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ADJUSTMENT PROCESSES ON THE MARKET WITH COUNTABLE NUMBER OF AGENTS AND COMMODITIES[2]

1. INTRODUCTION

Innovations are the driving force for the economic development (see Schumpeter, 1912), hence the modelling the structures convenient to analyzing innovative processes remains at the core of interest of the economic theory.

The introducing of new products, new technologies, new ways of production etc., can be easily noticeable during the analysis of commodity bundles and producers' plans of action in different points of time. Economic agents, operating on the market, can observe and retrieve the diversity of feasible goods as well as the structure of the supply and the demand. If we want to focus on producers' and consumers' characteristics, then it is convenient to use the Arrow and Debreu apparatus (see Arrow, Debreu, 1954; Debreu, 1959) to model the economic dependencies on such a period of time on which the activities of economic agents are not changed. Such set-up is also useful in formulating and proving the sufficient conditions for existence equilibrium in the private ownership economy (see Arrow, Debreu, 1954; Mas-Colell et al., 1995). However, modeling economic processes resulting in equilibrium needs to involve time.

Since years, many researches have been done to explain how an economy evolves over time. Evolution of economic structures can be caused, among others, by modification activities of economic agents, revealing in introducing new commodities, in increasing or decreasing in amounts of existed commodities, or in eliminating some goods from the market. The set of economic agents may be changed on the observable period of time as some of economic agents might enter or exit the market now or in the future. The above are taken into consideration in the model presented in the current paper.

Generally, the models of evolution of an economy can be divided into two groups: the models where time is the discrete valuable and the models with continuous time. To the first group belong the two-periods and the multi-periods economies under and without risk, as well as the models in which economic processes are modeled

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by difference equations. Some results the reader can find, for example in Radner (1972), Magill, Quinzii (2002), Mas-Colell et al. (1995), Acemoglu (2009), Arrow, Intriligator (1987), Chiang (1992). The second group consists of the models in which the economic processes are examined by the use of differential equations. They are, among others, the Domar model, the classical Solow model, the Romer model as well as their modifications (see for example Romer, 2012; Acemoglu, 2009; Chiang, 1992; Malawski, 1999). Such approach is typical for the models studied in the growth theory.

Some results on the analysis of transitions of economic systems the reader can also find in Lipieta, Malawski (2016) and in Lipieta (2013). The examples of using difference equations in modelling some economic processes are presented, for instance, in Lipieta (2015, 2016).

The specific mathematical properties of the topological apparatus used by Kenneth Arrow and Gérard Debreu and separately by Lionel W. McKenzie (see also Panek, 1993) encourage to consider time in an economy defined with similar tools, especially that the analysis of Schumpeter's conceptions of the economic evolution (see for instance Schumpeter, 1912), leads to the Walras's approach in modelling innovative mechanisms (see also Shionoya, 2015; Lipieta, Malawski, 2016). The implementation of the Arrow and Debreu stationary economy into dynamic processes is not new as there are lots of papers devoted to that problem as well as lots of its solutions (for example Arrow, Intriligator, 1987; Ciałowicz, Malawski, 2011, 2017; Panek, 1997). However, there is no a coherent and unified model of economic evolution in the scientific literature, in which the innovative changes in an economy, could be model by the use of the Arrow and Debreu topological apparatus.

In this context, the aim of this paper is to determine a system of difference equations (see for instance Chiang, 1984) defined in the environment of an economy with countable number of agents and commodities. The above could be useful in modeling some aspects of economic life, especially innovative changes as well as so called adapting processes (see Andersen, 2009), which moves an economic system to equilibrium. In difference to the multi-periods economies, we model the situation in which the sets of commodities, consumers and producers (firms) can be changed on the analyzed time interval. In difference to the models in the growth theory, where the strong mathematical properties of the economic objects under study are required, the model presented in the current paper does not require additional mathematical assumptions. Therefore, the model of economic evolution presented in the paper can be used for exploration of many discontinuous processes such as innovative processes or the processes of bankruptcy of firms.

The paper consists of five parts: in the second part, the private ownership economy with countable number of agents and commodities is defined, the third part is devoted to modelling transformations of the above economy on a given time interval, defined by the use of the specific kind of dynamic system with discrete time. In the fourth part some qualitative properties of adjustment processes are specified while the five part contains conclusions.

2. THE PRIVATE OWNERSHIP ECONOMY WITH COUNTABLE NUMBER OF AGENTS AND COMMODITIES

The activities of two kinds of economic agents: producers and consumers are under our consideration. To emphasize the fact that the number of economic agents as well as that the number of commodities can be changed on the analyzed time interval, we construct the modification of the private ownership economy (see Debreu, 1959; Mas-Colell et al., 1995), to the economy with countable number of agents and commodities, defined in the form of the multi-range relational system (see Adamowicz, Zbierski, 1997; Malawski, 1999), analogously to the definition of the economy presented in Lipieta (2010). Let

– $A = (a_i)_{i\in\mathbb{N}}$ – be a countable set of consumers,
– $B = (b_j)_{j\in\mathbb{N}}$ – be a countable set of producers.

Hence $K \overset{\text{def}}{=} A \cup B$ is the countable set of economic agents. Let $\ell \in \{1,2,\dots\}$,

$$\mathcal{R}^\ell \overset{\text{def}}{=} \mathbb{R}^\ell \times \{0\} \times \{0\} \times \dots$$

as well as

$$\mathcal{R} = \{(x_n)_{n\in\mathbb{N}} \colon \exists n_0 \in \mathbb{N}\ \forall n > n_0\ x_n = 0\}. \tag{1}$$

For every $\ell \in \{1,2,\dots\}$, $\mathcal{R}^\ell \subset \mathcal{R}$. Suppose that $\ell \in \{1,2,\dots\}$ commodities are on the market. Producers' activities in space $\mathcal{R}^\ell$ with respect to achievable technologies are demonstrated by correspondence of production sets

$$y \colon B \ni b \to Y^b \subset \mathcal{R}^\ell,$$

which to every producer assigns his feasible plans of action. Moreover, it is assumed that

$$\exists n \in \{1,2,\dots\}\ \forall j > n\ \ y(b_j) \overset{\text{def}}{=} \{0\},$$

what illustrates the assumption that a finite number of producers operate on the market while every producer b_j, for $j > n$, is an inactive producer at the given moment. Due to such set-up, it is underlined that an unknown number of producers might enter or exit the market in the future.

Assume that a price vector $p \in \mathcal{R}^\ell$ is given.

Definition 1. A two-range relational system

$$P_q^\ell = (B, \mathcal{R}; y, p)$$

is called the ℓ-dimensional quasi-production system.

In the quasi-production systems, the aim of producers is not specified, hence quasi-production systems could be regarded as the area for modeling the producers' activities under the perfect or the bounded rationality assumption (see Simon, 1955; Lipieta, Malawski, 2016).

Definition 2. The three-range relational system

$$C_q^\ell = (A, \mathcal{R}, \Xi; \chi, \epsilon, \varepsilon, p),$$

where:
– $\mathcal{R}$ is of the form (1),
– $\Xi \subset \mathcal{R}^\ell \times \mathcal{R}^\ell$ is the family of all preference relations in $\mathcal{R}^\ell$,
– $\chi: A \ni a \to \chi(a) = X^a \subset \mathcal{R}^\ell$ is the correspondence of consumptions sets, where

$$\exists\, m \in \{1,2,\ldots\}\ \forall i > m\ \chi(a_i) \stackrel{\text{def}}{=} \{0\},$$

which analogously means that every consumer a_i, for $i > m$, is an inactive consumer,
– $\epsilon: A \ni a \to \omega^a \in \mathcal{R}^\ell$ is the initial endowment mapping,
– $\varepsilon \subset A \times (\mathcal{R}^\ell \times \mathcal{R}^\ell)$ is the correspondence, which to every consumer $a \in A$ assigns a preference relation $\preccurlyeq^a$ from set Ξ restricted to set $\chi(a) \times \chi(a)$,

is called the ℓ-dimensional quasi-consumption system.

Definition 3. The structure

$$\mathcal{E}_q^\ell = \left(\mathcal{R}, P_q^\ell, C_q^\ell, \theta, \omega\right),$$

where:
– $\mathcal{R}$ is of the form (1),
– P_q^ℓ is the ℓ-dimensional quasi-production system,
– C_q^ℓ is the ℓ-dimensional quasi-consumption system,
– $\omega = \sum_{a\in A} \omega^a \in \mathcal{R}^\ell$,
– for $a \in A$ and $b \in B$, number $\theta(a,b)$ is the share of consumer a in the profit of producer b as well as mapping $\theta: A \times B \to [0,1]$ satisfies $\forall b \in B\ \sum_{a\in A} \theta(a,b) = 1$,

is called the ℓ-dimensional private ownership economy.

The number ℓ is called the dimension of the private ownership economy, the number of economic agents active on the market is not greater than $m + n$.

Definition 4. If $P_q^\ell = (B, \mathcal{R}; y, p)$ is the ℓ-dimensional quasi-production system, where

$$\forall\, b \in B\quad \eta^b(p) \stackrel{\text{def}}{=} \{y^{b*} \in y(b): p \circ y^{b*} = \max\{p \circ y^b: y^b \in y(b)\}\} \neq \emptyset,$$

then

– $\eta: B \ni b \to \eta^b(p) \subset Y^b$ is called the correspondence of supply at price system p,
– $\pi: B \ni b \to \pi(b) = p \circ y^{b*} \in \mathbb{R}$ is called the maximal profit function at price system p,
– the quasi-production system P_q^ℓ is called the ℓ-dimensional production system and denoted by

$$P^\ell = (B, \mathcal{R}; y, p, \eta, \pi).$$

Let us notice that in contrast to quasi-production systems, in production systems the aim of producers is the profit maximization.

Definition 5. If, for every $a \in A$, at the given price vector $p \in \mathcal{R}^\ell$

$$\beta^a(p) = \{x \in \chi(a): p \circ x \leq p \circ \omega^a + \sum_{b \in B} \theta(a,b) \cdot \pi^b(p)\} \neq \emptyset$$

and

$$\varphi^a(p) = \{x^{a*} \in \beta^a(p): \forall x^a \in \ \beta^a(p)\ x^a \preccurlyeq^a x^{a*}, \preccurlyeq^a \in \Xi\} \neq \emptyset,$$

then

– $\beta: A \ni a \to \beta^a(p) \subset \mathcal{R}^\ell$ is the correspondence of budget sets at price system p, which to every consumer $a \in A$ assigns his set of budget constrains $\beta^a(p) \subset \chi(a)$ at price system p and initial endowment ω^a; number

$$w^a = p \circ \omega^a + \sum_{b \in B} \theta(a,b) \cdot \pi^b(p) \tag{2}$$

is called the wealth of consumer a,
– $\varphi: A \ni a \to \varphi^a(p) \subset \mathcal{R}^\ell$ is the demand correspondence at price system p, which to every consumer $a \in A$ assigns the consumption plans maximizing his preference on the budget set $\beta^a(p)$,
– the ℓ-dimensional quasi-consumption system C_q^ℓ is the ℓ-dimensional consumption system and is denoted by

$$C_q^\ell = C^\ell = (A, \mathcal{R}, \Xi; \chi, \epsilon, \varepsilon, p, \beta, \varphi).$$

We assume that consumers aim in the maximization of preferences on budget sets, however in quasi-consumption systems there may be no upper bound for a consumer's preference relation on the adequate budget set.

Definition 6 (see also in Lipieta, 2010). If P_q^ℓ is the ℓ-dimensional production system ($P_q^\ell = P^\ell$) and C_q^ℓ is the ℓ-dimensional consumption system ($C_q^\ell = C^\ell$), then the ℓ-dimensional private ownership economy $\mathcal{E}_q^\ell$ is called the ℓ-dimensional Debreu economy.

If economy $\mathcal{E}_q^\ell$ is the ℓ-dimensional Debreu economy, then we will write $\mathcal{E}_p^\ell = (\mathcal{R}, P^\ell, C^\ell, \theta, \omega)$ instead of $\mathcal{E}_q^\ell = \left(\mathcal{R}, P_q^\ell, C_q^\ell, \theta, \omega\right)$ or $\mathcal{E}_q^\ell = \mathcal{E}_p^\ell$. The commodity space of every ℓ-dimensional private ownership economy is the subset of the space of real sequences. In this meaning, every economy $\mathcal{E}_q^\ell$ can be viewed as the economy with the countable number of commodities. If $x^a \in X^a$ for every $a \in A$, $y^b \in Y^b$ for every $b \in B$ as well as

$$\sum_{a\in A} x^a - \sum_{b\in B} y^b = \omega,$$

then the sequence (x, y), where $x = (x^{a_1}, x^{a_2}, \dots)$, $y = (y^{b_1}, y^{b_2}, \dots)$, is called the feasible allocation. The sequence

$$(x^*, y^*, p), \tag{3}$$

where $x^* = (x^{a_1*}, x^{a_2*}, \dots)$, $y^* = (y^{b_1*}, y^{b_2*}, \dots)$, for which

- $\forall a \in A\ x^{a*} \in \varphi^a(p)$,
- $\forall b \in B\ y^{b*} \in \eta^b(p)$,
- $\sum_{a\in A} x^{a*} - \sum_{b\in B} y^{b*} = \omega$,

is called the state of equilibrium in economy $\mathcal{E}_p^\ell$. If there exists a state of equilibrium in economy $\mathcal{E}_p^\ell$, then we say that the economy $\mathcal{E}_p^\ell$ is in equilibrium as well as the price vector p is called the equilibrium price vector and is denoted by p^*.

3. ADJUSTMENT PROCESSES IN THE ℓ-DIMENSIONAL PRIVATE OWNERSHIP ECONOMY

The definitions presented below are borrowed from Arrow, Intriligator (1987) and are adapted to the private ownership economy with the countable number of commodities. Let $\tau \in \{1,2,\dots\}$ be the number of points of time indexed by t, $t = 0,1,2,\dots,\tau$. As in Lipieta (2015) and (2016), we say that the economic process is the sequence of actions of economic agents on time interval $[0, \tau]$, resulting in offered goods and services. The set of possible resource allocations will be denoted by Z.

The sequence of characteristics, determining an individual as agent $k \in K$ in the given economic process, is called the environment of that agent. The environment of agent k is denoted by e^k, whereas symbol E^k stands for the set of all his feasible environments. The set

$$E \overset{\text{def}}{=} E^{k_1} \times E^{k_2} \times \dots$$

is called the set of environments.

From now, if $\tau > 1$, then every natural number t such that $0 < t < \tau$, is identified with time interval $[t-1, t)$ on which the activities of producers and consumers are constant. The lengths (ranges) of times intervals do not have to be equal. Saying "at time t", we mean "at time interval $[t-1, t)$" for $0 < t < \tau$, or at the moment of time $t = 0$, or at the moment of time $t = \tau$.

By the fact that activities of producers and consumers are constant on the considered time intervals, we assume that the environment of every agent k is also constant on every time interval t. The environment of agent k at time interval t is denoted by $e^k(t)$. By the above

$$e^k(t) \in E^k,\ e^k(t) = const \ \text{ for } \ k \in K \ \text{ and } \ t \in \{1,2,\dots,\tau\}.$$

The set of messages (information) to be used on the market by agent k is denoted by M^k. The messages of agent k are denoted by m^k ($m^k \in M^k$). Moreover, $m^k(t)$ means the message of agent k at time t. As in case of environments,

$$m^k(t) \in M^k,\ m^k(t) = const \ \text{ for } \ k \in K \ \text{ and } \ t \in \{1,2,\dots,\tau\}.$$

The vector

$$m = (m^{k_1}, m^{k_2}, \dots)$$

is called the message, if $m^k \in M^k$ for every $k \in K$. Now, we can put the following definition:

Definition 7. The structure

$$(M, f, h), \tag{4}$$

where:
– $M \subset M^{k_1} \times M^{k_2} \times \dots$ is the set of messages,
– $f = (f^{k_1}, f^{k_2}, \dots): M \times E \to M$ is the response function, while $f^k: M \times E \to M^k$ is the response function of agent k,
– $m^k(t+1) = f^k(m(t), e(t))$, $t = 1, \dots, \tau - 1$; $k \in K$ is the process of exchanging messages, where $m(t) = (m^{k_1}(t), m^{k_2}(t), \dots)$,
– $h: M \to Z$ is the outcome function, which to every message m assigns the allocation which are the result of analysis of the message m by economic agents,

is called the adjustment process on time $[0, \tau]$.

Definition 8. Let the structure (M, f, h) be an adjustment process. A message

$$\bar{m} = (\bar{m}^{k_1}, \bar{m}^{k_2}, \dots) \in M$$

is said to be stationary if, for every $k \in K$, it satisfies the equation $\bar{m}^k = f^k(\bar{m}, e)$.

Let (M, f, h) be an adjustment process of the form (4). If the components of the environment at time $t = 0$

$$e(0) = (e^{k_1}(0), e^{k_2}(0), \dots) \in E$$

form the Debreu economy, then the adjustment process (4) is called the adjustment process in the Debreu economy (see also Lipieta, 2016).

We aim in modeling adjustment processes of the economic evolution using the Arrow-Debreu apparatus (see for example Arrow, Debreu, 1954), however we admit that the number of commodities, the number of active economic agents as well as the plans of action of economic agents can be changed in time.

Let $\tau \in \{1,2,\dots\}$. Number $\ell_t \in \{1,2,\dots\}$ means the dimension of the commodity-price space at time $t \in \{0,1,\dots,\tau\}$. We admit that if a commodity $l \in \{1,\dots,\ell_t\}$ is not used in production or consumption at time $t+1$, then in every producers' and consumers' plans of action at time $t+1$, l-th coordinate is equal to zero. Under such assumption, we can assume that $\ell = \ell_0 \leq \ell_1 \leq \dots \leq \ell_\tau$.

Let $\mathcal{E}_q^{\ell_t} = \left(\mathcal{R}, P_q^{\ell_t}, C_q^{\ell_t}, \theta, \omega\right)$ be the ℓ_t-dimensional private ownership economy. If an economic agent active at time t disappears from the market at time $t+1$, then he becomes the inactive agent (a producer or a consumer) with zero plans of action at time $t+1$. If an economic agent with zero plan of action at time t (so the inactive agent at time t) enters the market at time $t+1$, then he becomes the active agent with non-zero plans of action. For every t and ℓ_t:

– $Y^b(t) \subset \mathcal{R}^{\ell_t} \subset \mathcal{R}^{\ell_\tau} \subset \mathcal{R}$ means the set of plans of action of producer $b \in B$, feasible to realization at time t,
– $y^b(t)$ – the plan of producer b realized at time t, $y^b(t) \in Y^b(t)$.

In the same way, the characteristics of consumers: $X^a(t) \subset \mathcal{R}^{\ell_\tau}$ and $\epsilon_t(a) \in \mathcal{R}^{\ell_t} \subset \mathcal{R}^{\ell_\tau}$ at time t, for $a \in A$, are defined. The correspondence of preference relations at time t is denoted by $\varepsilon_t(a) = \preccurlyeq_{\ell_t}^a$, where $\preccurlyeq_{\ell_t}^a \subset X^a(t) \times X^a(t) \subset \mathcal{R}^{\ell_t} \times \mathcal{R}^{\ell_t} \subset \mathcal{R}^{\ell_\tau} \times \mathcal{R}^{\ell_\tau} \subset \mathcal{R} \times \mathcal{R}$ means the preference relation of consumer a at time t.

On the basis of the above notation, the environment $e^k(t)$ of every economic agent $k \in K = A \cup B$ at time t is defined. Namely

$$e^k(t) = \left(\tilde{y}_t(k), \tilde{\chi}_t(k), \tilde{\epsilon}_t(k), \tilde{\varepsilon}_t(k), \tilde{\theta}_t(k,\cdot)\right), \tag{5}$$

where:

– $\tilde{y}_t(k) = Y^k(t)$ for $k \in B$, $\tilde{y}_t(k) = \{0\}$ for $k \notin B$,
– $\tilde{\chi}_t(k) = X^k(t)$ for $k \in A$, $\tilde{\chi}_t(k) = \{0\}$ for $k \notin A$,
– $\tilde{\epsilon}_t(k) = \omega^k$ for $k \in A$, $\tilde{\epsilon}(k) = 0$ for $k \notin A$,
– $\tilde{\varepsilon}_t(k) = \preccurlyeq_{\ell_t}^a$ for $k \in A$, $\tilde{\varepsilon}(k) = \{\emptyset\}$ for $k \notin A$,
– the mapping $\tilde{\theta}: K \times K \to [0,1]$ satisfies:
  $\tilde{\theta}(k,\cdot) \equiv 0$ for $k \notin A$, $\tilde{\theta}(\cdot,k) \equiv 0$ for $k \notin B$ and $\forall b \in B \sum_{a \in A} \tilde{\theta}(a,b) = 1$;

moreover, for $a \in A$ and $b \in B$, number $\tilde{\theta}(a,b)$ is the share of consumer a in the profit of producer b.

By the above, the set of environments E^k of every agent $k \in K$ is of the form

$$E^k = P(\mathcal{R}) \times P(\mathcal{R}) \times \mathcal{R} \times P(\mathcal{R} \times \mathcal{R}) \times \mathcal{F}(K,[0,1]),$$

with $\mathcal{F}(K,[0,1]) \stackrel{\text{def}}{=} \{f \mid f: K \to [0,1]\}$. The set of environment is given by

$$E \stackrel{\text{def}}{=} E^{k_1} \times E^{k_2} \times \dots \,.$$

The rest of components of the adjustment process in the meaning of Definition 7 is defined in the standard way (compare to Arrow, Intriligator, 1987). Namely, the message of every agent $k \in K$ at time $t = 0,1,\dots\tau$, is understood as:

$$m^k(t) \stackrel{\text{def}}{=} (p(t), y^k(t), x^k(t)), \tag{6}$$

where $x^k(t) = 0 \in \mathcal{R}$ for $k \notin A$ and $y^k(t) = 0 \in \mathcal{R}$ for $k \notin B$.

Consequently, $M^k = \mathcal{R} \times \mathcal{R} \times \mathcal{R}$. Define $M \subset M^{k_1} \times M^{k_2} \times \dots$ by the formula

$$M \stackrel{\text{def}}{=} \left\{ \begin{matrix} (m^{k_1}(t), m^k(t), \dots): \sum_{k \in K} x^k(t) - \sum_{k \in K} y^k(t) = \omega(t) \\ \wedge \forall t = 0,1,\dots,\tau \; \exists \, p(t) \in \mathcal{R} \; \forall k \in K: m^k(t) = (p(t), x^k(t), y^k(t)) \end{matrix} \right\}.$$

Suppose that $M \neq \emptyset$. Every message $m \in M$ has to be a feasible message at any time $t \in \{0,1,\dots,\tau\}$. Hence $m = m(t)$ for $t \in \{0,1,\dots,\tau\}$. The response function of every agent k to the message $m(t) \in M$, for $t = 0,1,\dots,\tau - 1$, is of the form:

$$f^k(m(t), e(t)) = (p(t+1), y^k(t+1), x^k(t+1)). \tag{7}$$

As a reply to prices $p(t)$ at time t, every agent k chooses his plan of action at time $t = 0,1,\dots,\tau$. If $x(t) = (x^{a_1}(t), x^{a_2}(t), \dots)$ and $y(t) = (y^{b_1}(t), y^{b_2}(t), \dots)$, then

$$Z \stackrel{\text{def}}{=} \left\{ \begin{matrix} (x,y): \exists \; t = 0,1,\dots,\tau \;\; (x = x(t) \;\wedge\; y = y(t)) \\ \sum_{a \in A} x^a(t) - \sum_{b \in B} y^b(t) = \omega \end{matrix} \right\}. \tag{8}$$

In this situation, the outcome function $h: M \to Z$ is of the form:

$$\begin{aligned} h(m) &= h(m^{k_1}(t), m^k(t), \dots) = \\ &= h\Big(\big(p(t), y^{k_1}(t), x^{k_1}(t)\big), \big(p(t), y^{k_2}(t), x^{k_2}(t)\big), \dots\Big) \stackrel{\text{def}}{=} \\ &= \Big((x^{k_1}(t), x^{k_2}(t), \dots), (y^{k_1}(t), y^{k_2}(t), \dots)\Big). \end{aligned} \tag{9}$$

Precisely, function h assigns to a message at time t the sequence of feasible allocation of economic agents transferred by this messages.

If a message $m(t)$, for a $t \in \{0,1,\dots,\tau-1\}$, is the stationary one, then $p(t) = p(t+1)$, and the state

$$(x(t), y(t), p(t))$$

has to be the state of equilibrium in economy $\mathcal{E}_q^{\ell_t}$. Consequently, the economy $\mathcal{E}_q^{\ell_t}$ is the private ownership Debreu economy ($\mathcal{E}_q^{\ell_t} = \mathcal{E}_p^{\ell_t}$). If there is $t_0 \in \{0,1,\dots,\tau-1\}$ such that message $m(t_0)$ defined in (6) is stationary, then for every $t \in \{t_0,\dots,\tau\}$ messages $m(t),\dots,m(\tau-1)$ are stationary as well as $m(t) = \dots = m(\tau)$. Consequently, the economies $\mathcal{E}_q^{\ell_t}$, for $t \in \{t_0,\dots,\tau\}$, are the Debreu economies in equilibrium ($\mathcal{E}_q^{\ell_t} = \mathcal{E}_p^{\ell_t}$).

Definition 9. An adjustment process (4) with the environments (5), the messages of the form (6), the response functions defined in (7) and the outcome function (8), is called the transformation process of economy $\mathcal{E}_q^{\ell_0}$.

If ℓ-dimensional private ownership economy $\mathcal{E}_q$, is built by the components of an environment $e(\tau)$ of the transformation process (4) of economy $\mathcal{E}_q^{\ell_0}$, then $\ell = \ell_\tau$, $\mathcal{E}_q = \mathcal{E}_q^{\ell_\tau}$ as well as $\mathcal{E}_p$ is said to be the transformation (or the evolution) of economy $\mathcal{E}_q^{\ell_0}$. This relationship will be noted by $\mathcal{E}_q^{\ell_0} \subset \mathcal{E}_q^{\ell_\tau} = \mathcal{E}_q$.

The transformation process of private ownership economy $\mathcal{E}_q^{\ell_0}$ can be used for modelling the Schumpeterian vision of economic development. Namely, if

$$\exists b_0 \in B\ \exists y^{b_0}(\tau)\ \forall b \in B\ y^{b_0}(\tau) \notin \cup_{b \in B} Y^b(0), \tag{10}$$

then the innovative changes are noticeable during the transformation process. If $\ell_0 = \ell_\tau$, then the above condition means that at least one new technology reveals in producers' activities in the framework of the economy $\mathcal{E}_q^{\ell_\tau}$ in comparison to economy $\mathcal{E}_q^{\ell_0}$. If $\ell_0 < \ell_\tau$, then at least one new product or new technology appear in the final economy $\mathcal{E}_q^{\ell_\tau}$ in comparison to initial economy $\mathcal{E}_q^{\ell_0}$. The producer b_0 satisfying condition (10) is called the innovator. If the profit of innovator b_0 realized in the economy $\mathcal{E}_q^{\ell_\tau}$ is greater than in initial economy $\mathcal{E}_q^{\ell_0}$, then it is said that innovator b_0 is the successful innovator (see also Lipieta, 2013). More about innovations and innovative changes modeled in the Arrow-Debreu apparatus, the reader can find, for example in Malawski (2013) or in Lipieta, Malawski (2016).

The above defined transformation process in the private ownership economy can also be used to model the procedure of adjustment producers' or consumers' plans of action as well as prices to equilibrium, without changing the set of commodities. Such an adjustment process can be viewed as the adapting process (see Andersen, 2009), during which economic agents adapt innovations and which results in a new state of equilibrium in the final transformation of the economy under study. Formally, the transformation process (4) of economy $\mathcal{E}_q^{\ell_0}$ is called the adapting process, if $\mathcal{E}_q^{\ell_\tau}$ is the Debreu economy and $\ell_0 = \ell_\tau$.

4. COMPARATIVE ANALYSIS OF TRANSFORMATIONS OF A DEBREU ECONOMY

Now we face the challenge of formulating criteria for comparing the transformation processes of a given initial private ownership economy $\mathcal{E}_q^{\ell_0}$. The transformation processes on the same time interval can be compared on the basis of qualitative properties of the final private ownership economies – built by components of environments at time $t = \tau$.

As earlier, the moment of time $t = 0$ is the starting point, number $\tau \in \{1,2, \dots\}$ – the ending point of two transformation processes (M, f, h) and $(\widetilde{M}, \tilde{f}, \tilde{h})$ of given economy $\mathcal{E}_q^{\ell_0}$. Assume that the components of environments $e(\tau)$ and $\tilde{e}(\tau)$ of transformation processes (M, f, h) and $(\widetilde{M}, \tilde{f}, \tilde{h})$ form the private ownership economies $\mathcal{E}_q^{\ell_\tau}$ and $\mathcal{E}_q^{\tilde{\ell}_\tau}$. Suppose firstly that economies $\mathcal{E}_q^{\ell_\tau}$ and $\mathcal{E}_q^{\tilde{\ell}_\tau}$ are the Debreu economies with states of equilibrium (see (3)) denoted by (x^*, y^*, p^*) and $(\tilde{x}^*, \tilde{y}^*, \tilde{p}^*)$, adequately.

Similarly as in Lipieta (2013), we say that a producer $b \in B$ is better off in Debreu economy $\mathcal{E}_p^{\tilde{\ell}_\tau}$ than in Debreu economy $\mathcal{E}_p^{\ell_\tau}$ if and only if,

$$p^*(\tau) \circ y^{b*}(\tau) < \tilde{p}^*(\tau) \circ \tilde{y}^{b*}(\tau). \tag{11}$$

Condition (11) means that the maximal profit of producer b in economy $\mathcal{E}_p^{\tilde{\ell}_\tau}$ is greater than in economy $\mathcal{E}_p^{\ell_\tau}$. In contrast to Lipieta (2013) it is said that a consumer $a \in A$ is better off in Debreu economy $\mathcal{E}_p^{\tilde{\ell}_\tau}$ than in Debreu economy $\mathcal{E}_p^{\ell_\tau}$ if and only if,

$$p^*(\tau) \circ x^{a*}(\tau) < \tilde{p}^*(\tau) \circ \tilde{x}^{a*}(\tau). \tag{12}$$

If there are the states of equilibrium in Debreu economies (see (3)) $\mathcal{E}_p^{\ell_\tau}$ and $\mathcal{E}_p^{\tilde{\ell}_\tau}$, then

$$p^*(\tau) \circ x^{a*}(\tau) = w^a(\tau) \text{ as well as } \tilde{p}^*(\tau) \circ \tilde{x}^{a*}(\tau) = \widetilde{w}^a(\tau)$$

(see (2)). Hence, condition (12) means that the wealth of consumer a in economy $\mathcal{E}_p^{\tilde{\ell}_\tau}$ is greater than in economy $\mathcal{E}_p^{\ell_\tau}$. The wealth of the Debreu economy $\mathcal{E}_p^{\ell_\tau}$, namely number

$$w(\tau) = \sum_{a \in A} w^a(\tau)$$

can be viewed as the result of consumers' wealths. Hence, we say that economy $\mathcal{E}_p^{\tilde{\ell}_\tau}$ is better off than economy $\mathcal{E}_p^{\ell_\tau}$ if

$$\sum_{a \in A} w^a(\tau) < \sum_{a \in A} \widetilde{w}^a(\tau). \tag{13}$$

Let (M,f,h) and $(\widetilde{M},\tilde{f},\tilde{h})$ be two adjustment processes of given Debreu economy $\mathcal{E}_p^{\ell_0}$ on time interval $[0,\tau]$. It is said that the adjustment process $(\widetilde{M},\tilde{f},\tilde{h})$ is more effective than adjustment process (M,f,h), if economy $\mathcal{E}_p^{\tilde{\ell}_\tau}$ is better off than economy $\mathcal{E}_p^{\ell_\tau}$.

By (2), condition (13) is equivalent to the following:

$$p^* \circ \omega + \sum_{b\in B} p^* \circ y^{b*} < \tilde{p}^* \circ \widetilde{\omega} + \sum_{b\in B} \tilde{p}^* \circ \tilde{y}^{b*}.$$

The above inequality means that the consumers' wealth depends on the size of producers' profits and the wealth of total endowment in the final economy.

If $\mathcal{E}_q^{\ell_\tau}$ is not the Debreu economy, then we put in criterion (12) the realized allocation $x^a(\tau)$ instead of equilibrium consumption plan $x^{a*}(\tau)$. Similarly in (11), equilibrium production plan $y^{b*}(\tau)$ is replaced by realized production plan $y^b(\tau)$. If $\mathcal{E}_q^{\tilde{\ell}_\tau}$ is not the Debreu economy, then it is done in the same way.

On the basis of the above, it is said that

– a producer $b \in B$ is better off in an economy $\mathcal{E}_q^{\tilde{\ell}_\tau}$ than in an economy $\mathcal{E}_q^{\ell_\tau}$, if and only if,

$$p(\tau) \circ y^b(\tau) < \tilde{p}(\tau) \circ \tilde{y}^b(\tau),$$

– a consumer $a \in A$ is better off in an economy $\mathcal{E}_q^{\tilde{\ell}_\tau}$ than in an economy $\mathcal{E}_q^{\ell_\tau}$, if and only if,

$$p(\tau) \circ x^a(\tau) < \tilde{p}(\tau) \circ \tilde{x}^a(\tau).$$

Now

$$p(\tau) \circ x^a(\tau) = w^a(\tau) \text{ as well as } \tilde{p}(\tau) \circ \tilde{x}^a(\tau) = \widetilde{w}^a(\tau),$$

and, as above, we say that economy $\mathcal{E}_q^{\tilde{\ell}_\tau}$ is better off than economy $\mathcal{E}_q^{\ell_\tau}$, if

$$\sum_{a\in A} w^a(\tau) < \sum_{a\in A} \widetilde{w}^a(\tau).$$

At the end let us notice that the adjustment process and consequently the transformation process of the economy $\mathcal{E}_p^{\ell_0}$ are also the economic mechanism in the sense of Hurwicz (see also Arrow, Intriligator, 1987; Hurwicz, Reiter, 2006). Hence in the same way as for economic mechanisms, we can say about qualitative properties for adjustment processes (see Lipieta, 2013; Lipieta, Malawski, 2016). Namely:

Definition 10. An adjustment process, in which prices of commodities are elements of the message space is called the price adjustment process. If in the given adjustment process at least one agent from the given set will be better off due to a given criterion, without making the rest of agents (from this set) worse off, then this adjustment process will be called the qualitative one with respect to the given set.

On the basis of the above, we can say that the transformation process of a private ownership economy (see Definition 9) is the price adjustment process. Moreover, if the final economy is the innovative extension of the initial economy (see Lipieta, 2013), then the transformation process of the initial economy is the qualitative adjustment process with respect to the set of successful innovators.

5. CONCLUSIONS

The modifications of the definitions of production and consumption systems – the components of the Debreu economy presented in part 2 can simplify comparing of two Debreu economies with the same set of economic agents. Such two structures can be interpreted as the mathematical models of a real economy in two points of time where "the subsequent" economy can be understood as the transformation of "the earlier" economy.

On the other hand, the transformation process of a private ownership economy defined in part 3 is an attempt to put the initial stationary model "in motion" to make it possible to study changes in the economy modeled in the Arrow-Debreu apparatus.

The part three seems to be the basis for further studying of properties of transformation processes as it contains the criteria for the choice of the best or at least "good enough" process from the point of view of producers or consumers.

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PROCESY DOSTOSOWAWCZE NA RYNKU Z PRZELICZALNĄ LICZBĄ AGENTÓW I TOWARÓW

Streszczenie

Większość składowych ekonomii z własnością prywatną to odwzorowania niezależne od czasu, choć opisują działania podmiotów gospodarczych rozgrywające się w czasie. Dlatego struktura ta jest interpretowana jako stacjonarny model gospodarki, w której działalność podmiotów ekonomicznych na rynkach jest stała w analizowanym przedziale czasu. Matematyczne własności przestrzeni towarów i cen ekonomii z własnością prywatną mogłyby być przydatne w analizie zmian działalności agentów ekonomicznych. Stąd potrzeba określenia w jaki sposób ekonomia z własnością prywatną mogłaby ewoluować w czasie.

W tym kontekście celem artykułu jest modelowanie ewolucji gospodarki zdefiniowanej w aparacie pojęciowym Arrowa i Debreu, z wykorzystaniem równań różnicowych. W rezultacie otrzymujemy spójny i jednolity opis tej ewolucji, który może być zastosowany, m.in., do analizy mechanizmów schumpeterowskiego rozwoju gospodarczego, odmiennie od metod używanych zwykle w teorii wzrostu.

Słowa kluczowe: ekonomia z własnością prywatną, procesy dostosowawcze

ADJUSTMENT PROCESSES ON THE MARKET
WITH COUNTABLE NUMBER OF AGENTS AND COMMODITIES

Abstract

Most components of the private ownership economy are the mappings independent on time, although they model activities of economic agents which take place in time. Therefore this structure is interpreted as the stationary model in which actions of economic agents on the market are constant on the analyzed time interval. The mathematical properties of the commodity-price space of the private ownership economy could be convenient in analyzing changes in the activities of economic agents. Hence, there is a need to determine how a private ownership economy could evolve over time.

In this context, the aim of the paper is to model evolution of the economy defined in the Arrow and Debreu apparatus by the use of difference equations. As a result, we get a coherent and unified description of the evolution of an economy that can be used, among others, in the analysis of the mechanisms of Schumpeter's economic development, differently from the methods usually used in the growth theory.

Keywords: private ownership economy, adjustment process