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Statistical Methods Used in the Assessment of the Influence of the Al-Si Alloy's Chemical Composition on its Properties

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Abstract

The paper presents the results of the application of a statistical analysis to evaluate the effect of the chemical composition of the die casting Al-Si alloys on its basic mechanical properties. The examinations were performed on the hypoeutectic Al-Si alloy type EN AC-46000 and, created on its basis, a multi-component Al-Si alloy containing high-melting additions Cr, Mo, W and V. The additions were introduced into the base Al-Si alloy in different combinations and amounts (from 0,05% to 0,50%). The tensile strength R_m; the proof stress $R_{p0,2}$; the unit elongation A and the hardness HB of the examined Al-Si alloys were determined. The data analysis and the selection of Al-Si alloy samples without the Cr, Mo, W and V additions were presented; a database containing the independent variables (Al-Si alloy's chemical composition) and dependent variables (R_m; $R_{p0,2}$; A and HB) for all the considered variants of Al-Si alloy composition was constructed. Additionally, an analysis was made of the effect of the Al-Si alloy's component elements on the obtained mechanical properties, with a special consideration of the high-melting additions Cr, Mo, V and W. For the optimization of the content of these additions in the Al-Si alloy, the dependent variables were standardized and treated jointly. The statistical tools were mainly the multivariate backward stepwise regression and linear correlation analysis and the analysis of variance ANOVA. The statistical analysis showed that the most advantageous effect on the jointly treated mechanical properties is obtained with the amount of the Cr, Mo, V and W additions of 0,05 to 0,10%.

Keywords: Mechanical properties, Statistical methods, Multi-component Al-Si alloys, Pressure die casting

1. Introduction

The use of elements of a relatively high melting point, such as Cr, Mo, V and W, as additions to Al-Si alloys can cause the formation of intermetallic phase precipitates containing these elements in the alloy's microstructure. These phases precipitate in the Al-Si alloys as a result of the lack of or very limited solubility of these elements in aluminium in the solid state [1-9]. They can cause an increase of brittleness and a decrease of the strength properties of the Al-Si alloy. This problem is especially important with relatively large sizes and a complex morphology of the intermetallic phase precipitates. To the highest extent, the intermetallic phases can occur in Al-Si alloys crystallizing under the conditions of relatively slow heat removal, that is ones cast into sand and ceramic moulds. Increasing the intensity of heat removal from the crystallizing cast through the use of a metal mould can lead to oversaturation of the Al-Si alloy solid solution α(Al) with high-melting elements. This can cause an increase of the mechanical properties of the Al-Si alloy. The studies [10-12] present the results of the investigations of a pressure cast hypoeutectic Al-Si alloy containing different combinations of high-melting elements selected from the group: Cr, Mo, V and W. The presented test results point to the possibility of increasing the basic mechanical properties of the Al-Si alloy by way of applying specific combinations of these elements. In general, introducing relatively low amounts of the examined high-melting element combinations caused an increase of selected mechanical properties of the Al-Si alloy. In turn, higher amounts of these elements caused a secondary decrease of the mechanical properties. The increase of the properties is connected with oversaturation of the Al-Si alloy solid solution α with high-melting elements, whereas the secondary decrease of the properties is caused by the precipitation of intermetallic phases containing high-melting elements of relatively large sizes and a disadvantageous morphology.

The aim of this study is to apply a statistical analysis to present the effect of Cr, Mo, V and W additions introduced into a pressure cast hypoeutectic Al-Si alloy on its jointly considered basic mechanical properties.

2. Test methodology

The initial Al-Si alloy was the EN AC-46000 type. The scope of the chemical composition of the initial Al-Si alloy used in the tests is presented in Table 1. In this table there is a chemical composition tested by Authors.

Table 1.
Scope of the chemical composition of the initial test Al-Si alloy type EN AC-46000

Chemical composition, % wt.									
Si	Cu	Zn	Fe	Mg	Mn	Ni	Ti	Cr	Al
8,66 ÷ 9,43	2,09 ÷ 2,53	0,86 ÷ 1,09	0,70 ÷ 0,97	0,20 ÷ 0,35	0,18 ÷ 0,27	0,05 ÷ 0,13	0,038 ÷ 0,053	0,023 ÷ 0,031	Base

The initial Al-Si alloy was melted in a gas heated shaft furnace with the capacity of 1.5 Mg. Inside the shaft furnace, the Al-Si alloy was refined with the solid refiner Ecosal Al113.S. After the melting and refinement, the Al-Si alloy was deslagged and transported to a heating furnace placed near the die cast machine. In the heating furnace, high-melting additions Cr, Mo, V and W were introduced into the initial Al-Si alloy. The elements were added into the Al-Si alloy:

- separately (each of them individually),
- in combinations of two: CrMo, CrV, CrW, MoV, MoW and VW;
- in combinations of three: CrMoV, CrMoW, CrVW and MoVW;
- simultaneously (all four at the same time).

The examined high-melting additions were introduced into the Al-Si alloy in the form of master alloys: AlCr15, AlMo8, AlV10 and AlW8. The master alloys were introduced in the amount which made it possible to obtain the assumed amount of the particular additions in the melt. For the individually introduced additions, their content in the Al-Si alloy was applied in the scope of 0,0-0,5%; for the combinations of two, the scope was 0,0-0,4%, while in the case of combinations of three and the one of four, the scope was 0,00-0,25%. The amount of chromium, molybdenum, vanadium and tungsten introduced into the Al-Si alloy individually and in combinations of two was diversified every 0,1%; whereas in the case of combinations of three and the one of four – every 0,05%. For each variant of the Al-Si alloy's chemical composition, casts of a roller blind casing cover were made, with the dominant cast wall thickness of 2 mm. The casts were made on the die cast machine with a cold horizontal pressure chamber Idra 700S.

The strength tests were performed on samples cut out of the cover pressure casts. For each examined Al-Si alloy's chemical composition, 3 samples were cut out of one cast. The samples had a rectangular section of 2/10 mm, which is recommended by the standard [13]. The tensile tests were conducted on the tester Instron 3382 with the rate of 1 mm/min. During the tensile test, the following were determined: the tensile strength R_m, the proof stress $R_{p0,2}$ and the unit elongation A.

The hardness HB measurements were made on the universal hardness tester HPO-2400. The nodule of the diameter d = 2,5 mm and the load of 613 N were applied. Hardness was tested according to PN-EN ISO 6506-1:2014-12.

For the elaboration of the database, the Excel worksheet was used, while for the statistical calculations, the licensed packets: Statistica v. 7.1.PL by Statsoft and MedCalc Statistical Software v.14.10.2 (MedCalc Software bvba, Ostend, Belgium) were applied.

3. Statistical analysis results

In this chapter a selection of samples of the initial Al-Si alloy are presented. Also, a database was constructed for the statistical analysis, considering the independent and dependent variables for all the examined variants of Al-Si alloy composition. Next, an analysis was made of the effect of the Al-Si alloy's component elements on the obtained mechanical properties, with a special consideration of the high-melting additions: Cr, Mo, V and W.

In all the statistical analyses, the significance level (type 1 error) **$p(\alpha) = 0{,}05$** was assumed, which means that, if $p(\alpha) < 0{,}05$, the zero hypothesis should be rejected.

3.1. Creation of data bases

The data in the created database were divided into **independent (explanatory) variables** and **dependent (response) variables.** The independent variables are composed of:

- ✓ contents of ***basic elements,*** expressed in [%], that is the elements included in the standard defining the chemical composition of the initial Al-Si alloy EN AC-46000. The concentration of the elements: ***Si, Fe, Cu, Mn, Mg, Ni, Zn*** and ***Ti***, for all the performed melts and concentrations in the Al-Si alloy (according to the assumption) should vary slightly,
- ✓ expressed in [%], contents of the examined high-melting additions introduced into the initial Al-Si alloy, that is a varying concentration of the elements: ***Cr, Mo, V*** and ***W***.

The dependent variables constitute the basic mechanical properties of the examined Al-Si alloys: $\boldsymbol{R_m}$ [MPa], $\boldsymbol{R_{p0,2}}$ [MPa], ***A*** [%] and ***HB***. For the purpose of the statistical analysis, the dependent variables (element concentration) ***Cr, Mo, V*** and ***W*** were coded into **quality variables** (so called, factors), which had variables ***Cr_1, Mo_1, V_1*** and ***W_1*** **nine** levels (from **1** to **9**): 0,00% = **1**; 0,05% = **2**; 0,10% = **3**; 0,15% = **4**; 0,20% = **5**; 0,25% = **6**; 0,30% = **7;** 0,40% = **8;** 0,50% = **9**.

The database was supplemented with ***the standardized values of dependent variables***, which can be used to optimize the effect of the content of ***the elements*** in the examined Al-Si alloy on the dependent variables ***Rm; Rp0,2***; ***A*** and ***HB*** treated separately and **jointly**, by way of denoting them as ***S_Rm; S_Rp0,2***; ***S_A, S_HB*** and ***S_Sum.***

The standardized variable characterizes in its mean value being equal to zero, and the standard deviation equals one. The standardization procedure makes it possible to transform the denominate variable into an abstract number. Owing to this, the mean values obtained from different sources (in different units) can be compared with each other. The standardized value can be calculated according to the relation:

standardized value =
= (empirical value – mean value)/standard deviation

A fragment of the obtained database is presented in Table 2.

Table 2.
Fragment of the obtained database

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	LP	Si	Fe	Cu	Mn	Mg	Ni	Zn	Ti	Cr	Mo	V	W
2	1	9,37	0,77	2,15	0,19	0,20	0,06	1,04	0,04	0,00	0,00	0,00	0,00
3	2	9,37	0,77	2,15	0,19	0,20	0,06	1,04	0,04	0,00	0,00	0,00	0,00
4	3	9,37	0,77	2,15	0,19	0,20	0,06	1,04	0,04	0,00	0,00	0,00	0,00
5	4	9,29	0,70	2,11	0,18	0,19	0,06	1,04	0,04	0,10	0,00	0,00	0,00
6	5	9,29	0,70	2,11	0,18	0,19	0,06	1,04	0,04	0,10	0,00	0,00	0,00
7	6	9,29	0,70	2,11	0,18	0,19	0,06	1,04	0,04	0,10	0,00	0,00	0,00

	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	Cr_1	Mo_1	V_1	W_1	*Rm* [Mpa]	*Rp0,2* [Mpa]	*A* [%]	*HB*	*S_Rm*	*S_Rp0,2*	*S_A*	*S_HB*	*S_Sum*
2	1	1	1	1	211	97	4,4	100	-1,149	-1,403	0,262	-2,403	-4,694
3	1	1	1	1	199	106	3,2	102	-1,500	-0,832	-0,683	-1,993	-5,008
4	1	1	1	1	182	99	2,7	101	-2,031	-1,263	-1,009	-2,198	-6,501
5	3	1	1	1	226	95	4,0	107	-0,662	-1,589	-0,041	-0,968	-3,259
6	3	1	1	1	194	98	3,0	109	-1,678	-1,348	-0,815	-0,557	-4,399
7	3	1	1	1	216	99	3,5	108	-0,983	-1,282	-0,428	-0,762	-3,456

The database contains independent and dependent variables referring to 84 casts, that is 252 measurements of the mechanical properties. Of all the casts included in the database, 15 were made of the initial Al-Si alloy EN AC-46000, that is the one which does not contain an addition of ***Cr, Mo, V*** or ***W***. The casts were characterized with 45 results of the ***Rm; Rp0,2; A*** and ***HB*** measurements. For those results, the characteristics of the descriptive statistics of the dependent variables were elaborated, which are presented in Table 3.

It can be inferred from the data presented in Table 3 that, despite the aim of maintaining the normal (usual) composition of the initial Al-Si alloy (that is the concentration of ***Si, Fe, Cu, Mn, Mg, Ni, Zn*** and ***Ti***), very large scatters of the levels of dependent variables (explanatory features) were established: ***Rm; Rp0,2; A*** and ***HB***. Figure 1 shows an exemplary scatter of the values of tensile strength ***Rm*** of the initial Al-Si alloy, with the marked quartile 1 and 3 and the median.

Table 3.
Characteristics of the descriptive statistics of the dependent variables for samples of the initial Al-Si alloy EN AC-46000

Variable	N	Arithmetic mean	Median	Min	Max	Quartile 25%	Quartile 75%	SD	Standard error of the mean
Si	45	9,02	9,07	8,66	9,43	8,74	9,32	0,275	0,041
Fe	45	0,84	0,86	0,70	0,97	0,81	0,88	0,066	0,010
Cu	45	2,28	2,28	2,09	2,53	2,16	2,37	0,128	0,019
Mn	45	0,21	0,21	0,18	0,27	0,19	0,23	0,027	0,004
Mg	45	0,27	0,25	0,20	0,35	0,23	0,32	0,048	0,007
Ni	45	0,09	0,10	0,05	0,13	0,06	0,11	0,026	0,004
Zn	45	0,98	0,97	0,86	1,09	0,91	1,04	0,071	0,011
Ti	45	0,04	0,05	0,04	0,05	0,04	0,05	0,004	0,001
Rm	**45**	**235,06**	**239,32**	**176,85**	**296,45**	**203,34**	**260,25**	**36,527**	**5,445**
Rp0,2	**45**	**120,29**	**120,00**	**84,18**	**160,56**	**106,03**	**128,72**	**19,652**	**2,930**
A	**45**	**3,32**	**3,11**	**1,37**	**6,73**	**2,50**	**4,06**	**1,136**	**0,169**
HB	**45**	**113,29**	**114,00**	**100,00**	**122,00**	**112,00**	**118,00**	**6,247**	**0,931**
S_Rm	45	-0,394	-0,262	-2,193	1,503	-1,374	0,384	1,129	0,168
S_Rp0,2	45	0,107	0,088	-2,271	2,760	-0,832	0,663	1,294	0,193
S_A	45	-0,548	-0,714	-2,062	2,090	-1,187	0,021	0,880	0,131
S_HB	45	0,322	0,468	-2,403	2,109	0,058	1,289	1,281	0,191
S_Sum	45	-0,512	-0,467	-6,501	8,159	-3,363	1,801	3,254	0,485

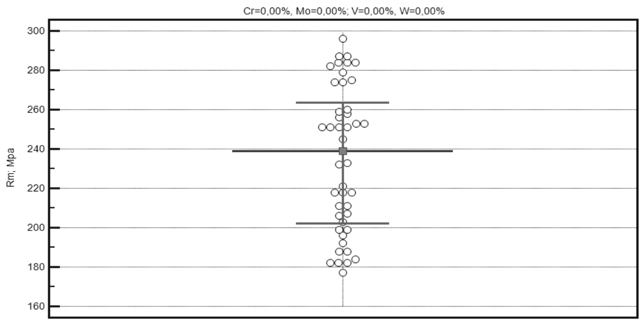

Fig. 1. Values of ***Rm*** for samples of the initial Al-Si alloy type EN AC-46000

Due to the large scatters in the levels of dependent variables, it was verified whether the different concentrations of ***Si, Fe, Cu, Mn, Mg, Ni, Zn*** and ***Ti*** (despite being in the normal scope) significantly affect the level of dependent variables ***Rm; Rp0,2; A*** and ***HB***. To that end, the multivariate (multiple) backward stepwise regression and linear correlation analysis was applied, by way of eliminating the insignificant variables in each step, until the final (end) model was obtained, in which all the independent variables significantly affect the dependent variable. Figure 2 and Table 4 show the initial model and the result of the search for the final model of the analysis of the ***Si, Fe, Cu, Mg, Ni, Zn*** and ***Ti*** concentration's effect on the level of the dependent variable ***Rm*** for samples of the initial Al-Si alloy EN AC-46000.

Multivariate linear regression - analysis for the content: Cr = 0,00; Mo = 0,00; V = 0,00 i W = 0,00						
Dependent variable: ***Rm***	R = 0,5028 R^2 = 0,2528		F(8,36)=1,52; p<0,1837; Standard error: 34,91			
Independent variables	**BETA**	**Std. error BETA**	**B**	**Std. error B**	**t(36)**	**p**
Constant			-300,65	545,36	-0,551	0,5848
Si	-0,010	0,276	-1,28	36,58	-0,035	**0,9724**
Fe	0,288	0,201	159,85	111,64	1,432	**0,1608**
Cu	0,549	0,516	156,84	147,50	1,063	**0,2947**
Mn	-0,046	0,271	-62,55	368,51	-0,170	**0,8662**
Mg	-0,803	0,624	-617,12	479,90	-1,286	**0,2067**
Ni	0,640	0,269	916,12	384,19	2,385	**0,0225**
Zn	0,084	0,422	43,14	217,57	0,198	**0,8439**
Ti	0,254	0,203	2451,60	1965,43	1,247	**0,2203**

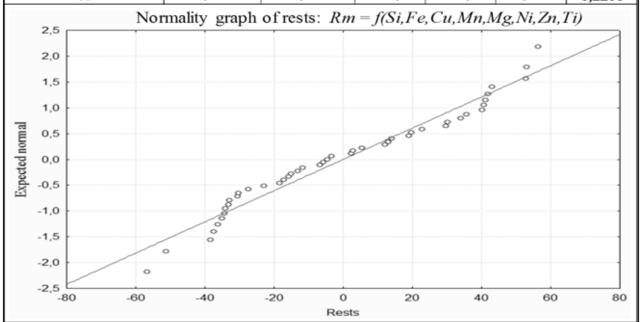

Fig. 2. Initial model of the analysis of the ***Si, Fe, Cu, Mg, Ni, Zn*** and ***Ti*** concentration's effect on the level of the dependent variable ***Rm*** for samples of the initial Al-Si alloy EN AC-46000

Table 4.
Result of the search for the final model

R= 0,2729 R^2= 0,0745		F(1,43)=3,4609; p<0,0697; Standard error: 35,54				
Dependent variable: ***Rm***	**BETA**	**Std. error BETA**	**B**	**Std. error B**	**t(43)**	**p**
Constant			200,37	19,39	10,335	0,0000
Ni	**0,27**	**0,15**	**390,41**	**209,86**	**1,860**	**0,0697**

After the elimination of the insignificant variables, a final (end) model with significant variables was not obtained (Tab. 4). In view of this fact, the changes in the concentration of ***Si, Fe, Cu, Mn, Mg, Ni, Zn*** and ***Ti*** do not significantly affect the level of ***Rm*** of the Al-Si alloy containing ***Cr, Mo, V*** and ***W***. Figure 3 shows the initial model of the analysis of the ***Si, Fe, Cu, Mn, Mg, Ni, Zn*** and ***Ti*** concentration's effect on the level of the dependent variable ***Rp0,2*** for samples of the initial Al-Si alloy. In turn, Figure 4 presents the final model of this analysis obtained after the elimination of the insignificant variables.

It can be inferred from the data included in Fig. 4 that the changes in the ***Si, Cu, Mg, Ni*** and ***Zn*** concentration significantly affect the level of ***Rp0,2*** of the Al-Si alloy containing ***Cr, Mo, V*** and ***W***. Due to this fact, the further analysis of ***Rp0,2*** should consider the variables: ***Si, Cu, Mg, Ni*** and ***Zn***. The complete analysis of the significance of the ***Si, Fe, Cu, Mg, Ni, Zn*** and ***Ti*** concentration's effect on the level of the dependent variables suggest the lack of a significant effect of these elements on the values of ***Rm*** and ***A***. In turn, in the further analysis of the yield point ***Rp0,2***, one should take into consideration the concentration of ***Si, Cu, Mg, Ni*** and ***Zn,*** whereas, the further analysis of the hardness ***HB*** should consider the effect of ***Si, Fe, Mn, Ni, Zn*** and ***Ti***.

Multivariate linear regression - analysis for the content: Cr = 0,00; Mo = 0,00; V = 0,00 i W = 0,00						
Dependent variable: ***Rp0,2***	R = 0,8182 R^2 = 0,6695		F(8,36)=9,12; p<0,0001; Standard error: 12,49			
Independent variables	**BETA**	**Std. error BETA**	**B**	**Std. error B**	**t(36)**	**p**
Constant			428,27	195,15	2,195	0,0347
Si	-0,641	0,183	-45,82	13,09	-3,501	**0,0013**
Fe	0,163	0,134	48,73	39,95	1,220	**0,2305**
Cu	1,055	0,344	162,11	52,78	3,071	**0,0040**
Mn	-0,088	0,180	-64,54	131,87	-0,489	**0,6275**
Mg	-1,997	0,415	-826,38	171,73	-4,812	**0,0000**
Ni	1,044	0,179	803,35	137,48	5,844	**0,0000**
Zn	-0,700	0,281	-193,88	77,86	-2,490	**0,0175**
Ti	0,212	0,135	1101,09	703,31	1,566	**0,1262**

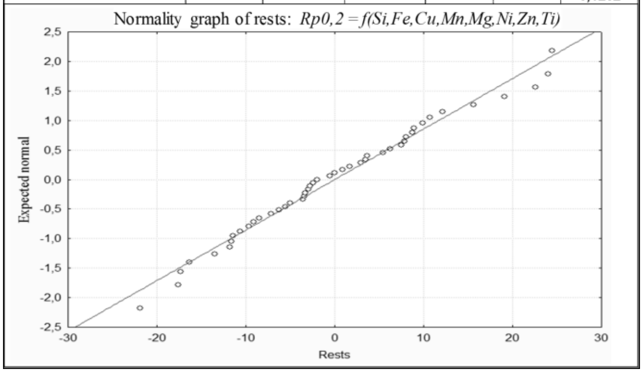

Fig. 3. Initial model of the analysis of the ***Si, Fe, Cu, Mg, Ni, Zn*** and ***Ti*** concentration's effect on the level of the dependent variable ***Rp0,2*** for samples of the initial Al-Si alloy EN AC-46000

R= 0,7986 R^2= 0,6378	**Final model**		F(5,39)=13,739 p<0,0001 Standard error: 12,562			
Dependent variable: ***Rp0,2***	**BETA**	**Std. error BETA**	**B**	**Std. error B**	**t(43)**	**p**
Constant			639,12	121,77	5,248	0,0000
Si	-0,780	0,152	-55,68	10,86	-5,127	**0,0000**
Cu	0,971	0,301	149,13	46,21	3,227	**0,0025**
Mg	-1,918	0,374	-793,49	154,61	-5,132	**0,0000**
Ni	0,969	0,149	745,60	114,30	6,523	**0,0000**
Zn	-0,770	0,252	-213,49	69,87	-3,056	**0,0040**

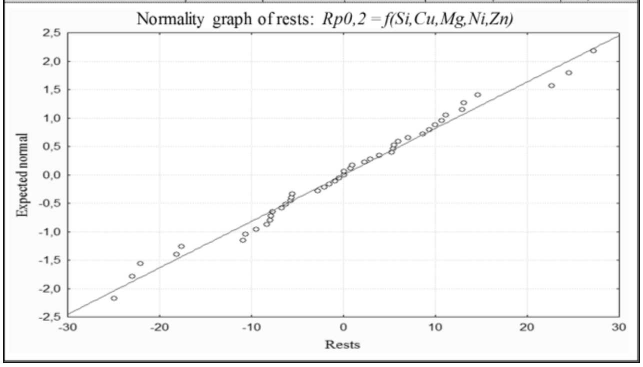

Fig. 4. Final model of the analysis of the ***Si, Fe, Cu, Mg, Ni, Zn*** and ***Ti*** concentration's effect on the level of the dependent variable ***Rp0,2*** for samples of the initial Al-Si alloy EN AC-46000

One of the methods of eliminating the significance of the effect of the ***Si, Fe, Cu, Mn, Mg, Ni, Zn*** and ***Ti*** concentration is the rejection of the extreme results of the examined dependent variables (that is the casts with such results). The extreme results were considered to be those below the quartile 25% (lower) and above the quartile 75% (upper). And so, only the casts for which all the four mean levels of the dependent variables ***Rm; Rp0,2; A*** and ***HB*** simultaneously lie in the scope of the interquartile range were assumed. Table 5 shows the criterion scores of the examined dependent variables assumed for the selection of casts of the initial Al-Si alloy assigned for the further analysis.

Table 5.
Data for the selection of casts of the initial Al-Si alloy EN AC-46000 for further analysis

Dependent variable	Arithmetic mean	Quartile 25%	Quartile 75%
Rm	235	203	261
Rp 0,2	120	106	129
A	3,3	2,5	4,1
HB	113	112	118

Of the 15 initial Al-Si alloy casts, the above criteria are fulfilled by 4 casts, to which 12 measurements of the values of the dependent variables correspond. And so, the final database takes into consideration 12 measurements of the value of each of the examined dependent variables for the initial Al-Si alloy, rather than 45, as was the case at the beginning.

The ultimate, corrected, database (3 measurement results of the values of ***Rm; Rp0,2; A*** and ***HB*** per each cast) subjected to a statistical analysis contains the results obtained from **73** casts, that is **219** measurements. For these results, the dependent variables were standardized again and marked as follows: ***Sost_Rm***; ***Sost_Rp0,2***; ***Sost_A***; ***Sost_HB*** and ***Sost_Sum***.

Preliminarily, it was established that the optimization of the chemical composition of the examined Al-Si alloy would be conducted with the aim to obtain the maximal values of R_m; $R_{p0,2}$; ***A*** and ***HB***. As the target (dependent, response) variable covering the joint result of the examined dependent variables (response features): ***Rm; Rp0,2; A*** and ***HB*** the sum of the standardized values of these variables was assumed, that is the variable with the name ***Sost_Sum***. The latter was calculated according to relation (1):

$$Sost\_Sum = Sost\_Rm + Sost\_Rp0,2 + Sost\_A + Sost\_HB \quad (1)$$

3.2. Evaluation of the effect of chemical composition on *Sost_Sum*

In the first stage, for the optimization of the joint effect of the explanatory features (independent variables), the multivariate (multiple) analysis of regression and correlation was applied. The so called backward stepwise regression was used, by way of eliminating, after each step, the independent variable characterizing in the lowest effect.

3.2.1. Analysis of the effect of Cr, Mo, V and W

Tables 6 and 7 show the obtained results of the evaluation of the effect of ***Cr, Mo, V*** and ***W*** on ***Sost_Sum*** for the first and last step.

Table 6.
Results of linear regression considering the effect of ***Cr, Mo, V*** and ***W*** on ***Sost_Sum*** - step 1

Dependent variable: ***Sost_Sum***						
$R=0,1738$; $R^2=0,0302$; $F(4,214)=1,6674$ $p<0,1587$; Standard error: 2,5248						
Independent variable:	**BETA**	**Std. error BETA**	**B**	**Std. error B**	**t(214)**	**p**
Constant			0,6567	0,3378	1,9440	**0,0532**
Cr	-0,1009	0,0676	-1,9714	1,3195	-1,4940	**0,1366**
Mo	-0,0858	0,0676	-1,6765	1,3195	-1,2705	**0,2053**
V	-0,0143	0,0676	-0,2796	1,3195	-0,2119	**0,8324**
W	-0,1262	0,0676	-2,4647	1,3195	-1,8679	**0,0631**

Table 7.
Results of linear regression considering the effect of ***Cr, Mo,*** V and ***W*** on ***Sost_Sum*** after the elimination of insignificant variables – last step

Dependent variable: ***Sost_Sum***						
$R=0,1167$; $R^2=0,0136$; $F(1,217)=2,9968$ $p<0,08485$; Standard error: 2,5287						
Independent variable:	**BETA**	**Std. error BETA**	**B**	**Std. error B**	**t(217)**	**p**
Constant			0,2342	0,2179	1,0745	**0,2838**
W	-0,1167	0,0674	-2,2793	1,3167	-1,7311	**0,0849**

It can be inferred from the data presented in Tables 6 and 7 that:

- no statistically significant (linear) effect on ***Sost_Sum*** was established,
- this result may come from the fact that these elements (in the considered concentrations) have no linear effect on the target feature,
- an analysis based on the quality features (factors) should be conducted,
- an analysis based on all the remaining alloy components should also be performed.

3.2.2. Analysis of the effect of Si, Fe, Cu, Mn, Mg, Ni, Zn, Ti, Cr, Mo, V and W

Tables 8 and 9 include the obtained results of the evaluation of the effect of ***Si, Fe, Cu, Mn, Mg, Ni, Zn, Ti, Cr, Mo, V*** and ***W*** on ***Sost_Sum*** for the first and last step.

Table 8.
Results of linear regression considering the effect of ***Si, Fe, Cu, Mn, Mg, Ni, Zn, Ti, Cr, Mo, V*** and ***W*** on ***Sost_Sum*** – step 1

Dependent variable: ***Sost_Sum***						
$R=0,4878$; $R^2=0,2379$; $F(12,206)=5,3596$ $p<0,0001$; Standard error: 2,2812						
Independent variable:	**BETA**	**Std. error BETA**	**B**	**Std. error B**	**t(206)**	**p**
Constant			-9,8808	9,2020	-1,0738	**0,2842**
Si	-0,0584	0,1272	-0,3537	0,7699	-0,4594	**0,6464**
Fe	**0,3355**	**0,1136**	**11,6215**	**3,9342**	**2,9540**	**0,0035**
Cu	0,1249	0,1020	1,8330	1,4964	1,2250	**0,2220**
Mn	-0,2231	0,1173	-23,3458	12,2740	-1,9021	**0,0586**
Mg	-0,1293	0,1179	-5,6770	5,1732	-1,0974	**0,2738**
Ni	**0,3626**	**0,0951**	**35,2608**	**9,2522**	**3,8111**	**0,0002**
Zn	0,1135	0,0900	2,2101	1,7523	1,2613	**0,2086**
Ti	0,0784	0,0874	25,1773	28,0903	0,8963	**0,3711**
Cr	-0,0983	0,0749	-1,9200	1,4621	-1,3131	**0,1906**
Mo	-0,0872	0,0930	-1,7036	1,8163	-0,9379	**0,3494**
V	0,0927	0,1114	1,8107	2,1753	0,8324	**0,4062**
W	-0,1743	0,0898	-3,4040	1,7545	-1,9402	**0,0537**

Table 9.

Results of linear regression considering the effect of ***Si, Fe, Cu, Mn, Mg, Ni, Zn, Ti, Cr, Mo, V*** and ***W*** on ***Sost_Sum*** – last step

Dependent variable: *Sost_Sum*						
R=0,4651 R^2=0,2163; F(6,212)=9,7545; p<0,0001; Standard error: 2,2803						
Independent variable:	BETA	Std. error BETA	B	Std. error B	t(212)	p
Constant			-11,9807	2,2013	-5,4424	0,0000
Fe	0,3536	0,0875	12,2482	3,0297	4,0427	0,0001
Mn	-0,1941	0,0907	-20,3174	9,4963	-2,1395	0,0335
Ni	0,2961	0,0618	28,7999	6,0110	4,7912	0,0000
Zn	0,2010	0,0638	3,9147	1,2416	3,1529	0,0019
V	0,1437	0,0685	2,8054	1,3369	2,0984	0,0371
W	-0,1823	0,0635	-3,5599	1,2400	-2,8709	0,0045

It can be inferred from the data presented in Tables 8 and 9 that:

- after the consideration of all the alloy components, a statistically significant effect of ***Fe, Ni, Zn*** and ***V*** (positive, that is, the more of them in the alloy, the better) as well as ***Mn*** and ***W*** (negative, that is, the less of them in the alloy, the better) on ***Sost_Sum*** was established,
- the determination coefficient R^2 is not too high (equaling 0,2163), and the standard error of estimation is quite high (equaling 2,28), which proves a low matching of the model (predicted values) with the empirical points (observed values). The matching has been presented in Figure 5.

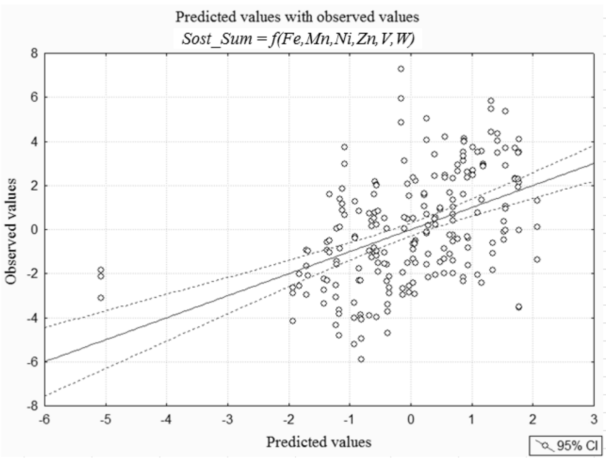

Fig. 5. Matching of the model with empirical points

3.3. Variance analysis test (*ANOVA*) for main effects

In the second stage, for the evaluation of the effect of the examined Al-Si alloy's chemical composition on the selected dependent variable: ***Sost_Sum,*** the variance analysis test (ANOVA - ANalysis Of VAriance) was applied for the main effects [14-16]. The variance analysis (ANOVA) for factorial designs is a set of statistical methods recommended for the analysis of only the main effects. Generally speaking, it is a technique of examining results (experiments, observations), which depend on many factors. Such factors, in our case, are the contents of the alloy elements: ***Cr, Mo, V*** and ***W*** in the Al-Si alloy, encoded with numbers from 0 to 9. And so, each factor has nine levels (variants): they are the variables ***Cr_1, Mo_1, V_1*** and ***W_1***, which have nine levels (from **1** to **9**): 0,00% = **1**; 0,05% = **2**; 0,10% = **3**; 0,15% = **4**; 0,20% = **5**; 0,25% = **6**; 0,30% = **7**; 0,40% = **8**; 0,50% = **9**.

3.3.1. Analysis of the effect of the *Cr_1*, *Mo_1*, *V_1* and *W_1* factors in the Al-Si alloy on *Sost_Sum*

Table 10 shows the results of the statistical evaluation of the effect of the ***Cr_1***, ***Mo_1***, ***V_1*** and ***W_1*** factors (i.e. the content of chromium, molybdenum, vanadium and tungsten) in the Al-Si alloy on the dependent variable ***Sost_Sum***.

Table 10.
Results of the statistical analysis of the effect of the ***Cr_1***, ***Mo_1***, ***V_1*** and ***W_1*** factors in the Al-Si alloy on ***Sost_Sum***

ANOVA of main effects. Dependent variable: *Sost_Sum*					
Parameter	Sum of squares	Degree of freedom	Mean square	F	p
Constant	0,2708	1	0,2708	0,0682	0,7942
Cr_1	112,0304	8	14,0038	3,5286	0,0008
Mo_1	67,6042	8	8,4505	2,1293	0,0350
V_1	166,9177	8	20,8647	5,2574	0,0000
W_1	116,3814	8	14,5477	3,6656	0,0005
Error	738,1702	186	3,9687		
Bartlett's test: χ^2 = 14,53; p = 0,0691					

For the verification of the basic assumption about uniformity (homogeneity) of variance in the scope of all the groups (levels of the examined factors), the Bartlett test was applied. It was established that the examined variances are homogeneous (p = 0,069) and a further analysis is possible to be performed.

The results of the ANOVA test for the main effects suggest that the mean values of ***Sost_Sum*** in all the groups (for all the levels) of the examined factors ***Cr_1***, ***Mo_1***, ***V_1*** and ***W_1*** differ in a statistically significant way (p: 0,0008; 0,0350, <0,0001; 0,0005, respectively). And so, all the examined initial variables (***Cr, Mo, V*** and ***W***) coded into factors significantly affect the target variable (***Sost_Sum***).

Another condition for the use of the results of the ANOVA test presented in Table 9 is the verification of the agreement of the examined target feature's distribution (***Sost_Sum***) with the normal distribution (Figure 6) as well as the agreement of the distribution of the residues with the normal distribution (Figure 7). The obtained results prove a sufficient matching of the examined target feature and the residues with the normal distribution.

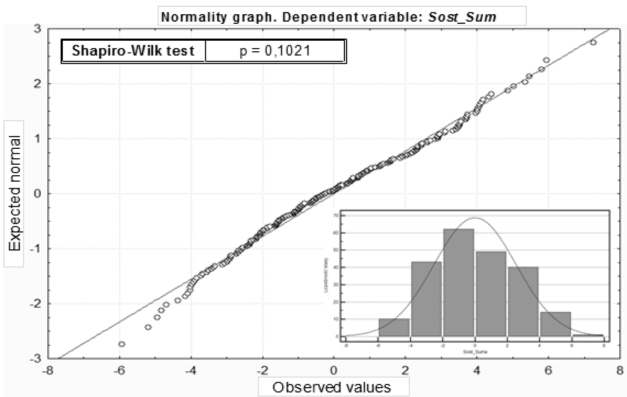

Fig. 6. Verification of the agreement of the examined target feature's distribution (***Sost_Sum***) with the normal distribution

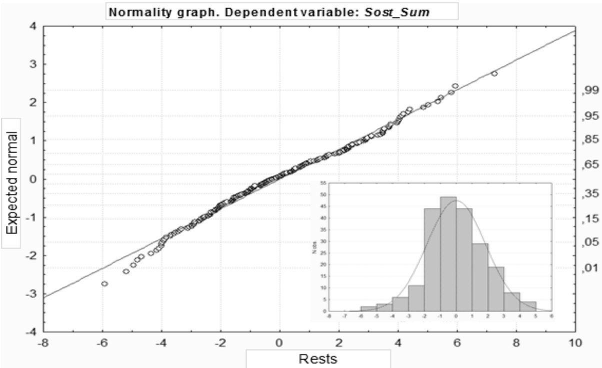

Fig. 7. Verification of the agreement of the residue distribution of variable ***Sost_Sum*** with the normal distribution

In order to determine the optimal level (concentration) of the examined factor, an estimation of the mean values of the dependent variable (***Sost_Sum***) was made for each examined level of one of the four factors (***Cr_1***, ***Mo_1***, ***V_1*** and ***W_1***) and NIR post-hoc tests were performed.

3.3.2. Analysis of the effect of the *Cr_1* factor in the Al-Si alloy on *Sost_Sum*

Fig. 8 shows the results of the statistical evaluation of the effect of the ***Cr_1*** factor (i.e. chromium content) in the Al-Si alloy on the dependent variable ***Sost_Sum***.

The condition for the use of the presented results in the statistical valuation is verification of the normality of the distribution of the examined target feature ***Sost_Sum*** (Figure 9).

As it can be inferred from the data presented in Fig. 8 and 9, the optimal level of chromium in the examined Al-Si alloy in respect of the total standardized value of ***Sost_Sum*** should be within the range of 0,05 - 0,1% Cr.

Dependent variable: *Sost_Sum*							
Factor	Level of the factor	N	Sost_Sum Mean	Sost_Sum SD	Sost_Sum Std. error	Sost_Sum -95,0% CI	Sost_Sum +95,0% CI
Cr_1	1 (0,00%)	108	-0,554	2,234	0,215	-0,980	-0,128
Cr_1	**2 (0,05%)**	**12**	**2,762**	**3,261**	**0,941**	**0,690**	**4,834**
Cr_1	**3 (0,10%)**	**24**	**1,819**	**2,808**	**0,573**	**0,633**	**3,005**
Cr_1	4 (0,15%)	12	0,835	2,012	0,581	-0,444	2,113
Cr_1	5 (0,20%)	24	0,917	1,885	0,385	0,121	1,713
Cr_1	6 (0,25%)	12	-0,986	2,445	0,706	-2,540	0,567
Cr_1	7 (0,30%)	12	-0,533	1,651	0,477	-1,582	0,516
Cr_1	8 (0,40%)	12	-1,519	1,438	0,415	-2,432	-0,605
Cr_1	9 (0,50%)	3	-4,174	0,947	0,547	-6,528	-1,821

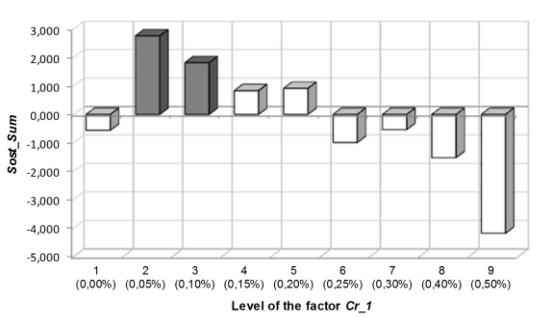

Fig. 8. Estimations of selected characteristics of the descriptive statistics of the ***Sost_Sum*** dependent variable's value for factor ***Cr_1***

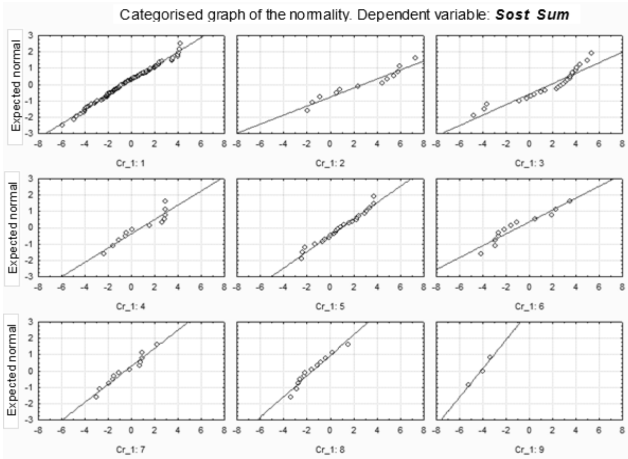

Fig. 9. Categorized diagrams of normality for the assumed levels of factor ***Cr_1***

Table 11 shows the results of the NIR post-hoc test.

Table 11.
Results of the NIR post-hoc test for the particular groups differing in the level of factor ***Cr_1***

Dependent variable: *Sost_Sum*									
Level of the factor Cr_1	1 (0,00%)	2 (0,05%)	3 (0,10%)	4 (0,15%)	5 (0,20%)	6 (0,25%)	7 (0,30%)	8 (0,40%)	9 (0,50%)
1 (0,00%)	p =	**0,0000**	**0,0000**	**0,0231**	**0,0013**	0,4768	0,9722	0,1132	**0,0022**
2 (0,05%)	**0,0000**	p =	0,1825	**0,0188**	**0,0096**	**0,0000**	**0,0001**	**0,0000**	**0,0000**
3 (0,10%)	**0,0000**	0,1825	p =	0,1638	0,1185	**0,0001**	**0,0010**	**0,0000**	**0,0000**
4 (0,15%)	**0,0231**	**0,0188**	0,1638	p =	0,9069	**0,0263**	0,0943	**0,0043**	**0,0001**
5 (0,20%)	**0,0013**	**0,0096**	0,1185	0,9069	p =	**0,0075**	**0,0409**	**0,0007**	**0,0000**
6 (0,25%)	0,4768	**0,0000**	**0,0001**	**0,0263**	**0,0075**	p =	0,5779	0,5135	**0,0141**
7 (0,30%)	0,9722	**0,0001**	**0,0010**	0,0943	**0,0409**	0,5779	p =	0,2270	**0,0051**
8 (0,40%)	0,1132	**0,0000**	**0,0000**	**0,0043**	**0,0007**	0,5135	0,2270	p =	**0,0403**
9 (0,50%)	**0,0022**	**0,0000**	**0,0000**	**0,0001**	**0,0000**	**0,0141**	**0,0051**	**0,0403**	p =

The presented results (Tab. 11) made it possible to demonstrate many statistically significant differences between the particular groups differing in the level of factor ***Cr_1***.

3.3.3. Analysis of the effect of individually treated factors *Mo_1, V_1* and *W_1* in the Al-Si alloy on *Sost_Sum*

Figures 10-12 show the results of the statistical evaluation of the effect of the factors ***Mo_1***, ***V_1*** and ***W_1*** (i.e. the contents of molybdenum, vanadium and tungsten) in the Al-Si alloy on the dependent variable ***Sost_Sum***.

Dependent variable: *Sost_Sum*							
Factor	Level of the factor	N	Sost_Sum Mean	Sost_Sum SD	Sost_Sum Std. error	Sost_Sum -95,0% CI	Sost_Sum +95,0% CI
Mo_1	1 (0,00%)	108	-0,423	2,506	0,241	-0,901	0,055
Mo_1	**2 (0,05%)**	**12**	**2,624**	**3,349**	**0,967**	**0,496**	**4,752**
Mo_1	**3 (0,10%)**	**24**	**1,417**	**2,162**	**0,441**	**0,504**	**2,330**
Mo_1	4 (0,15%)	12	1,263	1,628	0,470	0,229	2,298
Mo_1	5 (0,20%)	24	0,548	1,787	0,365	-0,207	1,302
Mo_1	6 (0,25%)	12	-1,055	2,714	0,783	-2,779	0,669
Mo_1	7 (0,30%)	12	-1,854	1,485	0,429	-2,798	-0,911
Mo_1	8 (0,40%)	12	-0,992	2,104	0,607	-2,329	0,344
Mo_1	9 (0,50%)	3	-0,424	1,416	0,818	-3,942	3,094

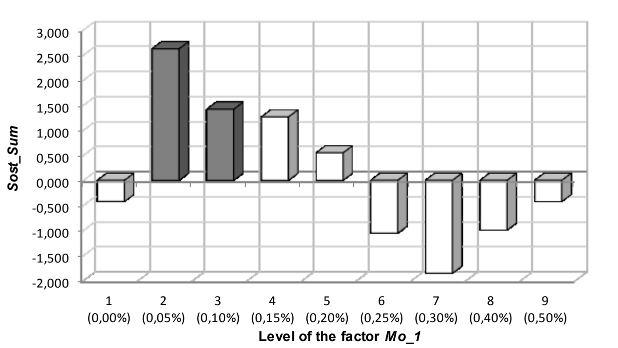

Fig. 10. Estimations of selected characteristics of the descriptive statistics of the ***Sost_Sum*** dependent variable's value for factor ***Mo_1***

Dependent variable: *Sost_Sum*							
Factor	Level of the factor	N	Sost_Sum Mean	Sost_Sum SD	Sost_Sum Std. error	Sost_Sum -95,0% CI	Sost_Sum +95,0% CI
V_1	1 (0,00%)	108	-0,794	2,267	0,218	-1,227	-0,362
V_1	**2 (0,05%)**	**12**	**3,006**	**2,976**	**0,859**	**1,115**	**4,897**
V_1	**3 (0,10%)**	**24**	**2,114**	**2,200**	**0,449**	**1,185**	**3,043**
V_1	4 (0,15%)	12	0,935	1,953	0,564	-0,306	2,176
V_1	5 (0,20%)	24	0,735	2,028	0,414	-0,122	1,591
V_1	6 (0,25%)	12	-0,423	2,399	0,693	-1,947	1,102
V_1	7 (0,30%)	12	-0,130	2,419	0,698	-1,667	1,407
V_1	8 (0,40%)	12	-1,086	1,464	0,423	-2,016	-0,155
V_1	9 (0,50%)	3	-3,404	0,941	0,543	-5,740	-1,067

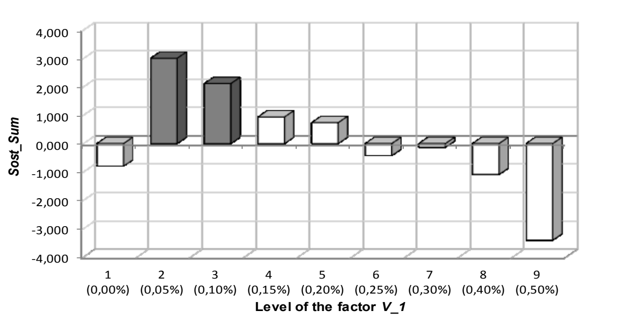

Fig. 11. Estimations of selected characteristics of the descriptive statistics of the ***Sost_Sum*** dependent variable' value for factor ***V_1***

Dependent variable: *Sost_Sum*							
Factor	Level of the factor	N	Sost_Sum Mean	Sost_Sum SD	Sost_Sum Std. error	Sost_Sum -95,0% CI	Sost_Sum +95,0% CI
W_1	1 (0,00%)	108	-0,136	2,459	0,237	-0,605	0,333
W_1	**2 (0,05%)**	**12**	**1,479**	**3,062**	**0,884**	**-0,466**	**3,425**
W_1	**3 (0,10%)**	**24**	**1,017**	**2,705**	**0,552**	**-0,125**	**2,160**
W_1	4 (0,15%)	12	0,213	1,665	0,481	-0,845	1,271
W_1	5 (0,20%)	24	0,476	2,889	0,590	-0,744	1,696
W_1	6 (0,25%)	12	-1,871	1,788	0,516	-3,007	-0,735
W_1	7 (0,30%)	12	0,160	2,100	0,606	-1,174	1,494
W_1	8 (0,40%)	12	-0,915	1,809	0,522	-2,064	0,235
W_1	9 (0,50%)	3	-3,317	0,939	0,542	-5,649	-0,986

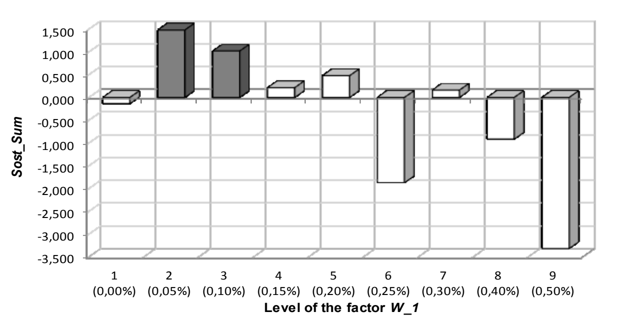

Fig. 12. Estimations of selected characteristics of the descriptive statistics of the ***Sost_Sum*** dependent variable's value for factor ***W_1***

For the presented results (Fig. 10-12), the normality of the distribution of the examined target feature (***Sost_Sum***) was verified and a NIR post-hoc test was performed. For all the examined factors (***Mo_1***, ***V_1*** and ***W_1***), the NIR post-hoc test made it possible to demonstrate many statistically significant differences between the particular groups differing in the level of the examined factors.

As it can be inferred from the presented data, the optimal level of molybdenum, vanadium and tungsten in the examined Al-Si alloy in respect of the total standardized value of ***Sost_Suma*** should be within the range of 0,05 - 0,10% W.

4. Conclusions

The following conclusions can be drawn from the data included in this study:

- The jointly treated mechanical properties ***Sost_Sum*** of the examined Al-Si alloy are the most beneficially affected by the content of 0,05 to 0,10% of the examined high-melting additions ***Cr***, ***Mo***, ***V*** and ***W***,
- It is, however, advisable to perform a statistical evaluation of the effect of the independent variables, that is the ***Cr_1***, ***Mo_1***, ***V_1*** and ***W_1*** factors, on the standardized dependent variables ***Sost_Rm***; ***Sost_Rp0,2***; ***Sost_A*** and ***Sost_HB***.
- A thorough analysis of the properties of the Al-Si alloys having the recommended content of ***Cr***, ***Mo***, ***V*** and ***W*** confirms the results of the statistical analysis.

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