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The Simulation of Vibrations of Railway Beam Bridges in the Object-oriented Environment Delphi

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Abstract

The peculiarities of combination of finite-element method and equations of solid dynamics, the basic stages of development of the program complex Belinda for calculation of statics and dynamics of the rods constructions as applied to railway bridges are described.

Keywords: bridge span dynamics, locomotive, Delphi, FEM, CAE, Belinda

1. Introduction

The introduction of computer technologies in the sphere of scientific, engineering and productive human activities became the in essence solution of a number of problems related to the automation of calculating processes. There is a great quantity of various programs, from comparatively small ones to difficult and expensive program complexes, meeting practically any needs of user. It is possible to assert the possibilities of modern software correspond in full measure to powers of computer techniques, opening progressively new prospects for the design, calculation and analysis of various physical processes and phenomena.

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In the field of calculation of building constructions the specialized software – computer-aided design systems – is used [1, 2] divided into three basic types: CAD (computer-aided design), CAM (computer-aided manufacturing) and CAE (computer-aided engineering).

In practice there is often a necessity for combination of the indicated technologies. E.g., an engineer-designer, engaged in the design of bridge constructions, frequently needs in the presence of not only a system of automated development of drafts (CAD) but also a system, by means of which for the elements of this construction the stressed-and-strained state can be determined and the verification on the limiting states is realizable (CAE).

Thus, during the last period it becomes reasonable the appearance of the so-called hybridcomputer-aided design systems, having available not only the developed visualization facilities but the high calculation potential as well (Ansys, Nastran, Adams, UM).

Nevertheless, many everyday engineer's tasks have narrowed enough specialization and do not require bringing in of universal calculation complexes for solving them. In such cases it is possible either to resort to the mathematical simulation or to take advantage of small profile software packages. The functionally profile CAD's are considerably simpler than universal calculation complexes and more flexibly take into account design features an end user needs. For example, the computer-aided design packages for building constructions, after determination of parameters of the system stressed-and-strained state offer to the user some prepared (library) solutions, e.g., assortment data, recommendations on mounting etc. Thus there is no need in long-term training at the special-purpose courses or studying programming languages by end user, because the interface of the profile programs, as a rule, is maximally adapted to application in a certain technical area and operates with characteristic for this area terms and concepts [3]. The calculation packages of both domestic (SCAD, Lyra, Monomah, Sapphire) and foreign (Abacus, Robot, Catia, and others) development can serve as examples.

In works [2, 4, 5] the fundamental approaches to development of computer-aided design systems (CAE) with the use of geometric modeling and numerical calculation methods are considered in detail. An original approach to realization of the CAD system for determination of stressed-and-strained state of a transport structure, wherein the finite-element method is used as a calculation basis, is proposed by authors in [4]. A module of geometrical design visualizing the designed object geometry is an integral part of every modern CAD system. The principles of construction of the modules of geometrical design on the basis of parametric curves, surfaces, bodies, static and dynamic models with the use of PLaSM programming language are considered in work [6]. Must be states the importance of presence of the specialized interface or high level problem-oriented language [7] in the calculation program as well as its close integration with other complexes through information-and-logical models [8].

However it is necessary noted that among modern computer-aided design systems, in particular, calculation CAE complexes there is a comparatively small number of electronic products oriented to the dynamic calculation of bridge constructions. In this article the authors present the basic results on development of software complex Belinda for calculation of statics and dynamics of the rod constructions (as it is applicable to railway bridges) based upon the finite-element method and the equations of solid dynamics.

2. Formation of Finite-Element Model of Span

Let us consider the metallic beam span of single-track railway bridge 33.6 m in length (Fig. 1, a). The cross-section of bridge span consists of two main beams combined by a system of longitudinal, transversal and diagonal connections. In the middle part of bridge span every beam has 2.61 m in height of a vertical sheet, horizontal sheets are situated symmetrically and have the following sizes: 25×490 mm (internal) and 40×590 mm(external).

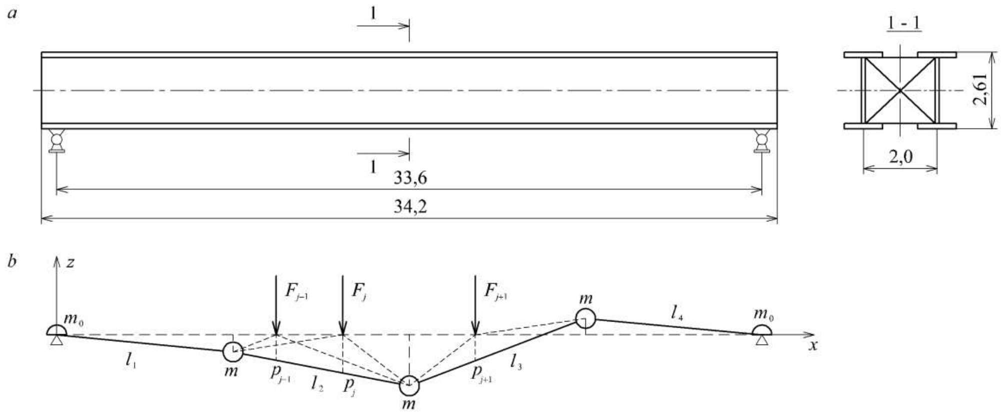

Fig. 1. Railway bridge span and its rod model

Let us accept the model of this span as a discrete design model (Fig. 1, b). We divide a construction into four sections of equal length, replace the main beams of span with a system of weightless elastic rods l_j = 8.4 m in length connected in nodes with numbers $i = 1,\ 2, \ldots,\ n$. We bind the local right-side coordinate system $O_{c,\,j}$ to every rod j, $j = 1, 2, \ldots, n-1$. For all the construction we define the global coordinate system O (Fig. 2, a).

The rod element presented in Fig. 2 works on deflection in planes xy, zx, on tension-compression and twisting in relation to a longitudinal axis x. The moments of inertia of cross section of rod j are as follows: J_y=0.131 m^4 in relation to main central axis y (vertical deflection), J_z = 7.44$\times$10^{-3}m^4 in relation to axis z (horizontal

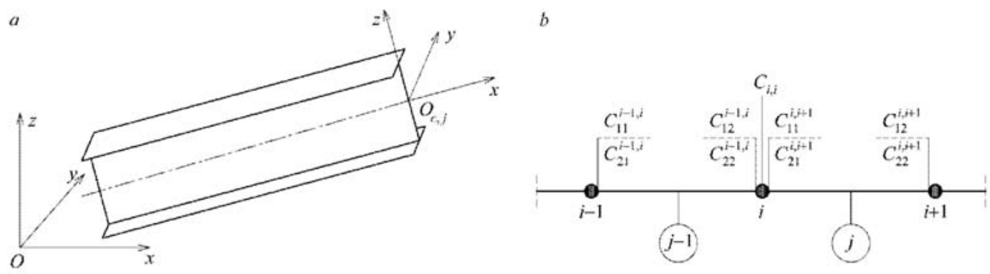

Fig. 2. Orientation of rod and assembling of stiffness matrix

deflection), $J_x = 3.2 \times 10^{-5}\text{m}^4$ in relation to axis x (twisting). The area of section A=0.102 m^2, beam material modulus of elasticity equals $E = 2.1\times10^{11}$ Pa, the shear modulus is $G = 7.8\times10^{10}$ Pa. It is assumed that in the system under consideration all the rods are geometrically and physically linear, the hypothesis of Bernoulli and the Saint-Venant principle are satisfied.

In the three-dimensional statement i-th node of construction, which is free of the kinematic fixings, possesses six degrees of freedom and its position in the global coordinate system at any time moment can be described by a vector with six elements $Z^{(i)}$. In the direction of these translations the main vector of node forces $R^{(i)}$ is formed:

$$Z^{(i)} = \left\{x \;\; y \;\; z \;\; \phi_x \;\; \phi_y \;\; \phi_z\right\}; \quad R^{(i)} = \left\{F_x \;\; F_y \;\; F_z \;\; M_x \;\; M_y \;\; M_z\right\}. \tag{1}$$

For determination of translations and forces in the end sections of rod j connecting nodes i and $(i + 1)$ we define the corresponding sectional matrices

$$Z_j = \begin{bmatrix} Z_j^{(i)} \\ \hline Z_j^{(i+1)} \end{bmatrix}; \quad R_j = \begin{bmatrix} R_j^{(i)} \\ \hline R_j^{(i+1)} \end{bmatrix}, \tag{2}$$

where an overhead index (in brackets) is related to the number of node, and a lower index – to the number of rod.

Each of matrices (2) is built by combining two corresponding matrices (1) and has an order 12×1. The connection between translations of the i-th end of rod and the forces in this section is described by linear equation

$$R_j^{(i)} = C_j^{(i)} Z^{(i)}, \tag{3}$$

where $C_j^{(i)}$ is a stiffness 6×6 square matrix for the i-th section of rod.

For both ends of rod we have

$$R_j = C_j Z_j, \tag{4}$$

where C_j is a stiffness 12×12 block square matrix in the form

$$C_j = \begin{bmatrix} C_{11} & | & C_{12} \\ ------ & | & ------ \\ C_{21} & | & C_{22} \end{bmatrix}. \tag{5}$$

Each of four embedded 6×6 matrices (5) contains the reactions to single translations of the end of rod j in its local coordinate system $O_{c,\,j}$. This is a stiffness matrix of joint (three-dimensional) element of rod [9] and matrix C_{11} can be represented in the following form:

$$C_{11} = \begin{bmatrix} \frac{EA}{l} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12EJ_z}{l^3} & 0 & 0 & 0 & \frac{6EJ_z}{l^2} \\ 0 & 0 & \frac{12EJ_y}{l^3} & 0 & -\frac{6EJ_y}{l^2} & 0 \\ 0 & 0 & 0 & \frac{GJ_x}{l} & 0 & 0 \\ 0 & 0 & -\frac{6EJ_y}{l^2} & 0 & \frac{4EJ_y}{l} & 0 \\ 0 & \frac{6EJ_z}{l^2} & 0 & 0 & 0 & \frac{4EJ_z}{l} \end{bmatrix}. \tag{6}$$

Every element of general stiffness matrix C of the rod system is calculated as an algebraic sum of separate stiffnesses of rods connected in the given node. E.g., for node i, adjacent to two nearby rods $j-1$ and j (Fig. 2, b), the corresponding element of general stiffness matrix looks like

$$C_{i,\,i} = \left[\begin{array}{c:c:c} \begin{matrix} C_{11}^{i-1,i} \\ --- \\ C_{21}^{i-1,i} \end{matrix} & 0 & 0 \\ \hdashline 0 & \begin{matrix} C_{12}^{i-1,i} + C_{11}^{i,i+1} \\ ----- \\ C_{22}^{i-1,i} + C_{21}^{i,i+1} \end{matrix} & 0 \\ \hdashline 0 & 0 & \begin{matrix} C_{12}^{i,i+1} \\ --- \\ C_{22}^{i,i+1} \end{matrix} \end{array}\right]. \tag{7}$$

In canonical equations of displacement method the external generalized forces, concentrated in the nodes of the discrete mechanical system, are considered as known [10]. However a system can also go into the stressed-and-strained state under the influence of such factors as the forced translations of its separate points (vertical, horizontal displacements of nodes, turns of sections, etc.). In this case the displacements of nodes are the sought-for quantities and must be set as unknown. Therefore let us divide the kinematic parameters of this mathematical model into two groups – the desired parameters determined by solution of system of displacement method canonical equations and the prescribed parameters determined by solution of motion equations system or by initial construction geometry. The first group parameters can be obtained, as a rule, as a result of static calculation of the system, while the second group ones - as a result of direct integration of the system of motion equations. These parameters belong to the factors of kinematic disturbance of the system and are possible transformed into equivalent node forces. Using dependence (4) and taking into account that the main vector of node reactions is equal by absolute value to the main vector of forces from the external load and is opposite to it by sign, for rod j we have

$$F_{j,\Delta} = -R_j = -C_j \cdot \Delta_j, \tag{8}$$

where Δ_j is a vector-column of the forced displacements of both ends of the j-th rod in the global coordinate system.

For all the rod system let us write down

$$F_{\Delta} = -C \cdot \Delta. \tag{9}$$

Then the forces F_{Δ}, which are equivalent to the forced displacements, are to be put to the nodes of construction together with other external loads and influences F_0, and the system of canonical equations of displacement method in matrix form gets the following form:

$$Z = L\,(F_{\Delta} + F_0)\,; \quad L = C^{-1}, \tag{10}$$

where L is a yielding matrix.

If we designate the vector of total node loads as:

$$F = F_{\Delta} + F_0, \tag{11}$$

then equation (10) is as follows:

$$Z = LF. \tag{12}$$

The order of matrices Z, C, L, F depends on the quantity of the system nodes n. In order the system of equations will cease to be degenerate, from all matrices (12) we will exclude rows and columns corresponding to the kinematic fixings of nodes:

$$\bar{Z} = \bar{L}\bar{F}; \quad \bar{L} = \bar{C}^{-1}, \tag{13}$$

where a line above a matrix means the reduced form of this matrix.

Using relations (4) and (13), let us find the values of internal forces (reactions) in the system nodes:

$$\tilde{R} = C\left(\bar{L}\bar{F}\right), \tag{14}$$

where in matrix $\tilde{R}$ adding to it the columns and rows eliminated before with completion of them by zero elements.

Dependences (12), (14) allow finding force and deformation factors in all nodes of the discrete system of rods, which corresponds to the static calculation of construction. For realization of dynamic calculation it is necessary to set a time interval in the range $t_0 \dots t_k$, to choose initial conditions and conduct a preliminary static calculation, to define the sizes of stiffness and yielding matrices, etc. Then it is possible to come to integration of the motion equations of the system nodes with application of the set static and dynamic forces to the construction. In the following section it is demonstrated how this approach is realized in the program complex Belinda.

3. Application of the Object-oriented Programming for Calculation of Statics and Dynamics of the Elastic System of Rods

The Delphi environment for object-oriented programming is a rapid and effective tool of software development for use in the wide range of different applications. Based on high-level algorithmic-strict language Object Pascal, the Delphi environment allows in full measure involving all potential of the modern personal computers and multiprocessor computer stations for solution of calculation tasks.

As other programming languages, Object Pascal operates the set of fundamental data types. Most essential for scientific and engineering calculations among them are numeric data, in particular, type of real number “Extended” having the range $3.6 \times 10^{-4951} \dots 1.1 \times 10^{4932}$; at design of mechanical systems it allows determining the objects of large mass and stiffness.

The program complex Belinda functions under the Microsoft Windows 32-bit operating system. In general case, there are no restrictions on the type of processor and operating memory used. However for effective realization of dynamic calculations the Intel Pentium III or AMD Athlon and higher processors, with no less than 1 GB of available RAM are recommended. The complex Belinda (Based on Euler-Lagrange equations Integrated for Non-linear Dynamics Application) serves for the calculation of nonlinear dynamics on the basis of intergrable equations of Euler-Lagrange.

Functionally the complex is divided into three blocks, characteristic for the majority of CAE programs of this class: design (preprocessor), calculation (processor), analysis of results (post-processor). The first block is served by the elements of interface responsible for development of structural model of the construction by means of geometrical modeling. An initial analytical model as a batch file can be prepared in any text editor.

The mathematical support of calculations is realized in the modules of complex Matrixes.pas, UnitMath.pas. The first module is responsible for forming, work and demounting of objects of data class TMatrix, TVector (matrix and vector of real numbers accordingly), different mathematical procedures and functions are declared in the second module.

The procedures for modeling and static calculation of the elastic system of rods are described in the FEM.pas module. The topology of data classes of this module is presented in Fig. 3.

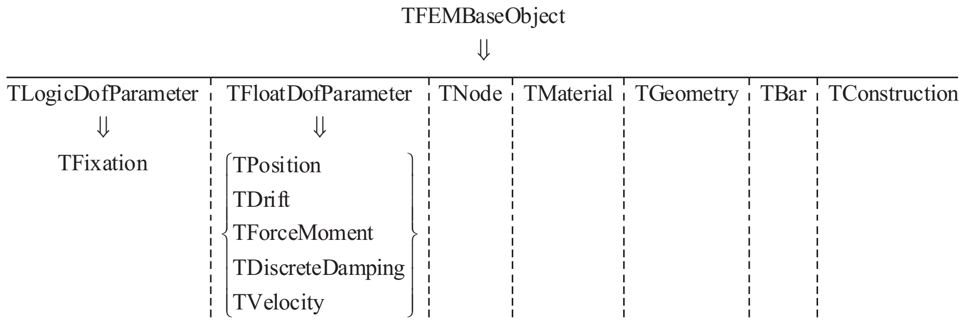

Fig. 3. FEM.pas topology of data classes

In the data class TFEMBaseObject the base procedures and functions are declared for work with the objects of structural model: creation of text mark of object, presentation of parameters of object as a matrix, etc. The classes TLogicDofParameter and TFloatDofParameter contain six Boolean and real components, respectively. The data classes for description of the kinematic fixings of node (TFixation class), position of node (TPosition), forced displacements of node (TDrift), generalized force factor (TForceMoment), matrix of viscous resistance coefficients (TDiscreteDamping), vector of node velocity (TVelocity) are based on these classes. The data classes TMaterial and TGeometry describe the properties of material and geometry of rod, respectively. The data class TNode designs the node of rods system, the class TBar is an elastic rod.

The independent TConstruction data class describes basic properties of the rods system, manages the lists of nodes (Nodes) and rods (Bars), and also realizes the functions and procedures of its static calculation by the finite-element method; their topology is presented in the following graph (Fig. 4).

The structural block of Matrixes in the TConstruction data class contains 10 basic matrices (TMatrix data class) for realization of static calculation of the rods system: StiffMatrixSource is a general stiffness matrix of the system without the account of the kinematic fixings; StiffMatrix is the same taking into account fixings; ComplMatrix is a yielding matrix of the system; NodalDisp is a vector of the node displacements; FM is a vector of the node loads; Dofs is a vector of Boolean constants for description of the kinematic fixings; Drift is a vector of the forced displacements of nodes; NodalReact is a vector of node reactions of the system; DriftToNodalFM is a vector of node forces equivalent to the forced displacements of the system nodes; Epure is a rod stress matrix. These matrices are formed in the corresponding procedures of structural block Calculation. The block Service is responsible for realization of some auxiliary operations: RecalcBarRotationMatrixes is to recalculate the rotation matrices of rods of the elastic system; SetActualFixations is to set the kinematic fixings of nodes; Cut is to transform the stiffness matrix for accounting in it the kinematic fixings by setting to zero of corresponding rows and columns with placing of one on their crossing.

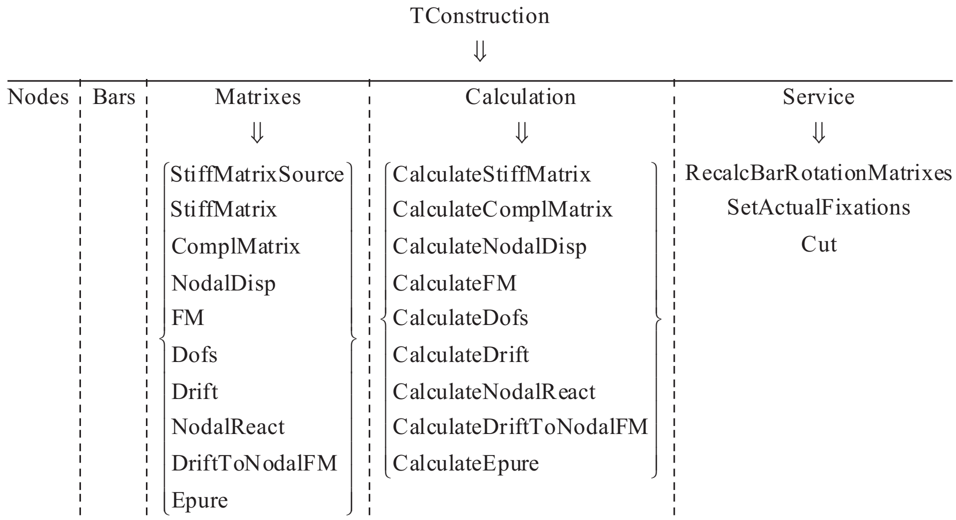

Fig. 4. TConstruction data class properties

The order of procedures call in the block Calculation is extremely important from the viewpoint of efficiency of realization of calculations and to minimize the time used. It does not matter in static calculation, time of which is, as a rule, from a few seconds to a few minutes (for systems with large number of degrees of freedom), but it is substantially meaningful in calculation of dynamics, when for determination of the stressed-and-strained state of the system a static calculation is to be executed repeatedly. In this context, the sequence of call of procedures of block Calculation in the program complex Belinda is determined by the following algorithm (Fig. 5).

For realization of static calculation it is enough to prepare information about the kinematic fixings of the system nodes (procedure SetStaticFixations) and to execute subsequently three stages of calculation: CalculateStage1 – for procedures RecalcBarRotationMatrixes, CalculateDofs, CalculateStiffMatrix, CalculateComplMatrix; CalculateStage2 – for CalculateDrift, CalculateDriftToNodalFM; CalculateStage3 – for DoLoad, CalculateFM, ApplyDriftToNodalFM, CalculateNodalDisp, CalculateNodalReact, CalculateEpure. Here the procedure DoLoad provides the application of the set external loads to the system before forming of them in a general vector in procedure CalculateFM. The procedure ApplyDriftToNodalFM applies to the system nodes the forces equivalent to the forced displacements of nodes, if such are set. On completion of calculation the tools of viewing of results (Postprocessor procedure) become accessible.

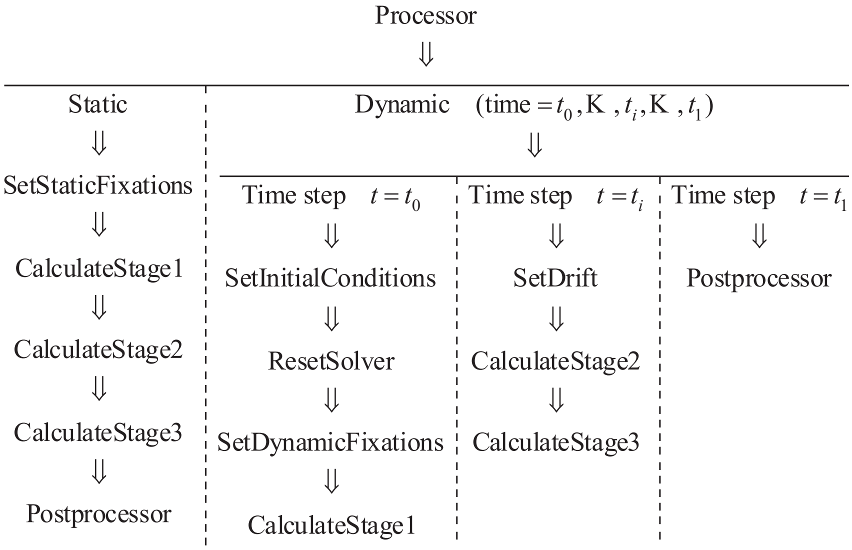

Fig. 5. The algorithm of static and dynamic calculations

As can be seen in Fig. 5, for dynamic calculations there is no need to conduct the first stage of calculations CalculateStage1 at every step of integration. After preparing of initial conditions (SetInitialConditions) and setting to zero for solutions of the system of differential equations (ResetSolver), the actions at the first step of dynamic calculation are analogous to ones actions in static calculation. The stages of calculation CalculateStage2, CalculateStage3 are to be executed at every step of integration.

The conditions, according to which kinematic fixations are superimposed on displacements of system nodes, are different for static and dynamic calculations. For example, in dynamics a node must have kinematic fixing if mass or mass moment of inertia are concentrated in this node. The last circumstance determines participation of kinematic parameters of the node in a general system of equations of motion.

The program complex Belinda is under continuous improvement. In the last version by means of the specialized program objects “contact group” and “dynamic loading” the interaction of the three-dimensional rods system with the series of moving harmonic forces is realized. In doing so, the concentrated force factors are objects independent on the construction, can possess linear velocity and move in the indicated direction. Other types of dynamic loads distributed, impulsive ones as well as the consideration of transient modes of their motion are planned for realization in further versions of the software.

The application of the program complex Belinda for analysis of the forced vibrations of metallic railway bridge spans is described in the following section.

4. Design of the Forced Vibrations of Beam Railway Bridge Spans

The rods model of beam (Fig. 1) is examined at different approaches to modeling of VL8 locomotive line. Such schemes of presentation of locomotive as systems of moving forces are studied: one, two, eight constant forces; one, two, eight permanent vertical forces with dynamic harmonic additions on natural frequency of vertical vibrations of VL8 locomotive are presented (Fig. 6).

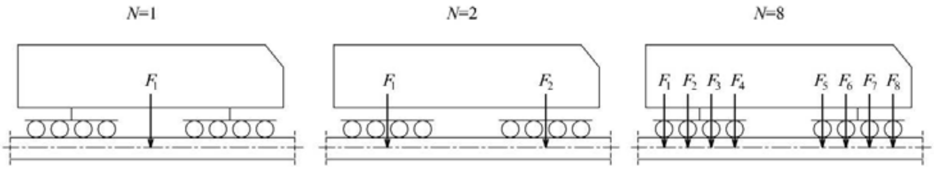

Fig. 6. The schemes of presentation of locomotive as systems of moving forces

The results of dynamic design of railway bridge span at different speed of motion obtained in the program complex Belinda are presented for static forces (Fig. 7) and for static forces with harmonic additions (Fig. 8). Because of motion time at different speeds the difference of axis x position of loads on length of bridge span is shown in Fig. 7, 8.

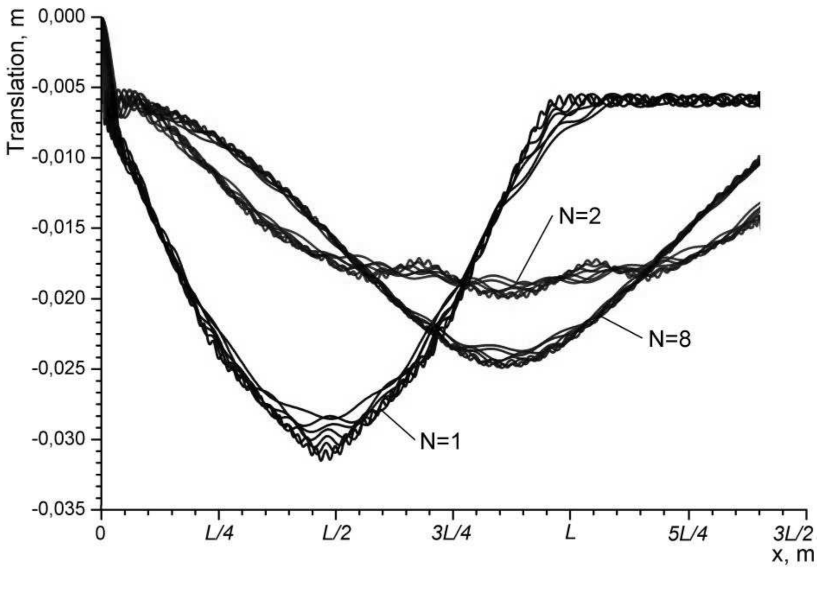

Fig. 7. Bending deflection of middle of span at different speeds of locomotive motion presented by the series of static forces, N quantity of forces

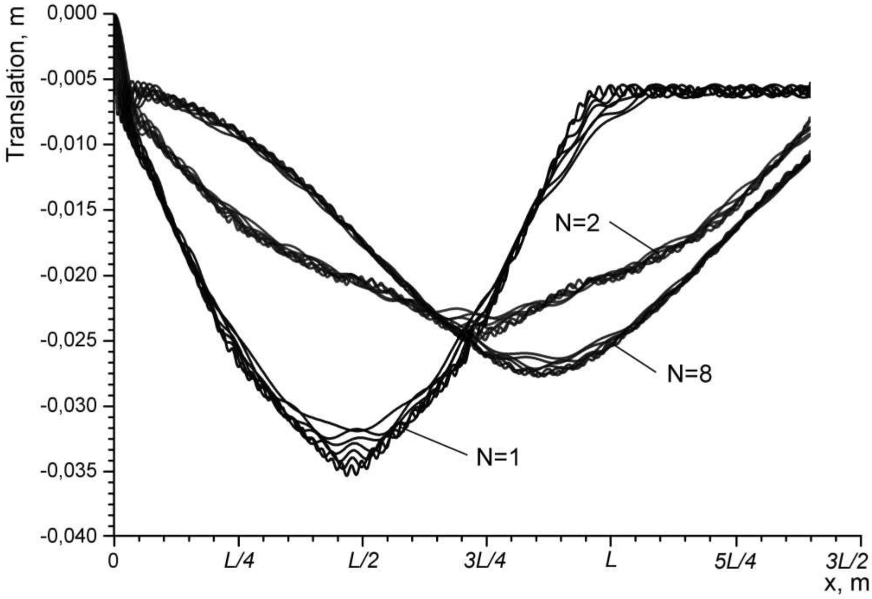

Fig. 8. Bending deflection of middle of span at different speeds of locomotive motion presented by the series of harmonic forces, N quantity of forces

The value of single static vertical moving force is accepted equal to the weight of single VL8 locomotive (F= 1840 kN), harmonic force is set in a form

$$F(t) = F_s (1 + A_1 \sin 2\pi v_1 t + A_2 \sin 2\pi v_2 t), \tag{15}$$

where F_s is a static component, A_i is a dynamic addition of i-th harmonic with linear frequency v_i.

The amplitude of vertical harmonic moving force is accepted equal to the weight of single locomotive VL8 with harmonics equal to vertical frequencies of the spring suspension of bogies. F_s=1840 kN, A_1=0.05, v_1=4.95 Hz, A_2=0.15, v_1=20.0 Hz.

Axis of ordinate reflects the location of load over the length of bridge. From the figures it is obvious that the chosen locomotive scheme affects substantially the maximal bending deflection of bridge span middle and it is necessary to accept a scheme with the quantity of forces equal to the number of wheel pairs. The estimations of error for the different considered schemes and motion speeds from 5 m/s to 35 m/s of the locomotive are given in Table 1.

The bending deflections of bridge span middle for the considered range of speeds depend more significantly on the chosen calculation scheme than on the speed of load motion.

Table 1

The estimations of error of the chosen calculation schemes for locomotive VL8

	Static Forces						Static Forces with Harmonic Additions					
	N=1		N=2		N=8		N=1		N=2		N=8	
Speed, m/s	deflac-tion, m	relative error, %	deflac-tion, m	relative error, %	deflac-tion, m	relative error, %	deflac-tion, m	relative error, %	deflac-tion, m	relative error, %	deflac-tion, m	relative error, %
5	0.033	65	0.025	25	0.020	–	0.036	33.33	0.025	7.41	0.027	–
10	0.031	55	0.025	25	0.020	–	0.034	25.93	0.024	11.11	0.027	–
15	0.030	57.89	0.025	31.58	0.019	–	0.034	25.93	0.024	11.11	0.027	–
20	0.030	66.67	0.024	33.33	0.018	–	0.033	17.86	0.024	14.29	0.028	–
25	0.029	52.63	0.024	26.32	0.019	–	0.034	21.43	0.024	14.29	0.028	–
30	0.029	52.63	0.024	26.32	0.019	–	0.032	18.52	0.024	11.11	0.027	–
35	0.028	47.37	0.023	21.05	0.019	–	0.032	18.52	0.024	11.11	0.027	–

5. Conclusions

The peculiarities of computing algorithms based on combination of finite-element method and equations of solid dynamics. The basic components of the program complex Belinda for calculation of statics and dynamics of the rods constructions (as applied to railway bridges) are described. The object-oriented topology of classes of objects and data used in the modules of complex is prepared. The differences in sequences of static and dynamic calculations are described in detail.

As a modeling example, the dynamics of typical metallic beam span 33.6 m in length at motion of single locomotive with various speeds is calculated. The importance of choice of external load calculation scheme to reduce the errors in calculation of joint dynamics railway bridge spans and locomotive is shown.

The series of analogical calculations were conducted for a freight train. The most acceptable results are got for the scheme of loading, consisting of moving forces group that equivalent axle loadings from rolling stock are given.

In further the research of three-dimensional dynamics of trussed girder span bridges under the action of moving loads is planned.

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