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THE USE OF THE SPATIAL CHARACTERISTICS TECHNIQUE WITH THE VIEW OF ESTIMATING THE EXPLOSION WAVE IMPACT ON THE STUCK DRILLING STRING ZONE

WYKORZYSTANIE TECHNIKI CHARAKTERYSTYK PRZESTRZENNYCH DO OKREŚLANIA WPŁYWU FALI UDERZENIOWEJ NA PRZYCHWYCONĄ KOLUMNĘ PRZEWODÓW WIERTNICZYCH

Studied here are the results of the asymmetric problem solution of the thick walled circular cylinder elasticity using the spatial characteristics technique. The practical implementation of the solution of the problem is based on the calculation of the stress-caused deformation state of the stuck drilling string zone affected by the explosion wave action upon the inner wall of the pipe.

Suggested here is the technique for determining axual σ_z and circular σ_θ stress on the drill pipe wall as well as the radial displacements u_r of the stuck drill pipe outer surface under the action of the explosion shock wave. The above technique enables to make a sound selection of the cylindrical explosive charge weight in order to avoid the residual strain during the drilling string shaping off and uncoupling the threaded joints or to prevent them from exceeding the admissible level.

Keywords: spatial characteristics technique, asymmetric problem of dynamic elasticity, stress caused deformation state of a thick walled circular cylinder, explosion wave.

W artykule przedstawiono wyniki uzyskane przy pomocy asymetrycznego rozwiązania zagadnienia związanego z elastycznością grubościennego walca o przekroju kołowym, w oparciu o metodę charakterystyk przestrzennych. Praktyczne zastosowanie tego rozwiązania opiera się na obliczeniu stanu odkształcenia strefy przy przychwyconej w otworze kolumnie przewodów wiertniczych spowodowanego naprężeniami wskutek oddziaływania fali uderzeniowej na wewnętrzne ściany przewodów.

W pracy zaproponowano metodę określenia naprężenia osiowego i obwodowego działającego na ściany przewodów wiertniczych oraz przemieszczenia promieniowego u umocowanego przewodu pod wpływem oddziaływania fali uderzeniowej po wybuchu. Technika powyższa umożliwia dokonanie właściwego doboru wielkości ładunku wybuchowego w kształcie walca w celu uniknięcia naprężeń resztkowych w kolumnie przewodów wiertniczych, które mogłyby doprowadzić do jego odkształcenia lub poluzowania połączeń gwintowanych lub dla utrzymania wielkości tych naprężeń w dopuszczalnych normach.

Słowa kluczowe: metoda charakterystyk przestrzennych, asymetryczne zagadnienie sprężystości dynamicznej, stan odkształcenia grubościennego walca o przekroju kołowym spowodowany działającymi naprężeniami, fala uderzeniowa

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1. Topicality of the Problem Under Study

As a rule, the elimination technology of the failures and complications arising in the course of the borehole construction envisages a successive application of a number of actions and means or their alternation. Such actions in the first head should be included: the drill string loosening and rotor aided turning, baths installation, the application of impulse action devices (jarring devices, vibration exciters, hydraulic hammers, and so forth), drilling string uncoupling and pulling it out by parts, drilling round the stuck zone of the pipe, the drilling mud circulation renewing and borehole flushing.

The predominant place among these techniques is taken by shot-firing operation and perforation that are based upon the use of explosion power. Paper (Moisyshyn, 1997) presents the analysis of the use of burning and explosion apparatus (BEA) designed for the stuck drilling string breakdown elimination by the Derzhkomgeologiya of Ukraine enterprises over the period of twenty-five years (1973-1997). Jack squibs and jack hammers have been used in 283 out of 506 wild cats where stuck drilling string occurred. It is established that the use of shot firing apparatus technique with the view of threaded joints uncoupling amounted to 78% of all cases of explosion power application, where as the shock firing technique application aimed at the drilling string "shaking" came to 18.4%, while the drilling string shear or breakage amounted to 5.7%, other shot firing technique application came to 5%, the perforation technique aimed at the drilling fluid circulation renewing or the next baths installation came to 2.8%. The total success in all cases of shot-firing technique application came to 89.5% while that of the perforation technique applied resulted in 87.5%.

Let's dwell in more detail on the use of the explosion technique aimed at pipes "shaking" and uncoupling.

The use of the explosion technique for drilling string "shaking" is practiced in case when the stuck drilling string is connected with pipes "sticking" to the borehole wall or the annular space partially filled up with loose rock. The above technique is based on the ability of shock waves to penetrate even though with attenuation through strong obstacles (in this case through the pipe wall) and, which is no less important, to exert the linear motion of the medium material in the direction of waves propagation. With a heterogeneous medium (and in his case the medium consists of a fluid and rock particles – sand and clay) during the braking time which occurs during wave propagation the particles are redistributed in the volume and small interstices are formed along them filled with fluid. In case of "sticking" the formation of these interstices may "take away" the forces that press the pipes to the rock equalizing the pressure around it, and with the circulation lost it may cause the renewing of the latter, and finally just brake its coupling with the rock, lessen the action of the forces necessary to pull out the drilling string on to the day surface.

The "shaking" technique is rather simple. After the turning operation, flushing and checking the pipes penetration ability the explosive is being lowered into the drilling string which is placed opposite the stuck zone. The drilling string is provided with a tension force aimed at pulling out the pipes and the torque in the direction of threaded joints tightening with the that during the explosion when the pipes are released of the holding forces to the maximum additional displacement freedom be supplied to them.

The above operation having been done the explosion is performed. The efficiency of shot-firing application to eliminate the similar failures is considerably greater if they are used instantaneously. With time, as a rule, the elimination of failures conditions become more complicated as a result of the chemical processes and the action of other factors which contribute to increasing the forces responsible for holding the drilling string.

Pipes explosion uncoupling technique also consists of the similar operations (*Technical Rules* ..., 1978). The stuck zone having been determined the jack squib is lowered in to the borehole at the pre-set depth after which the tension force and torque are applied to the pipes in the uncoupling direction. The tension will unload the clutch coupling, opposite to which the explosion is placed, of the pipes weight lessening the coupling friction forces to the minimum, and by means of the torque at the moment of explosion the turning is supplied to the drilling string. The tension value in the first approximation is determined by the weight of the drilling string upper section (to the stuck zone) while that of the torque by the one third of the torque number of revolutions which (according to the calculations) can be supplied to the drilling string of the given design and length when tightening the couplings.

The use of short jack squibs is reasonable (from 0.5 to 3 m long) which cover only one clutch coupling placed precisely opposite the object of operation. If the task is restricted by uncoupling only one coupling, then after the explosion the drilling string uncoupled section is pulled out.

But in case when the operations are carried out up to the complete elimination of failures, then without the drilling string being lifted through the uncoupled pipes a thorough flushing of borehole is done removing the forces that hold the pipes acting higher the uncoupling point after which the drilling string will be coupled again. The removing of a certain portion of forces after flushing and re-coupling enables to do the second uncoupling at a lower depth. The cycle of operation termed as "uncoupling – flushing – coupling – loosening – and the next uncoupling at a greater depth" sometimes repeated for several *times* under the favorable conditions will enable to completely release the stuck tool.

Therefore, the study of the drill pipes tension state an the determination of their radial displacements is an urgent task when carrying out explosion operations in the borehole with the view of stuck drilling string problems elimination without destructing the parts of the latter.

2. The Purpose and Problem Definition

The purpose of the given study is the estimation of the stress caused deformation state (SDS) of the stuck drill pipe under the above mentioned conditions.

The annular space rock that interacts with the drilling string outer wall of r_2 radius (Fig. 1) will be simulated with the rock pressure force p_1 and the elastic stress elements. The distributed force acting upon the outer wall of the pipe is expressed as the sum of two items

$$p_3 = p_1 + p_2 \tag{1}$$

where:

$p_1 = \rho_0 HA$ — the pressure force rocks (*Construction of*..., 1990), Pa;

ρ_0 — rock specific weight, kg/m^3;

H — depth, m;

A — horizontal burst opening factor;

$p_2 = \dfrac{k_0 u_r}{2\pi r_2 l}$ — force component p_3, is connected with elastic properties of the rock, Pa;

k_0 — rock rigidity factor, H/m;
l — length of the stuck drilling string section, m;
u_r — radial stuck pipe displacement.

Shock wave pressure $p(t)$ onto the pipe inner wall of r_1 radius is assumed using the equation

$$p(t) = \begin{cases} p_m \dfrac{t}{\theta_1}, 0 \le t \le \theta_1; \\ p_m e^{-\frac{t-\theta_1}{\theta_2}}, t \le \theta_1 \end{cases}$$

where

p_m — is the impulse pressure amplitude, Pa;
θ_1 — impulse increase time, sec;
θ_2 — the typical loading decrease time, sec.

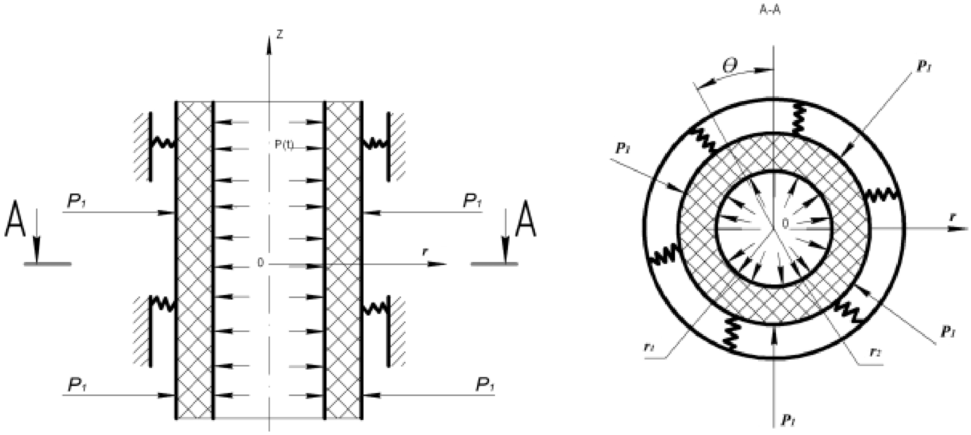


Fig. 1. Calculation diagram

In the cylindrical system of coordinates r, z, θ for rotationally symmetrical case when the variables are the function of coordinates r, z, the motion equation can be given as:

$$\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} = \rho \frac{\partial^2 u_r}{\partial t^2} \tag{2}$$

$$\frac{\partial \tau_{rz}}{\partial r} + \frac{\partial \sigma_2}{\partial z} + \frac{\tau_{rz}}{r} = \rho \frac{\partial^2 u_z}{\partial t^2} \tag{3}$$

To obtain closed combined to equations (2), (3) we shall add the following equations written on the basis of Hooks law:

$$\begin{aligned} \sigma_r &= \lambda\Omega + 2\mu\frac{\partial u_r}{\partial r}, \\ \sigma_z &= \lambda\Omega + 2\mu\frac{\partial u_z}{\partial z}, \\ \sigma_\theta &= \lambda\Omega + 2\mu\frac{u_r}{r}, \\ \tau_{rz} &= \mu\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right) \end{aligned} \tag{4}$$

where

λ, μ — of steel are the Lame's constants;

u_r, u_z — the axes r and z longitudinal displacements respectively;

Ω — volumetric deformation which is expressed by the formula

$$\Omega = \frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z} \tag{5}$$

As is seen, the problem is defined in two-dimensional mode. Its solution will be sought in a mixed form (in the stresses σ_r, σ_z, σ_θ, τ_{rz} and displacements in u_r, u_z).

The initial conditions will be assumed as follows:

– $u_r(r,0) = u_z(r,0) = 0$ (6)

– stress distribution σ_r and σ_θ along the cylinder thickness is taken from the Lame's problem solution (*Handbook*..., 1988) for a thick walled cylinder loaded all over its outer surface

$$\left.\begin{aligned} \sigma_r(r,0) &= -\frac{r_2^2}{r_2^2 - r_1^2}\left(1 - \frac{r_1^2}{r^2}\right)p_1, \\ \sigma_\theta(r,0) &= -\frac{r_2^2}{r_2^2 - r_1^2}\left(1 + \frac{r_1^2}{r^2}\right)p_1, \end{aligned}\right\} \tag{7}$$

$$\sigma_z(r,0) = 0, \quad \tau_{rz}(r,0) = 0 \tag{8}$$

The boundary conditions are assumed to be as follows:

– for the drill pipe inner side $(r = r_1, 0 \le z \le l)$

$$\sigma_r(r_1,t) = p(t), \quad \tau_{rz}(r_1,t) = 0 \tag{9}$$

– for the drill pipe outer side $(r = r_2, 0 \le z \le l)$

$$\sigma_r(r_2,t) = p_3, \quad \tau_{rz}(r_2,t) = 0 \tag{10}$$

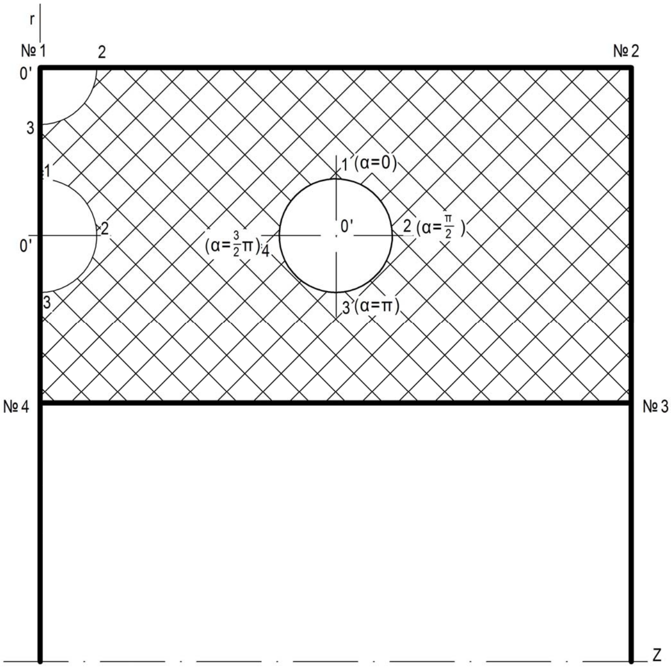


Fig. 2. The drill pipe section under test for which the boundary conditions are set

– for the edge surface ($z = 0$, $r_1 \le r \le r_2$ and $z = l$, $r_1 \le r \le r_2$)

$$\left.\begin{aligned} \sigma_r(0,t) &= 0, \tau_{rz}(0,t) = 0 \\ \sigma_r(l,t) &= 0, \tau_{rz}(l,t) = 0 \end{aligned}\right\} \tag{11}$$

– for angular points 1, 2, 3, 4 (fig. 2) observed are the boundary conditions for the respective surfaces (the inner or the outer) and edges. So, for instance, for point 1 (fig. 2) we obtain

$$\sigma_r(r_2,t) = p_3, \quad \sigma_z(0,l) = 0, \quad \tau_{rz}(0_1,t) = 0 \tag{12}$$

3. Deducing Distinctive Equation

To clear up the distinctive features of stress caused deformation state (SDS) the drill pipe zones were made stuck in the conditions under study, problem (3)-(12) will be solved by means of a spatial properties technique (Moisyshyn, 2009; Sabodash, 1971) which was for the first time suggested by R. Clifton (Cliffton, 1968) for solving plane problems related to elasticity dynamic theory. For which the combined equations (2)-(5) together with the starting (6)-(8) and final (8)-(12) conditions should be expressed as distinctive equations in the dimensionless form.

First of all, let's write down the ratios (2)-(5) in the dimensionless form. For this we shall add and subtract the expression used for elasticity σ_r and σ_z; the R. Hook's law equation obtained will be differentiated according to time; the dimensionless values will be introduced according to the formula:

$$\bar{r} = \frac{r}{h}, \ \bar{z} = \frac{z}{h}, \ \bar{t} = \frac{at}{h},$$
$$p = \frac{\sigma_r + \sigma_z}{2\rho a^2}, \ \rho = \frac{\sigma_r - \sigma_z}{2\rho a^2}, \ \sigma = \frac{\sigma_\theta}{\rho a^2}, \ \tau = \frac{\tau_{rz}}{\rho a^2};$$
$$\bar{u}(\bar{r}, \bar{z}, \bar{t}) = \frac{1}{a}\frac{\partial u_r}{\partial t}, \ \bar{V}(\bar{r}, \bar{z}, \bar{t}) = \frac{1}{a}\frac{\partial u_z}{\partial t};$$
$$a = \sqrt{\frac{\lambda + 2\mu}{\rho}}, \ b = \sqrt{\frac{\mu}{\rho}}, \ \ \gamma = \frac{a}{b} > 1$$

where

h — the typical dimension (in this case the cylinder thickness),
a — waves expansion velocity,
b — waves shifting velocity,
ρ — drill pipe material density.

After performing the above mentioned operation combined equations will be obtained in the dimensionless form:

$$\begin{aligned} & u_t - p_r - q_r - \tau_z = \frac{p + q - \sigma}{r}, \\ & v_t - p_z + q_z - \tau_r = \frac{\tau}{r}, \\ & \frac{\gamma^2}{\gamma^2 - 1} p_t - u_r - v_z = \frac{\gamma^2 - 2}{\gamma^2 - 1}\frac{u}{r}, \\ & \gamma^2 q_t - u_r - v_z = 0, \\ & \frac{\gamma^2}{\gamma^2 - 2}\sigma_t - u_r - v_z = \frac{\gamma^2}{\gamma^2 - 2}\frac{u}{r}, \\ & \gamma^2 \tau_t - u_z - v_r = 0 \end{aligned} \tag{13}$$

where the indices denote a partial differentiation according to the above mentioned argument.

Let's re-arrange the combined equations (13) to acquire a symmetric form. For which we shall find the difference between equation five and equation three of the combined equations. After determining the expressions for u/r, let's substitute it in equation three. Then we shall obtain the combined equations whose matrixes according to t, r, z prove to be symmetrical:

$$
\begin{aligned}
& u_t - p_r - q_r - \tau_z = \frac{p+q-\sigma}{r}, \\
& v_t - p_z + q_z - \tau_r = \frac{\tau}{r} \\
& \frac{\gamma^4}{3\gamma^2-4} p_t - \frac{\gamma^2(\gamma^2-2)}{3\gamma^2-4} \sigma_t - u_r - v_z = 0, \\
& \gamma^2 q_t - u_r + v_z = 0, \\
& -\frac{\gamma^2(\gamma^2-2)}{3\gamma^2-4} p_t + \frac{\gamma^2(\gamma^2-1)}{3\gamma^2-4} \sigma_t = \frac{u}{r}, \\
& \gamma^2 \tau_t - u_z - v_r = 0
\end{aligned}
\tag{14}
$$

The next stage in distinctive equations development is fairly completely presented in the paper (Sabodash, 1971). Here we shall put down only the results of the above stage – combined equations in functions increase:

$$
\begin{aligned}
& \delta u - \frac{k}{2r}\delta p - \frac{k}{2r}\delta\sigma + \frac{k}{2r}\delta\sigma = \frac{a_1 - a_3}{2} - \frac{b_2 - b_4}{2} - \frac{k}{2}(p_r + q_r + \tau_z), \\
& \delta v - \frac{k}{2r}\delta\tau = \frac{a_2 - a_4}{2} + \frac{b_1 - b_3}{2} - \frac{k}{2}(p_z - q_z + \tau_r), \\
& -\frac{\gamma^2 - 2}{\gamma^2 - 1}\frac{k}{2r}\delta u + \frac{\gamma^2}{\gamma^2 - 1}\delta p = \frac{a_1 + a_2 + a_3 + a_4}{2} - \frac{\gamma^2 - 2}{\gamma^2}\frac{k}{2}(u_r + v_z) + \frac{\gamma^2 - 2}{\gamma^2(\gamma^2 - 1)}\frac{k}{r}u, \\
& \gamma^2 \delta q = \frac{a_1 + a_2 + a_3 + a_4}{2} + \frac{\gamma^2 - 2}{\gamma^2}\frac{k}{2}(u_r - v_z), \\
& \frac{k}{2r}\delta u + \frac{\gamma^2(\gamma^2 - 2)}{3\gamma^2 - 4}\delta p - \frac{\gamma^2(\gamma^2 - 1)}{3\gamma^2 - 4}\delta\sigma = -\frac{k}{r}u, \\
& \gamma\delta\tau = \frac{b_1 - b_2 + b_3 - b_4}{2} - \frac{k}{2\gamma}(u_z + v_r)
\end{aligned}
\tag{15}
$$

where

$$
a_i = -\frac{k}{2} S_1(\alpha_i) - W_1(\alpha_i),
$$

$$
b_i = -\frac{k}{2} S_2(\alpha_i) - W_2(\alpha_i)
$$

$S_1(\alpha_i)$, $W_1(\alpha_i)$, $S_2(\alpha_i)$, $W_2(\alpha_i)$ — the expressions that are determined by the correlation along the b-characteristics (Cliffton, 1968; Sabodash, 1971).

Let's write down these expressions in detail:

$$W_1(O) = (u' - u_1) + (p' - p_1) + (q' - q_1),$$
$$W_1\left(\frac{\pi}{2}\right) = (v' - v_2) + (p' - p_2) - (q' - q_2),$$
$$W_1(\pi) = -(u' - u_3) + (p' - p_3) + (q' - q_3),$$
$$W_1\left(\frac{3}{2}\pi\right) = -(v' - v_4) + (p' - p_4) - (q' - q_4) \tag{16}$$

$$S_1(0) = -\frac{\gamma^2 - 2}{\gamma^2} v_{z1} - \tau_{z1} - \frac{\gamma^2 - 2}{\gamma^2}\left(\frac{u}{r}\right)_1 + \left(\frac{p + q - \sigma}{2}\right)_1,$$
$$S_1\left(\frac{\pi}{2}\right) = -\frac{\gamma^2 - 2}{\gamma^2} u_{r2} - \tau_{r2} - \frac{\gamma^2 - 2}{\gamma^2}\left(\frac{u}{r}\right)_2 - \left(\frac{\tau}{r}\right)_2,$$
$$S_1(\pi) = -\frac{\gamma^2 - 2}{\gamma^2} v_{z3} + \tau_{z3} - \frac{\gamma^2 - 2}{\gamma^2}\left(\frac{u}{r}\right)_3 + \left(\frac{p + q - \sigma}{r}\right)_3,$$
$$S_1\left(\frac{3}{2}\pi\right) = -\frac{\gamma^2 - 2}{\gamma^2} u_{r4} + \tau_{r4} - \frac{\gamma^2 - 2}{\gamma^2}\left(\frac{u}{r}\right)_4 + \left(\frac{\tau}{r}\right)_4 \tag{17}$$

Indices 1, 2, 3, … denote that the variable values are to be taken in the appropriate point (fig. 2). Mention should be made that it is necessary to draw the attention to the fact that r value is not to be considered constant in points 1, 3.

Further, we find the expressions $a_1 + a_3$, $a_1 - a_3$, $a_2 + a_4$, …

$$a_1 + a_3 = -\frac{k}{2}\left[S_1(0) + S_1(\pi)\right] - \left[W_1(0) + W_1(\pi)\right]$$

After performing the corresponding substitutions and grouping we shall obtain the follov-ing:

$$a_1 + a_3 = \frac{\gamma^2 - 2}{\gamma^2} - \frac{k}{2}(v_{z1} + v_{z3}) + \frac{\gamma^2 - 2}{\gamma^2} - \frac{k}{2}\left[\left(\frac{u}{r}\right)_1 + \left(\frac{u}{r}\right)_3\right] + (u_1 - u_3) +$$
$$+ \frac{k}{2}\left[\left(\frac{p + q - \sigma}{r}\right)_1 - \left(\frac{p + q - \sigma}{r}\right)_3\right] + (p_1 + p_3) + (q_1 + q_3) + \frac{k}{2}(\tau_{z1} - \tau_{z3}) - 2(p + q) \tag{18}$$

In the same way other expressions can also be obtained ($a_2 + a_4$, $a_1 - a_3$, ...).

The correlation of the type (18) contains the value of the function and their derivatives in points 1, 2, 3, 4 (fig. 2). It is necessary to express them through the functions and their derivatives in point $O'(r, z)$.

The correlation included in (18) we shall re-arrange in the expressions given below:

$$\begin{aligned}
&f(r+ck,z)-f(r-ck,z),\\
&f(r+ck,z)+f(r-ck,z),\\
&f_r(r,z+ck)-f_r(r,z-ck),\\
&f_r(r,z+ck)+f_r(r,z-ck)
\end{aligned} \tag{19}$$

where $c=\begin{cases}1-\text{for the outer cone,}\\ \dfrac{1}{\gamma}-\text{for the inner cone}\end{cases}$

It is also possible to find the expression obtained as a result of the – commutation of z and r. Function $f(r,z)$ up to Taylor's series round point $O'(r,z)$ chows that expression (19) differ from the values given below in the terms of $O(k^3)$ order:

$$2ckf_r(r,z),\ 2f(r,z)+(ck)^2 f_{rr}(r,z),\ 2ck^2 f_{rz}(r,z),\ 0 \tag{20}$$

After performing the necessary re-arrangements we obtain combined equations for the inner point that differ from the one given in paper (Sabodash, 1971):

$$\begin{aligned}
&\delta u-\frac{k}{2r}\delta p-\frac{k}{2r}\delta q+\frac{k}{2r}\delta\sigma=-\frac{\gamma^2-2}{\gamma^2}\frac{k^2}{2r^2}u+\frac{\gamma^2-2}{\gamma^2}\frac{k^2}{2r}u_r+\frac{k^2}{2}u_{rr}+\\
&+\frac{k^2}{2\gamma^2}u_{zz}+\frac{\gamma^2-1}{\gamma^2}\frac{k^2}{2}v_{rz}+\frac{k}{r}(p+q-\sigma)+k(p_r+q_r+\tau_z),\\
&\delta v-\frac{k}{2r}\delta\tau=\frac{\gamma^2-2}{\gamma^2}\frac{k^2}{2r}u_z+\frac{\gamma^2-1}{\gamma^2}\frac{k^2}{2}u_{rz}+\frac{k^2}{2\gamma^2}v_{rr}+\frac{k^2}{2}v_{zz}+\\
&+k(p_z-q_z+\tau_r)+\frac{k}{r}\tau,\\
&-\frac{\gamma^2+2}{\gamma^2-1}\frac{k}{2r}\delta u+\frac{\gamma^2}{\gamma^2-1}\delta p=\frac{\gamma^2-2}{\gamma^2-1}\frac{k}{r}u+k(u_r+v_z)-\frac{k^2}{2r^2}(p+q-\sigma)+\\
&+\frac{k^2}{2r}(p_r+q_r-\sigma_r+\tau_z)+\frac{k^2}{2}(p_{rr}+q_{rr}+p_{zz}-q_{zz}+2\tau_{rz}),\\
&\gamma^2\delta q=k(u_r-v_z)-\frac{k^2}{2r^2}(p+q-\sigma)+\frac{k^2}{2r}(p_r+q_r-\sigma_r-\tau_z)+\\
&+\frac{k^2}{2}(p_{rr}+q_{rr}-p_{zz}+q_{zz}),\\
&\frac{k}{2r}\delta u+\frac{\gamma^2(\gamma^2-2)}{3\gamma^2-4}\delta p-\frac{\gamma^2(\gamma^2-1)}{3\gamma^2-4}\delta\sigma=-\frac{k}{r}u,\\
&\gamma^2\delta\tau=k(u_z+v_r)+\frac{k^2}{2r}(p_z+q_z-\sigma_z+\tau_r)+\frac{k^2}{2}(2p_{rz}+\tau_{rr}+\tau_{zz})-\frac{k^2}{2r^2}\tau
\end{aligned} \tag{21}$$

While developing the distinctive equation for the surfaces and angular points it should be taken into consideration that some b-characteristics pass beyond the area of investigation and they are to be excluded from combined equations (15).

Given below are the combined equations for left edge surface (fig. 2). From equation (15) a_4 and b_4 must be excluded. For which we shall add the first and the sixth, the second and the third equation and subtract the fourth equation from the second one. And the fifth equation will be left without any changes.

Then we shall obtain:

$$\begin{aligned}
&\delta v-\gamma^2\delta q-\frac{k}{2r}\delta\tau=-\frac{a_1+a_3}{2}+\frac{b_1-b_3}{2}+a_2-\frac{\gamma^2-2}{\gamma^2}\frac{k}{2}(u_r-v_z)-\frac{k}{2}(p_z-q_z+\tau_r),\\
&-\frac{\gamma^2-2}{\gamma^2-1}\frac{k}{2r}\delta u+\delta v+\frac{\gamma^2}{\gamma^2-1}\delta p-\frac{k}{2r}\delta\tau=\frac{a_1+a_3}{2}+\frac{b_1-b_3}{2}+a_2+\\
&+\frac{\gamma^2-2}{\gamma^2(\gamma^2-1)}\frac{k}{r}u-\frac{\gamma^2-2}{\gamma^2}\frac{k}{2}(u_r+v_z)-\frac{k}{2}(p_z-q_z+\tau_r),\\
&\delta u-\frac{k}{2r}\delta p-\frac{k}{2r}\delta q+\frac{k}{2r}\delta\sigma+\gamma\delta\tau=\frac{a_1-a_3}{2}+\frac{b_1+b_3}{2}+\\
&+b_2-\frac{k}{2}(u_z+v_r)-\frac{k}{2}(p_r+q_r+\tau_z),\\
&\frac{k}{2r}\delta u+\frac{\gamma^2(\gamma^2-2)}{3\gamma^2-4}\delta p-\frac{\gamma^2(\gamma^2-1)}{3\gamma^2-4}\delta\sigma=-\frac{k}{r}u
\end{aligned}\tag{22}$$

As the values of a_1+a_3, a_1-a_3, ... are known it is possible to find a_1, a_2, a_3, a_4, … . In so doing it is necessary to bear it in mind that he re-arrangements must provide the accuracy of $O(k^3)$.

The combined equation s for an arbitrary point on the left edge surface (with the exclusion of angular points) are obtained from the equation (22) by substituting into them expressions a_1+a_3, a_1-a_3, ...:

$$\begin{aligned}
&\delta v-\gamma^2\delta q-\frac{k}{2r}\delta\tau=-k(u_r-v_z-p_z+q_z-\tau_r)+\frac{\gamma^2-2}{\gamma^2}\frac{k^2}{2r}u_z+\\
&+\frac{\gamma^2-1}{\gamma^2}\frac{k^2}{2}u_{rz}+\frac{k^2}{2}(v_{zz}-p_{rr}-q_{rr}+p_{zz}-q_{zz})+\frac{k^2}{2\gamma^2}v_{rr}+\\
&+\frac{k^2}{2r^2}(p+q-\sigma)-\frac{k^2}{2r}(p_r+q_r-\sigma_r-\tau_z)+\frac{k}{r}\tau,\\
&-\frac{\gamma^2-2}{\gamma^2-1}\frac{k}{2r}\delta u+\delta v+\frac{\gamma^2}{\gamma^2-1}\delta p-\frac{k}{2r}\delta\tau=\frac{\gamma^2-2}{\gamma^2-1}\frac{k}{r}u+k(u_r+v_z+p_z-q_z+\tau_r)+\\
&+\frac{\gamma^2-2}{\gamma^2}\frac{k^2}{2r}u_z+\frac{\gamma^2-1}{\gamma^2}\frac{k^2}{2}u_{rz}+\frac{k^2}{2\gamma^2}v_{rr}+\frac{k^2}{2}(v_{zz}+p_{rr}+q_{rr}+p_{zz}-q_{zz}+2\tau_{rz})-
\end{aligned}\tag{23}$$

$$-\frac{k^2}{2r^2}(p+q-\sigma)+\frac{k^2}{2r}(p_r+q_r-\sigma_r+\tau_z)+\frac{k}{r}\tau,$$

$$\begin{aligned}
&\delta u-\frac{k}{2r}\delta p-\frac{k}{2r}\delta q+\frac{k}{2r}\delta\sigma+\gamma\delta\tau=-\frac{\gamma^2-2}{\gamma^2}\frac{k^2}{2r}u+\frac{\gamma^2-2}{\gamma^2}\frac{k^2}{2r}u_r+\\
&+\frac{k^2}{2}u_{rr}+\frac{k^2}{2\gamma^2}u_{zz}+\frac{k}{\gamma}(u_z+v_r)+\frac{\gamma^2-1}{\gamma}\frac{k^2}{2}v_{rz}+\frac{k}{r}(p+q-\sigma)+\\
&k(p_r+q_r+\tau_z)+\frac{k^2}{2\gamma r}(p_z+q_z-\sigma_z+\tau_r)+\frac{k^2}{2\gamma}(2p_{rz}+\tau_{rr}+\tau_{zz})-\frac{k^2}{2\gamma r^2}\tau,\\
&\frac{k}{2r}\delta u+\frac{\gamma^2(\gamma^2-2)}{3\gamma^2-4}\delta p-\frac{\gamma^2(\gamma^2-1)}{3\gamma^2-4}\delta\sigma=-\frac{k}{r}u
\end{aligned}\tag{23}$$

In order to obtain closed combined equations it is necessary to write down two more equation that we shall equate under the edge conditions.

In the same way it is possible to obtain an equation for other surfaces.

When developing the combined equations for angular point number 1 (fig. 2) from equation (15), a_1, b_1, a_4, b_4 are necessary to be excluded, as these b-characteristics pass beyond the area of investigation. In so doing two equations are obtained which for some type of edge condition may be insufficient (Cliffton, 1968). An additional (the third) equation can be obtained directly from the ratios on b-characteristics.

After performing all the re-arrangements the combined distinctive equation for the first angular point will be like:

$$\begin{aligned}
&\left(1+\frac{\gamma^2-2}{\gamma^2-1}\right)\delta u-\delta v-\left(\frac{\gamma^2}{\gamma^2-1}+\frac{k}{2r}\right)\delta p-\frac{k}{2r}\delta q+\frac{k}{2r}\delta\sigma+\left(\gamma+\frac{k}{2r}\right)\delta\tau=\\
&=-\left(\frac{\gamma^2-2}{\gamma^2-1}\frac{k}{r}+\frac{\gamma^2-2}{\gamma^2}\frac{k^2}{2r^2}\right)u-k(u_r+v_z+p_z-q_z-p_r-q_r+\tau_r-\tau_z)+\\
&+\frac{\gamma^2-2}{\gamma^2}\frac{k^2}{2r}(u_r-u_z)+\frac{k}{\gamma}(u_z+v_r)+\frac{k^2}{2}(u_{rr}-p_{rr}-q_{rr}-v_{zz}-p_{zz}+q_{zz}-2\tau_{rz})+\\
&+\frac{k^2}{2\gamma^2}(u_{zz}-v_{rr})-\frac{\gamma^2-1}{\gamma^2}\frac{k^2}{2}(u_{rz}-v_{rz})+\frac{k}{r}\left(1+\frac{k}{2r}\right)\times(p+q-\sigma)-\\
&-\frac{k^2}{2r}(p_r+q_r-\sigma_r+\tau_z)+\frac{k^2}{2\gamma r}(p_z+q_z-\sigma_z+\tau_r)+\frac{k^2}{2\gamma}(2p_{rz}+\tau_{rr}+\tau_{zz})-\frac{k}{r}\left(1+\frac{k}{2\gamma r}\right)\tau,\\
&\delta u+\delta v-\frac{k}{2r}\delta p-\left(\gamma^2+\frac{k}{2r}\right)\delta q+\frac{k}{2r}\delta\sigma-\frac{k}{2r}\delta\tau=-\frac{\gamma^2-2}{\gamma^2}\frac{k^2}{2r^2}u-\\
&-k(u_r-p_r-q_r-\tau_r-v_z-p_z+q_z-\tau_z)+\frac{\gamma^2-2}{\gamma^2}\frac{k^2}{2r}(u_r+u_z)+
\end{aligned}\tag{24}$$

$$+\frac{k^2}{2}(u_{rr}-p_{rr}-q_{rr}+v_{zz}+p_{zz}-q_{zz})+\frac{k^2}{2\gamma^2}(u_{zz}+v_{rr})+\frac{\gamma^2-1}{\gamma^2}\frac{k^2}{2}(u_{rz}+v_{rz})+$$
$$+\frac{k}{r}\left(1+\frac{k}{2r}\right)\cdot(p+q-\sigma)-\frac{k^2}{2r}(p_r+q_r-\sigma_r-\tau_z)+\frac{k}{r}\tau-\frac{k^2}{\gamma}(\tau_{rr}-\tau_{zz}), \tag{24}$$
$$\frac{k}{2r}\delta u+\frac{\gamma^2(\gamma^2-2)}{3\gamma^2-4}\delta p-\frac{\gamma^2(\gamma^2-1)}{3\gamma^2-4}\delta\sigma=-\frac{k}{r}u$$

The next three (or four) equations we shall obtain under the edge condition. By analogy it is possible to equate for other angular points.

4. Numerical Presentation of Partial Derivatives

The left equations (21), (23), (24) members include partial derivatives which are to be expressed as a distinctive one. With the view of providing the accuracy of the problem solution of the order $O(h^3)$ it is necessary and sufficient that the remainder in the distinctive formulas for calculating the first and second derivatives be $O(h^2)$ and $O(h)$ respectively. Such accuracy will be achieved by means of a three-point diagram. It is better to use centered differences. In case if some of the points happen to appear on the boundary of the area of investigation it is necessary to use one-sided differences.

Let's write down the formulas for calculating partial derivatives using centered differences

$$f_r(m,n)=\frac{1}{2\Delta r}\left[f(m+1,n)-f(m-1,n)\right]-O(\Delta r^2),$$
$$f_{rr}(m,n)=\frac{1}{\Delta r^2}\left[f(m+1,n)-2f(m,n)+f(m-1,n)\right]-O(\Delta r^2), \tag{25}$$
$$f_{rz}(m,n)=\frac{1}{4\Delta r\Delta z}\left[f(m+1,n+1)-f(m-1,n+1)-f(m+1,n-1)+f(m-1,n-1)\right]$$

And with the use of one-sided differences

$$f_r(m,n)=\frac{1}{2\Delta r}\left[-3f(m,n)+4f(m+1,n)-f(m+2,n)\right]+O(\Delta r^2),$$
$$f_{rr}(m,n)=\frac{1}{\Delta r^2}\left[f(m,n)-2f(m+1,n)+f(m+2,n)\right]-O(\Delta r), \tag{26}$$
$$f_{rz}(m,n)=\frac{1}{2\Delta r\Delta z}\left[f(m+1,n+1)-f(m+1,n-1)-f(m,n+1)+f(m,n-1)\right]$$

for the points of surface $m=1$, $1<n<n_f$

$$f_{rz}(m,n) = \frac{1}{\Delta r \Delta z}\left[f(m+1,n+1) - f(m+1,n) - f(m,n+1) + f(m,n)\right]$$

for angular point $m = 1$, $n = 1$.

By changing r to z it is possible to write down the formulas for determining the derivatives using independent variable z.

It may seem that the higher is the order of the distinctive formulas used to find the derivatives the more accurate will the general solution be. However, as is seen from the numerous experiments, the use of distinctive diagrams of a higher accuracy order is not expedient. Equation (15) accuracy as well as the calculation accuracy of derivatives must correlate. The violation of this principle will lead to either instability or the lessening of problem solution accuracy.

5. Numerical Example. The Investigation of the Stress Caused Stuck Drill Pipe Zone

Equations (21), (23), (24) and others together with the boundary conditions form closed combined equations which can be solved relatively functions increase. Using the worked out algorithm the program has been created which enables to find the drill pipe stress caused state parameter at fixed time as well as stress and displacements (as a time function) at pre-set cylinder points. With the view of checking up the accuracy and stability of the problem solution at every step according to time the total cylinder strength and the work done which were compared, and in this way the error was determined.

The calculation was performed using the following initial data:

Pressure impulse amplitude $p_m = 0.2 \cdot 10^6$ Pa; impulse increase time $\theta_1 = 20 \cdot 10^{-6}$ s; representative pressure drop time $\theta_2 = 80 \cdot 10^{-6}$ s; Young module $E = 0.19 \cdot 10^{-6}$ MPa; Pausson coefficient $v = 0.27$; drill pipe material density $\rho = 0.774 \cdot 10^4$ kg/m^3; stuck medium material density $\rho_0 = 0.2 \cdot 10^4$ kg/m^3; upper stuck boundary $H = 1.000$ m; horizontal burst out open coefficient $A = 0.57$; stuck section length $l = 10$ m; rock rigidity coefficient $k_0 = 50 \cdot 10^6$ N/m.

Pipes geometric characteristics are given in Table 1.

TABLE 1

Pipes geometric characteristics

Cipher	External diameter *D*, mm	Internal diameter *d*, mm	Wall thickness Δ, mm
DC-178	178	80	49
DC-203	203	80	61.5
SDP-114x10	114	94	10
SDP-140x10	140	120	10

Calculation results show that the circular stress amplitudes lessening on the inner surface $(r = r_1)$ of the stuck pipe leads to the change in the dependence mode

During the impulse acting time (0.2 millisecond) stress amplitudes σ_θ are rather great and after the impulse action ceases the process become stable with the amplitude three times less.

The process is in the same way described by means of dependence diagrams $u_r(t)$. For the period of the impulse action the pipe "breathes" with the amplitudes 5-6 times greater than in the oscillations stable mode. This is what contributes to releasing the stuck drilling string zone.

Studying the diagrams in figures 3, 4 we can see pipe wall thickness Δ substantially effects the stress axial σz and circular σ_θ amplitudes on the pipe outer wall. Thus, for drill collars (DC) the above mentioned stresses prove to be 5-6 times less than the corresponding stresses for steel drill pipes (SDP).

The radial displacements u_r of the pipe outer surface with the pre-set excitation parameters (fig. 5) show considerably bigger values for SDP-114×10, SDP-140×10 if compared to DC-178, DC-203, which proves the possibility of destructing the stringy overlapping in these cases.

The appearance of subharmonics in the diagrams $\sigma_\theta(r_2, t)$, $u_r(r_2, t)$ may be accounted for the waves reflecting from the outer and inner surfaces of the pipe.

Time change diagrams of dimensionless cylinder strength (fig. 6) are used for the estimation of the wave process converging. It is clear that the less is the pipe rigidity the greater energy it is able to accumulate. By comparing the total cylinder self-energy with the work done by the shock wave to deform the cylinder the calculation error can be estimated (in our case it was not more than 2.3%). As diagrams $W(t)$ are given within the space of time equal to 1 msec., they so far

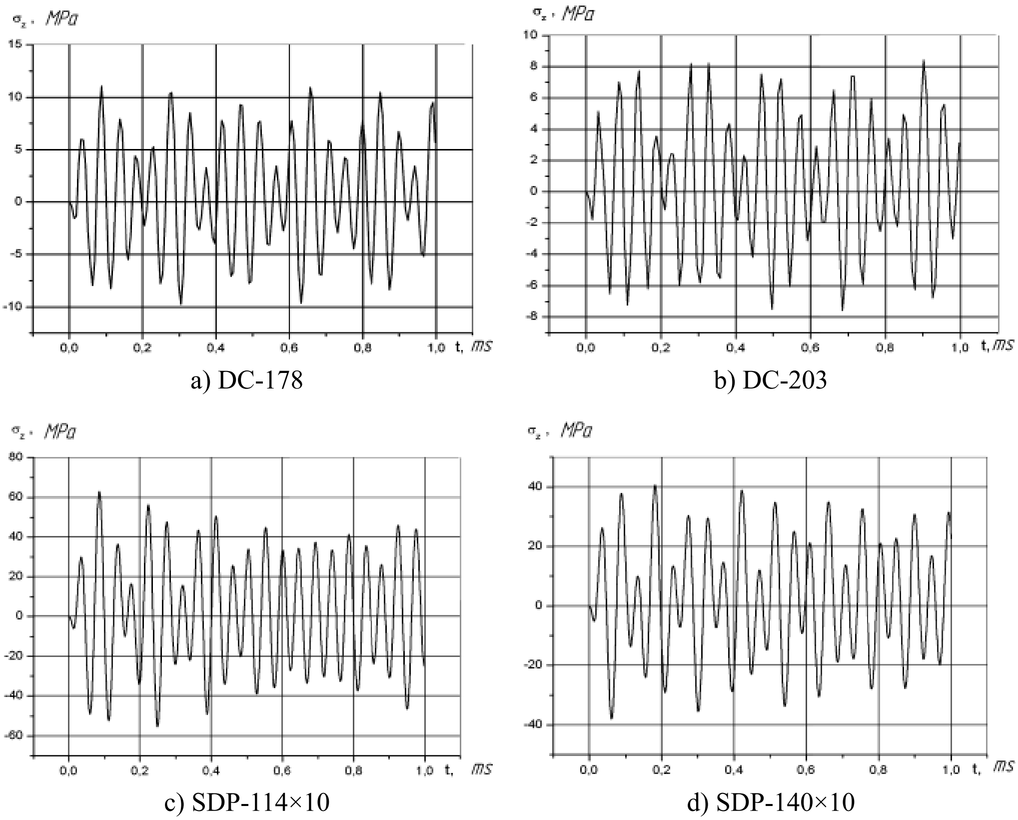
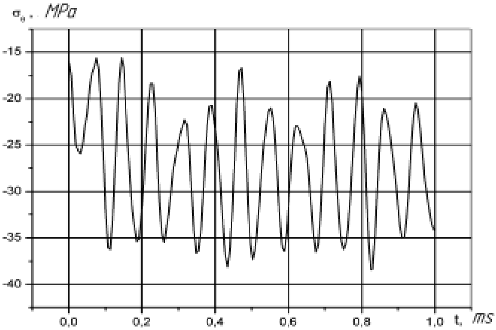
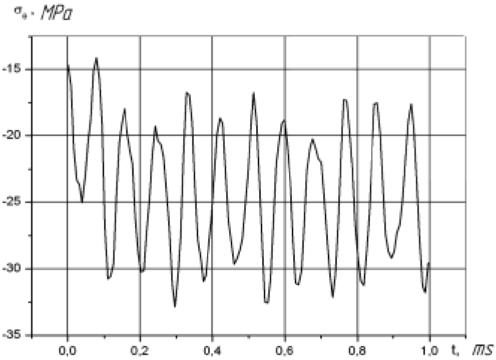


Fig. 3. Time change diagrams of axial stress σ_z on the stuck pipe outer surface

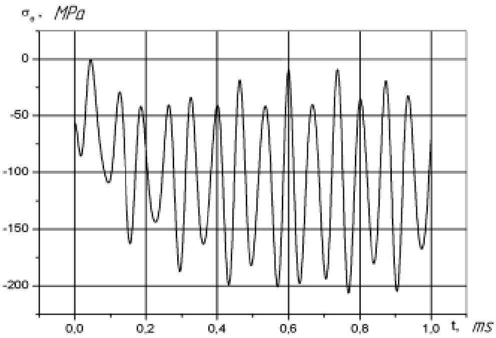
don's show the representative process damping which is stipulated by the partial loss of energy to do the work aimed at stringy lessening that is overlapped by stuck medium. And in case the process happens to be divergent, a sharp increase in the total self-energy and the availability of imbalance between energy and work is observed.



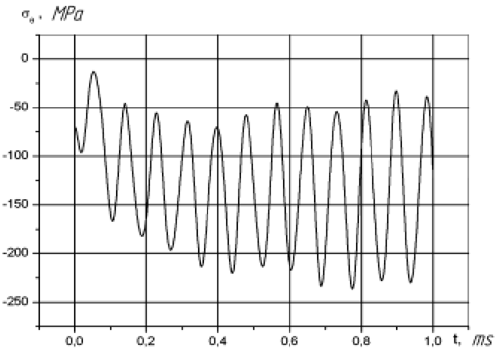
a) DC-178



b) DC-203

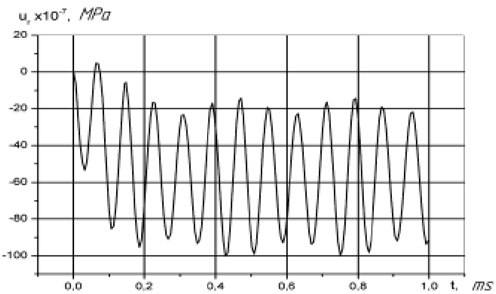


c) SDP-114×10

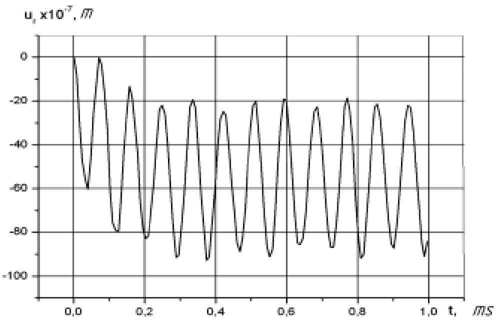


d) SDP-140×10

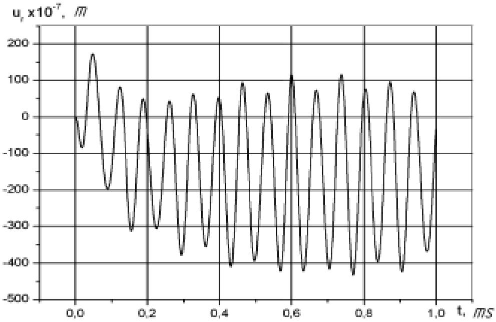
Fig. 4. Time change diagrams of circular stress σ_θ on the stuck pipe outer surface



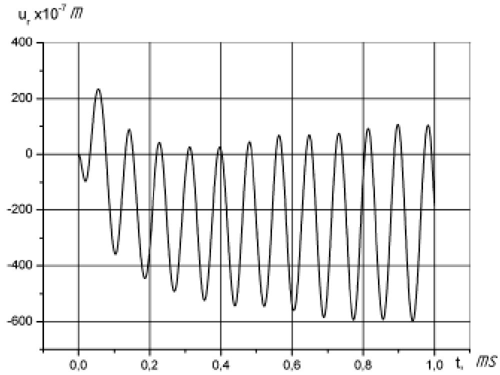
a) DC-178



b) DC-203

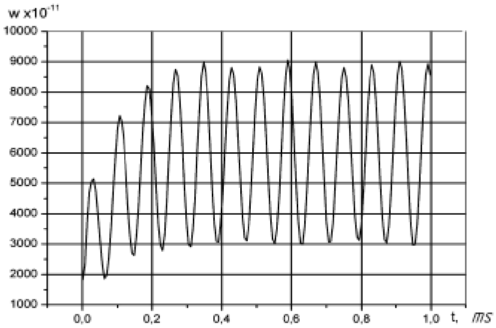


c) SDP-114×10

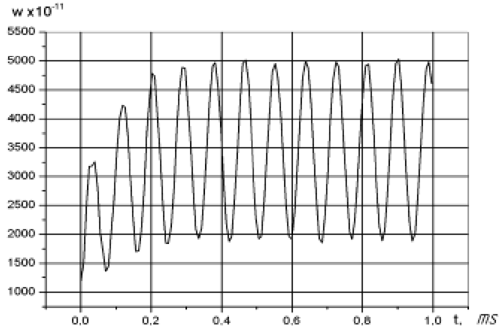


d) SDP-140×10

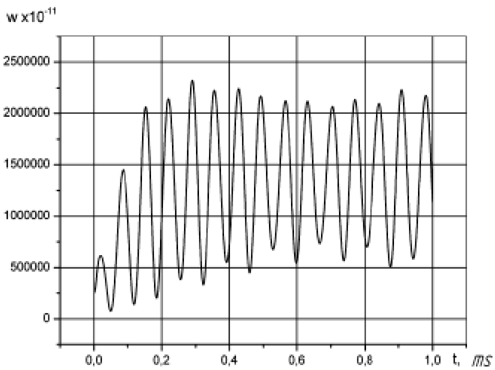
Fig. 5. Time change diagrams of radial displacements u_r of the stuck pipe outer surface



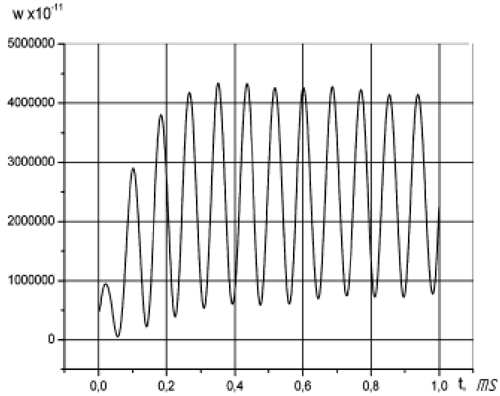
a) DC-178



b) DC-203



c) SDP-114×10



d) SDP-140×10

Fig. 6. Time change diagrams of dimensionless total self-energy W of the stuck pipe

6. Conclusions

The rotationally symmetrical problem of dynamic elasticity of circular thickwalled cylinder using the spatial characteristics technique is solved.

The stress-caused deformation state of the stuck drilling string zone under the action of shock wave upon the pipe inner wall is estimated. In the time change diagrams of the pipe outer surface radial displacements and the circular stress on the latter, the appearance of subharmonic is observed which is accounted for the elastic waves reflecting from the outer and inner surfaces of the pipe.

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