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Canonical Correlation Analysis in Panel Vector Error Correction Model. Performance Comparison

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Abstract

Small sample properties of unrestricted and restricted canonical correlation estimators of cointegrating vectors for panel vector autoregressive process are considered when the cross-sectional dependencies occur in the process generating nonstationary panel data. It is shown that the unrestricted Box-Tiao estimator is slightly outperformed by the unrestricted Johansen estimator if the dynamic properties of the underlying process are correctly specified. The comparison of performance of the restricted canonical correlation estimator of cointegrating vectors for the panel VAR and for the classical VAR applied independently for each cross-section reveals that the latter performs better in small samples when the cross-sectional dependence is limited to the error terms correlations, even though it is inefficient in the limit, but it falls short in comparison to the former when there are cross-sectional dependencies in the short-run dynamics and/or in the long-run adjustments.

Keywords: canonical correlation analysis, cointegration, panel VEC model, LCCA, Box-Tiao approach

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1 Introduction

During the last two decades, there was an increasing interest in an augmentation of the cointegration analysis of the integrated single-indexed processes for time-series data towards the nonstationary double-indexed processes for panel data. On the one hand, the focus was on the univariate analysis and the limit theory was developed by Phillips and Moon (1999). Among the surge of papers concerning univariate nonstationary panel analysis, Entorf (1997) has studied the spurious regression phenomenon in the panel framework, Levin *et al.* (2002) and Im *et al.* (2003) introduced the first generation of unit root tests for panels, the panel stationarity test was proposed by Hadri (2000), whereas the univariate cointegration tests were developed by Kao (1999), Pedroni (1999) and McCoskey and Kao (1998), see survey in Banerjee (1999). On the other hand, the multivariate cointegration analysis of panel data was proposed first by Groen and Kleibergen (2003) and Larsson and Lyhagen (2007). Anderson *et al.* (2006) advocated the use of both Box and Tiao (1977) and Johansen (1988) canonical correlation estimators of the cointegrating vectors in the context of panel data. It is noteworthy that Anderson *et al.* (2006) were able to find an additional cross-sectional cointegrating vector in empirical data using Box and Tiao approach, as opposed to the results suggested by Johansen's trace test.

Since the cointegration analysis of vector autoregressive processes is among the most successful methods used in empirical analyses and small sample performance of univariate cointegration analysis of panel data seems to be well-known, it is natural to investigate on properties of methods used in multidimensional analysis (time, cross-sections, multiple variables) of the nonstationary panel data. In this paper we focus on performance of Box-Tiao and Johansen canonical correlation estimators of the cointegrating vectors applied in the framework of panel data, since the existing small sample investigations that are based on the panel data framework are concentrated on the Johansen's approach only, see Larsson and Lyhagen (2000, 2007) and Larsson *et al.* (2001). The comparative investigations based on the purely time-series context are available in Bewley *et al.* (1994) and Bewley and Yang (1995).

Therefore, at first, using the framework of panel vector error correction model (panel VEC or PVEC henceforth) we compare the small sample properties of the unrestricted canonical correlation estimators proposed by Box and Tiao (1977) and Johansen (1988). With respect to the restricted canonical correlation analysis we show that under assumption of first order integratedness of the process the restricted canonical correlation estimator of the cointegrating vectors for both approaches are the same.

In this paper, we compare also the performance of the restricted canonical correlation estimator in case of the panel VEC model and in case of the usual VEC model when the cross-sectional dependencies occurs in the underlying process. The results show that when the cross-sectional dependencies are limited only to the non-zero error terms correlations between cross-sections, the asymptotically inefficient estimator of cointegrating vectors for the VEC model performs better in small samples. However, if there are cross-sectional dependencies in the short-run dynamics or in the long-run adjustments, which is often the case in practice, then the efficient estimator of cointegrating vectors for the panel VEC model performs significantly better even for moderate samples.

The rest of the paper is organized as follows. Section 2 describes the canonical correlation analysis for the vector autoregressive process integrated of order one in case of time-series. Sections 3 introduces the panel VEC model and the canonical correlation analysis for the panel framework. The design of experiment and the result are discussed in section 4. Section 5 contains some concluding remarks.

2 Canonical correlation analysis

To focus on the problem of unrestricted and restricted canonical correlation analysis, let us first consider briefly the canonical analysis in a purely time-series context.

Suppose y_t is a P-dimensional vector of observations and it follows a first-order vector autoregressive model

$$y_t = \Theta y_{t-1} + \varepsilon_t, \tag{1}$$

where Θ is a $P \times P$ matrix of coefficients, ε_t is an independently and identically distributed error term and the unit roots are allowed, thus the process can be non-stationary. Note that the first-order VAR model without any deterministic terms is considered here only for the sake of easy of the exposition, since the canonical analysis can be easily performed as well for higher order vector autoregressions and with deterministic components (after concentrating out short-run effects and deterministic terms according to Frisch-Waugh theorem). Following Box and Tiao (1977) and Bewley *et al.* (1994) the canonical transformation of the original process can be performed, by solving the eigenvalue problem

$$\left|\lambda YY' - \left(YY'_{-1}\right)\left(Y_{-1}Y'_{-1}\right)^{-1}\left(Y_{-1}Y'\right)\right| = 0, \tag{2}$$

where Y and Y_{-1} are $P \times T$ matrices of current and lagged observations, for eigenvalues λ_p and eigenvectors v_p such that

$$\lambda_p v_p = \left(YY'\right)^{-1}\left(YY'_{-1}\right)\left(Y_{-1}Y'_{-1}\right)^{-1}\left(Y_{-1}Y'\right)v_p, \tag{3}$$

where the eigenvalues λ_p are ranked in the ascending order and $\hat{V} = \begin{bmatrix} \hat{v}_1 & \dots & \hat{v}_P \end{bmatrix}$. The transformed process $\hat{V} y_t$ generates contemporaneously independent canonical variates that are ordered from the least to the most predictable according to the order of the eigenvalues. The eigenvectors associated with eigenvalues approaching unit boundary represent the non-stationary directions and span the non-stationary subspace. The eigenvectors corresponding to eigenvalues smaller than one describe the stationary long-run relationships and form the cointegration subspace.

As opposed to the levels canonical correlation analysis (LCCA henceforth) introduced by Box and Tiao (1977), the other approach to the canonical analysis, proposed by Johansen (1988), is to take explicitly into account the dynamic properties of the process. For the VAR process integrated of order one, the error correction form of model (1) shall be considered

$$\Delta y_t = \Pi y_{t-1} + \varepsilon_t, \tag{4}$$

where $\Pi = \Theta - I$. Therefore, the Johansen's approach constitutes the canonical correlation analysis of first differences and lagged levels and the canonical transformation of the process is achieved by solving the following eigenvalue problem

$$\left| \lambda Y_{-1} Y_{-1}^{'} - \left(Y_{-1} \Delta Y^{'} \right) \left(\Delta Y \Delta Y^{'} \right)^{-1} \left(\Delta Y Y_{-1}^{'} \right) \right| = 0, \tag{5}$$

where the eigenvalues are ranked in descending order and the eigenvectors associated with non-zero eigenvalues (in the limit) span the cointegration subspace.
Clearly, since estimators of cointegrating vectors in both canonical correlation analyses are derived as a solution to an eigenvalue problem, they represent system approaches with non-normalised long-run relationships, as opposed to methods based on the OLS regression. Moreover, Box-Tiao's estimator essentially diagonalise $\overset{\vee}{\Theta} \overset{\wedge}{\Theta}$ matrix, whereas Johansen's estimator diagonalise $\overset{\wedge}{\Pi} \overset{\vee}{\Pi}$, as it follows from (2) and (5) (the "hat" and the "reversed hat" denote the OLS estimators for a given regression, (1) or (4), and an associated reverse regression), see Bewley *et al.* (1994).
The property of cointegration implies the non-zero rank of matrix Π, say R, and enables to decompose Π into AB', where both matrices are full rank matrices, $\hat{B} = \left[\begin{array}{ccc} \hat{v}_1 & \dots & \hat{v}_R \end{array} \right]$ and A denotes the loading matrix. The restricted canonical correlation analysis for the Johansen's approach can be simply performed by employing the switching algorithm and sequentially concentrating out all but one stationary long-run relationships, as proposed by Johansen (1991). Therefore, for the VAR model (4) given j-th cointegrating vector can be iteratively estimated by conditioning on $B_{[j]}^{'} Y_{-1}$, where $\hat{B}_{[j]} = [\hat{v}_1 \, \dots \, \hat{v}_{j-1} \, \hat{v}_{j+1} \, \dots \, \hat{v}_R]$ and solving the eigenvalue problem for the residuals

$$\Delta Y._{B_{[j]}} = \Delta Y - \left(\Delta Y Y_{-1}^{'} \hat{B}_{[j]} \right) \left(\hat{B}_{[j]}^{'} Y_{-1} Y_{-1}^{'} \hat{B}_{[j]} \right)^{-1} \left(\hat{B}_{[j]}^{'} Y_{-1} \right), \tag{6a}$$

and

$$Y_{-1,}._{B_{[j]}} = Y_{-1} - \left(Y_{-1} Y_{-1}^{'} \hat{B}_{[j]} \right) \left(\hat{B}_{[j]}^{'} Y_{-1} Y_{-1}^{'} \hat{B}_{[j]} \right)^{-1} \left(\hat{B}_{[j]}^{'} Y_{-1} \right), \tag{6b}$$

as follows from the Frisch-Waugh theorem.
In case of Box-Tiao approach the restricted canonical correlation analysis shall follow the decomposition of matrix Θ, since $\Theta = AB' + I = a_1 v_1^{'} + \ldots + a_R v_R^{'} + I$. Conditioning on $B_{[j]}^{'} Y_{-1}$ can be easily performed, however, conditioning on IY_{-1} leads

to the singularity problem, since $Y_{\cdot IY_{-1}} = \Delta Y$ and $Y_{-1,\cdot IY_{-1}} = 0$. Therefore, instead of conditioning, subtraction of IY_{-1} is a natural solution. Nevertheless, subtracting IY_{-1} replaces in fact model (1) with model (4), which means that the restricted canonical correlation analysis for Box-Tiao approach leads to the same estimator of B as in the case of Johansen approach. Summing up, restricted canonical correlation analysis of the VAR process with cointegration property requires imposition of a priori assumptions about the dynamic properties of the process. As a result, the restricted Box-Tiao estimator for the cointegrated VAR process is the same as the restricted estimator proposed by Johansen (1991).

3 Canonical correlation analysis for panel vector error correction model

The panel cointegrated vector autoregression can be considered in two main strands of model's specification. The first one is a multivariate analysis of the double-indexed process that constitutes one form of augmentation of the VEC model towards panel data. Therefore, the following model can be considered at first

$$\Delta y_{it} = \Pi_i y_{i,t-1} + \sum_{k=1}^{K-1} \Gamma_{ki} \Delta y_{i,t-k} + \Phi_i d_t + \varepsilon_{it}, \tag{7}$$

where $y_{it} = \begin{bmatrix} y_{1it} & y_{2it} & \dots & y_{Pit} \end{bmatrix}'$ is a P-dimensional vector of observations for given cross-section i and period t, Π_i and Γ_{ki} are $P \times P$ matrices of coefficient, d_t and Φ_i denote a N-dimensional vector of (common) deterministic components and $P \times N$ matrix of their coefficients and ε_{it} is a P-dimensional independently and identically distributed error term with mean equal to zero and covariance matrix Ω_i for cross-section i.

Consider next the following VEC model for the panel VAR process

$$\Delta y_t = \Pi y_{t-1} + \sum_{k=1}^{K-1} \Gamma_k \Delta y_{t-k} + \Phi d_t + \varepsilon_t, \tag{8}$$

where $y_t = [y'_{1t}\ y'_{2t}\ \dots\ y'_{It}]'$ is a IP-dimensional vector of observations for period t, Π and Γ_k are $IP \times IP$ matrices of coefficients, Φ denotes $IP \times N$matrix of deterministic term coefficients and ε_t is a IP-dimensional independently and identically distributed error term with mean equal to zero and covariance matrix Ω. In case of the cointegrated panel VAR process matrix Π can be decomposed into $IP \times IR$ full rank matrices A and B, and the panel VEC model is

$$\Delta y_t = AB' y_{t-1} + \sum_{k=1}^{K-1} \Gamma_k \Delta y_{t-k} + \Phi d_t + \varepsilon_t. \tag{9}$$

The benefits of employing model (8) instead of model (7) can be quickly revealed by comparing their structures with respect to the cross-sectional dimension. The panel VEC model (9) can be rewritten as

$$\begin{aligned}\Delta y_t = & \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1I} \\ A_{21} & A_{22} & \cdots & A_{2I} \\ \vdots & \vdots & \ddots & \vdots \\ A_{I1} & A_{I2} & \cdots & A_{II} \end{bmatrix} \begin{bmatrix} \mathrm{B}'_{11} & \mathrm{B}'_{12} & \cdots & \mathrm{B}'_{1I} \\ \mathrm{B}'_{21} & \mathrm{B}'_{22} & \cdots & \mathrm{B}'_{2I} \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{B}'_{I1} & \mathrm{B}'_{I2} & \cdots & \mathrm{B}'_{II} \end{bmatrix}' \begin{bmatrix} y_{1t} \\ y_{2t} \\ \vdots \\ y_{It} \end{bmatrix} + \\ & + \sum_{k=1}^{K-1} \begin{bmatrix} \Gamma_{11,k} & \Gamma_{12,k} & \cdots & \Gamma_{1I,k} \\ \Gamma_{21,k} & \Gamma_{22,k} & \cdots & \Gamma_{2I,k} \\ \vdots & \vdots & \ddots & \vdots \\ \Gamma_{I1,k} & \Gamma_{I2,k} & \cdots & \Gamma_{II,k} \end{bmatrix} \begin{bmatrix} \Delta y_{1,t-k} \\ \Delta y_{2,t-k} \\ \vdots \\ \Delta y_{I,t-k} \end{bmatrix} + \\ & + \begin{bmatrix} \Phi_{11} \\ \Phi_{21} \\ \vdots \\ \Phi_{I1} \end{bmatrix} d_t + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \vdots \\ \varepsilon_{It} \end{bmatrix},\end{aligned} \tag{10}$$

and its covariance matrix is

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \cdots & \Omega_{1I} \\ \Omega_{21} & \Omega_{22} & \cdots & \Omega_{2I} \\ \vdots & \vdots & \ddots & \vdots \\ \Omega_{I1} & \Omega_{I2} & \cdots & \Omega_{II} \end{bmatrix}. \tag{11}$$

On the other hand, model (7) can be written in the same full notation as in (10) but with block-diagonal matrices A, B, Γ_k and Ω. Therefore, model (7) assumes lack of any cross-sectional dependencies, however this assumption can be clearly too restrictive for the underlying process. This is the main reason to use models nested in framework (9), see Groen and Kleibergen (2003), Larsson and Lyhagen (2007), and Jacobson *et al.* (2008), instead of straightforward panel augmentation for double-indexed processes given by (7). The only, yet significant, disadvantage of using model (9) is the potential dimensionality effect that can limit its application for small samples in case of a large number of cross-sections and variables simultaneously.

The unrestricted Box-Tiao estimator of the panel VEC model (9) is computed as follows. At first, the short-run effects are concentrated out and the concentrated regression is

$$\tilde{y}_t = \Theta \tilde{y}_{t-1} + \varepsilon_t, \tag{12}$$

where $\tilde{y}_t = y_t - y_t z_t' \left(z_t z_t'\right)^{-1} z_t$, $\tilde{y}_{t-1} = y_{t-1} - y_{t-1} z_t' \left(z_t z_t'\right)^{-1} z_t$ and $z_t = \begin{bmatrix} \Delta y_{t-1}' & \ldots & \Delta y_{t-K+1}' & d_t' \end{bmatrix}'$. Next, the canonical transformation is achieved by solving the eigenvalue problem

$$\left|\lambda\tilde{Y}\tilde{Y}' - \left(\tilde{Y}\tilde{Y}_{-1}'\right)\left(\tilde{Y}_{-1}\tilde{Y}_{-1}'\right)^{-1}\left(\tilde{Y}_{-1}\tilde{Y}'\right)\right| = 0, \tag{13}$$

for the eigenvalues $0 < \hat{\lambda}_1 < \ldots < \hat{\lambda}_{IP} < 1$ and eigenvectors $\hat{V} = \left[\begin{array}{ccc} \hat{v}_1 & \ldots & \hat{v}_{IP} \end{array}\right]$, of which the first IR constitute the cointegration subspace.
Similarly, the unrestricted Johansen estimator is calculated by concentrating out at first the short-run effects, thus the concentrated regression is

$$\Delta\tilde{y}_t = AB'\tilde{y}_{t-1} + \varepsilon_t. \tag{14}$$

Then the canonical transformation is performed by solving

$$\left|\lambda\tilde{Y}_{-1}\tilde{Y}_{-1}' - \left(\tilde{Y}_{-1}\Delta\tilde{Y}'\right)\left(\Delta\tilde{Y}\Delta\tilde{Y}'\right)^{-1}\left(\Delta\tilde{Y}\tilde{Y}_{-1}'\right)\right| = 0, \tag{15}$$

and $\hat{B} = \left[\begin{array}{ccc} \hat{v}_1 & \ldots & \hat{v}_{IR} \end{array}\right]$.
In order to enable performance comparison of both unrestricted estimators, the eigenvectors from both eigenvalue problems (13) and (15) shall be transformed into linear combinations that are as close as possible to subspaces spanned by design matrices. Therefore, the cointegrating vectors are found by solving the following eigenvalue problem

$$\left|\rho\hat{B}'\hat{B} - \hat{B}'H_j\left(H_j'H_j\right)^{-1}H_j'\hat{B}\right| = 0, \tag{16}$$

for the eigenvalues $\rho_1 > \ldots > \rho_{IR}$ and eigenvectors $u_1, \ldots, u_{IR}$, and choosing $\hat{b}_j = \hat{B}\hat{u}_1$, for each design matrix

$$H_j = \left[\begin{array}{ccccccc} 0 & \ldots & 0 & \tilde{I} & 0 & \ldots & 0 \end{array}\right]', \tag{17}$$

where H_j is $IP \times R$ matrix, $j = 1, \ldots, IR$, and $\tilde{I}$ denotes a submatrix that consist of zeros and ones if variable is present in the cointegrating vector, f.e. $\tilde{I} = \left[\begin{array}{ccccc} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$ if the third and the fourth variable form the cointegrating vector, see Johansen and Juselius (1994).

4 Design of experiment and results

Performance of both unrestricted estimators is compared within the framework of panel VEC model (9). To this end, Monte Carlo simulation is employed (simulations are carried out in Gauss 14). We consider second-order ($K = 2$) cointegrated panel VAR model with five variables and two cointegrating vectors for each cross-section. The number of cross-sections varies from one to eight. Since we are primarily interested in small sample properties of the high dimensional process, the sample size is $T \in \{100, 200, 400, 800\}$. The only deterministic term is a constant, which is considered in two cases, firstly, as an unrestricted constant, secondly, as a constant restricted to the cointegration space. However, the results for the latter case are not reported here (available upon request), since they do not alter the conclusions drawn from the former case. The number of replications is set as 10000.

Since the switching algorithm for the restricted estimation does not ensure that the global maximum of the likelihood function is reached, we truncate the empirical distribution. Therefore, a very rare replications (one or two for 10000 replications and only for $I > 5$) with clearly invalid numerical convergence are rejected. In order to identify these cases it was assumed that no individual element of cointegrating vectors should be higher than the true value by at least 10 times. The value of the truncation point is (enough) high in order to ensure that the distributions of both estimators that are fat-tailed (mixed Gaussian in the limit) are not affected, on the one hand. On the other hand, the cases of invalid numerical convergence are characterized by clearly extreme estimates.

We allow for three different sources of cross-sectional dependence: in the error term, in the short-run dynamics, and in the long-run adjustments. Cross-sectional dependence in the error term may occur for example due to common shocks affecting cross-sections. This sort of dependence can be easily observed in case of globalizing economies and integrated financial markets, see e.g. Groen and Kleibergen (2003) and Leuvensteijn *et al.* (2013). Cross-sectional short-run dependencies can be observed in case of interdependent economies. Cross-sectional long-run adjustments can be expected if an error-correction mechanism for given cross-section affects dynamics of other cross-sections. The cross-sectional short-run and long-run dependencies may arise for example in case of closely related economies and financial markets (e.g. members of economic and monetary unions or free trade agreements), see e.g. Larsson and Lyhagen (2007) and Beckmann *et al.* (2011). We do not consider here the cross-sectional cointegrating vectors. However they can occur in specific cases, often by definition of considered phenomenon, for example in case of purchasing power parity testing, see Banerjee *et al.* (2004) and Jacobson *et al.* (2008).

The data generating process (DGP henceforth) is as follows:

$$\Delta y_t = A\left(I_I \otimes B'_{11}\right)\left[\begin{array}{cc} y'_{t-1} & j' \end{array}\right]' + \sum_{k=1}^{K-1} \Gamma_k \Delta y_{t-k} + \varepsilon_t, \tag{18}$$

where the error term ε_t comes from the multivariate normal distribution, $\varepsilon_t \sim N_{IP}\left(0\,;\Omega\right)$, with covariance matrix from the inverse Wishart distribution – $\Omega \sim W_{IP}^{-1}\left(I\,;100\right)$. The initial values comes from the multivariate normal distribution and the first one hundred observation is truncated. With respect to the cointegration matrix we assume that

$$\underset{i=1,\ldots,I}{\forall} \mathrm{B}_{ii} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{bmatrix}', \tag{19a}$$

and

$$\underset{i \neq j}{\forall} \mathrm{B}_{ij} = 0. \tag{19b}$$

In case of cross-sectional dependence in the long-run adjustments we impose that

$$\underset{i=j}{\forall} A_{ij} = \begin{bmatrix} -0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.5 & 0 & 0 \end{bmatrix}', \tag{20a}$$

and

$$\underset{i \neq j}{\forall} A_{ij} = \begin{bmatrix} -0.1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.1 & 0 & 0 \end{bmatrix}'. \tag{20b}$$

For the cross-sectional dependence in the short-run adjustments we consider $\underset{i=j}{\forall} \gamma_{ij,1} = 0.5$ and $\underset{i \neq j}{\forall} \gamma_{ij,1} \sim U(-0.1\,; 0.1)$. Therefore, the DGP imposes for each cross-section two homogenous long-run relationships within cross-section (19a) and no cross-sectional cointegrating vectors (19b) and the error correction mechanisms affect not only the same cross-section (20a) but also the other cross-sections (20b), even though with rather small pulling force. Clearly, this setting does not mimic exactly any specific empirical phenomenon, however it is intended to reflect the cross-sectional features of empirical panel datasets, where cross-sectional co-movements are usually easily observed, see e.g. Kębłowski 2011. The roots of the autoregressive polynomial are computed in order to exclude explosive roots in the DGP.
With respect to the structure of cross-sectional dependence we consider three different cases:

case 1: cross-sectional dependence in the error term, in the short-run dynamics and in the long-run adjustments,

case 2: cross-sectional dependence in the error term and in the short-run dynamics,

case 3: cross-sectional dependence only in the error term.

Therefore, case 1 is the most general among aforementioned and it accounts for a significant information that comes from the cross-sectional dimension of the process, even though it does not allow for the special case of the cross-sectional cointegrating vectors. Cases 2 and 3 limit the cross-sectional dependencies to the short-run dynamics and error terms correlations respectively.
As can be seen from Tables 1 and 2, the unrestricted Johansen estimator (ML) slightly

Table 1: Standard deviations of unrestricted canonical correlation estimators of the cointegrating vectors, $T = 200$

I	1	2	3	4	5	6	7	8
case 1								
ML	0.064	0.159	0.278	0.512	0.736	0.991	1.535	1.949
LCCA	0.076	0.182	0.300	0.529	0.748	1.002	1.535	1.955
case 2								
ML	0.052	0.160	0.281	0.440	0.611	0.874	1.091	1.318
LCCA	0.072	0.175	0.303	0.453	0.648	0.902	1.116	1.371
case 3								
ML	0.056	0.143	0.279	0.414	0.609	0.880	1.178	1.479
LCCA	0.068	0.159	0.298	0.436	0.628	0.907	1.208	1.505

Table 2: Standard deviations of unrestricted canonical correlation estimators of the cointegrating vectors, $T = 800$

I	1	2	3	4	5	6	7	8
case 1								
ML	0.012	0.036	0.061	0.083	0.137	0.148	0.192	0.332
LCCA	0.016	0.041	0.065	0.089	0.141	0.156	0.197	0.337
case 2								
ML	0.013	0.028	0.048	0.084	0.119	0.139	0.203	0.237
LCCA	0.016	0.034	0.051	0.088	0.125	0.146	0.206	0.245
case 3								
ML	0.010	0.031	0.051	0.078	0.110	0.140	0.186	0.217
LCCA	0.015	0.035	0.056	0.082	0.116	0.145	0.194	0.223

outperforms the Box-Tiao estimator (LCCA) in short and in long samples for the DGP given by (15). This holds irrespective of the structure of cross-sectional dependence, the number of cross-sections as well as the sample size. The finding is consistent with the fact that the unrestricted Johansen estimator takes advantage of the true assumptions about the dynamic properties of the underlying process (integratedness of order 1), whereas the Box-Tiao estimator does not impose any specific conditions. However, this does not rule out the possibility that the Box-Tiao estimator may probably outperform the Johansen estimator for misspecified models or for other classes of cointegrated processes.

As mentioned above, the restricted canonical correlation analysis of the cointegrated processes requires imposing assumptions about the dynamic properties of the process, which equates the Box-Tiao and the Johansen's approach. However, the performance of the restricted canonical correlation estimator of the PVEC model (8) may be referred to the restricted estimator of the (misspecified) VEC model (4) applied for each cross-sections independently. This shall mimic the practical choice for small sample situations, i.e. whether to neglect the cross-sectional dependencies and consider the P-dimensional VAR model or to employ the (asymptotically) effective IP-dimensional panel VAR framework. In other words, it can be investigated, firstly, how the dimension of the process affects the performance of the restricted estimator in case of the PVEC model (dimensionality effect). Secondly, how the misspecified VEC model (4) deteriorates the performance of the restricted estimator. Thirdly, which approach performs better.

Table 3: Standard deviations of restricted maximum likelihood estimators of the cointegrating vectors, cross sectional dependence in the error term, in short-run dynamics and in long-run adjustments

I	1	2	3	4	5	6	7	8
				$T = 100$				
VEC	0.052	0.097	0.113	0.127	0.145	-	-	-
PVEC	0.052	0.090	0.158	0.373	0.592	-	-	-
				$T = 200$				
VEC	0.032	0.042	0.048	0.052	0.055	0.063	0.058	0.066
PVEC	0.032	0.039	0.042	0.043	0.033	0.046	0.032	0.050
				$T = 400$				
VEC	0.011	0.019	0.021	0.028	0.031	0.030	0.031	0.029
PVEC	0.011	0.017	0.018	0.022	0.021	0.019	0.017	0.011
				$T = 800$				
VEC	0.005	0.007	0.010	0.011	0.013	0.016	0.015	0.012
PVEC	0.005	0.006	0.007	0.008	0.009	0.009	0.007	0.004

As expected, the results of Monte Carlo simulations depend heavily on the structure of cross-sectional dependence and also on the sample size. In case of cross-sectional dependence in the error term, in the short-run dynamics and in the long-run adjustments (case 1), the restricted estimator for VEC models that neglects cross-sectional dependencies falls short in comparison to performance of the restricted estimator for the panel VEC model, see Table 3. The exception occurs for the sample size $T = 100$, which is simply too small for estimator of the panel VEC model.
In case of cross-sectional dependence in the error term and in the short-run dynamics (case 2), the asymptotically efficient restricted estimator for the panel VEC model in general still outperforms the restricted estimator for VEC models, even though for small number of cross-sections – $I < 5$, both estimators performs comparably (except for $T = 100$), see Table 4. As in the case 1, the more cross-sections the underlying process has and the longer time spans are used, the better performs the restricted estimator for the panel VEC model.
The reverse situation is observed when the cross-sectional dependence is limited to error terms correlations solely (case 3). Even for moderate samples, the restricted estimator for the VEC models performs better than the restricted estimator for the panel VEC model, see Table 5. The efficiency of the latter can be observed only in the limit, but even then, performance of both estimators is comparable. Therefore, in case of cross-sectional dependence limited to error terms correlations and for small samples, the dimensionality effect for the panel model prevails over the efficiency gains from proper specification of the panel framework.

Table 4: Standard deviations of restricted maximum likelihood estimators of the cointegrating vectors, cross sectional dependence in the error term and in short-run dynamics

I	1	2	3	4	5	6	7	8
				$T = 100$				
VEC	0.048	0.080	0.101	0.104	0.121	–	–	–
PVEC	0.048	0.095	0.215	0.413	0.669	–	–	–
				$T = 200$				
VEC	0.029	0.035	0.039	0.051	0.048	0.066	0.054	0.063
PVEC	0.029	0.037	0.039	0.051	0.043	0.061	0.055	0.058
				$T = 400$				
VEC	0.012	0.015	0.015	0.020	0.019	0.024	0.034	0.032
PVEC	0.012	0.014	0.015	0.018	0.015	0.017	0.020	0.019
				$T = 800$				
VEC	0.006	0.008	0.010	0.009	0.009	0.012	0.009	0.018
PVEC	0.006	0.008	0.009	0.008	0.008	0.009	0.003	0.004

Table 5: Standard deviations of restricted maximum likelihood estimators of the cointegrating vectors, cross sectional dependence in the error term

I	1	2	3	4	5	6	7	8
				$T = 100$				
VEC	0.050	0.092	0.098	0.121	0.132	–	–	–
PVEC	0.050	0.120	0.190	0.587	1.134	–	–	–
				$T = 200$				
VEC	0.029	0.036	0.047	0.048	0.056	0.061	0.067	0.070
PVEC	0.029	0.038	0.053	0.059	0.073	0.085	0.101	0.136
				$T = 400$				
VEC	0.013	0.015	0.019	0.025	0.026	0.030	0.030	0.033
PVEC	0.013	0.015	0.019	0.025	0.027	0.032	0.032	0.036
				$T = 800$				
VEC	0.006	0.009	0.010	0.011	0.013	0.015	0.015	0.018
PVEC	0.006	0.009	0.010	0.011	0.012	0.014	0.014	0.016

The results for cases 1 – 3 suggest that the error-term cross-sectional dependence shall not be considered as a decisive argument for employing the panel framework in the multivariate cointegration analyses. On the other hand, if the cross-sections are closely related (cross-sectional dependence in the short-run dynamics and/or in the long-run adjustments), then use of the panel framework leads to important efficiency gains and allows for a correct insight into the structure of dependencies in the time-series dimension as well as in the cross-sectional dimension.

5 Conclusions

In this paper we have examined small sample properties of the canonical correlation estimators of cointegrating vectors when the cross-sectional dependencies occur in

the underlying process generating panel data. We have carried out Monte Carlo simulations with the multidimensional panel VEC process generating the data. Three different sources of cross-sectional dependence were considered: in the error-terms, in the short-run dynamics and in the long-run adjustments. The results shows that the unrestricted canonical correlation estimator proposed by Box-Tiao is slightly outperformed by the unrestricted Johansen estimator. However, this is contingent upon proper specification of dynamic properties of the underlying process, which in turn is not required in case of the Box-Tiao canonical correlation analysis for levels of variables.
An interesting result has been obtained when the performance of the restricted canonical correlation estimator of the panel VEC model was compared to the restricted estimator of the classical VEC model applied for each cross-section independently. It was found that when the cross-sectional dependence is limited to the error terms correlations, the latter outperforms the former in small samples, even though it is inefficient in the limit. On the other hand, in case of cross-sectional dependence in the short-term dynamics and/or in the long-run adjustments the restricted estimator for the panel VEC model works significantly better than its time-series counterpart in small samples, except for very short samples, which are simply too short for the multidimensional panel framework.
This work may be extended in different directions. For example, it would be interesting to explore whether imposing the restriction of common cointegration space, i.e. that the cointegrating vectors in each cross-section span the same space, gives further essential gains to the panel framework. Therefore, the trade-off between the dimensionality effect and the homogeneity effect in small samples should be investigated. Another issue would be to examine the performance in case of near-I(2) processes e.g.

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