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Impulse Response Functions in the Dynamic Stochastic General Equilibrium Vector Autoregression Model

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Abstract

The model considered in the paper is defined as VAR with the prior distribution for parameters generated by the dynamic stochastic general equilibrium (DSGE) model. The degree of economic restrictions in the DSGE-VAR model is controlled by the weighting parameter. In the paper there is investigated the impact of the weighting parameter prior specifications for the posterior shape of impulse response functions (IRFs). In case of conditional models the paths of IRFs highly depend on the value of the weighting parameter that is set arbitrary. When considering full estimation with different prior types, means and gradual change in the dispersion the posterior time paths of IRFs are similar in models with high values of the marginal data density.

Keywords: dynamic stochastic general equilibrium vector autoregression, DSGE-VAR, impulse response functions, marginal data density

JEL Classification: C11, C18, C51, E17

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1 Introduction

The main purpose of the paper is to illustrate changes in time paths of impulse response functions (IRFs) in the dynamic stochastic general equilibrium and vector autoregression (DSGE-VAR) model. The model was proposed by Del Negro, et al. (2002), (2004a) and further developed by Del Negro, et al. (2007). The DSGE-VAR is an identified VAR with the prior distribution for parameters constructed with the information from DSGE model. The amount of the prior information is controlled by the weighting parameter that can be set arbitrary or estimated depending on the considered approach. In the paper there is investigated the impact of the prior specification for the weighting parameter on the posterior shape of IRFs. There are two main approaches present in the empirical research: the first one is to consider series of conditional models, in which the weighting parameter is treated as a constant with arbitrary chosen values, while the second approach is to fully estimate it. The estimation approach enables to obtain marginal posterior distribution of the optimal weight of the information from the DSGE model in the VAR and, as a consequence, the assessment optimal degree of economic restrictions. The posterior estimate of the weighting parameter indicates how strictly economic restrictions agree with the data and consequently indicate what is the shape of IRFs. The conditional model can be used to illustrate the systematic change of IRFs paths as a consequence of the gradual release of economic restrictions imposed by the DSGE model.

The DSGE-VAR model can be seen as one of the method of IRFs identification in VARs. Other methods including basic techniques such as sign restrictions were broadly discussed in the literature, see among others: Uhlig (2005), Dedola *et al.* (2006), for theoretical aspects: Mittnik *et al.* (1993), Baumeister *et al.* (2014). The identification procedures on the Bayesian ground through so called generalised impulse response functions, also adequate for non-linear models, were proposed by Koop (1996) and Koop *et al.* (1996). Uncertainty measures of IRFs were discussed by Sims *et al.* (1999). It is worth mentioning works of Rubio-Ramirez *et al.* (2010) and Liu *et al.* (2010) in the context of relation between the VAR and general equilibrium models. The method of estimation by minimising distance between IRFs from VAR and DSGE models were discussed by Ravenna (2006). Conditions were such IRFs are similar can be found in Fernández-Villaverde *et al.* (2007). Identification when there are more structural shocks in a model than endogenous variables were considered by Fukač (2007).

The contribution of the paper is as follows. First, for the considered set of assumptions the DSGE-VAR model was estimated conditionally to arbitrary chosen values of the weighting parameter in order to investigate the IRFs potential change. The analysis extends the original paper by Del Negro *et al.* (2002), (2004a), where IRFs were computed for three different values of the weighting (tightness) parameter for a simple monetary DSGE model. Such conditional approach was also considered by Del Negro *et al.* (2006) where the marginal likelihood was calculated for nine values of the weighting parameter for three competing DSGE specifications based on the work Del Negro *et al.* (2004b)). The same nine values were considered by Del Negro *et al.* (2007) for the graphic IRFs presentation, see also Christiano (2007). Adolfson *et al.* (2008) considered twelve values of the weighting parameter for the Laplace approximation of the log marginal likelihood for the open economy DSGE model. Watanabe (2007) calculated marginal likelihood of the Christiano *et al.* (2005) model for nine arbitrary chosen values of the weighting parameter. Lee *et al.* (2007) considered twelve values for the weighting parameter and Ghent (2008) used seven values. None of the aforementioned papers include presentation of the step by step IRFs paths changes as a result of gradual release of DSGE restrictions. They also used a different set of DSGE assumptions.

Second, there is considered the full estimation approach where the weighting parameter is estimated after assuming different priors. Since the conditional approach is dominant in the empirical literature there are not many cases where the weighting parameter was estimated within DSGE-VAR framework. Adjemian *et al.* (2008) assumed a uniform prior, over the interval from 0 to 10, for the weighting parameter as a consequence of lack of prior beliefs about the optimal degree of economic restrictions in the DSGE-VAR model. The same idea was followed by Kolasa *et al.* (2012). The research below starts with the uniform marginal prior distribution for the weighting parameter defined over the range of gradually changed intervals. Then other types of priors are considered such as moved gamma, truncated normal and modified beta distributions. For each of them the gradual change of prior means and dispersions was considered to assess the impact on posterior IRFs shapes. At the end of the article the misspecification analysis of the assumed DSGE model is presented.

The rest of the paper is organised as follows: part 2 outlines the methodological concept of the DSGE-VAR with details of the prior distribution, the posterior and IRFs identification. Part 3 summarises equations of the DSGE model used to construct the prior distribution for VAR. Part 4 contains results obtained for the conditional approach, that is the general assessment of IRFs and their shape adjustment when gradually changing the amount of the prior information from the DSGE model. The last part of section 4 presents the misspecification analysis. In part 5 there are presented results of the empirical research for the case of full estimation of the weighting parameter, that is the situation when the amount of the prior information from the DSGE model is estimated not set arbitrary. The following work continues three articles: Wróbel-Rotter (2013c), (2013b), (2013a) on DSGE-VAR approach and constitute a part of the wider work partly summarised in the monograph Wróbel-Rotter (2015).

2 Model construction

The unrestricted VAR model of order p for n observed variables at the moment t has the form:

$$Y_t = \Phi_0 + \Phi_1 Y_{t-1} + \Phi_2 Y_{t-2} + \dots \Phi_p Y_{t-p} + u_t, \tag{1}$$

which can be interpreted as the reduced form of the structural model:

$$\mathrm{B}_0 Y_t = \mathrm{B}_c + \mathrm{B}_1 Y_{t-1} + \mathrm{B}_2 Y_{t-2} + \cdots + \mathrm{B}_p Y_{t-p} + \varepsilon_t, \tag{2}$$

under condition that: $\Phi_i = \mathrm{B}_0^{-1}\mathrm{B}_i$ for $i = c, 1, \ldots, p$, and $u_t = \mathrm{B}_0^{-1}\varepsilon_t$; $\varepsilon_t \sim N^{(n)}(0, I_n)$, where I_n is the identity matrix of degree n, Y_t depicts column $(n \times 1)$ of the observed variables, matrices Φ_i for $i = 1, \ldots, p$, of dimension $(n \times n)$ contain parameters, Φ_0 is a vector $(n \times 1)$ of constants, $u_t \sim N^{(n)}(0, \Sigma_u)$ is $(n \times 1)$ vector of random errors of the reduced form, $N^{(n)}(0, \Sigma_u)$ means n-dimensional normal distribution, with mean equal to zero vector, $E(u_t) = 0$, and the $(n \times n)$ covariance matrix $E(u_t u_t') = \Sigma_u$, U_t and X_t are assumed to be independent, $E(u_t u_{t-j}') = 0$. In a matrix notation it is written:

$$Y = X\Phi + U, \tag{3}$$

where Y is $(T \times n)$ matrix build of rows Y_t', X is $(T \times (1 + np))$ matrix with rows $X_t' = [1\ Y_{t-1}'\ Y_{t-2}'\ \ldots\ Y_{t-p}']$, U is $(T \times n)$ matrix with rows u_t', $\Phi = [\Phi_0'\ \Phi_1'\ \ldots\ \Phi_p']'$ is $((1 + np) \times n)$ matrix of coefficients.
The linear solution of the DSGE model is written in the state space representation:

$$s_{t+1} = As_t + B\varepsilon_t \text{ and } Y_t = Cs_t + D\varepsilon_t, \tag{4}$$

where s_t is the state vector, elements of matrices A and B are nonlinear functions of structural parameters θ, ϵ_t depicts vector of innovations, $\varepsilon_t \sim N^{(n)}(0, \Sigma_\varepsilon)$, elements of matrices C and D are functions of structural parameters (see: Fernández-Villaverde *et al.* (2007)).
The prior distribution of all parameters has a hierarchical form:

$$p(\Phi, \Sigma_u, \theta, \lambda) = p\left(\Phi, \Sigma_u \middle| \theta, \lambda\right) p(\theta) p(\lambda), \tag{5}$$

where the weighting parameter λ defines the amount of the prior information from DSGE in the DSGE-VAR model, $p(\theta)$ and $p(\lambda)$ are independent. The joint distribution of the parameters and the data is defined by:

$$p(Y, \Phi, \Sigma_u, \theta, \lambda) = p\left(Y \middle| \Phi, \Sigma_u\right) p(\Phi, \Sigma_u, \theta, \lambda). \tag{6}$$

The posterior distribution is of the form:

$$p\left(\Phi, \Sigma_u, \theta, \lambda \middle| Y\right) = \frac{\ell\left(\Phi, \Sigma_u \middle| Y\right) p\left(\Phi, \Sigma_u \middle| \theta, \lambda\right) p(\theta) p(\lambda)}{p(Y)}, \tag{7}$$

where $p(Y)$ denotes the marginal data density.
The prior distribution $p(\Phi, \Sigma_u | \theta, \lambda)$ of parameters Φ and Σ_u, conditional to θ and λ, is specified through the formal inference in the auxiliary VAR model:

$$\tilde{Y} = \tilde{X}\tilde{\Phi} + \tilde{U}, \tag{8}$$

written for the one vector observation in the form:

$$\tilde{Y}_t = \tilde{\Phi}_0 + \sum_{i=1}^{p} \tilde{\Phi}_i \tilde{Y}_{t-i} + \tilde{u}_t = \tilde{\Phi}' \tilde{X}_t + \tilde{u}_t, \tag{9}$$

where $\tilde{Y}_t$ is $(n \times 1)$ vector containing endogenous variables that correspond to observed variables, appearing in the observation equation of the state space representationof the DSGE model, $\tilde{Y}_{(\lambda T \times n)}$ consists of rows $\tilde{Y}_t'$, $\tilde{\Phi}' = [\tilde{\Phi}_0' \ \tilde{\Phi}_1' \ \dots \ \tilde{\Phi}_p']$, $\tilde{\Phi}$ is $((1+np) \times n)$ matrix of the coefficients, $\tilde{X}$ is $(\lambda T \times (1+np))$ matrix with rows $\tilde{X}_t' = [1 \ \ \tilde{Y}_{t-1}' \ \tilde{Y}_{t-2}' \ \dots \tilde{Y}_{t-p}' \ ]$, $\tilde{X}_t$ has dimensions $((1+np) \times 1)$, $\tilde{U}$ is $(\lambda T \times n)$ matrix consisting of rows $\tilde{u}_t'$, $\tilde{u}_t \sim iidN^{(n)}(0, \tilde{\Sigma}_u)$, $t = 1, \dots, \lambda T$. The auxiliary VAR approximates the linear solution of the DSGE model by projecting parameters θ onto the matrix of coefficients $\tilde{\Phi}$ and the covariance $\tilde{\Sigma}_u$. The posterior distribution in the auxiliary model (8), after assuming noninformative prior: $p(\tilde{\Phi}, \tilde{\Sigma}_u) \propto \det(\tilde{\Sigma}_u)^{-\frac{n+1}{2}}$, is of the form:

$$\begin{aligned} & p\left(\tilde{\Phi}, \tilde{\Sigma}_u \middle| \theta, \lambda\right) \propto \det\left(\tilde{\Sigma}_u\right)^{-\frac{\lambda T+n+1}{2}} \\ & \quad \exp\left\{ -\frac{1}{2} tr \left[\lambda T \tilde{\Sigma}_u^{-1} \left(\Gamma_{yy}^*(\theta) - \tilde{\Phi}' \Gamma_{xy}^*(\theta) - \Gamma_{yx}^*(\theta) \tilde{\Phi} + \tilde{\Phi}' \Gamma_{xx}^*(\theta) \tilde{\Phi} \right) \right] \right\} , \end{aligned} \tag{10}$$

where: $\Gamma_{xx}^*(\theta) = E_\theta(\tilde{X}_t \tilde{X}_t')$, $\Gamma_{xy}^*(\theta) = E_\theta(\tilde{X}_t \tilde{Y}_t')$, $\Gamma_{yy}^*(\theta) = E_\theta(\tilde{Y}_t \tilde{Y}_t')$ and $\Gamma_{yx}^*(\theta) = \Gamma_{xy}^{*\prime}(\theta)$ are expected values $E_\theta(.)$, with respect to the probability distribution of θ.

The marginal posterior distribution of the covariance matrix $\tilde{\Sigma}_u$, given θ, has the form of inverted Wishart, $IW(.)$, Zellner (1971):

$$p\left(\tilde{\Sigma}_u \middle| \theta, \lambda\right) = IW\left(\lambda T \hat{\tilde{\Sigma}}_u(\theta), \lambda T - (1+np), n\right), \tag{11}$$

while conditionally to $\tilde{\Sigma}_u$, θ and λ, the posterior distribution of coefficients $\tilde{\Phi}$ is of the form of the matrix normal, $MN(.)$, Poirier (1995):

$$p\left(\tilde{\Phi} \middle| \tilde{\Sigma}_u, \theta, \lambda\right) = MN_{(1+np)\times n}\left(\hat{\tilde{\Phi}}(\theta), \ \tilde{T}\Gamma_{xx}^*(\theta), \ \tilde{\Sigma}_u^{-1}\right), \tag{12}$$

where: $\hat{\tilde{\Phi}}(\theta) = [\Gamma_{xx}^*(\theta)]^{-1} \Gamma_{xy}^*(\theta)$ and $\hat{\tilde{\Sigma}}_u(\theta) = \Gamma_{yy}^*(\theta) - \Gamma_{yx}^*(\theta) [\Gamma_{xx}^*(\theta)]^{-1} \Gamma_{xy}^*(\theta)$.
Assuming in the place of the prior $p(\Phi, \Sigma_u | \theta, \lambda)$ the posterior distribution in the auxiliary VAR (10), the final form of the posterior distribution (7) is obtained. It can be decomposed as:

$$p(\Phi, \Sigma_u, \theta, \lambda | Y) = p(\Phi, \Sigma_u | Y, \theta, \lambda) \, p(\theta, \lambda | Y) \tag{13}$$

where $p(\theta, \lambda | Y)$ is of the nonstandard form and $p(\Phi, \Sigma_u | Y, \theta, \lambda)$ is the matrix normal – inverted Wishart (see: Adjemian *et al.* (2008)):

$$p(\Phi, \Sigma_u | Y, \theta, \lambda) \propto \det(\Sigma_u)^{-\frac{(1+\lambda)T+n+1}{2}} \exp\left\{-0.5tr\left\{\Sigma_u^{-1}\left[(Y-X\Phi)'(Y-X\Phi) + \lambda T\left(\Gamma_{yy}^*(\theta) - \Phi'\Gamma_{xy}^*(\theta) - \Gamma_{yx}^*(\theta)\Phi + \Phi'\Gamma_{xx}^*(\theta)\Phi\right)\right]\right\}\right\} \quad (14)$$

and leads to the conditional posterior distributions:

$$p(\Sigma_u | Y, \theta, \lambda) = IW\left((\lambda+1)T\hat{\Sigma}_u^m(\theta), (1+\lambda)T - (1+np), n\right), \quad (15)$$

$$p(\Phi | Y, \Sigma_u, \theta, \lambda) = MN_{(1+np)\times n}\left(\hat{\Phi}^m(\theta), (\lambda T\Gamma_{xx}^*(\theta) + X'X), \Sigma_u^{-1}\right), \quad (16)$$

where:

$$\hat{\Phi}^m(\theta) = (\lambda T\Gamma_{xx}^*(\theta) + X'X)^{-1}\left(\lambda T\Gamma_{xy}^*(\theta) + X'Y\right), \quad (17)$$

$$\hat{\Sigma}_u^m(\theta) = [(\lambda+1)T]^{-1}\left[\left(\lambda T\Gamma_{yy}^*(\theta) + Y'Y\right) - \left(\lambda T\Gamma_{yx}^*(\theta) + Y'X\right)\hat{\Phi}^m(\theta)\right]. \quad (18)$$

The weighting parameter λ expresses the degree of strictness of economic restrictions introduced to the DSGE-VAR model via the prior $p(\Phi, \Sigma_u | \theta, \lambda)$. The optimal value of the weighting parameter λ is the one that leads to maximum value of the marginal likelihood. When the parameter λ is estimated the mean of its marginal posterior distribution is a point estimate and the standard deviation is taken as an uncertainty measure. In case of the conditional models the optimal value of λ is typically chosen from the set $\lambda \in \Lambda = \{\lambda_1, \ldots, \lambda_q\}$, in such a way that: $\hat{\lambda} = \arg\max_{\lambda>0} p(Y|\lambda)$, given that $\lambda_i > \lambda_{min} = (n+k^*)/T$, where k^* is the rank of $\tilde{X}$.

The identification of the vector of structural shocks in the DSGE-VAR model is as follows, (for details see Del Negro *et al.* (2007)). The partial effect of a shock on endogenous variables in the VAR model is defined by the $(n \times n)$ matrix of partial derivatives:

$$\frac{\partial Y_t}{\partial \varepsilon_t'} = B_0^{-1} = \Sigma_{tr}\Omega \quad (19)$$

assuming that elements of the vector ϵ_t are orthogonal. The identification of vector autoregression shocks (19) is obtained by application of QR decomposition to B_0^{-1} and exchanging the orthonormal matrix Ω in equation (19) with the matrix $\Omega^*(\theta)$ from the equation (20), assuming that lower triangular matrix Σ_{tr} is the Cholesky decomposition of covariance matrix Σ_u. The DSGE model is identified in the sense that for each value of parameter vector θ there exists a unique matrix $D(\theta)$, derived from the state space representation of the structural model (4), which consists of effects of structural shocks on the endogenous variables:

$$\frac{\partial Y_t}{\partial \varepsilon_t'} = D(\theta) = \Sigma_{tr}^*(\theta)\Omega^*(\theta), \quad (20)$$

where the QR decomposition of $D(\theta)$ leads to lower triangular matrix $\Sigma_{tr}^*(\theta)$ and orthonormal $\Omega^*(\theta)$. The matrix $\Sigma_{tr}^*(\theta)$ is approximated by the Cholesky decomposition of the restrictions functions: $\Sigma_u^*(\theta) = \Gamma_{yy}^*(\theta) - \Gamma_{yx}^*(\theta)\hat{\Phi}^*(\theta)$, where $\Phi^*(\theta) = [\Gamma_{xx}^*(\theta)]^{-1}\Gamma_{xy}^*(\theta)$, see Del Negro *et al.* (2004a). The identification procedure implies that IRFs are a priori centred round their DSGE counterparts even for small values of λ. The identified vector autoregression approximation of the linear solution of the DSGE model is given by the triple $\Phi^*(\theta)$, $\Sigma_u^*(\theta)$ and $\Omega^*(\theta)$. The matrix Ω can be treated as an additional parameter to be estimated what leads to the hierarchical prior:

$$p\left(\Phi, \Sigma_u, \theta, \lambda, \Omega\right) = p\left(\Phi, \Sigma_u \right| \theta, \lambda) \, p(\theta) p(\lambda) p(\Omega) \tag{21}$$

where $p(\theta)$, $p(\lambda)$ and $p(\Omega)$ are independent. The joint distribution of the data, structural parameters and the weighting parameter takes the form:

$$p\left(Y, \Phi, \Sigma_u, \theta, \lambda, \Omega\right) = p\left(Y \left| \Phi, \Sigma_u \right.\right) p\left(\Phi, \Sigma_u, \theta, \lambda, \Omega\right) \tag{22}$$

The posterior distribution:

$$p\left(\Phi, \Sigma_u, \theta, \lambda, \Omega \left| Y \right.\right) = \frac{\ell\left(\Phi, \Sigma_u \right| Y) \, p\left(\Phi, \Sigma_u \left| \theta, \lambda \right.\right) p(\theta) p(\lambda) p(\Omega)}{p(Y)}, \tag{23}$$

can be decomposed as:

$$p\left(\Phi, \Sigma_u, \theta, \lambda, \Omega \left| Y \right.\right) = p\left(\Phi, \Sigma_u \left| Y, \theta, \lambda \right.\right) p\left(\Omega \mid \theta\right) p\left(\theta, \lambda \right| Y), \tag{24}$$

where $p(\Omega \mid \theta, Y, \lambda) = p(\Omega \mid \theta)$ what indicates that the matrix Ω depends only on the structural parameters of the DSGE model. The marginal posterior distribution of Ω is updated indirectly through updating θ, which changes with parameters Φ and Σ_u. The inference about θ is conditional with respect to the weighting parameter λ:

$$p\left(\theta, \lambda \left| Y \right.\right) = p\left(\theta \left| \lambda \right., Y\right) p(\lambda), \tag{25}$$

The conditional posterior distribution of the vector autoregression parameters Φ and Σ_u do not depend on Ω:

$$p\left(\Phi, \Sigma_u \left| Y, \theta, \lambda, \Omega \right.\right) = p\left(\Phi, \Sigma_u \left| Y, \theta, \lambda \right.\right). \tag{26}$$

3 The general equilibrium model

The prior specification for the DSGE-VAR in the following analysis is based on the fundamental New-Keynesian model with the wage mechanism proposed by Erceg *et al.* (2000), which was later used to illustrate the econometric issues by Rabanal *et al.* (2005b), (2005a). From now on denoted as EHL. The model was also treated as a starting point for building more complicated systems, for instance: Christiano *et al.* (2005). The derivation of equations and discussion of microeconomic optimisation problems of the chosen model were presented by: Wróbel-Rotter (2011a), (2011b), (2012b).

The DSGE structural equations, written in a form of percentage deviations from steady state, consists of the following equations:

1. The consumption Euler equation:

$$y_t = E_t y_{t+1} - \sigma \left(r_t - E_t \pi_{t+1} + E_t g_{t+1} - g_t \right) \tag{27}$$

where: y_t denotes output, r_t is the nominal interest rate, g_t is the preference shifter shock, π_t is inflation, σ is the elasticity of intertemporal substitution, E_t is the conditional expectation operator with information up to time t.

2. The production function and the real marginal cost function:

$$y_t = a_t + (1-\delta) n_t \text{ and } mc_t = w_t^r + n_t - y_t, \tag{28}$$

where: n_t is the amount of hours worked, a_t is a technology shock, mc_t is real marginal cost, w_t^r is real wage, δ is capital share of output.

3. The marginal rate of substitution between consumption and working hours:

$$mrs_t = \sigma^{-1}\, y_t + \gamma\, n_t - g_t, \tag{29}$$

where: γ is the inverse elasticity of labour supply with respect to real wages.

4. The price inflation assuming the mechanism proposed by Calvo (1983):

$$\pi_t = \beta E_t \left(\pi_{t+1} \right) + \frac{(1-\alpha)(1-\theta\beta)(1-\theta)}{\theta \left(1 + \alpha(\bar{\varepsilon} - 1) \right)} \left(mc_t + \varepsilon_t^{\pi} \right) \tag{30}$$

where: $\bar{\varepsilon}$ is the steady state value of elasticity of substitution between different categories of goods, ε_t^{π} is the price markup shock, θ is the probability of the price non-optimisation and β discount factor.

5. The wage inflation assuming the mechanism proposed by Calvo (1983):

$$\pi_t^w = \beta E_t \pi_{t+1}^w + \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w(1+\gamma\varepsilon_w)} (mrs_t - w_t^r) \tag{31}$$

where: θ_w is the Calvo probability of wage nonoptimisation, ϵ_w is the elasticity of substitution of different between different kinds of labour.

6. The Taylor rule:

$$r_t = \rho_r r_{t-1} + (1-\rho_r)(\gamma_\pi \hat{\pi}_t + \gamma_y y_t) + \varepsilon_t^r \tag{32}$$

where γ_π and γ_y are the long-run responses of the monetary authority to deviations of inflation and output from their steady-state values, and ε_t^r is the monetary shock, ρ_r is an interest rate smoothing parameter.

7. The equation that relates real wage growth, nominal wage growth and price inflation:

$$w_t^r = w_{t-1}^r + \pi_t^w - \pi_t. \tag{33}$$

8. The stochastic processes for the technology change and preferences:

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \text{ and } g_t = \rho_g g_{t-1} + \varepsilon_t^g \tag{34}$$

where: ε_t^a and ε_t^g denote innovations.

The equations constitute the linear rational expectation system with four innovations: $\varepsilon_t^* = [\varepsilon_t^a \ \varepsilon_t^g \ \varepsilon_t^r \ \varepsilon_t^\pi]'$, where each of them follows a normal distribution with mean equal to zero and standard deviations: σ_a, σ_g, σ_r and σ_π respectively. The vector $\theta = [\alpha \ \sigma \ \beta \ \gamma \ \varepsilon \ \theta \ \rho_r \ \gamma_\pi \ \gamma_y \ \rho_a \ \rho_g \ \theta_w \ \varepsilon_w]'$ contains all structural parameters of the DSGE model. The data contain 74 observations on price inflation, real wages, interest rates and output for the United States at a quarterly frequency, and were originally prepared for the work: Rabanal *et al.* (2005b). The original source is the Bureau of Labour Statistics and the Federal Reserve System. The numerical approximation of marginal posterior distributions is done by the Metropolis - Hastings algorithm, the marginal likelihood is the modified harmonic mean proposed by Geweke (1999). The estimation was accomplished by Dynare package (see: Adjemian *et al.* (2011)), which is the standard tool to estimation and verification of rational expectation models. All considered models were verified to assure numerical stability. For each of all considered models there was run 250000 iteration of Metropolis-Hastings with burn-in 20% of the first passes. Total number of iterations was divided to five parallel chains, where starting points were obtained after independent drawing from the normal distribution, whose mode and covariance matrix approximate numerically the central tendency and the covariance matrix of the posterior distribution. More empirical issues related to numerical side of the considered DSGE model were discussed by Wróbel-Rotter (2012a).

4 Conditional models

The conditional models were estimated for two competing sets of values for the weighting parameter λ: the first one contains ten dispersed numbers from the set: $\Lambda_1 = \{0.3, 0.43, 0.67, 1, 1.5, 2.33, 4, 9, 19, 1000\}$, while the second set is specified to estimate precisely the marginal data density in regions of its high value:

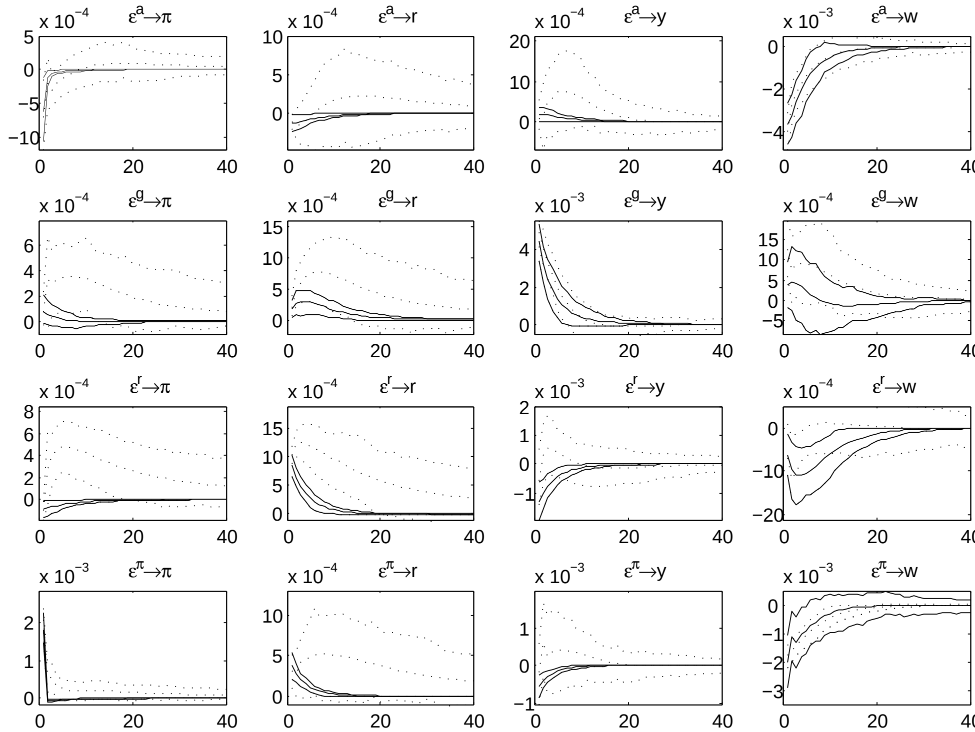
$$\Lambda_2 = \{0.32, 0.35, 0.39, 0.43, 0.47, 0.52, 0.56, 0.61, 0.67, 0.72, \\ 0.79, 0.85, 0.92, 1, 1.08, 1.17, 1.27, 1.38, 1.5, 1.63\}$$

The first set of values for λ corresponds with ten different values of the optimal weight $W_i = \lambda_i/(1+\lambda_i)$ of the DSGE model in the DSGE-VAR, approximately from 0.1 to 1 with step 0.1. The second set contains values of λ which indicate weights W form 0.24 to 0.62, with step 0.02. Lower bound for the possible values of λ, which guaranties existence of the posterior distribution, is equal to $\lambda_{min} = 0.183$ for lag $p = 2$. The maximum value of the logarithm of the marginal data density in the first set is obtained for $\lambda = 0.679$ for $p = 2$, what indicates the optimal weight of the DSGE model equal to 0.4. The logarithm of marginal likelihood reveals tendency to increase at first, as values of λ increase, and after obtaining the maximum value, become to decrease. That behaviour indicates that there exists optimal value of the degree of economic restrictions in the DSGE-VAR, which corresponds in that case with $\lambda = 0.79$, precisely estimated from the set Λ_2. Similar behaviour is observed for other lags of VAR which was considered from one to twelve by Wróbel-Rotter (2013b). As the lag increases, and consequently the number of free parameters, the optimal quantity of prior information from the DSGE model also increases. Obtained results indicate that stylised DSGE model is too tightening and is not completely approved by the data. The logarithm of marginal data density for EHL is equal to 1180. However the level of marginal likelihood of the unrestricted VAR is even lower clearly indicating that there are advantages of incorporating prior information into VAR and considering the DSGE-VAR model. Such specification delivers a tool to flexibly adjust the optimal amount of information from the prior distribution leading to higher marginal likelihood than the separate approaches, as DSGE model is usually misspecified and VAR is typically too flexible in the data fit. All IRFs in the paper are calculated as means of their posterior distributions generated after drawing values of the structural parameters from the posterior distribution.

4.1 General assessment

As a starting point to deeper analysis of IRFs their time paths are presented for the DSGE-VAR obtained for $\lambda = 0.79$ in the conditional approach, which leads to maximum value of marginal likelihood, see graph 1. Dotted lines indicate time paths and 90% high posterior density (HPD) intervals for IRFs estimated within the DSGE-VAR while solid lines indicate values and 90% HPDs of IRFs obtained from DSGE model, estimated within the DSGE-VAR model. Notation: $\epsilon^a \to \pi$, $\epsilon^a \to r$, $\epsilon^a \to y$ and $\epsilon^a \to w$ depicts respectively: impact of technology shock on inflation, interest rate, production and real wage. Analogously: $\epsilon^g \to \pi$, $\epsilon^g \to r$, $\epsilon^g \to y$ and $\epsilon^g \to w$ indicate the impact of the preference shock on respective endogenous variables, $\epsilon^r \to \pi$, $\epsilon^r \to r$, $\epsilon^r \to y$ and $\epsilon^r \to w$ describe effects of the monetary policy shock and $\epsilon^\pi \to \pi$, $\epsilon^\pi \to r$, $\epsilon^\pi \to y$ and $\epsilon^\pi \to w$ illustrate influence of the price shock. The general tendency in shape of IRFs in both models is preserved only in a few cases what means that the shape of responses and their economic interpretation depend on which model, VAR or DSGE, we choose. The uncertainty of the estimation of IRFs in the DSGE-VAR model is quite strong in comparison with the ones obtained from the DSGE model, estimated within the DSGE-VAR model and individually. The overall empirical results concerning the shape and posterior uncertainty are similar to that published by Adjemian *et al.* (2008).

Figure 1: IRFs based on VAR and DSGE model

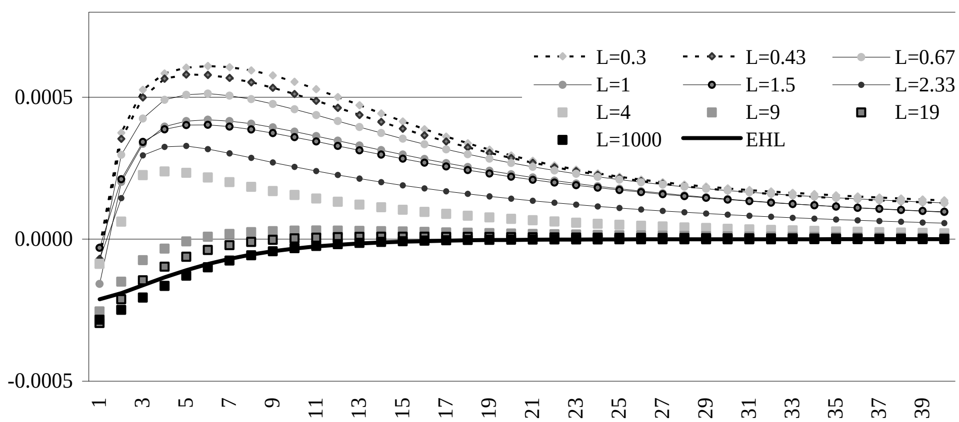


Dotted lines indicate time paths and 90% high posterior density (HPD) intervals for IRFs estimated within the DSGE-VAR. Solid lines indicate values and 90% HPDs of IRFs obtained from DSGE model.

4.2 Changing value of the weighting parameter

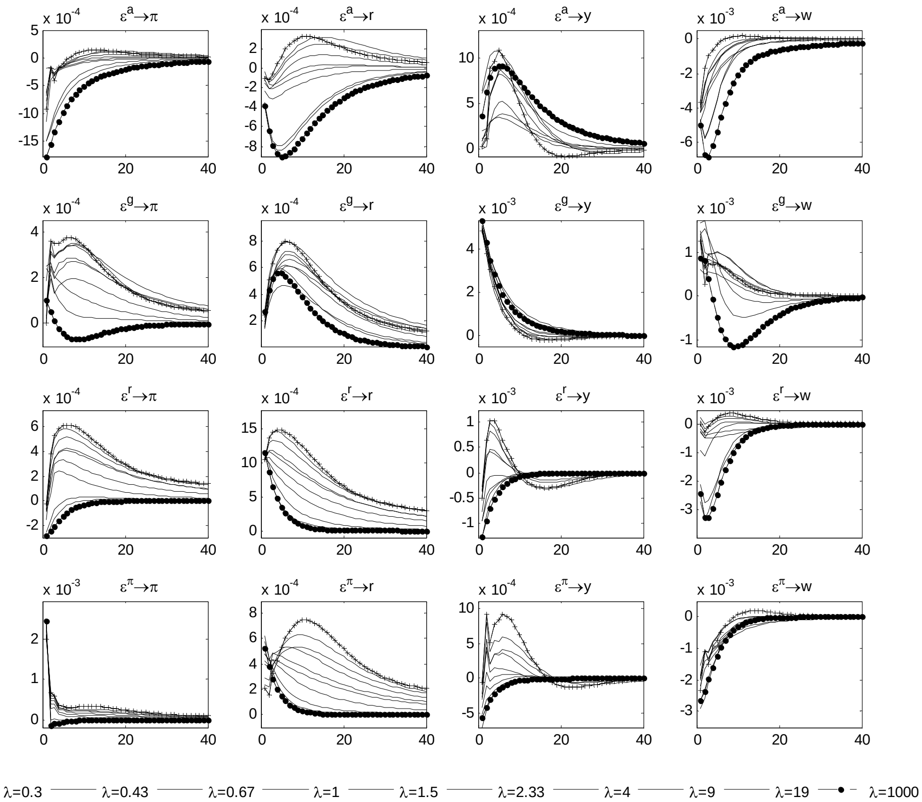
The gradual change in values of the weighting parameter λ, interpreted as step by step release of strong economic assumptions of the DSGE specification, leads to systematic change in shapes of IRFs, what illustrates graph 2 for the impact of an interest rate shock on inflation ($\epsilon^r \to \pi$) and graph 3 for the rest of shocks and variables for $\lambda \in \Lambda_1$. The systematic change in shape of IRFs is strongly visible for the weighting parameter from the set Λ_1, as it covers wide range of values, comprising almost unrestricted VAR from one side and practically DSGE model from the other side. For the relatively narrow range of the weighting parameter values taken from the set Λ_2 the shape of IRFs is quite similar as conditional models for $\lambda \in \Lambda_2$ lead to relatively high values of the marginal data density. The systematic shape change for $\lambda \in \Lambda_1$ mimics gradual reduction of the strictness of the prior coming from the DSGE model what can be seen as slow reduction of potentially misspecified economic restrictions. The reaction of inflation to stochastic shock for low values of the weighting parameter λ is stronger that for the high values and for $\lambda \to \infty$ is almost flat the same as for the DSGE model estimated individually. Such regularity is also visible for the rest of responses to shocks, especially for the influence of price and monetary shocks to production and inflation ($\epsilon^\pi \to y$, $\epsilon^r \to y$ and $\epsilon^\pi \to r$, $\epsilon^r \to r$). In the rest of the cases the path is independent of the value of λ (graph 3). Such independence especially concerns the change of inflation and real wage after the price shock ($\epsilon^\pi \to \pi$ and $\epsilon^\pi \to w$), and the adjustment of production to households' preference shock ($\epsilon^g \to y$). The observed behaviour is also present for other lags of the DSGE-VAR. The analysis suggests that restrictions on the parameter space coming from the DSGE model, exactly: from the solution of the linearised set of first order conditions and other constraints, derived from the microeconomic optimisation problems of households and producers, are strict and they are not entirely approved by the observed data, but also they are not completely rejected. The DSGE-VAR model could better fit the data as it is elastic in deciding of the correct degree of economic restrictions.

Figure 2: Impact of the interest rate shock on inflation ($\epsilon^r \to \pi$)



The value of the weighting parameter has influence on high posterior probability intervals for impulse responses. In case of low values of λ their borders are quite away indicating low precision of inference, which has tendency to rise as the value of λ increases, leading to the narrowest intervals for the case of $\lambda \to \infty$. That means that increasing the amount of prior information from the DSGE model and increasing the strength of economic restrictions lead to better precision of the posterior inference. The highest uncertainty, in the conditional models for low λ, is present in case of influence of all shocks on interest rates while the highest precision of HPD is observed for impact of the price shock on inflation and real wage. The observed behaviour of IRFs is also visible for other lags of VAR assuming that we take into account models

Figure 3: IRFs from DSGE-VAR estimated for values of weighting parameter from the set Λ_1



Notation: $\epsilon^a \to \pi$, $\epsilon^a \to r$, $\epsilon^a \to y$ and $\epsilon^a \to w$ depicts respectively: impact of technology shock on inflation, interest rate, production and real wage. Analogously: $\epsilon^g \to \pi$, $\epsilon^g \to r$, $\epsilon^g \to y$ and $\epsilon^g \to w$ indicate the impact of the preference shock on respective endogenous variables, $\epsilon^r \to \pi$, $\epsilon^r \to r$, $\epsilon^r \to y$ and $\epsilon^r \to w$ describe effects of the monetary policy shock and $\epsilon^\pi \to \pi$, $\epsilon^\pi \to r$, $\epsilon^\pi \to y$ and $\epsilon^\pi \to w$ illustrate influence of the price shock.

with the highest marginal likelihood within given order p.

The DSGE-VAR model allows to inference about the parameters of the DSGE model indirectly via the autoregression parameters. It means that parameters θ and λ could display the posterior dependence if the covariance matrix of all parameters $[\theta\, \lambda]$ is not diagonal. That implies the influence of the prior specification for λ on the posterior inference of θ and further on IRFs obtained from the DSGE model. The most visible change in the shape of the posterior IRFs is present in the case of the technological shock influence on all endogenous variables, the impact of the preference shock on the interest rate and the real wage, also in the initial path of the monetary shock to inflation. In the rest of the cases the time path of responses is quite independent of the assumed value for the weighting parameter in conditional models. For $\lambda = 1000$ their shape is identical to ones obtained from the DSGE model estimated individually.

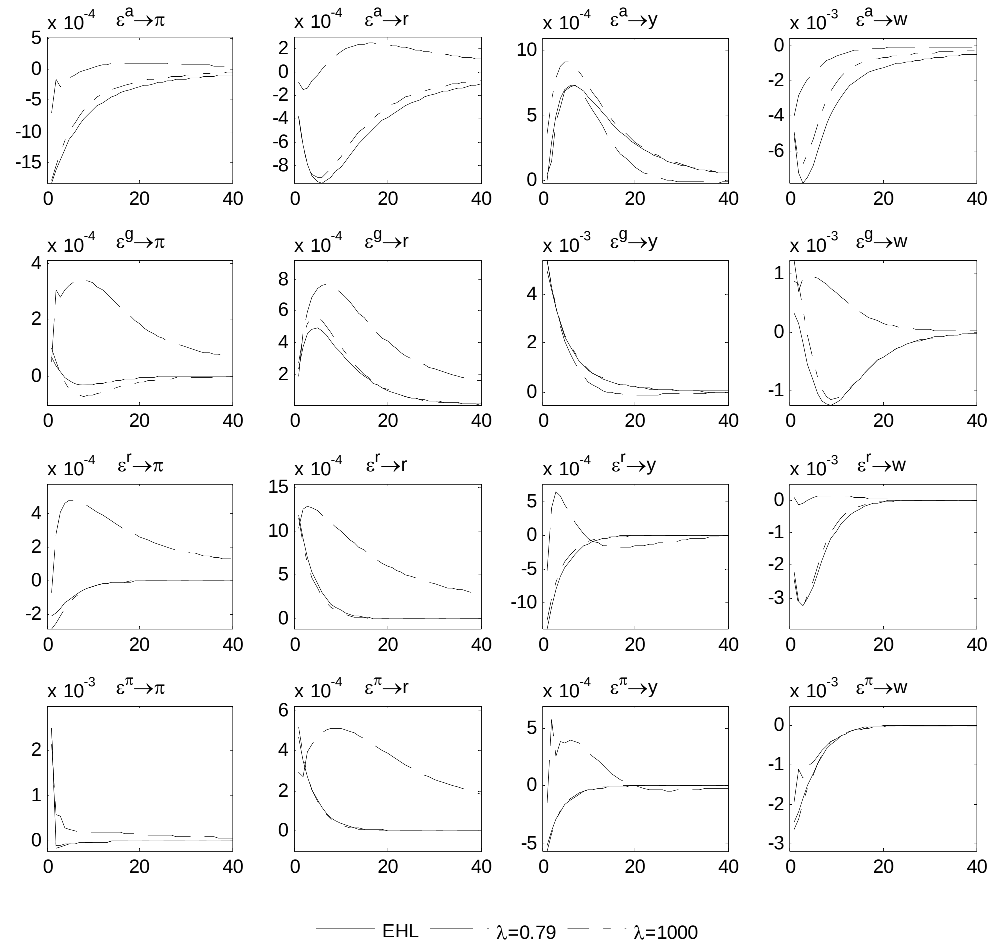
4.3 Misspecification analysis

The empirical analysis of misspecification degree of DSGE assumptions is possible by comparing paths of IRFs for certain values of the weighting parameter λ. The important comparison concerns cases for $\lambda = 1000$, which is treated as an approximation of $\lambda \to \infty$, with the IRFs for λ_{max} and the DSGE model estimated separately. The model with the highest marginal data density, conditional to λ_{max}, is regarded as one which optimally releases economic restrictions. Similarity of IRFs for $\lambda \to \infty$ and λ_{max} indicates correct specifications of the economic assumptions, while the discrepancy signalises the area which potentially could be modified to obtain better harmony. The IRFs from the DSGE-VAR of lag two, estimated conditional to $\lambda_{max} = 0.79$ and $\lambda = 1000$, the results obtained from the DSGE model estimated separately, depicts graph 4.
Comparison of results for $\lambda_{max} = 0.79$ and $\lambda = 1000$ indicates that there are differences in the shape of the impact of the technology shock on the interest rate, and to some extent, on inflation and real wage. Discrepancies are also observed for effects of the preference shock on inflation and real wage and the influence of monetary shock on all endogenous variables. The influence of the price shock on the interest rate and production also depend on the value of the weighting parameter. The similarity of IRFs paths for the conditional model for $\lambda_{max} = 0.79$ and $\lambda = 1000$ is visible for the impact of the technology and preference shock on production and also in effects of the price shock on inflation and the real wage. It means that in this area the economic assumptions meet the data, while in the rest of the cases DSGE assumptions are strongly modified. For the limiting case ($\lambda \to \infty$) there exists agreement in the paths of IRFs with IRFs obtained from the DSGE model estimated individually what indicates that the solution of the linearised DSGE model, which takes the form of the vector autoregression moving average process, is well approximated by the finite order VAR with restricted number of lags. In the analysed case the IRFs are identical for all variables and shocks.

5 Estimation of weighting parameter

The impact analysis of competing prior distributions for the weighting parameter on posterior IRFs shapes was considered in two ways: firstly different types of the prior is taken, secondly the dispersion of them is gradually changed to illustrate consequences on IRFs' shapes. It is common in the empirical research to assume a priori uniform distribution for λ, which seems to be uninformative about the quantity of prior information from the DSGE model. Unfortunately it does not imply uniform

Figure 4: IRFs from DSGE-VAR conditional to λ_{max}, $\lambda \to \infty$ and form DSGE model estimated individually (EHL)



distribution for the weight W. What distribution for W is implied by the assumed distribution for λ can be assessed in a simulation way, see Wróbel-Rotter (2013c).

5.1 The prior type

The impact analysis of different types of the prior distribution for the weighting parameter on posterior shapes of IRFs is considered after assuming the following prior specifications for λ:

1. uniform from 0 to λ_G, where $\lambda_G = 1, 4, 9, 19, 49$ and 99, which corresponds to

maximal possible weights W equal to: 50%, 80%, 90%, 95%, 98% and 99% respectively,

2. gamma: $f_G(\lambda|1.4, 0.3)$, $f_G(\lambda|1.5, 1)$, $f_G(\lambda|5, 1)$ and $f_G(\lambda|5, 4)$, where for $f_G(\lambda|g_1, g_2)$, parameters $g_1 > 0$, $g_2 > 0$, $E(\lambda) = g_1g_2$ and the standard deviation: $D(\lambda) = g_1^{1/2}g_2$,

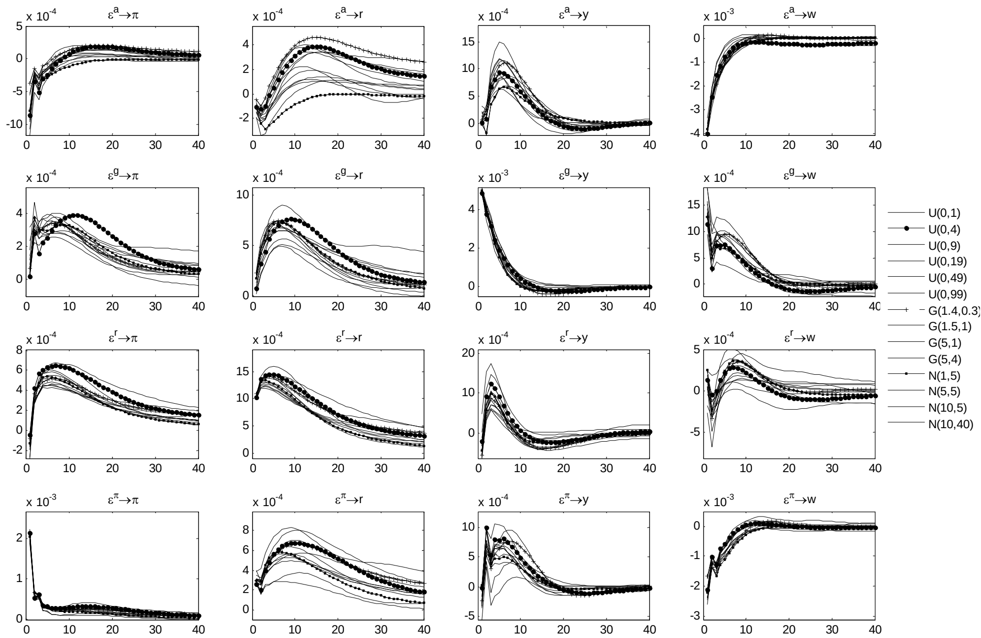
3. normal: $f_N(\lambda|1, 5)$, $f_N(\lambda|5, 5)$, $f_N(\lambda|10, 5)$ and $f_N(\lambda|10, 40)$, where n_1 denotes mean, n_2 standard deviation, the distribution is truncated to fulfil requirements: $\lambda > \lambda_{min}$,

The specified distributions cover a wide range of prior beliefs about possible values of the weighting parameter. The maximum value of logarithm of the marginal data density was obtained for the gamma distribution $f_G(\lambda|1.4, 0.3)$, equal to 1221.5. Among uniform distributions the best model is found to be for $U(0, 4)$ where logarithm of the modified harmonic mean is equal to 1217.3, while for normal distributions the maximum value of the marginal likelihood is 1218.9 for $f_N(\lambda|1, 5)$. The graph of marginal posterior distributions for weighting parameter λ and weights W for abovementioned priors can be found in the work Wróbel-Rotter (2013c). The paths of the IRFs are quite similar and independent of the assumed type of the prior distribution for λ, what can be considered as an indication of the robustness of posterior inference (see graph 5). The slight difference is visible for the impact of the technological and price shocks on the interest rate.

5.2 The prior dispersion

The impact assessment of the gradual change in the dispersion of the prior distribution for λ on posterior IRFs paths is analysed after assuming three types of the prior: uniform, gamma (moved to assure $\lambda > \lambda_{min}$) and the beta extended to interval (a, b), which is the same as for uniform distributions. Values for the prior mean $E(\lambda)$ are set from 0.5 to 25 with step 0.5. The lower bound a of the interval (a, b) for the uniform distribution is $\lambda_{min} = 0.19$ for $p = 2$, the upper bound b was set as to guarantee the assumed mean $E(\lambda)$ of the prior distribution. For the gamma distribution $f_G(\lambda|g_1, g_2)$ we set $g_2 = 2$, which means that we assume χ^2 distributions with $2g_1$ degrees of freedom, what indicates $g_1 = E(\lambda)/2$. Assumed values for the weighting parameter prior indicate that the central tendencies of distributions are similar but the dispersion of them is not directly comparable as uniform and modified beta distributions have restricted support while the gamma and truncated normal can take any positive number in that case.
The highest value of logarithm of the marginal data density for the lag $p = 2$ is obtained for the uniform distribution on the interval 0.19 to 0.81 ($E(\lambda) = 0.5$) and is equal to 1225. For the modified beta distributions the maximum value of the marginal likelihood is registered for the same interval and it is equal to 1224. That means that the DSGE-VAR model with the uniform distribution is more than two

Figure 5: IRFs for alternative priors of the weighting parameter



$U(a,b)$ means uniform distribution on the interval (a,b), $G(c,d)$ – Gamma distribution with parameters c and d and $N(e,f)$ denotes Normal distribution with mean e and standard deviation f, truncated to $\lambda > \lambda_{min}$.

times better that the model with the beta prior. In the group of gamma distributions the maximum level of log marginal data density is calculated for the case $E(\lambda) = 1.5$ and is equal to 1221. This quantity is lower than for uniform and beta distributions, possibly as a consequence of the fact that the gamma prior is much more dispersed $(\lambda \geq \lambda_{min})$ than the beta and uniform, which are defined for intervals. In all cases the marginal likelihood has tendency to decrease as the dispersion of prior distributions increases. Within the given value of $E(\lambda)$ the uniform distributions are better than the gamma in 36 cases and better than the beta distribution in 21 cases. In 40 cases beta distributions lead to higher marginal likelihoods than the gamma distribution. More detailed discussion of the results can be found in the work Wróbel-Rotter (2013b).

As previously the results indicate that the shape of IRFs is quite robust to the prior distribution for the weighting parameter. The weighting parameter estimation approach allows freely the data to decide about posterior mean of λ and allows to avoid the problem of arbitrary specifications of the set of values for it. The case of estimated λ leads directly to maximum values of the marginal likelihood while in the conditional approach one has to choose set of arbitrary values that do not guarantee that the optimal one is present among them. Further, IRFs could be affected by the choice of λ. Analogous results were also found for different lags of the DSGE-VAR for which optimal value of the weighting parameter is higher as number of free parameters increases.

6 Summary

In the paper there was examined impact on the posterior inference of the shape of IRFs after assuming competing prior distributions for the weighting parameter, which defines the strength of economic restrictions in the DSGE-VAR model. The first approach was to consider the set of conditional models obtained for arbitrary chosen values for the weighting parameter. The second approach was to fully estimate the weighting parameter what requires to assume for it a prior probability distribution. At the beginning it was considered the most typical distribution: an uniform, and then less common in such applications: gamma, modified beta to assure the same support as for the uniform distribution, and normal, truncated to values that guarantee existence of the prior from the DSGE model. For considered distributions the analysis of IRFs posterior paths was conducted for different prior means and gradual change in the prior dispersion.

In the conditional approach obtained results of the marginal likelihood confirmed the inverse U-shape presented in the literature for different models. The IRFs time paths in the conditional approach illustrated gradual change of their shape as a consequence of gradual change in the degree of economic restrictions from the DSGE model. Such comparison illustrate how the economic conclusions can be affected by incorrect choice of the weighting parameter values. The uncertainty of the estimation of IRFs in the DSGE-VAR model is quite strong in comparison with the ones obtained from the DSGE model, estimated within the DSGE-VAR model and individually. HPDs depend on the weighting parameter values. The values for the weighting parameter have influence on the posterior IRFs obtained from the DSGE model. The solution of the considered DSGE model is well approximated by the finite order VAR with restricted number of lags. Misspecification analysis of the considered DSGE model indicated areas when the economic assumption do not agree with the data and revealed that restrictions on the parameter space coming from the solution of the DSGE are strict and they are not entirely approved by the observed data, but also they are not strongly rejected.

Estimation with different priors for the weighting parameter enabled comparison between similarly specified prior and quite diversified. The paths of the IRFs are quite independent of the assumed type of the prior distribution for the weighting parameter, what can be considered as the robustness of economic inference based on the DSGE-VAR model, even though the values of marginal likelihood reveal differences. Comparison of the IRFs paths for a wide range of prior means and dispersion for the weighting parameter and for different VAR lags confirm the results. The estimation of the weighting parameter allows freely the data to decide about its posterior mean. This allows to omit the issues of correct choice of the weighting parameter values in conditional models. The case of full parameters estimation leads directly to the maximum values of the marginal likelihood while in the conditional approach there is a two stage procedure. As a consequence, IRFs could be affected by the choice of values for the weighting parameter. Both approaches confirm that there exists advantages of incorporating economic restrictions into the VAR.

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References

[1] Adjemian A., DarracqPariès M., Moyen S. (2008), Towards a monetary policy evaluation framework, *European Central Bank Working Paper* 942, Frankfurt am Main, Germany.

[2] Adjemian S., Bastani H., Juillard M., Mihoubi F., Perendia G., Ratto M., Villemot S. (2011), Dynare: Reference manual, version 4, *Dynare Working Papers* 1, Paris.

[3] Adolfson M., Laseén S., Lindé J., Villani M. (2008), Evaluating an estimated New Keynesian small open economy model, *Journal of Economic Dynamics and Control*, vol. 32(8), 2690–2721.

[4] Baumeister C., Hamilton J. D. (2014), Sign restrictions, structural vector autoregressions and useful prior information, *Econometrica*, 83(5), 1963–1999.

[5] Christiano L. J. (2007), Comment on Marco Del Negro, Frank Schorfheide, Frank Smets, and Raf Wouters, 'On the Fit of New-Keynesian Models', *Journal of Business & Economic Statistics*, 25(2), 143–151.

[6] Christiano L. J., Eichenbaum M., Evans C. (2005), Nominal rigidities and the dynamic effects of a shock to monetary policy, *Journal of Political Economy*, vol. 113(1), 1–45.

[7] Dedola L., Neri S. (2006), What does a technology shock do? A VAR analysis with model-based sign restrictions, *Working Paper Series* 705, European Central Bank.

[8] Del Negro M., Schorfheide F. (2002), Priors from General Equilibrium models for VARs, Penn Institute for Economic Research, *Working Paper*, No. 02-024.

[9] Del Negro M., Schorfheide F. (2004a), Priors from General Equilibrium models for VARs, *International Economic Review*, vol. 45(2), 643–673.

[10] Del Negro M., Schorfheide F. (2006), How good is what you've got? DSGE-VAR as a toolkit for evaluating the DSGE models, *Economic Review*, Federal Reserve Bank of Atlanta, Q2, 21–37.

[11] Del Negro M., Schorfheide F., Smets F., Wouters R. (2004b), On the fit and forecasting performance of New Keynesian models, *Federal Reserve Bank of Atlanta Working Paper* 2004-37, December.

[12] Del Negro M., Schorfheide F., Smets F., Wouters R. (2007), On the fit of New-Keynesian models, *Journal of Business & Economic Statistics*, 25(2), 123–143.

[13] Erceg C. J., Henderson D. W., Levin A. T. (2000), Optimal monetary policy with staggered wage and price contracts, *Journal of Monetary Economics*, vol. 46(2), 281–313.

[14] Fernández-Villaverde J., Rubio-Ramírez J. F., Sargent T. (2007), A, B, C's (and D's) for understanding VARs, *American Economic Review*, vol. 97(3), 1021–1026.

[15] Fukač M. (2007), Impulse Responses Identifcation in DSGE Models, Reserve Bank of Kansas City, *Research Working Paper* RWP 10-07.

[16] Geweke J. (1999), Using simulation methods for Bayesian econometric models: inference, development and communication, *Econometric Reviews*, vol. 18(1), 1–73

[17] Ghent A. C. (2008), Comparing DSGE-VAR forecasting models: How big are the differences?, *Journal of Economic Dynamics and Control* 33, 864–882.

[18] Kolasa M., Rubaszek M., Skrzypczyński P. (2012), Putting the New Keynesian DSGE model to the real-time forecasting test, *Journal of Money, Credit and Banking*, vol. 44(7), 1301–1324.

[19] Koop G. (1996), Parameters uncertainty and impulse response analysis, *Journal of Econometrics*, vol. 72(1-2), 135–149.

[20] Koop G., Pesaran M. H., Potter S. M. (1996), Impulse response analysis in nonlinear multivariate models, *Journal of Econometrics*, vol. 74(1), 119–147

[21] Lee K., Matheson T., Smith C. (2007), Open economy DSGE-VAR forecasting and policy analysis: Head to head with the RBNZ published forecasts, *Reserved Bank of New Zealand Discussion Paper Series*, DP2007/01.

[22] Liu P., Theodoridis C. (2010), DSGE model restrictions for structural VAR identification, *Bank of England Working Paper* 402.

[23] Mittnik S., Zadrozny P. (1993), Asymptotic distributions of impulse responses, step responses, and variance decompositions of estimated linear dynamic models, *Econometrica*, vol. 61(4), 857–870.

[24] Poirier D. J. (1995), *Intermediate Statistics and Econometrics: A Comparative Approach*, MIT Press, Hong Kong.

[25] Rabanal P., Rubio-Ramírez J. F. (2005a), Comparing New Keynesian models in the Euro Area: A Bayesian approach, *Journal of Monetary Economics* 52, 1151–1166.

[26] Rabanal P., Rubio-Ramírez J. F. (2005b), Comparing New Keynesian models of the Business cycle: A Bayesian approach, *Journal of Monetary Economics*, vol. 52(6), 1151–1166.

[27] Ravenna F. (2006), Vector autoregressions and reduced form representations of DSGE models, *Documentos de Trabajo*, No. 0619, Banco de Espania.

[28] Rubio-Ramirez J. F., Waggoner D. F., Zha T. (2010), Structural vector autoregressions: theory of identyfication and algorithms for inference, *Review of Economic Studies*, 77(2), 665–696.

[29] Sims C. A., Zha T. (1999), Error bands for impulse responses, *Econometrica*, 67(5), 1113–1156.

[30] Uhlig H. (2005), What are the effects of monetary policy on output? Results from an agnostic identification procedure, *Journal of Monetary Economics*, 52(2), 381–419.

[31] Watanabe T. (2007), The application of DSGE-VAR model to macroeconomic data in Japan, *ESRI Discussion Paper Series* 225-E.

[32] Wróbel-Rotter R. (2011a), Empiryczne modele równowagi ogólnej: gospodarstwa domowe i producent finalny, *Zeszyty Naukowe Uniwersytetu Ekonomicznego w Krakowie, seria Ekonomia*, 869, 109–128.

[33] Wróbel-Rotter R. (2011b), Sektor producentów pośrednich w empirycznym modelu równowagi ogólnej, *Zeszyty Naukowe Uniwersytetu Ekonomicznego w Krakowie, seria Ekonomia* 872, 73–93.

[34] Wróbel-Rotter R. (2012a), Empiryczne modele równowagi ogólnej: zagadnienia numeryczne estymacji bayesowskiej, *Zeszyty Naukowe Uniwersytetu Ekonomicznego w Krakowie, seria: Metody analizy danych*, 878, 143–162.

[35] Wróbel-Rotter R. (2012b), Struktura empirycznego modelu równowagi ogólnej dla niejednorodnych gospodarstw domowych, *Zeszyty Naukowe Uniwersytetu Ekonomicznego w Krakowie, seria Ekonomia*, 879, 107–126.

[36] Wróbel-Rotter R. (2013a), Estymowane modele równowagi ogólnej i autoregresja wektorowa. Aspekty teoretyczne, *Przegląd Statystyczny*, 3(60), 359–380.

[37] Wróbel-Rotter R. (2013b), Estymowane modele równowagi ogólnej i wektorowa autoregresja. Aspekty praktyczne, *Przegląd Statystyczny*, 4(60), 477–498.

[38] Wróbel-Rotter R. (2013c), Estymowane modele równowagi ogólnej i wektorowa autoregresja: model hybrydowy, *Bank i Kredyt* 44(5), 533–570.

[39] Wróbel-Rotter R. (2015), *Estymowane modele równowagi ogólnej. Podejście bayesowskie*, Wydawnictwo Uniwerstytetu Ekonomicznego w Krakowie, Zeszyty Naukowe, Seria specjalna: Monografie, 239

[40] Zellner A. (1971), *An introduction to Bayesian inference in econometrics*, John Wiley and Sons, Ltd., New York.