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Hammerstein Nonlinear Active Noise Control with the Filtered-Error LMS Algorithm

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Active Noise Control (ANC) of noise transmitted through a vibrating plate causes many problems not observed in classical ANC using loudspeakers. They are mainly due to vibrations of a not ideally clamped plate and use of nonlinear actuators, like MFC patches. In case of noise transmission though a plate, nonlinerities exist in both primary and secondary paths.

Existence of nonlinerities in the system may degrade performance of a linear feedforward control system usually used for ANC. The performance degradation is especially visible for simple deterministic noise, such as tonal noise, where very high reduction is expected. Linear feedforward systems in such cases are unable to cope with higher harmonics generated by the nonlinearities. Moreover, nonlinearities, if not properly tackled with, may cause divergence of an adaptive control system.

In this paper a feedforward ANC system reducing sound transmitted through a vibrating plate is presented. The ANC system uses nonlinear control filters to suppress negative effects of nonlinearies in the system. Filtered-error LMS algorithm, found more suitable than usually used Filtered-reference LMS algorithm, is employed for updating parameters of the nonlinear filters. The control system is experimentally verified and obtained results are discussed.

Keywords: active noise-vibration control, active structural acoustic control, adaptive algorithm, nonlinear system.

1. Introduction

Vibrating plates are very attractive for noise reduction systems. They can be used as secondary sources in ANC systems or as active acoustic barriers (Hansen, Snyder, 1997; Fahy, Gardonio, 2007). In the latter application the noise transmitted through a plate is reduced. Such active barriers can be much thinner than comparable passive barriers. Double panel systems with two plates separated by a cavity are even more beneficial, because they can provide improvement in transmission loss as compared to the single plate (Pietrzko, 2009).

The plate itself provides some passive noise reduction. The reduction can be improved by using shunt damping systems (Tawfik, Baz, 2006; Pietrzko, 2009). In such systems the vibration energy is converted to electrical energy, usually by piezoelectric patches, and then it is dissipated in appropriately designed RLC circuits. Noise reduction can be further improved by using semi-active shunt systems, where a control system modifies parameters of RLC circuits. This technique is very attractive because of a very low energy consumption of the control system.

The reduction of sound transmitted though a plate can also be improved by using active control. In such systems the plate vibrations are usually controlled by electromagnetic or piezoelectric actuators mounted on a plate. Such actuators can be used to reduce vibrations (active structural acoustic control) or to generated vibrations in order to reduce the sound pressure level at selected locations (active noise-vibration control).

For active control systems, the Filtered-reference FXLMS algorithm, which is especially popular in Active Noise Control systems, is commonly used. This algorithm, however, exhibits a slow convergence rate. Recursive Least Squares or other computationally exhausting algorithms are used to speed the system up (Leniowska, Kos, 2009; Leniowska, 2011).

In this paper reduction of sound pressure level at a single point, where the error microphone is located is investigated. However, presented algorithms can be easily extended to reduce sound pressure level or sound intensity in multiple points, and provide larger area of reduced sound. Also the location of the area can be shifted by using Virtual Microphone Control (Pawełczyk, 2003).

2. Filtered-reference LMS

For systems that should cope with possibly non-deterministic and wide-band noise, feedforward control is usually used. In case of noise reduction in rooms, lack of changes in acoustic paths cannot be usually assumed. When a vibrating plate is used, also temperature change may lead to changes in electroacoustic paths (Mazur, Pawełczyk, 2011a). Adaptive control is then the most appropriate technique. In such systems a signal correlated with the noise is acquired and used as the reference signal. The reference signal is then filtered by an adaptive linear (in terms of parameters) control filter to obtain the control signal and drive the secondary source. Filtered-reference LMS (FXLMS) algorithm is usually used for updating the control filter (Kuo, Morgan, 1996).

Figure 1 shows the block diagram of typical multichannel ANC system with the FXLMS algorithm (Kuo, Morgan, 1996; Elliott, 2001). This system is presented here in brief to give a reference for the remainder of the paper.

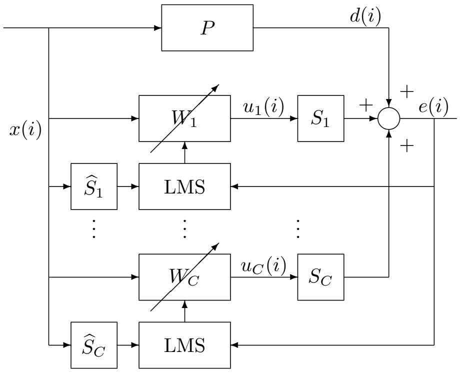

Fig. 1. Active Noise Control system diagram.

In Fig. 1 $x(i)$ is the reference signal, P is the primary path, $d(i)$ is the primary noise at the point of interest, $e(i)$ is the error signal, S_1–S_C are the secondary paths, the symbols with hats stand for models of respective paths and W_1–W_C are control filters, one for each secondary path.

The j-th control signal is obtained according to:

$$u_j(i+1) = \mathbf{w}_j(i)^{\mathrm{T}} \mathbf{x}_u(i), \qquad (1)$$

where $\mathbf{w}_j(i) = [w_{j,0}(i), w_{j,1}(i), \ldots, w_{j,N-1}(i)]^{\mathrm{T}}$ is a vector of parameters of the j-th control filter and $\mathbf{x}_u(i) = [x(i), x(i-1), \ldots, x(i-(N-1))]^{\mathrm{T}}$ is a vector of recent N reference signal samples.

When Normalized FXLMS algorithm is used control filter parameters are updated using the following formula (Elliott, 2001):

$$\mathbf{w}_j(i+1) = \mathbf{w}_j(i) - \mu \frac{\mathbf{r}_j(i)}{\sum\limits_{k=0}^{C} \mathbf{r}_k^{\mathrm{T}}(i)\mathbf{r}_k(i) + \zeta} e(i), \qquad (2)$$

where μ is the convergence coefficient, and ζ is a parameter protecting against division by zero in case of lack of excitation. In this equation $\mathbf{r}_j(i) = [r_j(i), r_j(i-1), \ldots, r_j(i-(N-1))]^{\mathrm{T}}$ is a vector of N regressors of the filtered-reference signal, with elements obtained as:

$$r_j(i) = \widehat{\mathbf{s}}_j(i)^{\mathrm{T}} \mathbf{x}(i), \qquad (3)$$

where $\widehat{\mathbf{s}}_j(i) = [\widehat{s}_{j,0}(i), \widehat{s}_{j,1}(i), \ldots, \widehat{s}_{j,M-1}(i)]$ is a model of the j-th secondary path filter impulse response, $\mathbf{x}(i) = [x(i), x(i-1), \ldots, x(i-(M-1))]^{\mathrm{T}}$ is a vector of regressors of the reference signal. Zero initial conditions, $\mathbf{w}_{j,k}(i) = 0$ for $k \in \mathbb{Z}$, $0 \leq k \leq (N-1)$, were used.

3. Nonlinear feedforward control

The FXLMS algorithm works satisfactorily for typical ANC systems, where primary and secondary paths are linear. Then, assuming that the primary source generates a tonal sound, tones of the same frequency, modified only in amplitude and phase arrive to the reference and error sensors, respectively. With classical loudspeakers used as secondary sources, the assumption concerning the secondary path is usually acceptable. However, in case of vibrating plates used as secondary sources, the secondary paths are nonlinear. Additionally, if sound transmission through the plates is concerned (what is the subject of this paper), the primary paths are also nonlinear. The nonlinearities can be caused due to vibrations of clamped plates (El Kadiri *et al.*, 1999; Saha *et al.*, 2005) and also by actuators (i.e. d33 effect of MFC patches) (Stuebner *et al.*, 2009).

Nonlinearity in the primary or/and the secondary path generates higher harmonics of the original signal and can degrade performance of such systems, because a linear control cannot generate frequencies that are not present in the reference signal. In the simplest case of reduction of tonal noise the nonlinearity in primary path will cause presence of additional harmonic frequencies at the point of interest. Introducing a nonlinearity in the secondary path will cause generation of additional harmonic frequencies of the control signal. In both cases, even when the fundamental tone were successfully reduced by the ANC system its harmonics would be present at the area, where noise reduction is demanded. The control system would not be able to reduce them because the linear control filter with sinusoidal input can only change phase and amplitude and cannot generate different frequencies. Feed-back systems have the potential to mitigate the effect of plant nonlinearity to some extent. However, they cannot be effectively used for non-deterministic wide-band disturbances. Also in case of nonlinearities in the system, the reduction of unwanted harmonics in linear feedback systems may be limited and worse than in nonlinear feedforward systems (Mazur, Pawełczyk, 2011b).

The output of general nonlinear adaptive finite impulse response filter can be written as:

$$\begin{aligned} u_j(i+1) = {} & f(x(i), x(i-1), \ldots, x(x-N_B), \\ & w_{j,0}, w_{j,1}, \ldots, w_{j,k}, \ldots, w_{j,K-1}), \end{aligned} \tag{4}$$

where $u_j(i+1)$ is the value of the control signal in the $i+1$ sample, f is an arbitrary nonlinear function, x is the reference signal and $w_{j,k}$ is the k-th coefficient for filter for j-th control signal. In such general form, the optimal nonlinear function f and filter coefficients are hard to find.

Because of this, some structured filters, e.g. artificial neural networks can be used (Hansen, Snyder, 1997). Second large group of filters used in Nonlinear Active Noise Control (NANC) are filters, which are linear with respect to parameters:

$$\begin{aligned} u_j(i+1) = {} & \sum_{k=0}^{K-1} w_{j,k} \\ & f_k(x(i), x(i-1), \ldots, x(x-N_B)). \end{aligned} \tag{5}$$

For such filters optimal coefficients can be more easily obtained. In case of adaptive control simple classical algorithms, like LMS or RLS, can be employed for adaptation. For ANC, where the secondary path is present, the algorithms are extended by filtering the reference signal through a model of that path. FXLMS has been found as an efficient algorithm for adaptation (Fig. 2). The past N_B+1 regressors of reference signal x are processed by a bank of K nonlinear f_k functions. The sum of results of each function multiplied by w_k coefficient gives the control signal.

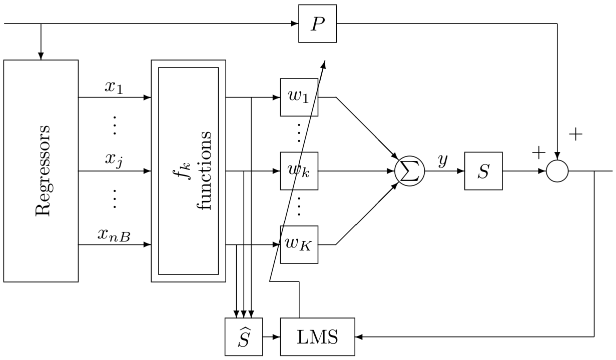

Fig. 2. Active Noise Control system with nonlinear filter linear with respect to parameters using FXLMS algorithm.

There are many possibilities for choosing f_k functions. They can be multivariable polynomials of a specified order with previous samples of reference signal x as independent variables. Such functions are used in the Volterra FXLMS algorithm (Tan, Jiang, 2001). Other common approach is to choose functional-link Artificial Neural Network (FLANN), what is behind the Filtered-s LMS algorithm (Das, Panda, 2004).

Also sum of Hammerstein models can be used as non-linear filter (Mazur, Pawełczyk, 2011b). The Hammerstein model combines nonlinear static function with linear dynamics:

$$u_j(i+1) = \sum_{k=0}^{K-1} W_{j,k}(z^{-1}) F_k(x(i)), \tag{6}$$

where $W_{j,k}(z^{-1})$ is the linear finite response filter for j-th control signal and k-th nonlinear function, z^{-1} is the one sample delay operator and F_k are functions.

Such model leads directly to a simpler and less computationally demanding implementation. This system can be seen as a linear K-channel ANC system of K reference inputs, generated by nonlinear F_k functions. Multichannel FXLMS algorithm (Tu, Fuller, 2000) can be used for adaptation of $W_{j,k}(z^{-1})$ filters (Fig. 3).

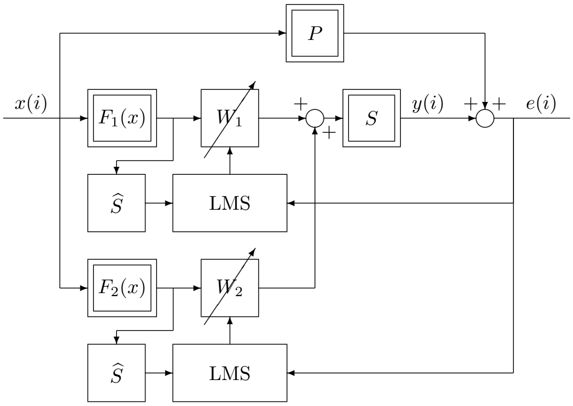

Fig. 3. ANC system with Hammerstein nonlinear control filters using FXLMS algorithm.

This model can be also extended by combining values of reference signal from different samples:

$$\begin{aligned} u_j(i+1) = {} & \sum_{k=0}^{K-1} W_{j,k}(z^{-1}) \\ & F_k(x(i), x(i-1), \ldots, x(x-R)). \end{aligned} \tag{7}$$

4. Filtered-error LMS

For the application concerned, the FXLMS algorithm involves two significant problems. Firstly, each reference signal must be filtered by a secondary path model. Because multiple virtual reference signals generated from the single reference signal are used, a large number of operations is needed. The second problem is that the FXLMS algorithm assumes linearity of the secondary path. Nonlinear secondary path model cannot be simply used in this structure. Both problems can be solved by using the Filtered-error (FELMS) structure, which is presented for a linear filter, e.g. in KUO, MORGAN (1996). The number of filters in the Filtered-error LMS does not depend on the number of reference signals.

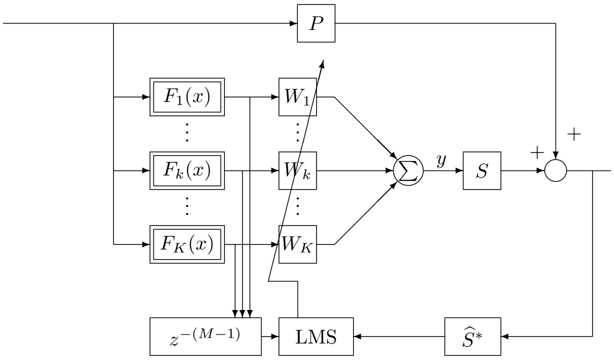

Fig. 4. ANC system with Hammerstein nonlinear control filters using FELMS algorithm.

For adaptation of filter coefficients the Normalized FELMS algorithm can be applied. Control filter parameters are updated using the following formula:

$$\mathbf{w}_j(i+1) = \mathbf{w}_j(i) - \mu \frac{\mathbf{x}_j^*(i)}{\sum\limits_{k=0}^{C} \mathbf{x}_k^{*\mathrm{T}}(i)\mathbf{x}_k^*(i) + \zeta} e^*(i), \qquad (8)$$

where $\mathbf{x}^*(i) = \mathbf{x}(i + (M-1))$ is a vector of regressors of the delayed reference signal and $e_j^*(i)$ stands for the filtered error obtained as:

$$e_j^*(i) = \widehat{\mathbf{s}}_j(i)^{\mathrm{T}} \mathbf{e}(i), \qquad (9)$$

where $\widehat{\mathbf{s}}_j(i) = [\widehat{s}_{j,M-1}(i), \widehat{s}_{j,M-2}(i), \ldots, \widehat{s}_{j,0}(i)]$ is a model of the j-th secondary path filter impulse response, $\mathbf{e}(i) = [e(i), e(i-1), \ldots, e(i-(M-1))]^{\mathrm{T}}$ is a vector of regressors of the error signal.

5. Experimental results

The noise transmitted through a fully clamped aluminum plate is considered (Fig. 5). The plate is 1 mm thick. The noise is generated by a loudspeaker in the enclosure on one side of the plate, and it is transmitted to laboratory room. The primary noise is monitored by a reference microphone. The sound transmitted from that enclosure through the plate is measured by an error microphone located 1.2 m away from the plate at the centre line in the laboratory room. The goal of the control system is to reduce sound pressure level at a given area, where the error microphone is located.

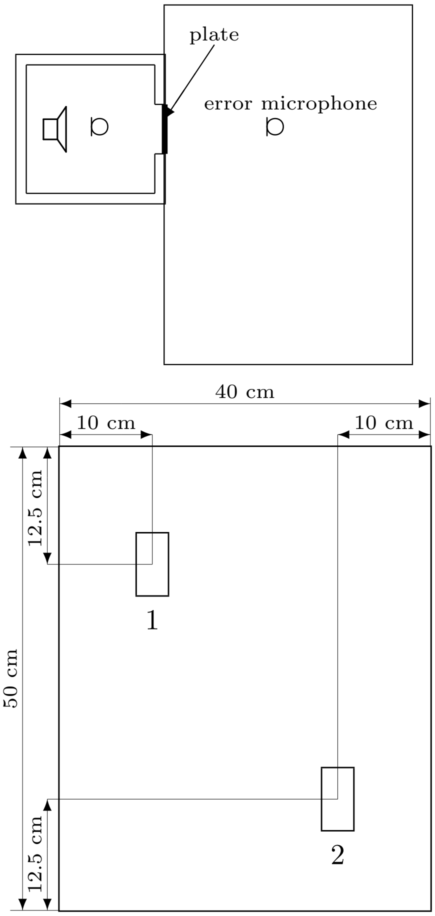

Fig. 5. Laboratory setup (top) and MFC patches on a plate (bottom).

Two d33-effect Macro-Fiber Composite (MFC) patches working in bending mode are used as actuators. MFC is a piezoelectric actuator. Such actuators are recently frequently used for active control of plates and other structures (SODANO *et al.*, 2004; GÓRSKI, KOZUPA, 2012). They provide higher power-to-weight ratio than PZT patches. The positions of actuators have been chosen experimentally to maximize sound radiation from the plate for the selected frequency band. An alternative approach would be to perform optimization using numerical methods to solve the model as in (BRUANT *et al.*, 2010; GÓRSKI, KOZUPA, 2012; KEDZIORA, MUC, 2012) or to guarantee controllability of vibration modes by maximizing eigenvalues of appropriate grammian matrix (WRONA, PAWEŁCZYK, 2013).

For all experiments the sampling frequency has been set to 4 kHz, and 8th order Butterworth low-pass analogue filters with 1200 Hz cut-off frequency have been used as antialiasing and reconstruction filters. The order of the FIR path models has been $M = 256$ for all experiments. This value has been chosen based on impulse response analysis. The order of the FIR control filters has been $N = 256$ for all experiments.

Filtered-error structure has been used. For nonlinear control, two functions $F_1 = x$ and $F_2 = x^3$ have been used.

For tests, two simple deterministic signals 382 Hz tone and sum of 382 Hz and 504 Hz tones have been selected due to high possible reduction of such noises by ANC system without causality problems. The frequencies for tonal signals have been roughly selected for the secondary path to yield a high gain. The exact choice has been dictated to have a high least common multiple with the other frequency and the sampling frequency, to avoid aliasing of harmonic frequencies. The third signal is a recorded real-world noise originating from ball-bearing pulverizers.

Figure 6 shows the PSD of A-weighted noise measured by the error microphone for 382 Hz tonal noise. The ANC system with linear control filter has been able to reduce the fundamental frequency to the noise floor level, however some clearly visible harmonic frequencies have been generated. The measured sound pressure level reduction is equal to 22.0 dB. However, the dominating third harmonic is in the band of a higher sensitivity of the human hearing system. With the A-weighting, the reduction level drops to 17.3 dB. The ANC system with nonlinear control filters has been able to reduce the third harmonic to the noise floor level and improve sound pressure level reduction to 28.7 dB, and to 27.9 dB with the A-weighting. It is in accordance with the design assumption, because it has been noticed that the highest noise reduction could be achieved by reducing the third harmonic, and hence the nonlinear function $F_2 = x^3$ has been selected. Further improvements can be obtained by adding more non-linear functions or, for simple signals, by using nonlinear functions, which would generate multiple harmonics.

Figure 7 shows the PSD of A-weighted noise measured by the error microphone for sum of two tones – 382 Hz and 504 Hz. Similarly to the single tone case, the third harmonic after reduction by the ANC system with linear control filter is dominant. By applying the ANC system with nonlinear control filters, the A-weighted noise reduction level was improved from 15.3 dB to 23.9 dB.

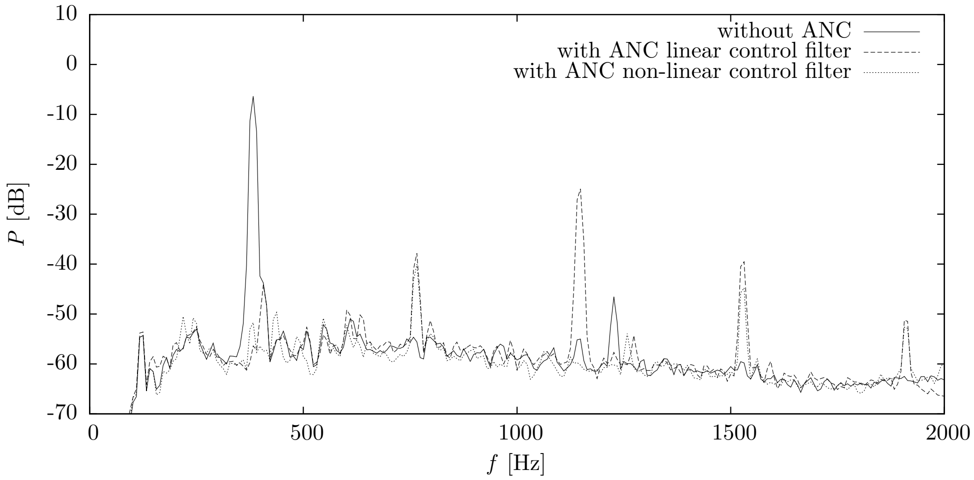

Fig. 6. PSD of A-weighted error microphone signal for different control strategies for 382 Hz tonal noise.

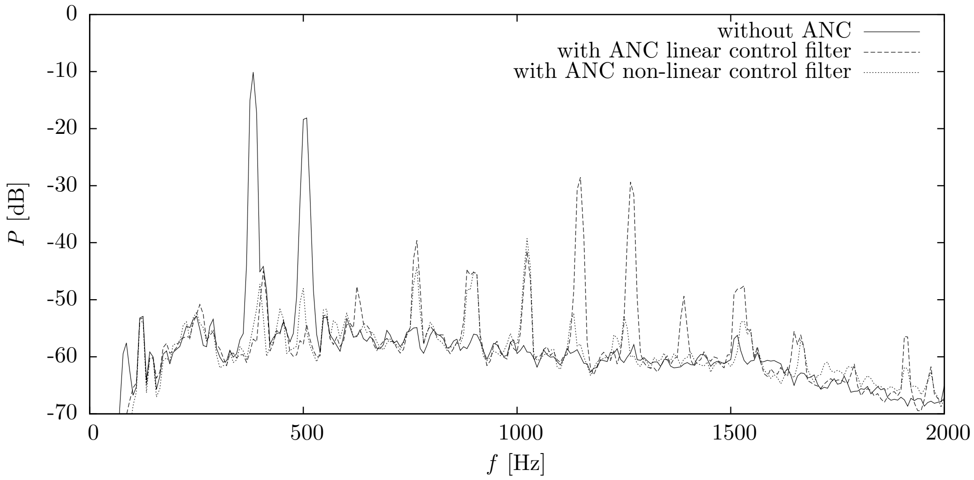

Fig. 7. PSD of A-weighted error microphone signal for different control strategies for sum of 382 Hz and 504 Hz tones.

For more complex noise signals such as recorded noise from ball-bearing pulverizers the improvement is much lower (see Table 1). In such case nonlinear distortions have lower power than residue from active reduction of primary noise.

Table 1. Noise Pressure Levels measured without active control and with different control systems operating (A-weighted).

Noise	without ANC [dB]	FXLMS [dB]	nonlinear FELMS [dB]
382 Hz tone	87.0	69.7	59.1
382 Hz + 504 Hz tones	82.8	67.5	58.9
ball-bearing pulverizers	74.7	64.6	64.1

6. Conclusions

In this paper the problem of Active Noise Control of sound transmitted though a vibrating plate has been considered. Nonlinearity of the vibrating plate significantly degrades performance of the ANC system. The human hearing system is more sensitive to higher harmonics than to fundamental frequencies of the noise usually tackled with by active means.

By using a nonlinear control filter the performance of ANC can be improved resulting in a high acoustic comfort. The nonlinear control filter, however, comes with the cost of huge increase of computational load. Application of the Filtered-error LMS algorithm instead of the Filtered-reference LMS algorithm is therefore beneficial, because the set of generated reference signals does not need filtering.

Significant improvements have been obtained only for simple signals where noise reduction level was high and nonlinear distortions were dominant. The idea needs further development for wide-band noises.

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