www.czasopisma.pan.pl PAN POLSKA AKADEMIA NAUK www.journals.pan.pl

PAN ARCHIVES OF ACOUSTICS
Vol. **37**, No. 2, pp. 171–180 (2012)

Copyright © 2012 by PAN – IPPT
DOI: 10.2478/v10168-012-0023-6

Computing Reverberation Time in a 3D Room Model Using a Finite Difference Method Applied for the Diffusion Equation

Jarosław KRASZEWSKI

Chair of Acoustics, Institute of Telecommunication, Teleinformatics and Acoustics, Wrocław University of Technology
Wybrzeże Wyspiańskiego 27, 50-370 Wrocław, Poland; e-mail: jaroslaw.kraszewski@pwr.wroc.pl

(*received August 7, 2011; accepted March 19, 2012*)

The article presents theoretical values of reverberation times calculated on the basis of the diffusion equation for three room models: flat, cubic, and long. The article shows that not only the average absorption coefficient, but also location of the absorbent material, as well as the place where the time is counted have an impact on the reverberation time, calculated on the basis of the diffusion equation. Despite that, the diffusion model is based on statistical assumptions. The primary goal of the article was to show that the model has geometrical features.

Keywords: diffusion method, reverberation time, finite differences method.

1. Introduction

Over the last decade, more than a dozen papers discussed the diffusion method. Articles referred to the basis of the method (Picaut *et al.*, 1997), concerned initially uncomplicated spaces (Picaut *et al.*, 1999b), and even streets (Picaut *et al.*, 1999a) with a low absorption coefficient. Then the model was equipped with the boundary conditions which allowed introducing different absorption coefficients (Valeau *et al.*, 2006). The model was also compared with the geometrical one and used to model the coupled space (Bilion *et al.*, 2006). Then Eyring's formula was introduced to the model to improve results in areas with higher absorption coefficients (Jing, Xiang, 2007). Further studies related to the applicability of the method in rooms with specular reflection (Valeau *et al.*, 2007; Xiang, Jing, 2009). The usage of the model in rooms separated by a partition wall has been reported (Billon *et al.*, 2008b), as well as introduction of the absorption factor associated with absorption by air (Billon *et al.*, 2008a). In later articles it was proposed to calculate the diffusion equation in an iterative process by using the finite differences' method, and it showed modifications which reduce additional energy generation and computational complexity (Kraszewski, 2010b; 2010a). Finally reverberation time calculations were performed for the modified model (Kraszewski, 2011; Dobrucki, Kraszewski, 2011).

2. Theory

Detailed theoretical basis can be found in the cited studies. Here we focus on the final forms of acoustical diffusion equations, to go on later to an iteration based form that will be the basis for calculating reverberation times. Particles density in the room can be estimated from the acoustical diffusion equation (Valeau *et al.*, 2006):

$$\frac{\partial w(\mathbf{r},t)}{\partial t} - D\nabla^2 w(\mathbf{r},t) + \sigma w(\mathbf{r},t) = 0. \tag{1}$$

The value of σ determines the energy absorption in gas.

Carrying out the analogy between the scattering particles in the gas and the energy absorption by the walls of the model α, we get (Valeau *et al.*, 2006):

$$D = \frac{\lambda c}{3} = \frac{4Vc}{3S}, \tag{2}$$

$$\sigma = \frac{\alpha c}{\lambda} = \frac{\alpha c S}{4V}, \tag{3}$$

where c is the speed of sound (particle's speed).

The mean free path is described by the relationship:

$$\lambda = \frac{4V}{S}, \tag{4}$$

V and S are respectively the volume and the room's surface.

Boundary conditions are necessary in the model because of walls limiting the room. We get two equations respectively for the V (volume) and on S (surface) – the first one defines the acoustical energy density in the room, and the second one ensures that the sound energy will not leave the room volume (Valeau *et al.*, 2006). Adding a source to the equation:

$$\frac{\partial w(\mathbf{r},t)}{\partial t} - D\nabla^2 w(\mathbf{r},t) + \sigma w(\mathbf{r},t) = q(\mathbf{r},t), \tag{5}$$

$$\frac{\partial w}{\partial \mathbf{n}} = 0, \tag{6}$$

where $q(\mathbf{r},t)$ is the strength of the sound source, define the number of particles in time and volume unit in point $\mathbf{r}$, and $\mathbf{n}$ is the outgoing normal vector of the wall surface.

Equation (6) is the Neumann boundary condition, used here because the factor responsible for the absorption σ was used in Eq. (5). This factor can be transferred from Eq. (5) to (6), to move the absorption factor into the wall equation. After assumption regarding the walls and outgoing energy flux, whose detailed description can be found in the articles listed in the references (Valeau *et al.*, 2006; 2007; Bilion *et.al.*, 2006), the two above equations can be written in the following form:

$$\frac{\partial w(\mathbf{r},t)}{\partial t} - D\nabla^2 w(\mathbf{r},t) = q(\mathbf{r},t), \qquad \text{for } V, \tag{7}$$

$$-D\frac{\partial w(\mathbf{r},t)}{\partial \mathbf{n}} = \frac{c\alpha}{4} w(\mathbf{r},t), \qquad \text{on } S. \tag{8}$$

3. Solving method

To solve the diffusion equation, the finite difference method has been used. It consists in finding the values of function $u(x)$ around a point x_i by expansion of this function in Taylor's series.

From Taylor's series, the forward and backward differential quotient can be obtained, respectively:

$$\left.\frac{\mathrm{d}u}{\mathrm{d}x}\right|_i = \frac{u_{i+1} - u_i}{\Delta x} + O(\Delta x), \tag{9}$$

$$\left.\frac{\mathrm{d}u}{\mathrm{d}x}\right|_i = \frac{u_i - u_{i-1}}{\Delta x} + O(\Delta x). \tag{10}$$

The average value leads to the central differential quotient:

$$\left.\frac{\mathrm{d}u}{\mathrm{d}x}\right|_i = \frac{u_{i+1} - u_{i-1}}{2\Delta x} + O(\Delta x^2). \tag{11}$$

Central differential quotient is characterized by a better accuracy. Δx is the distance between the net nodes in the direction of x, $O(\Delta x)$ – means the rest of Δx order.

The second derivative can be found by taking five beginning expressions of Taylor's series:

$$\left.\frac{\mathrm{d}^2 u}{\mathrm{d}x^2}\right|_i = \frac{u_{i+1} - 2u_i + u_{i-1}}{\Delta x^2} + O\left(\Delta x^2\right). \tag{12}$$

The diffusion equation belongs to the group of parabolic equations of the type:

$$\frac{\partial W}{\partial t} = D\frac{\partial^2 W}{\partial x^2}. \tag{13}$$

Replacing the time derivative with the forward differential quotient, and the spatial derivative with the central differential quotient we get in the i-th node:

$$\frac{W_i^{t+1} - W_i^t}{\Delta t} = D\frac{W_{i+1}^t - 2W_i^t + W_{i-1}^t}{\Delta x^2}, \tag{14}$$

which, after transformation, leads to the form:

$$W_i^{t+1} = W_i^t + \delta(W_{i+1}^t - 2W_i^t + W_{i-1}^t), \tag{15}$$

where Δt is the time step, Δx is the spatial step in direction of x, D is the diffusion coefficient

$$\delta = D\frac{\Delta t}{\Delta x^2}. \tag{16}$$

Basing on the values of the surrounding points (nodes) in the previous time step, it is possible to calculate the values in given nodes in the present time step. We get a so-called explicit scheme of equation, called also an FTCS scheme (Forward Time, Central Space). The method is numerical stable for $0 < \delta < 0.5$ for 1D case, which means that for these values, the error in following iterations is not growing.

4. One-dimensional model

For the calculations in all points in a volume, the central differential quotients have been used. The values on the wall have been calculated on the basis of a neighbouring point lying next to the wall and the neighbouring point lying outside the wall. Finally, the diffusion Eqs. (7) and (8) have the finite-difference form:

$$\frac{W_i^{t+1} - W_i^t}{\Delta t} - D\frac{W_{i+1}^t - 2W_i^t + W_{i-1}^t}{\Delta x^2} = Q_i^t, \quad \text{for } V, \tag{17}$$

$$-D\frac{W_{i+1}^t - W_{i-1}^t}{2\Delta x} = \frac{c\alpha}{4} W_i^t, \qquad \text{on } S. \tag{18}$$

Equation (17) can be written as:

$$W_i^{t+1} = W_i^t + \delta\left(W_{i+1}^t - 2W_i^t + W_{i-1}^t\right) + \Delta t Q_i^t, \tag{19}$$

and when the source is switched off:

$$W_i^{t+1} = W_i^t + \delta\left(W_{i+1}^t - 2W_i^t + W_{i-1}^t\right). \tag{20}$$

Finally the equations have the form:

$$W_i^{t+1} = W_i^t(1-2\delta) + \delta\left(W_{i+1}^t + W_{i-1}^t\right), \quad \text{for } V, \quad (21)$$

$$-D\frac{W_{i+1}^t - W_{i-1}^t}{2\Delta x} = \frac{c\alpha}{4}W_i^t, \qquad \text{on } S. \quad (22)$$

To calculate the value on the nodes on the wall, a value at the point lying outside the room can be calculated from Eq. (22):

$$W_{i+1}^t = -\frac{2\Delta x c\alpha}{4D}W_i^t + W_{i-1}^t, \qquad \text{on } S. \quad (23)$$

5. Three-dimensional model

In the case of the three-dimensional space Eqs. (21) and (22) take the form:

$$W_{i,j,k}^{t+1} = W_{i,j,k}^t(1-6\delta) + \delta\left(W_{i+1,j,k}^t + W_{i-1,j,k}^t + W_{i,j+1,k}^t + W_{i,j-1,k}^t + W_{i,j,k+1}^t + W_{i,j,k-1}^t\right), \quad \text{for } V, \quad (24)$$

$$-D\frac{W_{i+1}^t - W_{i-1}^t}{2\Delta x} = \frac{c\alpha}{4}W_i^t, \qquad \text{on } S_i, \quad (25)$$

$$-D\frac{W_{j+1}^t - W_{j-1}^t}{2\Delta y} = \frac{c\alpha}{4}W_j^t, \qquad \text{on } S_j, \quad (26)$$

$$-D\frac{W_{k+1}^t - W_{k-1}^t}{2\Delta z} = \frac{c\alpha}{4}W_k^t, \qquad \text{on } S_k, \quad (27)$$

where Δx, Δy, and Δz are the spatial step in the direction of the axes x, y, and z.

In order to calculate the energy values behind the walls, Eqs. (25), (26), and (27) have been transformed:

$$W_{i+1}^t = -\frac{2\Delta x c\alpha}{4D}W_i^t + W_{i-1}^t, \qquad \text{on } S_i, \quad (28)$$

$$W_{j+1}^t = -\frac{2\Delta y c\alpha}{4D}W_j^t + W_{j-1}^t, \qquad \text{on } S_j, \quad (29)$$

$$W_{k+1}^t = -\frac{2\Delta z c\alpha}{4D}W_k^t + W_{k-1}^t, \qquad \text{on } S_k. \quad (30)$$

6. Reverberation time calculations

Reverberation times were calculated for three room models: cubic, with the dimensions: $10\times10\times10$ m, flat: $12\times12\times6$ m, and long: $6\times22\times6$ m. Models were constructed respectively with separated by 1 m nodes of the following dimensions: $13\times13\times13$, $15\times15\times9$, and $9\times25\times9$. Note that along each dimension, additional points (outside room points) are present to calculate the energy values on the wall (28), (29), (30).

Reverberation times were calculated using Sabine's and Eyring's formulas for the diffusion equation proposed by Jing and Xiang (2007). Diffusion equations using Eyring's formulas are used on the wall. Formulas are analogical to (28), (29), (30) and take the form:

$$W_{i+1}^t = \frac{2\Delta x c\cdot\ln(1-\alpha)}{4D}W_i^t + W_{i-1}^t, \qquad \text{on } S_i, \quad (31)$$

$$W_{j+1}^t = \frac{2\Delta y c\cdot\ln(1-\alpha)}{4D}W_j^t + W_{j-1}^t, \qquad \text{on } S_j, \quad (32)$$

$$W_{k+1}^t = \frac{2\Delta z c\cdot\ln(1-\alpha)}{4D}W_k^t + W_{k-1}^t, \qquad \text{on } S_k. \quad (33)$$

Reverberation times taken from the classical Sabine's and Eyring's formulas (RT60 Sabine, RT60 Eyring), as well as times calculated from the diffusion equation (RT60 DIFF Sabine, RT60 DIFF Eyring) are presented in tables. RT60 taken from the classical and diffusion formulas were compared and shown in percentage as "accordance". The accordance was calculated as a ratio of the diffusion approach reverberation time to the classical formula reverberation time. Despite the fact that the classical Sabine formula is useful for absorption coefficient up to $\alpha = 0.2$, calculations are made up to $\alpha = 1.0$, to show the numerical tendency in the diffusion Sabine's behaviour. Since two cases were taken into account in the calculations: a uniform absorbers distribution and distribution when one of the XZ or YZ walls was totally absorbing, while other walls had a uniform coefficient (see Fig. 1), two absorption coefficients are presented in tables: "α" and "α average". Coefficient "α" tells what the absorption of the walls with a uniform absorption coefficient is, and "α average" tells what the average absorption coefficient in the whole room is. In the case where all the walls have a uniform absorption coefficient, "α" and "α average" are identical. In the case where Eyring's formula was used, the absorption coefficient $\alpha = 1.0$ was changed to $\alpha = 0.99$.

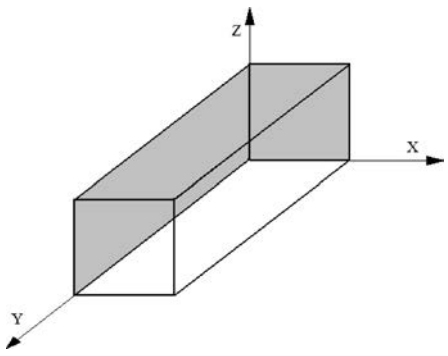

Fig. 1. Schematic drawing of the long room with the axis orientation and marked totally absorbing XZ, YZ walls.

RT60 for a uniform absorption distribution case are shown in Table 1. It can be seen that despite of the dimensions of the room, the accordance for Sabine diffusion equation varies approximately from 100% to 120%, and in Eyring case it varies from 100% up to 180%, and even over 200% for the absorption coefficient equal to unity.

Table 1. Reverberation times of three room models, for the uniform absorption coefficient varying from 0.1 to 1.0.

uniform absorption coefficient								
	α	α average	RT60 Sabine [s]	RT60 Eyring [s]	RT60 DIFF Sabine [s]	accordance Sabine [%]	RT60 DIFF Eyring [s]	accordance Eyring [%]
room 13, 13, 13 pts. (10 m, 10 m, 10 m) $x = 1$ $t = 0.0002$	0.10	0.10	2.68	2.53	2.72	101.49	2.59	102.37
	0.20	0.20	1.34	1.20	1.39	103.73	1.25	104.17
	0.30	0.30	0.89	0.75	0.94	105.62	0.80	106.67
	0.40	0.40	0.67	0.52	0.72	107.46	0.57	109.62
	0.50	0.50	0.54	0.38	0.58	107.41	0.43	113.16
	0.60	0.60	0.45	0.29	0.49	108.89	0.34	117.24
	0.70	0.70	0.38	0.22	0.43	113.16	0.27	122.73
	0.80	0.80	0.34	0.17	0.38	111.76	0.21	123.53
	0.90	0.90	0.30	0.12	0.35	116.67	0.16	133.33
	1.00	1.00	0.27	0.06	0.32	118.52	0.11	183.33
room 15, 15, 9 pts. (12 m, 12 m, 6 m) $x = 1$ $t = 0.0002$	0.10	0.10	2.42	2.28	2.45	101.24	2.33	102.19
	0.20	0.20	1.21	1.08	1.25	103.31	1.12	103.70
	0.30	0.30	0.81	0.67	0.85	104.94	0.72	107.46
	0.40	0.40	0.60	0.47	0.64	106.67	0.51	108.51
	0.50	0.50	0.48	0.35	0.52	108.33	0.39	111.43
	0.60	0.60	0.40	0.26	0.44	110.00	0.30	115.38
	0.70	0.70	0.35	0.20	0.39	111.43	0.24	120.00
	0.80	0.80	0.30	0.15	0.34	113.33	0.19	126.67
	0.90	0.90	0.27	0.10	0.31	114.81	0.15	150.00
	1.00	1.00	0.24	0.05	0.28	116.67	0.08	160.00
room 9, 25, 9 pts. (6 m, 22 m, 6 m) $x = 1$ $t = 0.0002$	0.10	0.10	2.13	2.00	2.16	101.41	2.05	102.50
	0.20	0.20	1.06	0.95	1.10	103.77	0.99	104.21
	0.30	0.30	0.71	0.59	0.74	104.23	0.63	106.78
	0.40	0.40	0.53	0.41	0.57	107.55	0.45	109.76
	0.50	0.50	0.43	0.30	0.46	106.98	0.34	113.33
	0.60	0.60	0.35	0.23	0.39	111.43	0.27	117.39
	0.70	0.70	0.30	0.18	0.34	113.33	0.21	116.67
	0.80	0.80	0.27	0.13	0.30	111.11	0.17	130.77
	0.90	0.90	0.24	0.09	0.27	112.50	0.13	144.44
	1.00	1.00	0.21	0.03	0.25	119.05	0.08	266.67

Tables 2–5 show a comparison of two situations: the first one when one XZ (Tables 2–5) or YZ (Table 5) wall is totally absorbing, while other have a uniform absorption coefficient, and the second case, when all walls have a uniform absorption coefficient but α values are identical with α average from the first case. In both situations average absorption in the room is identical but, as we can see from tables, reverberation times counted from the diffusion equation are different. This follows from the inequality of power inside the room caused by an unequal distribution of absorbing materials. Of course, differences disappear when the absorption coefficient is heading to unity. This situation leads us to a conclusion that the diffusion model has geometrical model features. That means that the distribution of the absorbing material has a significant influence on the model's behaviour.

Table 6 presents calculations in the long room with a uniform absorption coefficient made for different room locations. The difference between the previous and following tables is in the RT60 calculation. In the previous table, RT60 was calculated globally – the energy in the whole room had to fall by 30 dB. Then the reverberation time was calculated by multiplying by 2 the time corresponding to a reduction of energy in the room by 30 dB. In the following tables, RT60 is calculated in the same manner but not in the whole room, only in a specific point. At the beginning, at time $t = 0$, the energy in the room is uniform – every point in the model has the same energy. After that the time energy begins to fall (due to the absence of the energy source) because of the wall absorption. As it can be seen from Table 6, times are different for different room locations. Differences are not large because the

Table 2. Reverberation times of the cubic room model for two cases of the absorbers' arrangement.

the XZ wall is totally absorbing, other walls have a uniform absorption coefficient								
	α	α average	RT60 Sabine [s]	RT60 Eyring [s]	RT60 DIFF Sabine [s]	accordance Sabine [%]	RT60 DIFF Eyring [s]	accordance Eyring [%]
room 13, 13, 13 pts. (10 m, 10 m, 10 m) $x = 1$ $t = 0.0002$	0.10	0.25	1.07	0.93	1.30	121.50	0.74	79.57
	0.20	0.33	0.81	0.66	0.92	113.58	0.57	86.36
	0.30	0.42	0.64	0.49	0.72	112.50	0.46	93.88
	0.40	0.50	0.54	0.38	0.60	111.11	0.38	100.00
	0.50	0.58	0.46	0.30	0.52	113.04	0.32	106.67
	0.60	0.67	0.40	0.24	0.45	112.50	0.27	112.50
	0.70	0.75	0.36	0.19	0.41	113.89	0.23	121.05
	0.80	0.83	0.32	0.15	0.37	115.63	0.19	126.67
	0.90	0.92	0.29	0.11	0.34	117.24	0.15	136.36
	1.00	1.00	0.27	0.06	0.32	118.52	0.11	183.33
uniform absorption coefficient								
room 13, 13, 13 pts. (10 m, 10 m, 10 m) $x = 1$ $t = 0.0002$	0.25	0.25	1.07	0.93	1.12	104.67	0.98	105.38
	0.33	0.33	0.81	0.66	0.86	106.17	0.72	109.09
	0.42	0.42	0.64	0.49	0.68	106.25	0.54	110.20
	0.50	0.50	0.54	0.38	0.58	107.41	0.43	113.16
	0.58	0.58	0.46	0.30	0.51	110.87	0.36	120.00
	0.67	0.67	0.40	0.24	0.45	112.50	0.29	120.83
	0.75	0.75	0.36	0.19	0.40	111.11	0.24	126.32
	0.83	0.83	0.32	0.15	0.37	115.63	0.20	133.33
	0.92	0.92	0.29	0.11	0.34	117.24	0.15	136.36
	1.00	1.00	0.27	0.06	0.32	118.52	0.11	183.33

Table 3. Reverberation times of the flat room model for two cases of the absorbers' arrangement.

the XZ wall is totally absorbing, other walls have a uniform absorption coefficient								
	α	α average	RT60 Sabine [s]	RT60 Eyring [s]	RT60 DIFF Sabine [s]	accordance Sabine [%]	RT60 DIFF Eyring [s]	accordance Eyring [%]
room 15, 15, 9 pts. (12 m, 12 m, 6m) $x = 1$ $t = 0.0002$	0.10	0.21	1.14	1.01	1.40	122.81	0.92	91.09
	0.20	0.30	0.81	0.67	0.93	114.81	0.65	97.01
	0.30	0.39	0.62	0.49	0.70	112.90	0.49	100.00
	0.40	0.47	0.51	0.37	0.57	111.76	0.39	105.41
	0.50	0.56	0.43	0.29	0.48	111.63	0.32	110.34
	0.60	0.65	0.37	0.23	0.42	113.51	0.26	113.04
	0.70	0.74	0.33	0.18	0.37	112.12	0.22	122.22
	0.80	0.82	0.29	0.14	0.34	117.24	0.18	128.57
	0.90	0.91	0.26	0.10	0.31	119.23	0.14	140.00
	1.00	1.00	0.24	0.05	0.28	116.67	0.09	180.00
uniform absorption coefficient								
room 15, 15, 9 pts. (12 m, 12 m, 6 m) $x = 1$ $t = 0.0002$	0.21	0.21	1.14	1.01	1.19	104.39	1.06	104.95
	0.30	0.30	0.81	0.67	0.85	104.94	0.72	107.46
	0.39	0.39	0.62	0.49	0.66	106.45	0.53	108.16
	0.47	0.47	0.51	0.37	0.55	107.84	0.42	113.51
	0.56	0.56	0.43	0.29	0.47	109.30	0.34	117.24
	0.65	0.65	0.37	0.23	0.41	110.81	0.27	117.39
	0.74	0.74	0.33	0.18	0.37	112.12	0.22	122.22
	0.82	0.82	0.29	0.14	0.34	117.24	0.18	128.57
	0.91	0.91	0.26	0.10	0.31	119.23	0.14	140.00
	1.00	1.00	0.24	0.05	0.28	116.67	0.09	180.00

Table 4. Reverberation times of the long room model for two cases of the absorbers' arrangement.

the XZ wall is totally absorbing, other walls have a uniform absorption coefficient								
	α	α average	RT60 Sabine [s]	RT60 Eyring [s]	RT60 DIFF Sabine [s]	accordance Sabine [%]	RT60 DIFF Eyring [s]	accordance Eyring [%]
room 9, 25, 9 pts. (6 m, 22 m, 6 m) $x = 1$ $t = 0.0002$	0.10	0.15	1.38	1.26	1.70	123.19	1.44	114.29
	0.20	0.25	0.86	0.74	0.98	113.95	0.82	110.81
	0.30	0.34	0.62	0.50	0.69	111.29	0.56	112.00
	0.40	0.44	0.49	0.37	0.54	110.20	0.41	110.81
	0.50	0.53	0.40	0.28	0.45	112.50	0.32	114.29
	0.60	0.62	0.34	0.22	0.38	111.76	0.25	113.64
	0.70	0.72	0.30	0.17	0.33	110.00	0.20	117.65
	0.80	0.81	0.26	0.13	0.30	115.38	0.16	123.08
	0.90	0.91	0.23	0.09	0.27	117.39	0.12	133.33
	1.00	1.00	0.21	0.03	0.25	119.05	0.08	266.67
uniform absorption coefficient								
room 9, 25, 9 pts. (6 m, 22 m, 6 m) $x = 1$ $t = 0.0002$	0.15	0.15	1.38	1.26	1.45	105.07	1.34	106.35
	0.25	0.25	0.86	0.74	0.88	102.33	0.77	104.05
	0.34	0.34	0.62	0.50	0.66	106.45	0.55	110.00
	0.44	0.44	0.49	0.37	0.52	106.12	0.40	108.11
	0.53	0.53	0.40	0.28	0.44	110.00	0.32	114.29
	0.62	0.62	0.34	0.22	0.38	111.76	0.25	113.64
	0.72	0.72	0.30	0.17	0.33	110.00	0.20	117.65
	0.81	0.81	0.26	0.13	0.30	115.38	0.16	123.08
	0.91	0.91	0.23	0.09	0.27	117.39	0.12	133.33
	1.00	1.00	0.21	0.03	0.25	119.05	0.08	266.67

Table 5. Reverberation times of the long room model for two cases of the absorbers' arrangement.

the YZ wall is totally absorbing, other walls have a uniform absorption coefficient								
	α	α average	RT60 Sabine [s]	RT60 Eyring [s]	RT60 DIFF Sabine [s]	accordance Sabine [%]	RT60 DIFF Eyring [s]	accordance Eyring [%]
room 9, 25, 9 pts. (6 m, 22 m, 6 m) $x = 1$ $t = 0.0002$	0.10	0.30	0.71	0.60	0.84	118.31	0.41	68.33
	0.20	0.38	0.57	0.45	0.64	112.28	0.34	75.56
	0.30	0.45	0.47	0.35	0.52	110.64	0.29	82.86
	0.40	0.53	0.40	0.28	0.44	110.00	0.25	89.29
	0.50	0.61	0.35	0.22	0.39	111.43	0.22	100.00
	0.60	0.69	0.31	0.18	0.34	109.68	0.19	105.56
	0.70	0.77	0.28	0.15	0.31	110.71	0.16	106.67
	0.80	0.84	0.25	0.11	0.29	116.00	0.14	127.27
	0.90	0.92	0.23	0.08	0.26	113.04	0.11	137.50
	1.00	1.00	0.21	0.03	0.25	119.05	0.08	266.67
uniform absorption coefficient								
room 9, 25, 9 pts. (6 m, 22 m, 6 m) $x = 1$ $t = 0.0002$	0.30	0.30	0.71	0.60	0.74	104.23	0.63	105.00
	0.38	0.38	0.57	0.45	0.59	103.51	0.48	106.67
	0.45	0.45	0.47	0.35	0.51	108.51	0.39	111.43
	0.53	0.53	0.40	0.28	0.44	110.00	0.32	114.29
	0.61	0.61	0.35	0.22	0.38	108.57	0.26	118.18
	0.69	0.69	0.31	0.18	0.34	109.68	0.22	122.22
	0.77	0.77	0.28	0.15	0.31	110.71	0.18	120.00
	0.84	0.84	0.25	0.11	0.29	116.00	0.15	136.36
	0.92	0.92	0.23	0.08	0.26	113.04	0.12	150.00
	1.00	1.00	0.21	0.03	0.25	119.05	0.08	266.67

Table 6. Reverberation times of long room with uniform absorption coefficient calculated for different locations.

	α	α average	RT60 Sabine [s]	RT60 Eyring [s]	RT60 DIFF Sabine [s]	accordance Sabine [%]	RT60 DIFF Eyring [s]	accordance Eyring [%]
RT60 calculated in the center: 3 m, 11 m, 3 m								
room 9, 25, 9 pts. (6 m, 22 m, 6 m) $x = 1$ $t = 0.0002$; RT60 at the point: 5, 13, 5 (3 m, 11 m, 3 m)	0.10	0.10	2.13	2.00	2.17	101.88	2.06	103.00
	0.20	0.20	1.06	0.95	1.11	104.72	1.00	105.26
	0.30	0.30	0.71	0.59	0.76	107.04	0.64	108.47
	0.40	0.40	0.53	0.41	0.58	109.43	0.46	112.20
	0.50	0.50	0.43	0.30	0.47	109.30	0.35	116.67
	0.60	0.60	0.35	0.23	0.40	114.29	0.28	121.74
	0.70	0.70	0.30	0.18	0.35	116.67	0.22	122.22
	0.80	0.80	0.27	0.13	0.31	114.81	0.18	138.46
	0.90	0.90	0.24	0.09	0.28	116.67	0.14	155.56
	1.00	1.00	0.21	0.05	0.26	123.81	0.09	180.00
RT60 calculated 1m from the center of the XZ wall: 3 m, 1 m, 3 m								
room 9, 25, 9 pts. (6 m, 22 m, 6 m) $x = 1$ $t = 0.0002$; RT60 at the point: 5, 3, 5 (3 m, 1 m, 3 m)	0.10	0.10	2.13	2.00	2.15	100.94	2.04	102.00
	0.20	0.20	1.06	0.95	1.09	102.83	0.98	103.16
	0.30	0.30	0.71	0.59	0.74	104.23	0.62	105.08
	0.40	0.40	0.53	0.41	0.56	105.66	0.45	109.76
	0.50	0.50	0.43	0.30	0.45	104.65	0.34	113.33
	0.60	0.60	0.35	0.23	0.38	108.57	0.26	113.04
	0.70	0.70	0.30	0.18	0.33	110.00	0.21	116.67
	0.80	0.80	0.27	0.13	0.29	107.41	0.16	123.08
	0.90	0.90	0.24	0.09	0.27	112.50	0.12	133.33
	1.00	1.00	0.21	0.05	0.24	114.29	0.08	160.00
RT60 calculated 1m from the center of the YZ wall: 1 m, 11 m, 3 m								
room 9, 25, 9 pts. (6 m, 22 m, 6 m) $x = 1$ $t = 0.0002$; RT60 at the point: 3, 13, 5 (1 m, 11 m, 3 m)	0.10	0.10	2.13	2.00	2.17	101.88	2.06	103.00
	0.20	0.20	1.06	0.95	1.11	104.72	1.00	105.26
	0.30	0.30	0.71	0.59	0.75	105.63	0.64	108.47
	0.40	0.40	0.53	0.41	0.58	109.43	0.46	112.20
	0.50	0.50	0.43	0.30	0.47	109.30	0.35	116.67
	0.60	0.60	0.35	0.23	0.40	114.29	0.28	121.74
	0.70	0.70	0.30	0.18	0.35	116.67	0.22	122.22
	0.80	0.80	0.27	0.13	0.31	114.81	0.17	130.77
	0.90	0.90	0.24	0.09	0.28	116.67	0.13	144.44
	1.00	1.00	0.21	0.05	0.26	123.81	0.09	180.00
RT60 calculated in the corner: 1 m, 1 m, 1 m								
room 9, 25, 9 pts. (6 m, 22 m, 6 m) $x = 1$ $t = 0.0002$; RT60 at the point: 3, 3, 3 (1 m, 1 m, 1 m)	0.10	0.10	2.13	2.00	2.15	100.94	2.04	102.00
	0.20	0.20	1.06	0.95	1.08	101.89	0.97	102.11
	0.30	0.30	0.71	0.59	0.73	102.82	0.62	105.08
	0.40	0.40	0.53	0.41	0.55	103.77	0.44	107.32
	0.50	0.50	0.43	0.30	0.45	104.65	0.33	110.00
	0.60	0.60	0.35	0.23	0.38	108.57	0.26	113.04
	0.70	0.70	0.30	0.18	0.33	110.00	0.20	111.11
	0.80	0.80	0.27	0.13	0.29	107.41	0.15	115.38
	0.90	0.90	0.24	0.09	0.26	108.33	0.12	133.33
	1.00	1.00	0.21	0.05	0.24	114.29	0.07	140.00

room has a uniform absorption coefficient. Differences are more evident in Tables 7–9, where the case when one of the XZ walls is entirely absorbing is considered. Because of a strong influence on the energy, caused by a strongly absorbing wall, differences in the energy distribution are bigger than in the uniform absorption case. In Table 10 reverberation times for the long room with the totally absorbing YZ wall and other walls hav-

Table 7. Cubic room RT60; the XZ wall is totally absorbing; other walls have a uniform absorption coefficient.

RT60 calculated in the center: 5 m, 5 m, 5 m								
	α	α average	RT60 Sabine [s]	RT60 Eyring [s]	RT60 DIFF Sabine [s]	accordance Sabine [%]	RT60 DIFF Eyring [s]	accordance Eyring [%]
room 13, 13, 13 (10 m, 10 m, 10 m) $x = 1$ $t = 0.0002$; RT60 at the point: 7, 7, 7 (5 m, 5 m, 5 m)	0.10	0.25	1.07	0.93	1.31	122.43	0.75	80.65
	0.20	0.33	0.81	0.66	0.94	116.05	0.58	87.88
	0.30	0.42	0.64	0.49	0.74	115.63	0.47	95.92
	0.40	0.50	0.54	0.38	0.61	112.96	0.39	102.63
	0.50	0.58	0.46	0.30	0.53	115.22	0.33	110.00
	0.60	0.67	0.40	0.24	0.47	117.50	0.28	116.67
	0.70	0.75	0.36	0.19	0.42	116.67	0.24	126.32
	0.80	0.83	0.32	0.15	0.38	118.75	0.20	133.33
	0.90	0.92	0.29	0.11	0.35	120.69	0.17	154.55
	1.00	1.00	0.27	0.06	0.33	122.22	0.12	200.00
RT60 calculated 1 m from the center of the XZ wall: 5 m, 1 m, 5 m								
room 13, 13, 13 (10 m, 10 m, 10 m) $x = 1$ $t = 0.0002$; RT60 at the point: 7, 3, 7 (5 m, 1 m, 5 m)	0.10	0.25	1.07	0.93	1.26	117.76	0.67	72.04
	0.20	0.33	0.81	0.66	0.90	111.11	0.52	78.79
	0.30	0.42	0.64	0.49	0.71	110.94	0.42	85.71
	0.40	0.50	0.54	0.38	0.59	109.26	0.35	92.11
	0.50	0.58	0.46	0.30	0.51	110.87	0.30	100.00
	0.60	0.67	0.40	0.24	0.45	112.50	0.25	104.17
	0.70	0.75	0.36	0.19	0.41	113.89	0.22	115.79
	0.80	0.83	0.32	0.15	0.37	115.63	0.19	126.67
	0.90	0.92	0.29	0.11	0.35	120.69	0.15	136.36
	1.00	1.00	0.27	0.06	0.32	118.52	0.11	183.33

Table 8. Flat room RT60; the XZ wall is totally absorbing; other walls have a uniform absorption coefficient.

RT60 calculated in the center: 6 m, 6 m, 3 m								
	α	α average	RT60 Sabine [s]	RT60 Eyring [s]	RT60 DIFF Sabine [s]	accordance Sabine [%]	RT60 DIFF Eyring [s]	accordance Eyring [%]
room 15, 15, 9 (12 m, 12 m, 6 m) $x = 1$ $t = 0.0002$; RT60 at the point: 8, 8, 5 (6 m, 6 m, 3 m)	0.10	0.21	1.14	1.01	1.41	123.68	0.94	93.07
	0.20	0.30	0.81	0.67	0.94	116.05	0.67	100.00
	0.30	0.39	0.62	0.49	0.72	116.13	0.51	104.08
	0.40	0.47	0.51	0.37	0.58	113.73	0.41	110.81
	0.50	0.56	0.43	0.29	0.49	113.95	0.33	113.79
	0.60	0.65	0.37	0.23	0.43	116.22	0.27	117.39
	0.70	0.74	0.33	0.18	0.38	115.15	0.23	127.78
	0.80	0.82	0.29	0.14	0.35	120.69	0.19	135.71
	0.90	0.91	0.26	0.10	0.32	123.08	0.15	150.00
	1.00	1.00	0.24	0.05	0.30	125.00	0.11	220.00
RT60 calculated 1m from the center of the XZ wall: 6 m, 1 m, 3 m								
room 15, 15, 9 (12 m, 12 m, 6 m) $x = 1$ $t = 0.0002$; RT60 at the point: 8, 3, 5 (6 m, 1 m, 3 m)	0.10	0.21	1.14	1.01	1.34	117.54	0.81	80.20
	0.20	0.30	0.81	0.67	0.90	111.11	0.58	86.57
	0.30	0.39	0.62	0.49	0.69	111.29	0.44	89.80
	0.40	0.47	0.51	0.37	0.56	109.80	0.36	97.30
	0.50	0.56	0.43	0.29	0.47	109.30	0.29	100.00
	0.60	0.65	0.37	0.23	0.42	113.51	0.24	104.35
	0.70	0.74	0.33	0.18	0.37	112.12	0.20	111.11
	0.80	0.82	0.29	0.14	0.34	117.24	0.17	121.43
	0.90	0.91	0.26	0.10	0.31	119.23	0.14	140.00
	1.00	1.00	0.24	0.05	0.29	120.83	0.10	200.00

Table 9. Long room RT60; the XZ wall is totally absorbing, other walls have a uniform absorption coefficient.

RT60 calculated in the center: 3 m, 11 m, 3 m								
	α	α average	RT60 Sabine [s]	RT60 Eyring [s]	RT60 DIFF Sabine [s]	accordance Sabine [%]	RT60 DIFF Eyring [s]	accordance Eyring [%]
room 9, 25, 9 (6 m, 2 2m, 6 m) $x = 1$ $t = 0.0002$; RT60 at the point: 5, 13, 5 (3 m, 11 m, 3 m)	0.10	0.15	1.38	1.26	1.73	125.36	1.47	116.67
	0.20	0.25	0.86	0.74	1.00	116.28	0.84	113.51
	0.30	0.34	0.62	0.50	0.71	114.52	0.58	116.00
	0.40	0.44	0.49	0.37	0.56	114.29	0.43	116.22
	0.50	0.53	0.40	0.28	0.46	115.00	0.34	121.43
	0.60	0.62	0.34	0.22	0.39	114.71	0.27	122.73
	0.70	0.72	0.30	0.17	0.35	116.67	0.22	129.41
	0.80	0.81	0.26	0.13	0.31	119.23	0.18	138.46
	0.90	0.91	0.23	0.09	0.28	121.74	0.14	155.56
	1.00	1.00	0.21	0.05	0.26	123.81	0.09	180.00
RT60 calculated 1m from the center of the XZ wall: 3 m, 1 m, 3 m								
room 9, 25, 9 (6 m, 22 m, 6 m) $x = 1$ $t = 0,0002$; RT60 at the point: 5, 3, 5 (3 m, 1 m, 3 m)	0.10	0.15	1.38	1.26	1.57	113.77	1.16	92.06
	0.20	0.25	0.86	0.74	0.91	105.81	0.67	90.54
	0.30	0.34	0.62	0.50	0.65	104.84	0.46	92.00
	0.40	0.44	0.49	0.37	0.51	104.08	0.35	94.59
	0.50	0.53	0.40	0.28	0.42	105.00	0.27	96.43
	0.60	0.62	0.34	0.22	0.36	105.88	0.22	100.00
	0.70	0.72	0.30	0.17	0.32	106.67	0.18	105.88
	0.80	0.81	0.26	0.13	0.29	111.54	0.15	115.38
	0.90	0.91	0.23	0.09	0.26	113.04	0.11	122.22
	1.00	1.00	0.21	0.05	0.24	114.29	0.08	160.00

Table 10. Long room RT60; YZ wall is totally absorbing, other walls have a uniform absorption coefficient.

RT60 calculated in the center: 3 m, 11 m, 3 m								
	α	α average	RT60 Sabine [s]	RT60 Eyring [s]	RT60 DIFF Sabine [s]	accordance Sabine [%]	RT60 DIFF Eyring [s]	accordance Eyring [%]
room 9, 25, 9 (6 m, 22 m, 6 m) $x = 1$ $t = 0.0002$; RT60 at the point: 5, 13, 5 (3 m, 11 m, 3 m)	0.10	0.30	0.71	0.60	0.85	119.72	0.42	70.00
	0.20	0.38	0.57	0.45	0.66	115.79	0.35	77.78
	0.30	0.45	0.47	0.35	0.54	114.89	0.30	85.71
	0.40	0.53	0.40	0.28	0.46	115.00	0.26	92.86
	0.50	0.61	0.35	0.22	0.40	114.29	0.23	104.55
	0.60	0.69	0.31	0.18	0.36	116.13	0.20	111.11
	0.70	0.77	0.28	0.15	0.33	117.86	0.17	113.33
	0.80	0.84	0.25	0.11	0.30	120.00	0.15	136.36
	0.90	0.92	0.23	0.08	0.28	121.74	0.12	150.00
	1.00	1.00	0.21	0.05	0.26	123.81	0.09	180.00
RT60 calculated 1m from the center of the YZ wall: 1 m, 11 m, 3 m								
room 9, 25, 9 (6 m, 22 m, 6 m) $x = 1$ $t = 0.0002$; RT60 at the point: 3, 13, 5 (1 m, 11 m, 3 m)	0.10	0.30	0.71	0.60	0.84	118.31	0.39	65.00
	0.20	0.38	0.57	0.45	0.64	112.28	0.33	73.33
	0.30	0.45	0.47	0.35	0.53	112.77	0.28	80.00
	0.40	0.53	0.40	0.28	0.45	112.50	0.24	85.71
	0.50	0.61	0.35	0.22	0.39	111.43	0.21	95.45
	0.60	0.69	0.31	0.18	0.35	112.90	0.19	105.56
	0.70	0.77	0.28	0.15	0.32	114.29	0.16	106.67
	0.80	0.84	0.25	0.11	0.30	120.00	0.14	127.27
	0.90	0.92	0.23	0.08	0.27	117.39	0.12	150.00
	1.00	1.00	0.21	0.05	0.26	123.81	0.09	180.00

ing a uniform absorption coefficient are presented. The situation with the totally absorbing YZ wall is shown only for the long room, because of the symmetry of the flat and cubic rooms. For these two rooms, the case with the totally absorbing YZ wall is identical with the case with the totally absorbing XZ wall.

7. Conclusions

The first observation which emerges from the first table is that despite of the room dimensions, the reverberation times accordance for Sabine diffusion equation varies approximately from 100% to 120%, and is consistent at about 100%–180%, and even over 200%, for the coefficient equal to unity, for the Eyring's case. In Eyring's case such big differences are probably caused by the nonlinear expression ln(1-α). Nevertheless, first comparisons with Ease with aura module – geometrical simulation software – shows that discrepancies between diffusion model calculations and Ease results are not as high as those showed in the tables.

It should be noted that the diffusion model formulas have in their bases parts of the classical Sabine and Eyring formulas. That is why comparisons with the classical Sabine and Eyring formulas were made – to show tendencies of reverberation times taken from the diffusion approach. It also should be emphasized that the method is statistically based, however, it shows the characteristics of geometric methods. As it can be seen from the tables, not only average absorption coefficient, but also the location of the absorber and of the point where the reverberation time was calculated has influence on the diffusion model. The presented results are theoretical, so they have to be compared with actual measurements. Also other investigations, like influence of obstacles on the sound field, showed for instance in Weyna (2010), should be made. The purpose of the article was primarily to present geometrical features of the diffusion model solved by using the finite difference method. That is why a discussion of the model's accuracy and necessary modifications was moved to other articles.

Acknowledgment

This research was supported by the European Social Fund, under the programme: Operational Programme Human Capital, Priority VIII, Activity 8.2 Knowledge Transfer, Sub activity 8.2.2 Regional Innovation Strategies.

References

1. Billon A., Foy C., Picaut J., Valeau V., Sakout A. (2008a), *Introducing atmospheric attenuation within a diffusion model for room-acoustic predictions*, Journal of Acoustical Society of America, 4040–4043.
2. Billon A., Foy C., Picaut J., Valeau V., Sakout A. (2008b), *Modeling the sound transmition between rooms coupled through partition walls by using a diffusion model*, Journal of Acoustical Society of America, **123**, 6, 4261–4271.
3. Bilion A., Valeau V., Sakout A., Picaut J. (2006), *On the use of a diffusion model for acoustically coupled rooms*, Journal of Acoustical Society of America, **120**, 4, 2043–2054.
4. Dobrucki A., Kraszewski J. (2011), *Application Modified numerical algorithm for diffusion equation applied in room acoustics*, 58th Open Seminar on Acoustics joined with 2nd Polish-German Structured Conference on Acoustics, 181–202.
5. Jing Y., Xiang N. (2007), *A modified diffusion equation for room acoustic predication*, Journal of Acoustical Society of America, **121**, 6, 3284–3287.
6. Kraszewski J. (2011), *Application of diffusion model for calculation of reverberation time*, 6th European Congress on Acoustics: Forum Acusticum, 1511–1516.
7. Kraszewski J. (2010a), *Boundary term modification in room acoustic field modeling using diffusion equation method, based on finite difference method*, 57th Open Seminar on Acoustics OSA, 99–102.
8. Kraszewski J. (2010b), *Dealing of the acoustic field in rooms using diffusion equation method based on finite difference method- numerical problems*, EAA Euroregio 1st European Congress of Sound and Vibration, paper index: 055.
9. Picaut J., Simon L., Hardy J. (1999a), *Sound field modeling in streets with diffusion equation*, Journal of Acoustical Society of America, **106**, 5, 2638–2645.
10. Picaut J., Simon L., Polack J. (1997), *A mathematical model of diffuse sound field based on a diffusion equation*, Acustica. Acta Acustica, **83**, 614–621.
11. Picaut J., Simon L., Polack J. (1999b), *Sound field in long rooms with diffusely reflecting boundaries*, Applied Acoustics, **56**, 217–240.
12. Valeau V., Hodgson M., Picaut J. (2007), *A diffusion-based analogy for the prediction of sounds fields in fitted rooms*, Acta Acustica united with Acustica, **93**, 94–105
13. Valeau V., Picaut J., Hodgson M. (2006), *On the use of a diffusion equation for room-acoustic predictions*, Journal of Acoustical Society of America, **119**, 3, 1504–1513.
14. Weyna S. (2010), *Acoustic intensity imaging methods for in-situ wave propagation*, Archives of Acoustics, **35**, 2, 265–273.
15. Xiang N., Jing Y., Bockman A.C. (2009), *Investigation of acoustically coupled enclosures using a diffusion-equation model*, Journal of Acoustical Society of America, **126**, 3, 1187–1198.