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# STATISTICAL ESTIMATION AND MATHEMATICAL MODELLING OF TROPOSPHERIC RADIO REFRACTIVITY BASED ON METEOROLOGICAL DATA

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**Abstract**

To obtain optimal performance, estimation of radio refractivity is essential in planning and design of radio links/systems. The dependence of radio refractivity on different climatic parameters such as temperature, pressure, humidity and scale height is studied. The method proposed by International Telecommunication Union (ITU) is used in calculation of tropospheric radio refractivity. The main objective of this work is to formulate a mathematical equation/expression which can be used to predict/estimate the value of radio refractivity for an arbitrary day of any year, with a low error of estimation, in a local environment. Sensitivity of radio refractivity to temperature, pressure and humidity has been evaluated for a period of 5 years from 2008 to 2012, in a particular area under consideration. The results presented for tropospheric radio refractivity take into consideration both the location height and scale height parameters. The results are analysed in terms of statistical measures such as the moving averages, probability density function, monthly mean values, and their corresponding standard deviations. Finally, a mathematical model is formulated to calculate the radio refractivity for any day of a whole year. Reliability of error analysis in respect to accuracy is also shown. It was implemented in the industrial enterprise.

Keywords: radio refractivity, statistical estimation, ITU, mathematical modelling.



## 1. Introduction

Propagation of electromagnetic (EM) waves is affected by the properties of atmosphere. EM waves can be reflected, refracted, scattered, and absorbed by different atmospheric constituents. The troposphere is the lowest portion of Earth's atmosphere with the average depth of approximately 17 km in the middle latitudes and deeper in the tropics, up to 20 km, and shallower near the polar-regions, almost 7 km [1]. The intensity of atmospheric effects on radio waves depends

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mainly upon the signal power, the frequency of operation and on the state of the troposphere through which it propagates. The characterization of tropospheric state has a great significance to wireless communications systems, air avionics, environmental monitoring, disaster forecasting etc. Worse propagation conditions result in increased attenuation / fading in communication links and – consequently – errors occur and problems arise in signal detection at the receiver [2].

Quality of propagation of EM waves between the receiver and transmitter depends on the link's performance and consistency. Normally, in the radio link design, the link-planning engineers require measuring the signal strength data. Consequently, developing a radio propagation model needs the statistical analysis of a whole year, in order to evaluate signal magnitude variations that occur at various locations of interest over different periods of the year.

The radio refractive index is an important element of a propagation model to evaluate radio propagation in the troposphere [3]. It is observed that sometimes microwave/radio systems could become inaccessible due to the seasonal variation of refractive index [2, 4]. Therefore, the precise information on radio refractive index is important in the planning and design of terrestrial radio links for communication networks, radar and propagation applications [5–7]. Therefore, the refractive index variations in time and space are required in building such a model.

The refractive index, $n$, of a medium is known as a ratio of the signal velocity in a free space and its velocity in a specified medium [8]. The refractive index, $n$, of a medium, is given by:

$$n = \frac{V_{fs}}{V_m}, \tag{1}$$

where '$V_{fs}$' and '$V_m$' are signal velocities in a free space and in the other medium, respectively.

Consider a plane wave propagating in a medium of a uniform refractive index, $n$; its amplitude $E(r,t)$ varies in space $(r)$ and time $(t)$ and can be expressed as:

$$E(r,\tau) = E_o e^{[i(k_o n.r - \omega\tau)]}, \tag{2}$$

where $E_o$ is an amplitude which decays with the inverse square law for $r$, $\omega = 2\pi f$, $\omega$ is an angular frequency ($f$ denotes frequency) and $k_o$ is a free-space wave vector specifying the direction in which the wave propagates. The refractive index varies in the atmosphere and its space-time distribution results in such phenomena like fading, sub-refraction, super-refraction, scattering, ducting and absorption etc. [9]. The refractive index depends on the properties of medium and the boundary conditions, and – at the radio frequencies – on the wavelength scale. So, in general, the magnitude of $E(r,t)$ will vary with $r$ and it may be expressed as

$$E(r,\tau) = E_o e^{[i(k_o n(r).r - \omega\tau)]}. \tag{3}$$

Creating a radio refractive index database is essential because the knowledge of radio refractive index values is prerequisite when measurements are made in air [10–12]. In the absence of reliable local data, the wireless service providers use the radio refractive index and other data from the world charts and global numerical maps provided by International Telecommunication Union (ITU). However, in Pakistan, no local reliable data are available on the atmospheric radio refractivity. This paper estimates the refractive index values and presents their thorough statistical analysis. The variations of radio refractive index are derived from the radiosonde data over a 5-year period from 2008 to 2012 for Islamabad, Pakistan.

## 2. Atmospheric Parameter Analysis

The refractive index $n$ depends upon three factors: temperature, pressure and humidity (water vapours). While moving vertically upwards in the atmosphere, all these parameters show deviation from the value measured at the Earth's surface and – consequently – the values of refractive index $n$ also change. Near the Earth's surface it is slightly greater than unity and then diminishes to about $10^{-5}$. There are two major factors responsible for this deviation [2]:

(i) Polarization of the air molecule ingredients (i.e. nitrogen, oxygen, carbon dioxide and water vapours) of lower atmosphere; mainly due to the interaction of the electromagnetic signal with air molecules up to millimetre waves at varying values of pressure, humidity/water vapours and temperature.

(ii) The quantum mechanical molecular resonance effect, which is restricted to narrow frequency bands, around 22 GHz and 60 GHz.

A derived quantity referred to as the radio refractivity, $N$, is used for convenience in most scientific studies, and is mathematically expressed as:

$$n = (1+N)10^{-6}. \tag{4}$$

Here, $N$ is a dimensionless quantity, but it is expressed in $N$-units. The radio refractivity $N$ depends on the absolute air temperature $T(K)$, the atmospheric pressure $P$ (mbar), and the vapour pressure $e$ (mbar).

The radio refractivity $N$ can be computed by the following formula (ITU, 1970–1986–1990 –1992–1994–1995–1997–1999–2001–2003) [13]:

$$N = N_{dry} + N_{wet} = \frac{77.6}{T}\left(P + 4810\frac{e}{T}\right). \tag{5}$$

The "Dry Term $(N_{dry})$" of radio refractivity is calculated as:

$$N_{dry} = 77.6 \times \frac{P}{T}. \tag{6}$$

Also, the "Wet Term $(N_{wet})$" is given by:

$$N_{wet} = 3.732 \times 10^5 \frac{e}{T^2}, \tag{7}$$

where: $T$ is an absolute temperature (K), $P$ is an atmospheric pressure (hPa), $e$ is a water vapour pressure (hPa).

This expression can be useful for all radio frequencies; i.e. up to 100 GHz, with an error tolerance of 0.5%. The relationship between the water vapour pressure $e$ and the relative humidity $H$ is given by:

$$e = \frac{H.e_s}{100}, \tag{8}$$

$$es = a\exp\left(\frac{bt}{t+c}\right), \tag{9}$$

where $H$ is a relative humidity (%), $t$ is a Celsius temperature (°C), $es$ is a saturation vapour pressure (hPa) at a temperature $t$ (°C) and constants $a$, $b$, $c$, are listed in Table 1.

The vapour pressure $e$ can also be attained from the water vapour density $\rho$ using the equation:

$$e = \rho\frac{T}{216.7}, \tag{10}$$

where $\rho$ is given in g/m$^3$. Typical values of $\rho$ are given in Recommendation ITU-R P.836 [14].

Table 1. The values of different constants and boundary conditions along with accuracy.

| | *a* | *b* | *c* | Valid between | Accuracy |
|---|---|---|---|---|---|
| For water | 6.1121 | 17.502 | 240.97 | −20° to 50° | 0.20% |
| For ice | 6.1115 | 22.452 | 272.55 | −50° to 0° | 0.20% |

### *2.1. Surface Refractivity and its Relation to Height*

It has been found that the long-term average dependency of the refractive index $n$ upon the height $h$ is well expressed by an exponential law:

$$n(h) = 1 + N_o \times 10^{-6} \times \exp\left(\frac{-h}{h_s}\right), \tag{11}$$

where: $N_o$ is an average value of atmospheric refractivity extrapolated to the sea level, $h_s$ is a scale height in kilometres. $N_o$ and $h_s$ can be calculated statistically for different environments.

This reference profile may be used to compute the value of refractivity $N_s$ at the Earth's surface from $N_o$ as follows:

$$N_s = N_o \times \exp\left(\frac{-h_s}{h_0}\right), \tag{12}$$

where $h_s$ is the height of the Earth's surface above the sea level in kilometres.

### *2.2. Input Statistics*

The geographic area under analysis is the city of Islamabad, the capital of Pakistan. Table 2 provides information about its location. Radio refractivity deviations are calculated using the local trustworthy data acquired from the Islamabad airport.

Table 2. The key information about the area under observation [15, 16].

| | |
|---|---|
| Country | Pakistan |
| Region | Punjab |
| Location City | Islamabad |
| Latitude | 33° 43′ 0″ N |
| Longitude | 73° 4′ 0″ E |
| Elevation | 490–620 m |
| Avg Max Temperature | 38.1°C |
| Highest Temperature | 46.5°C |
| Lowest Temperature | −4°C |

The records used in this study were based on the results of radiosonde measurements of pressure, relative humidity and temperature taken every hour within the period of 5 years (2008–2012), obtained from the Pakistan Meteorological Department. Fig. 1 shows the complete measured input data profile for the 5-year period (2008–2012), Islamabad temperature used in the computation of radio refractivity and other parameters. The average value of temperature for each day is represented by a small rectangle with colour scale values defined by the bar (scale) attached on the right of the datasets. Such 31 small rectangles make a month in a column, as shown in Fig. 1. Then 365 small rectangles are joined to form a column of 12 months, i.e. one column represents data of one complete year. The horizontal scale shows years from 2008 to 2012. A large rectangle is formed from 5 columns (each column equivalent to 1 year) and 365 rows.

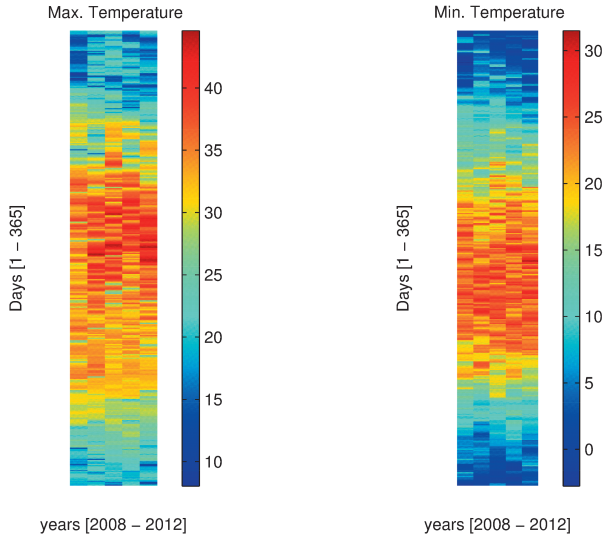


Fig. 1. The 5-year [2008–2012] measured input dataset of temperature of Islamabad used to calculate the refractivity.

## 3. Results and analysis

Generally, the changes of radio refractivity $N$ are caused by variations in temperature $(T)$, atmospheric pressure $(P)$ and water vapour pressure $(e)$ [3, 17]. The relative importance of these constraints $(T, P, e)$, especially water vapour contents, could also be perceived from the differentials of (5) [2] as:

$$\partial N = 77.6\frac{\partial P}{T} - \left(77.6 \times \frac{P}{T^2} + 7.46 \times 10^5 \times \frac{e}{T^3}\right)\partial T + 3.73 \times 10^5 \frac{\partial e}{T}. \tag{13}$$

For typical atmospheric conditions, (i.e. temperature $T = 290$ K, pressure $P = 1000$ hPa, relative humidity $RH = 60\%$ and vapour pressure $e = 13.7$ hPa) the above equation is reduced to:

$$\partial N = 0.268\partial P - 1.289\partial T + 4.435\partial e. \tag{14}$$

It is observed that the impact of $e$ on the gradients of radio refractivity is large in comparison with that of $T$ and $P$. This is due to the fact that water vapour molecules become polarized during interaction with the radio signal. As a consequence, the dielectric constant of water vapour rises, resulting in a relatively greater impact on $N$ than that of $T$ and $P$. In standard atmosphere conditions, the $P$, $T$, and $e$ values also decrease with height, resulting in decreasing $N$ by a gradient of $-40$ N/km.

The pressure remains constant at around $1000 \pm 10$ hPa. Therefore, its contribution to the variation of radio refractivity $N$ is not significant. It is customary to analyse the sensitivity of $N$ to temperature and water vapour pressure (i.e. humidity). Fig. 2 shows the sensitivity to atmospheric temperature and relative humidity variations, if pressure is kept constant. As the relative humidity increases, the value of $N$ also increases. From Fig. 2 it is also clear that a 1°C change in temperature results in about $4N$-unit change in radio refractivity $N$ in standard atmospheric conditions.

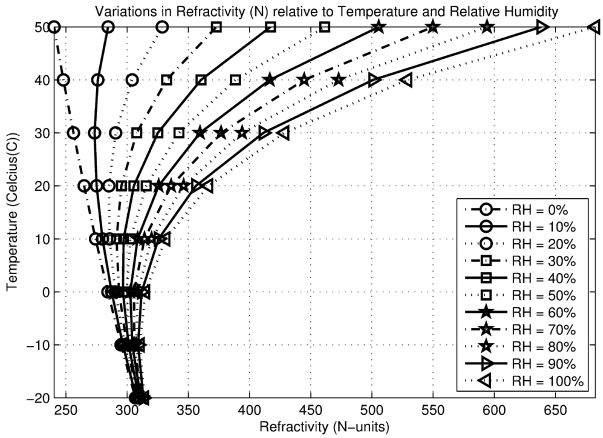


Fig. 2. The dependency of refractivity on temperature for different values of relative humidity at a constant pressure.

In (5), the dry and wet terms in the radio refractivity depend on both temperature and water vapour pressure. In the case of normal water vapour content in the atmosphere, an increase in the temperature would decrease the wet term faster than the dry term in (5). Hence, the contribution of wet term to the value of $N$ is relatively small. However, in the case of high water content in the atmosphere, the contribution of $N_{wet}$ to $N$ is higher. This is due to the fact that the decrease in its value due to the temperature is more than compensated by the increase due to the higher water vapour content. Consequently, the contribution of $N_{wet}$ to the value of $N$ is relatively larger over a period when atmosphere has a high vapour content. This effect can be also observed in Fig. 3, where the contribution of the wet term to the value of $N$ for the months between June and September is relatively higher than for the rest of months in the year. It is obvious that the wet part $N_{wet}$ contributes a smaller portion to $N$ than the dry part $N_{dry}$. The largest contribution of $N_{wet}$ to $N$ is in July and August; however, its value does not exceed about 30% of $N$. This is due to the humid and hot season from June to September of each year of the period under study, as is evident in Fig. 1.

### *3.1. Statistical Distribution of Radio Refractivity N and Scale Height Hs*

In a standard atmosphere, temperature, pressure, and relative humidity decrease with altitude above the ground surface. Thus, the radio refractivity decreases as well [18–20]. The application of a data smoothing/moving average system (a low-pass filter) to time-dependent datasets removes fluctuations and provides de-noised values of $N$ and $H_s$. The statistical estimates of $N$ and $H_s$ are acquired by applying the second order 31-days' smoothing. These parameters, $N$ and $H_s$, have shown periodic variations over the 5-year period under consideration.

Fig. 3 depicts the radio refractivity without and with the second order (31 days) moving average of $N$ versus a period from 1st January 2008 (day 1) to 31st December 2012 (day 1825). This variation of $N$ through the whole year can be correlated with relative changes in temperature, pressure and relative humidity depicted in Fig. 1. It is observed in Fig. 3 that the radio refractivity decreases from January reaching a minimum value of the year in May. From June on, the value of refractivity starts to upturn and attains a peak value in August. Then, it starts declining until September, after which it shows an upsurge and a small peak is obtained in October, where it reaches a second peak, smaller than the previous one. After October, for the rest of the winter period, the radio refractivity shows a decreasing trend.

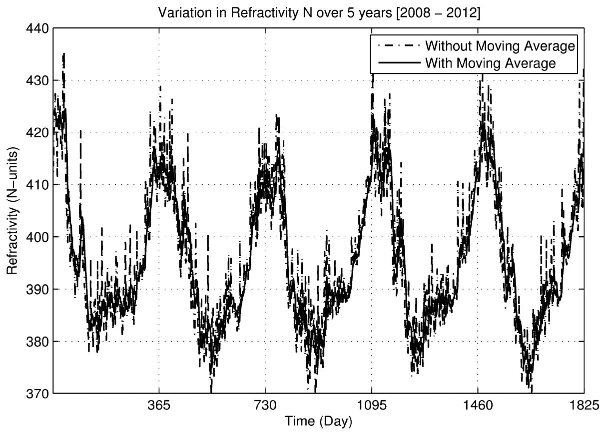


Fig. 3. The calculated refractivity without and with the second order moving of 31 days versus time (days).

Figure 4 shows the scale height $H_s$ with and without the second order moving average (31 days) versus days of a year (2008–2012). The scale height values (Fig. 4) follow a similar trend, but with lower variation than the values of $N$ (Fig. 3).

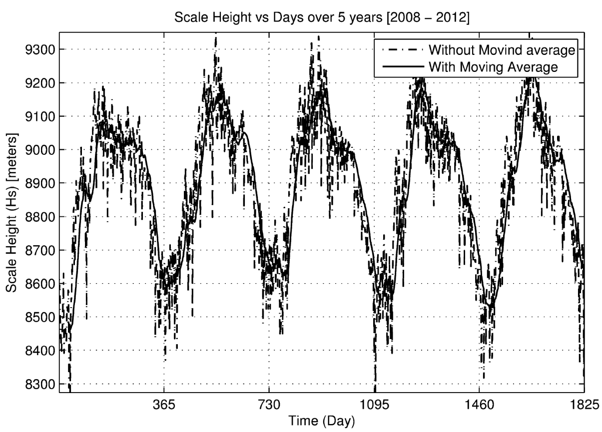


Fig. 4. The variation of monthly mean values of refractivity versus time (Month) for the entire period of study [2008–2012].

Figure 5 depicts the difference between the maximum and minimum values of mean refractivity for each month of a year over the 5-year study period. This difference for December is the smallest. In June this difference has the largest value, approximately 5 times greater than that of December. This fact can also be linked with datasets shown in Fig. 1. Thus, a year can be divided into the following three time windows for the study of $N$: (i) October to May, (ii) May to August, and (iii) August to October. More insight about variations in refractivity could be revealed from the examination of Fig. 5.

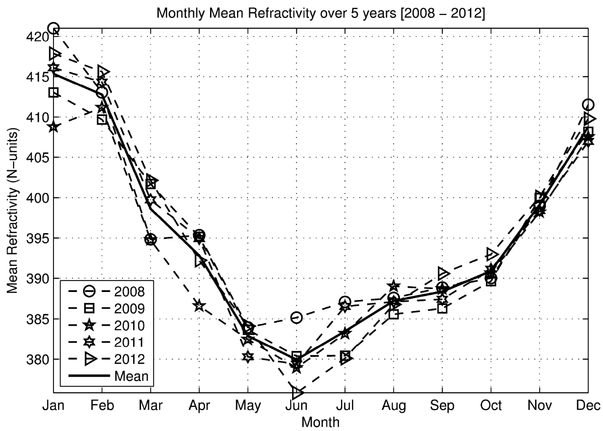


Fig. 5. The scale height values with and without the second order moving average of 31 days versus time (days) over [2008–2012].

Figure 6 provides the standard deviation of monthly radio refractivity versus each month for the 5-year period (2008–2012). A significant change is observed during the months from May to August, with the highest variation being experienced in June and July. The large deviation of radio refractivity from mean has a significant impact on propagation of radio signals and needs to be properly taken care of.

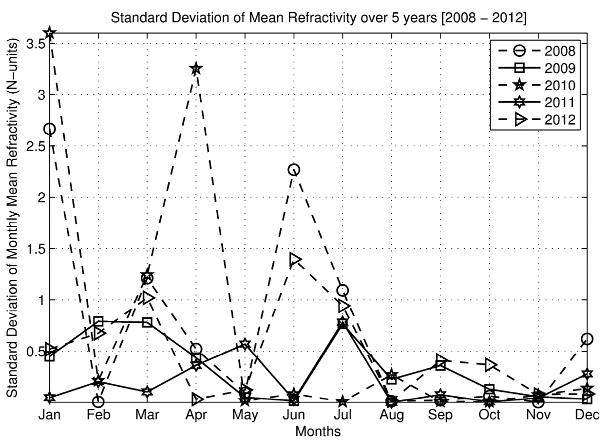


Fig. 6. The standard deviation of mean refractivity $(N)$ versus time (months) over the study period [2008–2012].

Figure 7 provides the monthly mean values of scale height $H_s$ versus time in months for the 5-year period under study [2008–2012]. The variations in monthly values of $H_s$ from year to year are negligible, being equal to approximately 5%. However, the value of $H_s$ during a typical year varies significantly. It increases from January to May, remains stable within a certain range during May–July, attaining the maximum value of the year in this period, and then decreases from August to December like waterfall curves. Fig. 9 depicts the standard deviation of $H_s$ versus time in months for the 5-year period under study (2008–2012). It is seen that the behaviour is quite sporadic over the months of a typical year and from year to year over the 5-year period under consideration. Primarily, the $H_s$ incorporates the temperature variation due to the elevation (or change of height above the sea level) that subsequently has an impact on the value of radio refractivity $N$.

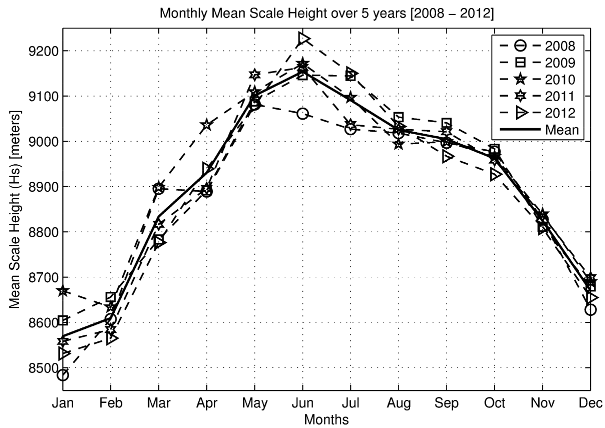


Fig. 7. The monthly values of mean Scale Height over the 5-year study period [2008–2012].

### *3.2. Calculation of Radio Refractivity Parameters and Determination of Mathematical Expressions*

The four-year data [2008–2011] are used to study the behaviour of fundamental parameters (Temperature, Atmospheric Pressure, Relative Humidity) affecting the radio refractivity. As the earlier analysis has shown, the behaviour of these parameters follows a specific pattern. A mathematical equation using the mean square error and curve-fitting methods is developed. This equation represents the behaviour of radio refractivity and it can be used to obtain the refractivity for any day of a year without the knowledge of ITUR recommendations. This model is believed to fit the actual radio refractivity with a reasonable accuracy.

As the behaviour of refractivity is periodic and repeats the values for each year with a slight difference, so the regeneration of such behaviour is possible with the help of weighted summation of *Sine* functions. Generally, increasing the number of *Sine* functions for weighted summation will lead to a more accurate solution at the cost of a greater number of coefficients. A general form of *Sine* function is given as:

$$a_n Sin(b_n x + c_n). \tag{15}$$

Here, $x$ is an independent variable and can be considered as the representation of any day of a year. The total number of terms is represented by $n$, whereas $a_n$, $b_n$ and $c_n$ are coefficients representing amplitude, frequency and initial phase of *Sine* function.

The estimated equation involves eight *sine* functions with three coefficients for each *sine* function. The values calculated for different parameters of fitness functions like SSE, R-square, Adjusted R-square and RMSE are listed in Table 4. It is noted that the error values are close to the ideal ones for eight *sine* functions and deviate from ideal values by decreasing the number of *sine* terms. The fitness analysis is shown in Fig. 8. The mean value extracted from the data is being analysed by *sine* functions with different numbers of terms. It is shown that eight *sine* functions will regenerate the data of refractivity with the smallest error.

$$N = f(D) = a_1\,Sin(b_1 D + c_1) + a_2\,Sin(b_2 D + c_2) + a_3\,Sin(b_3 D + c_3) + a_4\,Sin(b_4 D + c_4) + a_5\,Sin(b_5 D + c_5) + a_6\,Sin(b_6 D + c_6) + a_7\,Sin(b_7 D + c_7) + a_8\,Sin(b_8 D + c_8). \qquad (16)$$

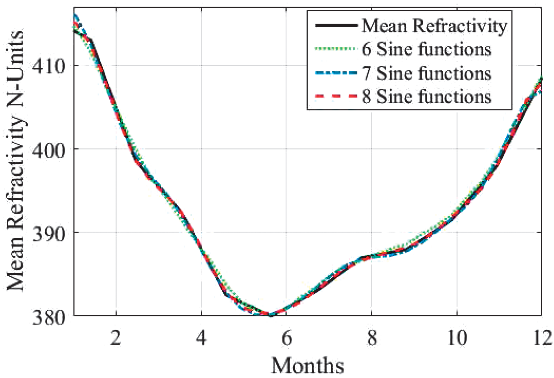


Fig. 8. Comparison of different numbers of terms in the *sine* function with the mean value of refractivity.

The Relative Refractivity ($N$) is represented by (16) as a function of days ($f(D)$). Note, that $D$ is a day of a complete year [1–365]. One can find the value of radio refractivity for a particular day of any year by substituting the Day number for $D$. Thus, 1 represents the 1st day (1st Jan) and 365 – the last day of a year (31st Dec).

The values of coefficients for (16) are calculated using the curve-fitting tool of MATLAB. The coefficient values of amplitude, frequency and initial phase are listed in Table 3.

Table 3. The values of parameters extracted with the method of least square errors using (16) in order to estimate the radio refractivity.

| | Value | | Value | | Value |
|---|---|---|---|---|---|
| $a_1$ | 44.93 | $b_1$ | 0.006914 | $c_1$ | 0.7255 |
| $a_2$ | 19.3 | $b_2$ | 0.01379 | $c_2$ | 3.443 |
| $a_3$ | 7.018 | $b_3$ | 0.02393 | $c_3$ | 4.815 |
| $a_4$ | 1.488 | $b_4$ | 0.05169 | $c_4$ | 1.723 |
| $a_5$ | 1.938 | $b_5$ | 0.05595 | $c_5$ | 4.609 |
| $a_6$ | 0.433 | $b_6$ | 0.08841 | $c_6$ | 1.494 |
| $a_7$ | 17.5 | $b_7$ | 0.1143 | $c_7$ | −2.063 |
| $a_8$ | 17.43 | $b_8$ | 0.1148 | $c_8$ | 0.9594 |

Table 4. The fitness parameter values of functions for increasing number of *sine* functions.

| | 6 Sine functions | 7 Sine Functions | 8 Sine Functions |
|---|---|---|---|
| SSE | 10.41 | 19.58 | 6.832 |
| R-square | 0.9974 | 0.9951 | 0.9983 |
| Adjusted R-square | 0.9946 | 0.9875 | 0.9943 |
| RMSE | 0.8065 | 1.227 | 0.8265 |

Other possible candidate functions for regression – like linear, quadratic, exponential and polynomial ones – produced less accurate results in comparison with the *sine* function.

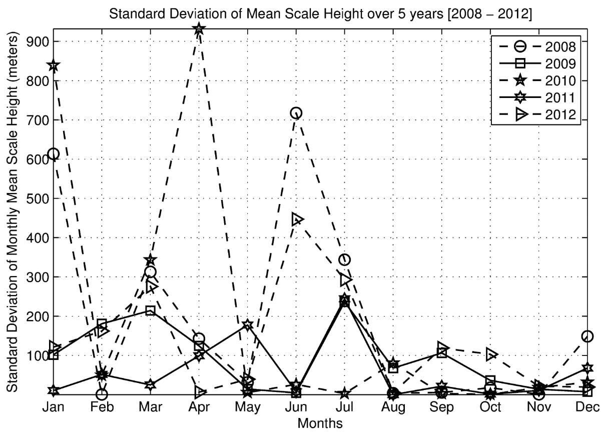


Fig. 9. Standard Deviation of mean Scale Height over the 5-year study period [2008-2012].

It is evident from the statistical data that the overall standard deviation for year 2012 is large in comparison with other years 2008–2011, regarding temperature, refractivity and scale height. Therefore, the proposed model extracted from 2008 to 2011 only and validated for 2012, will show a worse possible error in comparison with those obtained for the other years. However, the proposed model is valid for the entire period of 2008–2012. Fig. 10 compares the measured refractivity of year 2012 with the estimated refractivity obtained by the equation of estimation (16). It is clear from this figure that the estimated/modelled refractivity closely follows the real refractivity and thus the equation (16) bears a reasonable accuracy for almost all days of a year. This is confirmed by plotting the percentage error in estimation of radio refractivity for year 2012, as depicted in Fig. 11.

From Fig. 11 it is clear that the magnitude of percentage error is very low and is never greater than 6% over the whole study year of 2012. It can also be noted that the percentage error is randomly distributed over the whole year and the error is mostly lower than 2%.

The probabilistic analysis of percentage error in estimation of refractivity is shown in Fig. 12. The PDF of percentage error in refractivity for year 2012 shows a peak of slightly over 0.5 (actually 0.5257) at a 1% error. It means that, for an arbitrary day there is a 52% chance that the error obtained in calculation of refractivity is less than 1%. The PDF of error decreases exponentially by relaxing the criteria of percentage error to more than 1%. It becomes practically

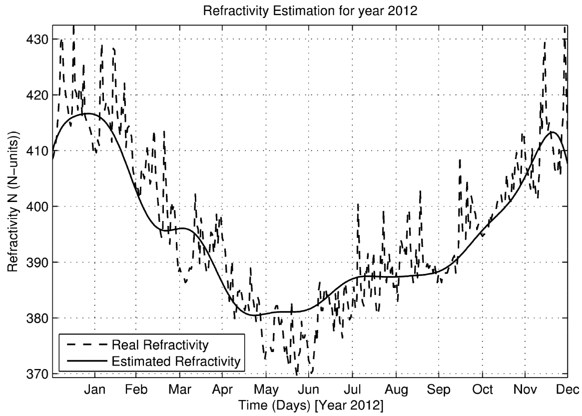


Fig. 10. Comparison of the measured (real) and estimated/modelled Refractivity (5.2) for year 2012.

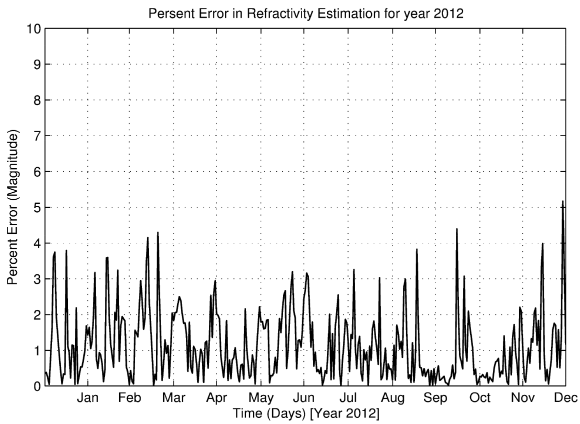


Fig. 11. The percentage error in estimation of Refractivity for year 2012.

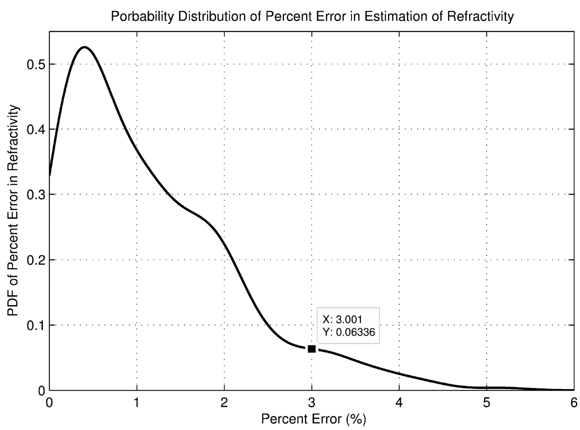


Fig. 12. The PDF of percentage error in estimation of Refractivity for year 2012.

zero at a 6% percentage error. Fig. 12 – the percentage error acceptance of 3% reduces the probability of error to 0.063, which is an extremely low value. This confirms the fact that a reasonable accuracy has been achieved in calculation of radio refractivity and (16) can be used as a reliable source to generate refractivity.

## 4. Conclusions

This work has addressed the issue of calculation of radio refractivity in troposphere in varying climatic conditions along with the effect of location height. The tropospheric radio refractivity is calculated by applying the method recommended by ITU. The presented results indicate a strong dependence of the radio refractivity on temperature and humidity (i.e. water vapour pressure). The monthly mean values exhibit a dip during June–July and then increase on both sides. The peak values of refractivity appearing mostly during Jan–Feb are mainly due to moist and colder year. Further, a significant deviation from the mean value of radio refractivity noticed during Jan-Feb and has a substantial impact on radio signal propagation and hence needs to be properly outfitted. This variation trend remained almost identical across all years of the 5-year period under consideration. The $H_s$ (scale height) parameter has also been computed to evaluate the elevation effect and consequently its impact on radio refractivity. The sensitivity analysis of radio refractivity in terms of statistical measures has also been performed using the datasets of temperature, pressure and humidity for the 5-year period (2008–2012). The statistical procedures used in this analysis include moving average, monthly mean, standard deviation, and percentage error ones. An analysis of parameters affecting radio refractivity is presented together with formulation of mathematical expression/equation. This mathematical formula/expression bears a reasonable accuracy and thus it can serve as a handy tool in calculation of radio refractivity.

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