Active Noise Control Using a Fuzzy Inference System Without Secondary Path Modelling

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For many adaptive noise control systems the Filtered-Reference LMS, known as the FXLMS algorithm is used to update parameters of the control filter. Appropriate adjustment of the step size is then important to guarantee convergence of the algorithm, obtain small excess mean square error, and react with required rate to variation of plant properties or noise nonstationarity. There are several recipes presented in the literature, theoretically derived or of heuristic origin.

This paper focuses on a modification of the FXLMS algorithm, were convergence is guaranteed by changing sign of the algorithm steps size, instead of using a model of the secondary path. A Takagi-Sugeno-Kang fuzzy inference system is proposed to evaluate both the sign and the magnitude of the step size. Simulation experiments are presented to validate the algorithm and compare it to the classical FXLMS algorithm in terms of convergence and noise reduction.

Keywords: Active Noise Control, adaptive control, fuzzy inference system, FXLMS, sign-varying step size.

1. Introduction

Active noise control (ANC) systems have been found attractive for acoustic noise reduction in industrial environments (ELLIOTT, NELSON, 1993). Many different algorithms have been proposed in the literature. However, still the most popular is the Filtered-reference LMS algorithm, also known as Filtered-x LMS (FXLMS) (TAKENOUCHI *et al.*, 2006).

For many ANC algorithms (including FXLMS) a sufficiently accurate model of the secondary path is required in order to guarantee convergence (PAWELCZYK, 2008). The secondary path if defined as the acousto-electric path between adaptive filter digital output and sampled error signal (effect of acoustic sound interference). If the path is subject to change, what is a common case during ANC system operation, the model should be accordingly updated. Online identification of the secondary path often requires an additional excitation, which reduces effectiveness of residual noise reduction.

In the literature some ANC algorithms are presented, which do not require secondary path modeling. The method introduced by ZHOU and DEBRUNNER (2007), consists of four main steps: initialization, di-

rection search, adaptive filter update, and performance monitoring. During direction search, prior to adaptation, energy quantities of reference and error signals are estimated. After a few samples, during adaptation, they are estimated again. If the adaptation deteriorates the error-to-reference signal ratio, the step size sign is changed. This method was cleverly modified by Wu and Qiu (WU *et al.*, 2008), to search for adaptive filter update direction by using imaginary values of the step size. It improves system convergence, especially for secondary path phase modeling error close to ±90°. A similar approach, based on best parameters search, is presented by CHANG and CHEN (2010). However, in that paper genetic algorithm is used to update adaptive filter coefficients. A fitness function is defined based on the residual error, and therefore each individual set of adaptive filter coefficients must be simulated.

The delayed-x LMS (DxLMS) algorithm with a phase shifter can also be successfully used for reducing narrowband noise. For that algorithm the secondary path delay needs only to be known, instead of a full model. For wideband noise that algorithm was successfully used in a multi-bank structure by PAWELCZYK (2002).

A fuzzy algorithm for step size adjustment was proposed by KUNCHAKOORI *et al.* (2008). It uses information about error signal energy and its rate of change. That algorithm, however, is dependent on secondary path modeling.

In the classical FXLMS structure a model of the secondary path is needed with absolute phase modeling error smaller than 90° (WANG, REN, 1999). Then, the step size should be greater than 0 but smaller than a certain upper bound [10]. In this paper the authors motivated by the research of ZHOU and DEBRUNNER (2007), are developing further that modification to the FXLMS algorithm. To make the algorithm more appropriate for time varying plants, a model of the secondary path is avoided. To guarantee convergence in case of absolute phase errors greater than 90°, the sign of the step size is switched and the value of the step size is adjusted to improve convergence rate. A fuzzy inference system is proposed to evaluate both sign and magnitude of the step size. The inference is based on estimated of control and error signal energy quantities, and an appropriate rule base. The fuzzy rule base follows from properties of the FXLMS algorithm and its behaviour.

2. Modification to the FXLMS algorithm

The classical structure of the FXLMS algorithm is presented in Fig. 1. This algorithm minimizes a cost function defined as a instantaneous square value of the error signal:

$$\min e^2(i) = \min(d(i) + y(i))^2, \qquad (1)$$

where i is the sample index. The adaptive filter coefficient update equation takes the form (see e.g., (ELLIOTT, NELSON, 1993)):

$$\mathbf{w}(i + 1) = \mathbf{w}(i) - \mu(i)e(i)\mathbf{r}(i),$$
$$\mathbf{r}(i) = \widehat{s}^{\mathrm{T}}\mathbf{x}(i), \qquad (2)$$

where

$$\mathbf{w}(i) = [w_0(i)w_1(i)\ldots w_{N-1}(i)]^{\mathrm{T}}$$

is a vector of N filter parameters, μ is the step-size

$$\widehat{s} = [\widehat{s}_0\widehat{s}_1\ldots\widehat{s}_{M-1}]^{\mathrm{T}}$$

is a vector of M parameters of the secondary path model,

$$\mathbf{x}(i) = [r(i)r(i-1)\ldots r(i-M+1)]^{\mathrm{T}}$$

is a reference signal vector,

$$\mathbf{r}(i) = [r(i)r(i-1)\ldots r(i-N+1)]^{\mathrm{T}}$$

is a filtered-reference signal vector. According to TAKENOUCHI *et al.* (2006), for stability of (2) it is required that:

$$0 < \mu(i) < \frac{2}{\|\mathbf{r}(i)\|_{\max}^2}, \qquad (3)$$

where $\|\mathbf{r}(i)\|_{\max}$ is maximum norm of vector $\mathbf{r}(i)$. Thus, the upper limit of the step size varies, dependent on reference signal values, what should be accounted for in the defuzzification stage of the fuzzy inference system. Therefore to mitigate this problem the normalized FXLMS version is used:

$$\mathbf{w}(i + 1) = \mathbf{w}(i) - \frac{\mu(i)}{\mathbf{r}^{\mathrm{T}}(i)\mathbf{r}(i) + \gamma}e(i)\mathbf{r}(i), \qquad (4)$$

where γ is a small constant to ensure that denominator in (4) is not equal to zero. There are also other upper bounds on the step size developed by different authors to improve convergence properties, e.g. (BISMOR, 2012), although their general outcome in this term is similar. Analysis of FXLMS convergence, proposed by ZHOU and DEBRUNNER (2007), for a single-frequency input signal gives the following upper bound of the step size:

$$\mu < \frac{2\cos[\sphericalangle S(f)]}{c_f P_x(f)\,|S(f)|}, \qquad (5)$$

where $P_x(f)$ is the power spectrum of the input signal, c_f is a real constant representing the amplitude estimation error $|S(f)|$ is the magnitude response of the secondary path, and $S(f)$ is the phase estimation error, all for the given frequency f. When $|S(f)| \geq 90°$, FXLMS diverges (ZHOU, DEBRUNNER, 2007; WU *et al.*, 2008).
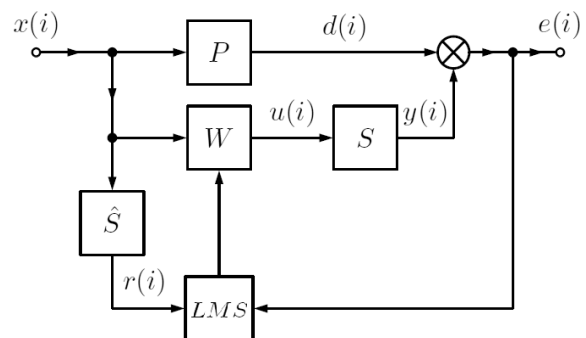

Fig. 1. Classic FXLMS structure.

The idea in this paper for control of tonal or narrowband noise is defined in the following steps:

a) apply a simplified version of the FXLMS algorithm – the Delayed-x LMS algorithm (PAWELCZYK, 2002), i.e.

$$\mathbf{w}(i + 1) = \mathbf{w}(i) - \frac{\mu(i)}{\mathbf{r}^{\mathrm{T}}(i)\mathbf{r}(i) + \gamma}e(i)\mathbf{x}(i - k), \quad (6)$$

where $\mathbf{x}(i) = [r(i)r(i-1)\ldots r(i-N+1)]^{\mathrm{T}}$ is a reference signal vector of the same number of regressors as the control filter order, N.

b) evaluate value and sign of the step size $\mu(i)$. A change of the sign is equivalent to phase shift of the delayed-reference signal by 180°. The step

size update is proposed to be performed using the Takagi-Sugeno-Kang fuzzy approach presented in Sec. 3;

c) in case of impossibility to converge, which is only possible for a special case of phase modeling error being close to $\pm 90°$, increase or decrease the time delay k (this will increase/decrease the phase error, what will automatically be corrected by change of the sign of the step size).

3. Takagi-Sugeno-Kang fuzzy model

The Takagi-Sugeno-Kang (TSK) fuzzy model uses functional dependency to describe consequences. A typical TSK rule has the form:

$$\text{if} \quad x_1 \text{ is } A \quad \text{and} \quad x_2 \text{ is } B \quad \text{then} \quad y = f(x_1 x_2), \quad (7)$$

where x_1, x_2 are system inputs, A and B are fuzzy sets, y is the system output. Usually $f(x_1, x_2)$ is an n-th order polynomial (JANG *et al.*, 1997).

Since first introduction of fuzzy logic by Zadeh in 1965, many fuzzy operators have been suggested. A generic structure of the fuzzy inference system and description of fuzzy operators used in fuzzy controllers can be found in (CORDÓN, *et al.*, 1997).

In this paper the PROD operator is preferred as the T-norm operator. The PROD is defined as:

$$\text{PROD}\left[\chi_A(x_1), \chi_B(x_2)\right] = \chi_A(x_1) \cdot \chi_B(x_2), \quad (8)$$

where $\chi_A(x_1)$ (and $\chi_B(x_2)$, respectively) is a degree of membership input x_1 to fuzzy set A. As the T-conorm operator the PROBOR operator is used, which is defined as:

$$\text{PROBOR}\left[\chi_A(x_1), \chi_B(x_2)\right] = \chi_A(x_1) + \chi_B(x_2)$$
$$- \chi_A(x_1) \cdot \chi_B(x_2) \quad (9)$$

assuming maximum degree of membership limited to one.

The implication operator is 'min', and the aggregation operator is 'max'. A weighted average is used for deffuzification.

4. Fuzzy step size adjustment

In process of developing the inference system and the rule base, the following assumptions are made to simplify the notation and analysis:

- Assumption 1: the noise stationary.
- Assumption 2 the noise is tonal or narrowband.

Nonstatiory noise would result in a more complex inference system taking into account energy of the reference signal. In turn, in case of a broader (or multi-tonal noise), it can be split into narrowband components, processed with this algorithm in parallel channels, and finally combined to produce a common control signal. Thus, the assumptions do not restrict considerations presented below.

The step size is updated according to the following law:

$$\mu = \beta \cdot \mu_{\max}, \quad (10)$$

where $\mu_{\max}$ is a user-defined bound, and the auxiliary parameter β is tuned within the range [-1;1] based on monitoring control and error signals energy estimates The proposed fuzzy inference system for step size adjustment is presented in Fig. 2. For energy estimation of the control signal (and for the error signal, respectively) an inertial equation is used:

$$E_u(i) = \frac{\alpha}{1 - \left(1 - \dfrac{1}{\alpha}\right) z^{-1}} u^2(i), \quad (11)$$

where $0 \ll \alpha < 1$ is a constant.

The gradient of the control signal energy is estimated as (and respectively for the error signal):

$$\Delta E_u(i) = \log\left[\frac{E_u(i)}{E_u(i-L)}\right], \quad (12)$$

where L is a time-span parameter.

Membership functions experimentally found suitable for the system inputs are shown in Fig. 3. The primary path gain and input signal gain are conditioned to ensure that amplitude of signal $d(i)$ is restrained to [0; 1]. Therefore error signal energy membership functions are nonsymmetrical, and saturated for $E_e(i) \geq 1$.
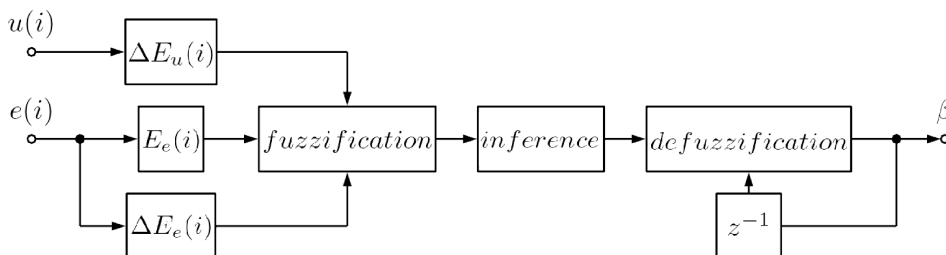
The rule base is presented in Table 1.

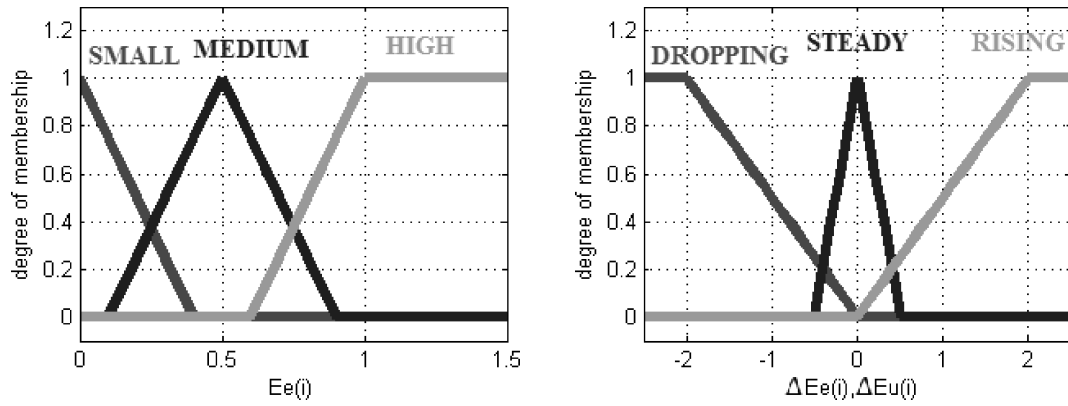Fig. 2. Fuzzy inference structure for the auxiliary parameter β.

Fig. 3. Membership functions for system inputs.

Table 1. Rule base for the proposed algorithm. In cases not mention in this table $\beta = \beta_{i-1}$.

	ΔE_u is 'steady'		ΔE_u is 'rising'	
	ΔE_e is 'steady'	ΔE_e is 'dropping'	ΔE_e is 'rising'	ΔE_e is 'dropping'
E_e is 'high'	R1: $\beta = 1 \cdot \mathrm{sgn}(\beta_{i-1})$	R4: $\beta = 1 \cdot \mathrm{sgn}(\beta_{i-1})$	R5: $\beta = -1 \cdot \mathrm{sgn}(\beta_{i-1})$	R4: $\beta = 1 \cdot \mathrm{sgn}(\beta_{i-1})$
E_e is 'medium'	R2: $\beta = 0.6 \cdot \mathrm{sgn}(\beta_{i-1})$	$\beta = \beta_{i-1}$	R6: $\beta = -0.6 \cdot \mathrm{sgn}(\beta_{i-1})$	$\beta = \beta_{i-1}$
E_e is 'low'	R3: $\beta = 0.1 \cdot \mathrm{sgn}(\beta_{i-1})$	$\beta = \beta_{i-1}$	R7: $\beta = -0.1 \cdot \mathrm{sgn}(\beta_{i-1})$	$\beta = \beta_{i-1}$

Rules R1–R3 are responsible for adjusting the step size, when the system reduces noise. In turn, rules R5–R7 are active in case of large phase error, and they detect the situation when an increase of control signal energy causes an increase of error signal energy.

5. Simulation experiment

All simulations are conducted for data recorded in a real power plant environment. For the secondary path, S, in Fig. 1 an FIR filter of 250 parameters is used. Magnitude and phase responses of the secondary path are presented in Fig. 4. No model is used for filtering

the reference signal as required by the classic FXLMS algorithm in Fig. 1, i.e. $\widehat{S} = 1$. The sampling frequency is 2 kHz.

The first simulation concerns FXLMS with fuzzy step size adjustment for a single-frequency input signal (tone freq. 532 Hz, $S(532\text{ Hz}) = 105.2°$, $\mu_{\max} = 0.01$, $L = 500$). Obtained results are presented in Fig. 5.

Narrowband random noise has also been simulated. Figure 6 presents power spectral density (PSD) of the reference signal, x ($|S| > 90°$, $\mu_{\max} = 0.1$, $L = 200$). Obtained results are presented in Fig. 7.

A comparison between classic FXLMS algorithm and the proposed modified FXLMS with fuzzy step
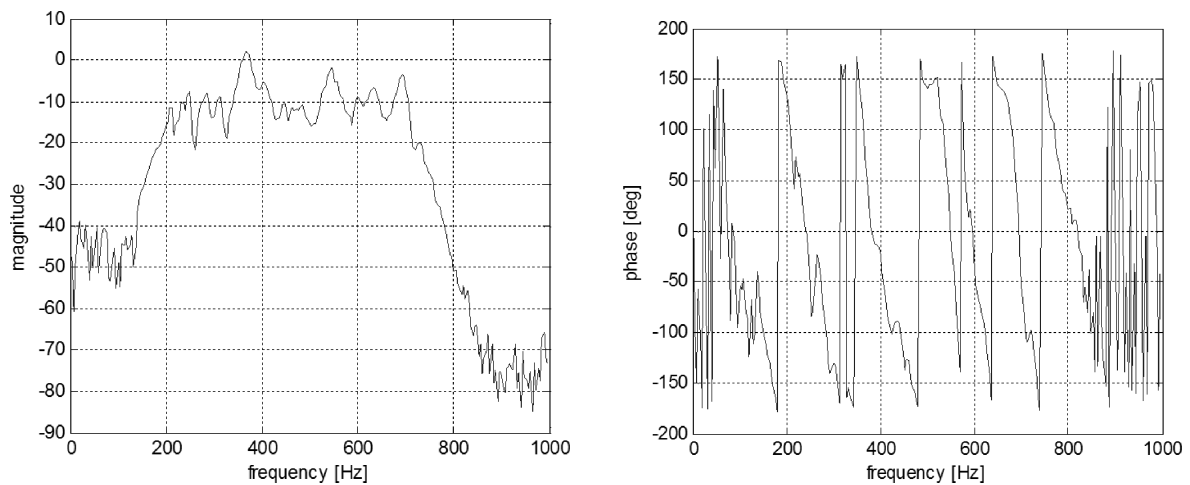

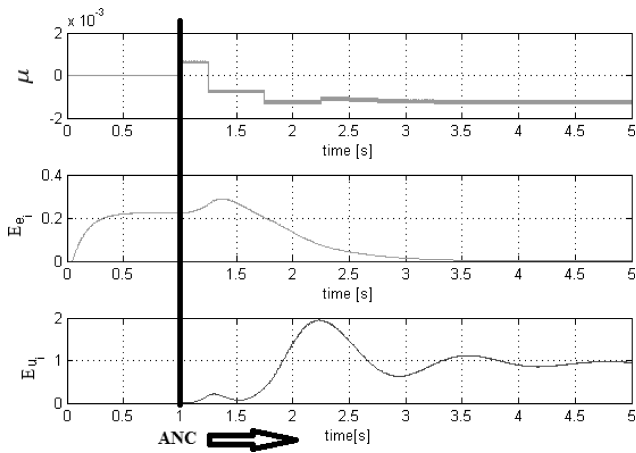Fig. 4. Magnitude and phase of frequency response of the secondary path.

Fig. 5. ANC results for a single-tone noise, $f = 532$ Hz, obtained using FXLMS with fuzzy step size adjustment, without a secondary path model. ANC starts at 1 s.
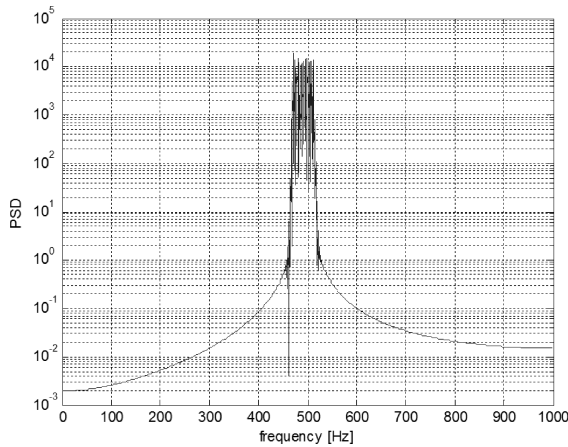

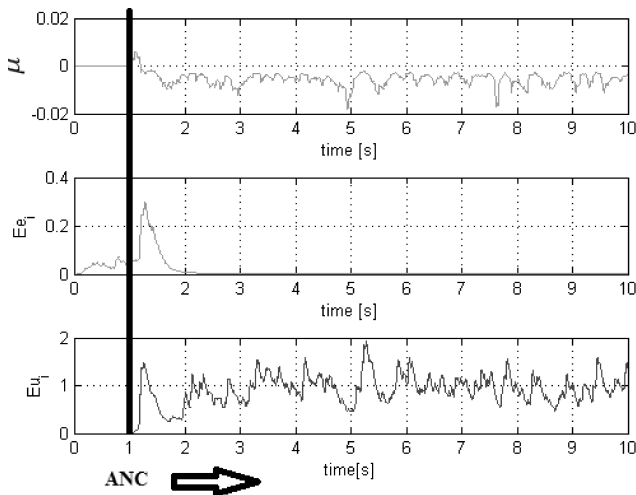Fig. 6. Power spectral density of narrowband input signal.

Fig. 7. ANC results for a narrowband noise, obtained using FXLMS with fuzzy step size adjustment, without a secondary path model. ANC starts at 1 s.

size adjustment for narrowband noise is presented in Fig. 8. The proposed algorithm increases the error signal energy at the beginning of simulation. This effect
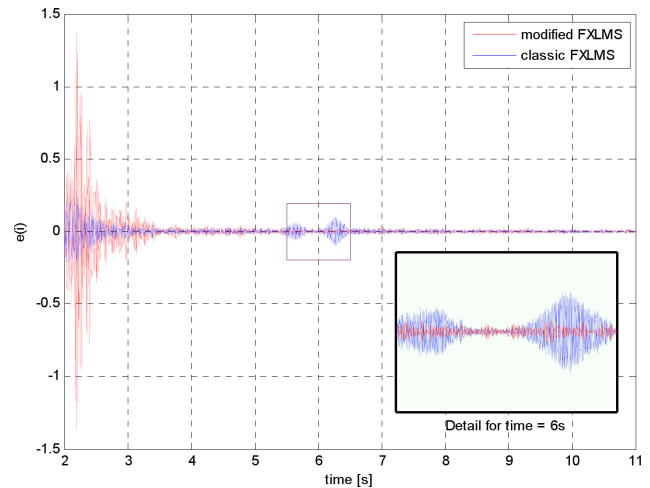

Fig. 8. Comparison between classic FXLMS (47.9 dB reduction in last 5 s) with ideal secondary path model and modified FXLMS without secondary path model (56.6 dB reduction in last 5 s).

is due to initially and on purpose chosen positive value of μ, whereas the phase modeling error was greater than $90°$. Then, a certain time is needed for the inference system to make decision. Algorithm convergence properties can be improved by using an estimate of the delay of the secondary path instead of $\widehat{S} = 1$, as it was mentioned in Sec. 2. System performance would be then similar to that for FxLMS, although without necessity to fully model the secondary path with a high-order filter. In Fig. 8, a transient short-time convergence problem is observed for the classic FXLMS algorithm, (Bismor, 2014), whereas the proposed algorithm operates successfully all the time thanks to the signal energy based criterion used in the update procedure for the step size. This exhibits another positive property of this algorithm.

6. Summary

This paper presents a modification of the FXLMS algorithm allowing avoiding necessity of modelling the secondary path. Instead, the sign and value of the step size are changed to provide algorithm convergence. Both the sign and value of the step size follow from fuzzy inference. The proposed method requires two parameters only – bound of the adaptation step size, and time span for the signal energy estimator. Simulation results for data recorded in a power plant, for tonal and narrowband noises have been presented in the paper Although the plant response is demanding, the algorithm is convergent and results in noise reduction level as that for the FXLMS algorithm based on a full plant model. Moreover, it avoids transient convergence problems thanks to signal energy based criterion for updating the step size. If the noise is multitonal or broadband, it should be split into a number of subbands

and processed individually in parallel channels. The algorithm can be then directly applied for each channel. The bandwidth in each channel is dependent on the phase response in terms of convergence itself, and magnitude response in terms of convergence rate. Processing narrowband components also helps to satisfy causality requirements for successful operation of the feedforward system (Zhang, Qiu, 2014). Presented approach can be coherently employed for problems, where the secondary path is subject to change and current solutions involve additional heuristics, based on other measured signals to update or switch between different models (Mazur, Pawelczyk, 2011) In case of nonstationary noise the rule base can easily be extended by including estimate of the reference signal energy.

Acknowledgments

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