a longitudinal axis parallel (a) and perpendicular (b) to the grain direction of the face plies; 1, 2 and 3 = geometric axes of the strips

Assuming that all compressed veneers are of the same thickness, one obtains:

$$I_1 = I_2, \qquad I_{1y} = I_{2x}, \qquad I_{1x} = I_{2y} \tag{5}$$

Using the following factors:

$$A = \frac{I_{2x}}{I_2} = \frac{I_{1y}}{I_1}, \qquad B = \frac{I_{2y}}{I_2} = \frac{I_{1x}}{I_1}, \tag{6}$$

and combining Eqs. (3)-(6) gives the relations:

$$E_1 = AE_x + BE_y \tag{7}$$

$$E_2 = AE_y + BE_x \tag{8}$$

which can be rearranged to expressions for the Young's moduli of the veneer:

$$E_x = \frac{AE_1 - BE_2}{A^2 - B^2} \tag{9}$$

$$E_y = \frac{AE_2 - BE_1}{A^2 - B^2} \tag{10}$$

Eqs. (9) and (10) permit the calculation of the veneer Young's moduli E_x and E_y through experimental determination of elastic moduli E_1 and E_2 of plywood strips.

Bodig and Goodman (1973) determined the elastic constants of many wood species. Using the obtained data and the data for other species presented by Hearmon (1948), they derived regression equations among the elastic constants of wood as orthotropic body. They proposed the function of a power-type for calculating other elastic constants (EC) on the basis of Young's modulus E_L in the grain direction:

$$(EC)_{\text{wood}} = \alpha E_L^{\beta} \tag{11}$$

where α and β are regression parameters determined separately for softwoods and hardwoods. The values of these parameters for hardwoods, for the Young's modulus E_R and shear moduli G_{LT}, G_{LR} and G_{RT}, when the modulus E_L is expressed in a pound force/inch^2 (psi), are listed in Table 1. The directions x, y and z for the rotary cut veneer agree with the wood directions longitudinal (L), tangential (T) and radial (R), respectively. Therefore, it was assumed that the beech veneer elastic moduli E_z, G_{xy}, G_{xz} and G_{zy} can be calculated by the regression equation (12), using the parameters α and β derived for hardwoods:

$$(EC)_{\text{veneer}} = \alpha E_x^{\beta}. \tag{12}$$

Table 1. Regression parameters α and β of Eq. (11) for individual elastic moduli of hardwoods (after Bodig and Goodman 1973)

Parameter	E_R	G_{LT}	G_{LR}	G_{RT}
α	0.278	0.017	0.104	0.0088
β	0.926	1.082	0.978	1.049

Because of limited correlations between Poisson's ratios and the modulus E_L Bodig and Goodman (1973) suggested not compute the values of Poisson's ratios by the equation (11) but their uniformity among softwoods and among hardwoods. They proposed to assume the values of these ratios as the averages found from own and Hearmon (1948) data.

MATERIALS AND METHODS

Plywood panels for this study were fabricated in a laboratory with the use of selected beech veneers 50-by 50-cm, without defects, of 1.50 mm thickness, 0.63 g/cm^3 average density and 7% moisture content. A phenol adhesive was used with a spread rate of 160 g/m^2. Three- and fiveply panels were made. They were pressed with 1.6 MPa and 135°C for 4 and 6 min for the 3-and 5-ply panels, respectively.

Two kinds of specimens were cut from the plywood panels, one with the longitudinal axis parallel and the other with that axis perpendicular to the grain of face veneers. These specimens were 50 mm wide, and 160 and 220 mm long for the 3- and 5-ply panels, respectively. Fifteen specimens for each number of plies, and specimen axis direction were prepared.

Prior to testing all the specimens were stored in controlled conditions (50% relative humidity and 20°C) for two weeks. The modulus of elasticity in bending was determined, as shown in Figure 3. The span l was equalled to 24 times the thickness t_p of plywood, the distance l_1 for measuring the specimen deflection being 5/6 of the distance between the loading heads. The bending speed was 2 mm/min.

The moduli of elasticity E_1 and E_2 of the specimens with the longitudinal axis parallel and perpendicular to the grain of face plies, respectively, were calculated with the formula:

$$E_i = \frac{l_1^2 l_2 \Delta F}{16 I \Delta w}, \quad i = 1,\ 2 \tag{13}$$

where: I = moment of inertia of the cross section of the specimen,
ΔF = increment of load on the straight line portion of the load-deflection curve,
Δw = increment of deflection corresponding to ΔF.

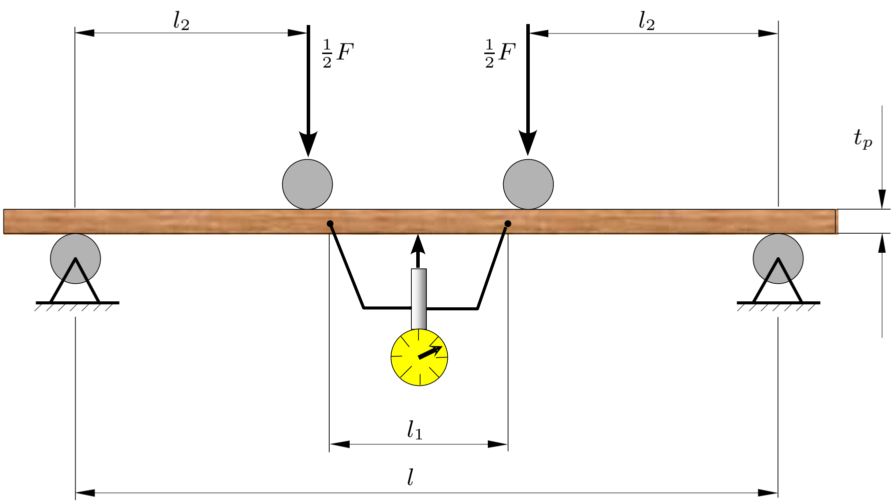

Fig. 3. Test arrangement for determining modulus of elasticity of plywood in bending

RESULTS

The results of bending tests, mean values and standard deviations of the elastic moduli of examined plywood, are given in Table 2. The values of the Young's moduli E_x and E_y of the veneer in plywood, calculated by Eqs (9) and (10) on the basis of the plywood moduli E_1 and E_2, are given in Table 3. The factors A and B, expressed by Eqs (6), were equalled to 0.963 and 0.037, respectively, for the 3-ply plywood, and 0.792 and 0.208, respectively, for the 5-ply plywood.

Table 2. Elastic moduli E_1 and E_2 of tested plywood

Number of plies	Modulus E_1 [MPa]		Modulus E_2 [MPa]	
	mean value	standard deviation	mean value	standard deviation
3	14 970	1 290	1 510	150
5	12 880	960	4 140	220

Table 3. Young's moduli E_x and E_y and compression ratio of veneers in plywood

Number of plies	E_x [MPa]	E_y [MPa]	E_x/E_y	Compression ratio (cr) of veneers* (%)
3	15 510	970	16.0	9.3
5	15 990	1 030	15.5	11.3

*$cr = (t_o - t_v) \cdot 100/t_o$, t_o = initial thickness of veneer (1.50 mm), t_v = average thickness of veneers in plywood (Table 5).

The moduli E_x and E_y of the veneer in the 5-ply plywood are about 4% greater than those in the 3-ply plywood. This was a result of a greater compression of veneers in 5-ply plywood (Table 3). The E_x/E_y ratio, characterizing anisotropy of elastic properties of the veneer in the xy-plane, amounts to about 16. It can be compared with the E_L/E_T ratio of the beech wood (with the density of 0.75 g/cm^3, moisture content of 11%, and modulus E_L of 13 700 MPa) which is equalled to 12 (Hearmon 1948). The greater ratio for the veneer is due to peeler checks caused by the rotary-cut processing.

Based on the obtained results, approximate values of the veneer Young's moduli: E_x = 16 000 MPa and E_y = 1 000 MPa were assumed. The Young's modulus E_z and the shear moduli G_{xy}, G_{xz}, G_{yz} were calculated by Eq. (12) for the assumed value of E_x. The Poisson's ratios ν_{xy}, ν_{xz} and ν_{zy} were taken from Bodig and Goodman (1973) as average for hardwoods, and the Poisson's ratios ν_{yz}, ν_{yx} and ν_{zx} were calculated from Eqs. (2). The complete set of elastic constants of beech veneer in plywood is listed in Table 4.

Table 4. Elastic constants of beech veneer in plywood

E_x	E_y	E_z	G_{xy}	G_{xz}	G_{yz}	ν_{xy}	ν_{xz}	ν_{yz}	ν_{zy}	ν_{yx}	ν_{zx}
[MPa]											
16 000	1 000	1 500	920	1 200	290	0.50	0.37	0.45	0.67	0.031	0.035

ASSESSMENT OF THE EFFECT OF GLUE LINES

Plywood is a panel built up of an odd number of veneers (plies) with the grain direction of adjacent plies oriented perpendicular to one another. These plies and the glue lines that bond them are a multilayered structures. When Young's moduli E_x and E_y were determined the glue lines were neglected as very thin when compared to veneers. Now they will be considered.

When the glue lines are considered Eqs. (3) and (4) take the following form:

$$E_1 I_2 = E_x^* I_{2x}^* + E_y^* I_{2y}^* + E_g I_g \tag{14}$$

$$E_2 I_1 = E_y^* I_{1y}^* + E_x^* I_{1x}^* + E_g I_g \tag{15}$$

where: E_x^* and E_y^* = Young's modulus of the veneer in the grain and perpendicular to grain directions, respectively,
I_{2x}^* and I_{1x}^* = moment of inertia of all the veneer plies with the longitudinal axis parallel to the grain about the neutral axis 2 and 1, respectively,
I_{2y}^* and I_{1y}^* = moment of inertia of all the veneer plies with the longitudinal axis perpendicular to the grain about the neutral axis 2 and 1, respectively,
E_g = Young's modulus of the glue line,
I_g = moment of inertia of all the glue lines about the neutral axis.

Assuming, as earlier:

$$I_1 = I_2, \qquad I_{1y}^* = I_{2x}^*, \qquad I_{1x}^* = I_{2y}^* \tag{16}$$

and using the following factors:

$$A^* = \frac{I_{2x}^*}{I_2} = \frac{I_{1y}^*}{I_1}, \qquad B^* = \frac{I_{2y}^*}{I_2} = \frac{I_{1x}^*}{I_1}, \qquad C^* = \frac{I_g}{I_2} = \frac{I_g}{I_1} \tag{17}$$

gives:

$$E_1 = A^* E_x^* + B^* E_y^* + C^* E_g \tag{18}$$

$$E_2 = A^* E_y^* + B^* E_x^* + C^* E_g \tag{19}$$

which can be rearranged to expressions for the Young's moduli of the veneer:

$$E_x^* = \frac{A^* E_1 - B^* E_2 - (A^* - B^*) C^* E_g}{A^{*2} - B^{*2}} \tag{20}$$

$$E_y^* = \frac{A^* E_2 - B^* E_1 - (A^* - B^*) C^* E_g}{A^{*2} - B^{*2}} \tag{21}$$

For the 3-ply plywood the factors A^*, B^* and C^* are given by:

$$A^* = 1 - (k_v + 2k_g)^3, \qquad B^* = k_v^3, \qquad C^* = (k_v + 2k_g)^3 - k_v^3 \tag{22}$$

and for the 5-ply-plywood:

$$A^* = 1 - (3k_v + 4k_g)^3 + k_v^3, \quad B^* = (3k_v + 2k_g)^3 - (k_v + 2k_g)^3,$$

$$C^* = (3k_v + 4k_g)^3 - (3k_v + 2k_g)^3 - (k_v + 2k_g)^3 - k_v^3 \tag{23}$$

where coefficients k_v and k_g depend on the thicknesses: t_v of veneer in plywood, t_g of the glue line, and t_p of the plywood (Fig. 4):

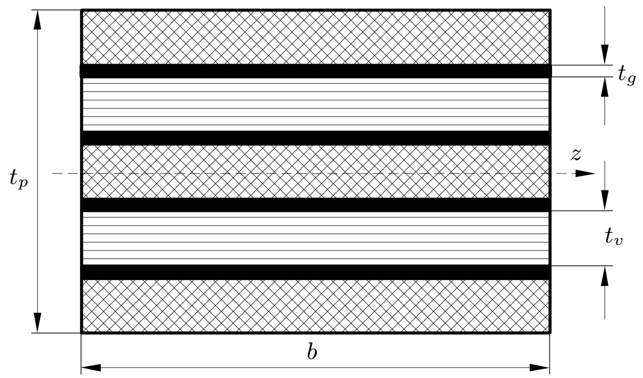

Fig. 4. Cross section of five-ply plywood strip with a mark of glue lines

$$k_v = \frac{t_v}{t_p}, \qquad k_g = \frac{t_g}{t_p} \tag{24}$$

The thicknesses t_v and t_g were determined by microscopic measurement in the way described in a previous paper (Wilczyński 2006). The average thicknesses of plywood, veneer in plywood, and glue line, and also the values of coefficients A^*, B^*, C^* calculated by Eqs (22) and (23) are listed in Table 5.

Table 5. Average thicknesses of plywood, veneer in plywood, and glue line, and coefficients A^*, B^*, C^*

Number of plies	t_p [mm]	t_v [mm]	t_g [mm]	A^*	B^*	C^*
3	4.22	1.36	0.07	0.955	0.033	0.012
5	6.93	1.33	0.07	0.773	0.202	0.025

Next the Young's moduli E_x^* and E_y^* of the veneer in plywood were calculated by Eqs (20) and (21), assuming that the values of Young's modulus of the glue line ranged from 1000 to 5000 MPa. The values of the veneer moduli E_x^* and E_y^* are given in Table 6 and are compared with the veneer moduli E_x and E_y (Table 3) obtained when the glue lines were neglected. The values of the E_x^* are greater than of the E_x but the relative difference between these moduli is very small and ranged from 0.4 to 0.8% for the veneer contained in the 3-ply plywood, and from 1.6 to 2.3% for the veneer in the 5-ply plywood. The relative difference between the

Table 6. Young's moduli E_x^* and E_y^* of veneer in plywood calculated for the assumed Young's modulus E_g of glue line, and their comparison with the veneer moduli E_x and E_y determined with neglecting glue lines

Number of plies	E_g [MPa]	E_x^* [MPa]	$\frac{E_x^* - E_x}{E_x} \cdot 100$ [%]	E_y^* [MPa]	$\frac{E_y^* - E_y}{E_y} \cdot 100$ [%]
3	1 000	15 630	0.8	1 030	6.2
	3 000	15 600	0.6	1 000	3.1
	5 000	15 580	0.4	980	1.0
5	1 000	16 360	2.3	1 050	1.9
	3 000	16 300	1.9	1 000	−2.9
	5 000	16 250	1.6	950	−7.8

veneer moduli E_y^* and E_y is slightly greater and ranged from 1.0 to 6.2% and from 1.9 to (−7.8%) for the veneer in the 3- and 5-ply plywood, respectively. Therefore, one can conclude that the effect of the glue lines on the results of determination of Young's moduli of veneer in plywood is negligible.

CONCLUSIONS

The veneers in plywood, under the conditions of assembly of the panel, can be regarded as orthotropic materials. The Young's moduli of the veneer, the modulus in the grain direction and the modulus in the direction perpendicular to the grain, can be determined by indirection method consisting in bending tests of two kinds of plywood strips, with the longitudinal axis parallel and perpendicular to the grain of the face plies. The effect of glue lines that bonded the veneers on their elastic properties is negligible.

The other elastic moduli of the veneer in plywood can be calculated using the regression functions derived for hardwoods by Bodig and Goodman (1973). These functions are based on the Young's modulus in the grain direction.

The veneer elastic constants data enables a strength analysis of structural members made of plywood, especially finite element analysis in the case in which orthotropic 3D elements within each ply of plywood are employed.

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