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Efficiency of the Reversed First-Price Sealed Bid Auctions with a Dynamic Run-Off. Results of Experiments

Paweł Kuśmierczyk*

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Abstract

In case of the private procurement auctions the entrepreneurs are not forced to limit themselves to the standard auction rules, and in practice one can observe many hybrid or quasi-auction mechanisms spontaneously introduced. The paper analyzes two of them, which start as a first-price sealed-bid auction, followed by a run-off in a form of an English auction, and which differ by the transparency of rules concerning the initiation of the second stage. The focus of the paper is on the analysis of the price and allocative efficiency of these mechanisms, in order to determine whether they can serve as an alternative to the standard auction rules. Theoretical analyses are followed by the laboratory experiments, which demonstrate that the mechanisms under study are characterized by both high price and allocative efficiency, and therefore could be considered an interesting substitute of the standard auction rules.

Keywords: auction, procurement auction, efficiency, experiments

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*Wrocław University of Economics; e-mail: pawel.kusmierczyk@ue.wroc.pl

1 Introduction

Auction theory has always focused on the sale auctions, i.e. auctions in case of which the auctioneer is either a seller or his representative, who wants to sell a given object at the highest possible price (henceforth referred to as *standard* auctions). The purchase auctions, in which a buyer (or her representative) carries out an auction in order to reach the lowest possible price, called the reverse auctions, have rarely been the case analyzed theoretically. With a good reason: the results from standard auctions theory can be easily extended over the reverse case, and thus there is no need for a special theoretical treatment of those.

But there does exist some practical reasons to devote a bit more time to the analysis of the reverse auctions. The first is: theoretical predictions quite often differ substantially from what is observed in real-life auctions, or laboratory experiments. Even though theoretical properties of the auction mechanisms can be "translated" into the reverse case, there are no grounds to believe that the same always holds for the real-life or experimental results, that could be prone to some other psychological phenomena. The second is: reverse auctions are very popular mechanisms for transaction making in case of procurement / tender, and the private entrepreneurs are by no means forced to apply the classical auction mechanisms. As a matter of fact, it can be observed, that the real-life reversed auctions quite often incorporate non-transparencies or some ad hoc designed hybrid solutions, that form new auction or quasi-auction mechanism.

This paper is devoted to the analysis of two hybrid auction mechanisms of this type, that have been modeled on the basis of some empirical observations of real procurement contracts. Both are two-stage (reverse) auctions, that start with the first-price sealed-bid auction. In FSR the two best bidders from the first stage advance to the English auction stage, whereas FSPR differs only by the uncertainty concerning the initiation of the second stage. The FSR and FSPR mechanisms are modeled, in order to describe the following strategy of the procurer: she requests the bidders to state their prices in the (reverse) first-price sealed-bid auction. Having collected the bids, she calls the second-best bidder, reveals the best price to him, and offers a chance to beat it. If he does so, the same chance is given to the best bidder from the first stage, and so it is as if an English auction stage was started with those two bidders involved. Naturally, if the second stage in the auction comes as a surprise to the unexpecting bidders, then it cannot hurt the procurer, as it will provide her with the price that is lower or equal to the one reached in the first stage. But the aim of the paper is to study, whether this auction mechanism could work as a market institution, i.e. be efficient, when its rules are known to bidders and incorporated in their strategies. The FSR auction is modeled as a two-stage auction mechanism, i.e. it is assumed that the second stage is always started, whereas FSPR is more of a quasi-auction: the decision whether to start a second stage or not is up to the auctioneer, and therefore is a risk factor from the bidders standpoint.

The paper focuses on the analysis and comparison of two efficiency criteria: the price

efficiency, and the allocative efficiency. The price efficiency is the most important criterion from the auctioneer's standpoint. It shows the extent to which a mechanism is capable of bringing the price down, which increases the buyer's surplus. The allocative efficiency shows how good is a given mechanism in maximizing the total surplus from transactions. In case of auctions of fixed quality, that are under the study in this article, an efficient transaction should be made with the lowest cost seller: only then the total surplus would be maximized.
The rest of the paper is arranged in the following form. Section 2 provides the main definitions and theoretical predictions concerning auction mechanisms under study. The efficiency of these hybrid rules was tested experimentally, and Section 3 describes the experimental design. Sections 4 and 5 provide the analyses of the results of the experiments, the former comparing auctions' efficiency, and the latter discussing the bidders' strategies. Section 6 concludes.

2 Definitions and theoretical predictions

The modeling of the economic agents within this paper follows a standard approach from the auction literature (e.g. McAfee, McMillan 1987, Krishna 2002), which was reversed though, to match the reverse auctions case. We assume that the auction is started by the buyer, who is searching for the lowest price. The market is homogeneous, therefore price is the only criterion. Sellers' costs are independently drawn from the identical distribution on the interval $[\alpha, \beta]$. The highest value of costs will be assumed to be the maximal price in any auction.
The profit of the winning seller is:

$$\pi = p - c, \tag{1}$$

where: π is the profit, p denotes the final price, and c stands for bidder's costs. No participation costs are assumed, and so the losing bidders make 0 profit.
In the classical auction theorem, called the Revenue Equivalence Principle (see Myerson 1981, and Riley, Samuelson 1981), it is additionally assumed that auction participants are risk neutral, thus maximize the expected value of the profit. Revenue Equivalence Principle claims that under such assumptions all auction mechanisms have the same expected value of final price, and bidders' payments, and it is easy to demonstrate that this rule can be extended over the reverse auctions (e.g. Kuśmierczyk, 2013). When the bidders are risk-averse, though, the Revenue Equivalence Principle's assumption do not hold, and so various auctions cannot be expected to have the same efficiency.
This paper is devoted to the analysis of a special class of hybrid auction mechanisms, that start with the sealed-bid auction, which is followed by the English auction stage. The definitions and all analyses hold for the reverse auctions case.

Definition 1. First-price sealed-bid auction with a dynamic run-off (FSR) is a two-stage auction mechanism. The first stage is the first-price sealed-bid auction. Two bidders with the lowest bids advance to the second stage, which is an English auction. The stage starts with the lowest bid made in the first stage, and continues as long as two participating bidders keep bidding the price down. It ends when one of them quits.

Definition 2. First-price sealed-bid auction with a possible dynamic run-off (FSPR) is a two-stage quasi-auction mechanism. The first stage is the first-price sealed-bid auction. After collecting the bids, the auctioneer picks the winner (the participant, that made the lowest bid), or starts a second stage for two bidders with the lowest bids in the first stage, that takes a form of an English auction. The rules concerning the probability of starting the English auction stage are non-transparent to the participants.

To the best author's knowledge these auction rules have not been studied in the literature, except for the author's own book Kuśmierczyk (2013). Let us start with the determination of the optimal bidder's strategy in FSR, by which we understand the price strategy, that maximizes the seller's profit. The optimal strategy is understood as a Bayesian Nash equilibrium of the game, and we limit ourselves to the equilibria that are symmetric and increasing with costs (see e.g. Krishna, 2002, p. 280).

Proposition 1. Let us assume that the number of auction participants is known, and bidders' costs are independently drawn from the identical, continuous distribution on the interval $[\alpha, \beta]$. Then the optimal bidders' strategy in the first-price sealed-bid auction with a dynamic run-off is:

1. bid the maximal price β in the first stage,
2. in the second stage bid down the price by the minimal decrement, as long as it is higher than costs, and quit when it gets lower.

Proof. The optimal strategy in the second stage of the auction is straightforward, as this is the unique, optimal strategy in the (reverse) English auction. Let us move to the first stage then. Let $b_1(c)$ denote the optimal seller's strategy in the first stage, i.e. the price that maximizes bidder's expected profit in the auction for the costs that equal c. Let us consider the following strategy in the first stage of FSR:

$$b_1(c) = c + (1 - \varepsilon) \cdot (\beta - c) \tag{2}$$

Assuming ε is a number close to 0, (2) describes a strategy of asking the price marginally below the maximal price. Price bid in (2) is the increasing function of c, and so if all participants followed it, then it would be the lowest cost bidder, and the second-lowest cost bidder, that would advance to the second stage. Notice now, that no one would gain anything by bidding the price below (2). For the lowest cost seller it would only decrease the starting price for the second stage, and therefore the expected value of the price, that he is paid. If bidder is not the lowest cost seller, then bidding below (2) would increase the probability of advancing to the second stage, but this bidder is going to lose this stage, as the lowest-cost bidder is going to advance to the second stage anyway (worst case scenario is that he advances with the second-best price, but it makes no difference for the English auction phase). The discussion above was true for any arbitrarily small ε, and so it holds also for $\varepsilon = 0$, when (2) turns to (3):

$$b_1(c) = \beta. \tag{3}$$

As the Proof of Proposition 1 demonstrated, by bidding the price equal to β we actually understand a price arbitrarily close to it. This is crucial for the determination of the two best bidders, because if all bidders bid literally β, then the choice of the participants for the second stage would be random. But even if we looked at the optimal strategy in the first stage of FSR from that perspective, it would still be difficult to believe, that any bidders would actually hold to (3). The point is that (3) is a Bayesian Nash equilibrium in the following sense: no one would gain anything by individually deviating from (3), as for the lowest-cost bidder it is enough that he is among the two best bidders in the first-stage (which guarantees him winning). But if two bidders deviated from (3) (even marginally), then it might change the whole situation. An equilibrium described by Proposition 1 is therefore not trembling-hand perfect (see Kreps, 1990, pp. 437-441), which to some extent explains, why it is difficult to believe, that anyone would act according to its predictions. Nevertheless (3) still has to be used as a reference point in the further analyses concerning FSR, for the mere fact, that it is the unique (even though unlikely to happen) Nash equilibrium of this game.

Let us now look into the Nash Bayesian equilibrium of the first-price sealed-bid auction with a possible dynamic run-off. The main difference between FSPR and FSR, described before, lies in the rule concerning the initiation of the second stage. Even though the auctioneer might actually follow a strict and deterministic rule, from the bidders' perspective it seems to be random, which introduces non-transparency into the auction. This non-transparency is deliberate; its aim is to make the sellers make better offers in the first stage. Even though the mechanism described in Definition 2 should be looked upon as a quasi-auction, rather than the auction, it actually meets the assumptions of the Revenue Equivalence Principle, and so its expected price and bidders' payments are the same as in the first-price sealed-bid auction, or the English auction. Unfortunately, the determination of the optimal bidder's strategy is impossible in the general case, even if one assumed a fixed probability of starting the second stage.

Example. Let us assume that n bidders take part in the first-price sealed-bid auction with a possible dynamic run-off, with their costs independently drawn from the uniform distribution on the interval $[\alpha, \beta]$. The second stage of the auction is started with the fixed probability q, that does not depend on the bids made by the auction participants.

The optimal strategy in the second stage, if it is started, is straightforward and consists in bidding the price down by the minimal decrements, and quitting when price gets below the costs. The only part that demands establishing is therefore the optimal strategy in the first stage, i.e. $b_1(c)$. In equilibrium the lowest price would be asked by the lowest cost seller, and the second-lowest price by the second-lowest cost seller, and so both of them would advance to the second stage (if it is started). Final price in case of FSPR would then be:

$$p_{FSPR} = \begin{cases} b_1(c_{1:n}), & \text{with probability } 1-q; \\ \min\{b_1(c_{1:n}), c_{2:n}\}, & \text{with probability } q. \end{cases} \tag{4}$$

where: $c_{1:n}$, $c_{2:n}$ are first- and second-order statistics respectively, i.e. the lowest and the second-lowest cost among n sellers.

Formula (4) results from the fact, that the auction has to be won by the lowest-cost bidder: he'd either outbid the second-lowest cost bidder in the second stage, asking the price marginally lower than his costs ($c_{2:n}$), or win with the price asked in the first stage ($b_1(c_{1:n})$), if it turns out to be lower than the second-best seller's costs or if the second stage is not started.

Let us consider a seller with costs c, who deviates from his optimal strategy by mimicking the optimal strategy of the seller with costs $\tilde{c}$. His expected profit is:

$$\begin{aligned} E\pi(\tilde{c}|c) \quad = \quad & P(\tilde{c} \le c_{1:n}) \cdot [b_1(\tilde{c}) \cdot (1-q) + b_1(\tilde{c}) \cdot q \cdot P(b_1(\tilde{c}) \le c_{2:n} | \tilde{c} \le c_{1:n}) + \\ & + c_{2:n} \cdot q \cdot P(b_1(\tilde{c}) > c_{2:n} | \tilde{c} \le c_{1:n}) - c]. \end{aligned} \tag{5}$$

If $b_1(c)$ is indeed the optimal strategy, then deviating from it must be suboptimal. Therefore the following must hold:

$$\frac{\partial}{\partial \tilde{c}} E\pi(\tilde{c}|c)_{\tilde{c}=c} = 0. \tag{6}$$

Solving (6) in case of the uniform distribution leads us to the following nonlinear differential equation:

$$\begin{aligned} & b'_1(c) \cdot (\beta - c)^{n-1} \cdot (1-q) + b'_1(c) \cdot (\beta - b_1(c))^{n-1} \cdot q + \\ & \quad - (n-1) \cdot (b_1(c) - c) \cdot (\beta - c)^{n-2} \cdot (1-q) = 0. \end{aligned} \tag{7}$$

Equation (7) does not have a general solution, but can be solved for $n = 2$, when it simplifies to:

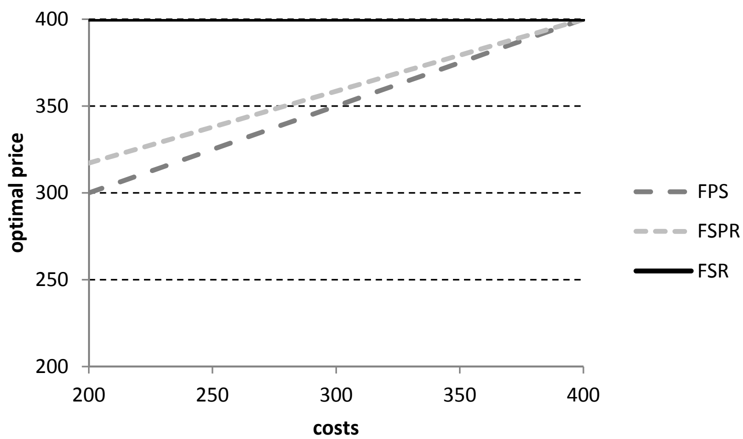
$$b'_1(c) \cdot (\beta - c) \cdot (1-q) + b'_1(c) \cdot (\beta - b_1(c)) \cdot q - (b_1(c) - c) \cdot (1-q) = 0. \tag{8}$$

Let us additionally assume, that $q = 0.5$. Solving (8) with the boundary condition $b_1(\beta) = \beta$, results with the following formula for the optimal strategy in the first stage of FSPR, when there are 2 participating bidders:

$$b(c) = c + \frac{\beta - c}{1 + \frac{1}{\sqrt{2}}} \tag{9}$$

Figure 1 compares the optimal prices depending on the cost level in case of two, risk-neutral bidders, with costs realizations coming from the uniform distribution on $[200, 400]$ in case of the first-price sealed-bid auction (FPS), and the first stages of FSR, and FSPR.

Figure 1: Optimal prices in FPS and first stages of FSR, and FSPR in case of an auction with two riskneutral bidders



As one can see the optimal price in FSPR is bigger than in FPS, which results from the Revenue Equivalence Principle and would also hold for $n > 2$. The second stage of FSPR, if started, would potentially make the price go down. The Revenue Equivalence Principle says that the expected value of final price in FPS and FSPR is the same, and so the expected value of price in the first stage of FSPR has to be higher than in FPS.

The aim of this paper is to compare the efficiency of the newly defined hybrid auctions with that of the standard auction mechanisms, for which the first-price sealed-bid auction, and the English auction would be used. If the assumptions of the Revenue Equivalence Principle held, then the expected efficiency of all those auctions would be the same. One of the reasons why the actual efficiencies are not the same is the risk-aversion. It is a known fact in auction theory (e.g. McAfee, McMillan 1987, Krishna 2002), that when the bidders are risk-averse the price-efficiency of FPS is higher than that of the English auction. Also it is known, that the dynamic bidding mechanism incorporated in the English auction helps reach a high allocative efficiency in case of this mechanism. These theoretical results (in case of standards auctions) have been confirmed in a number of experimental studies (e.g. Cox *et al.* 1982, Kagel, Levin 1993, Andreoni *et al.* 2007).

Designing a hybrid of the FPS and English auction mechanisms, with a hope of creating a mechanism of both high price and high allocative efficiency is not new to the auction theory. As an example one can recall the Anglo-Dutch auction designed by Klemperer (2002), which is a two stage mechanism, starting with the English auction, and followed by the first-price sealed-bid auction. The FSR mechanism can therefore be looked upon as a reversion of Anglo-Dutch auction rules.

We could use the known properties of FPS and English auction to form some hypotheses concerning the efficiency of the hybrid FSR, and FSPR mechanisms. And so the highest price efficiency is expected in case of FPS (pure, sealed-bid mechanism), then in case of FSPR (optionally an English auction), then FSR (always an English auction, but preceded by FPS), and the lowest in case of English auction (purely dynamic mechanism). The hypotheses concerning the allocative efficiency are the opposite: the highest allocative efficiency is expected in case of English auction, and then respectively in FSR, FSPR, and the lowest in case of FPS.

One has to remember, though, that risk-aversion is not the sole reason for which bidders deviate from their optimal strategies. From Proposition 1 we know that even the risk-averse bidders should bid the maximal price in the first stage of FSR, but it would not be surprising to observe that most of them do not. This is one of the reasons why the theoretical predictions, concerning the auctions' efficiency, should be always tested empirically. The experiment designed for that purpose is described in the next part.

3 Design of the experiment

The experimental method has become a standard in case of tests concerning the auctions' efficiency. Even if it has some weaknesses, that are well known, in practice it remains the unique method that can be applied to test the efficiency of the existing mechanisms, let alone the ones that are newly designed. The data coming from the real auctions are simply incomparable, as those auctions differ by the number of bidders, information, cost structure, and so on. Moreover, in order to calculate the price or the allocative efficiency one would need to know the values of costs of all participating bidders, which is impossible in real life. And above all, the experiments are the simplest and cheapest way of running a pre-test of the efficiency of the newly defined mechanisms, which are the subject of this paper.

The experiments described in this paper involved the 1st year students of the Wrocław University of Economics, that enrolled in the experiment voluntarily. As an incentive system the examination points were used. Points reached in the experiment augmented students' results from the exam on Microeconomics. The total number of points in the exam was 40, and in the experiment students could gain between 2 and 6 additional points. An additional study showed that the incentive system worked very well: when asked in a survey, students' median valuation of one point turned out to be 30 PLN. Students competed in the 4-person groups. Their costs were independently

Table 1: The most probable models in each analysed group

Auction design	No. of auctions	No. of participants
ENG	123 *a*	164 *a*
FPS	359	240
FSR	335	208
FSPR	128	76

a The numbers in case of the English auction come from the English auction stages played in case of FSR, and FSPR, where there was any potential to compete (i.e. the lowest price bid in the first stage was above the second-best bidder's costs).

drawn from a uniform distribution on $[200, 400]$, which was a common knowledge. An abstract frame was used, with the information provided formed in the following way: "*You take part in the procurement auction. The participant who asks the lowest price wins the auction (and is paid the price asked).* (...)" (detailed instructions available on request). The term "*procurement*" was used to help the students better understand the idea of the auction, but apart from that no additional information concerning the market, or commodity was used. The experiments involved 3 auction mechanisms: FPS, FSR, and FSPR. It was decided not to run the experiment in case of the English auction, as this mechanism has the well established properties, and such a trivial optimal strategy, that the theoretical predictions could be used instead of the actual results. Additionally the last stages of FSR, and FSPR are actually the English auctions (with the same optimal strategy), and so the strategic behavior of their participants can serve as an approximation of what would happen in the "pure" English auction. As for the non-transparent rules concerning the initiation of the second stage in FSPR, students were merely informed that "*Basing on the bids from the first stage, the auctioneer can either choose the winner (the participant that made the lowest bid) and finish the auction, or, if she's not satisfied with the bids made, she can start an additional stage in the form of the English auction*". Students were aware that the auctioneer bases her decision on some factors, but as those factors were not revealed, the rules concerning the initiation of the second stage were non-transparent to them. In fact the second stage was started with the following probability:

$$P = \min\left(1; \max\left(0; 0.5 + \frac{(b_{1:n} - 200) - 1.732 \cdot (b_{2:n} - b_{1:n})}{200}\right)\right) \tag{10}$$

i.e. it was increasing with the value of the lowest bid made ($b_{1:n}$), and decreasing with the difference between the best two bids ($b_{2:n} - b_{1:n}$). In other words, the second stage was the most likely when the lowest bid was on the high level, and when the

difference between the two best bidders was small, which, supposedly, corresponds with the real-life strategy of the buyers.
Each auction was first run in a trial mode (those results are not reported in the paper), and then repeated 7 to 10 times, so that the participants could learn the optimal strategies. The experiment was controlled for the learning effects, and because the first rounds' results were significantly different than later made decisions, they were excluded from the analyses. Table 1 provide the aggregate information on the number of auctions taken into analysis and the number of participating students.
All experiments were designed and carried out using zTree (Fischbacher, 2007).

4 Results: the efficiency measures

Price efficiency has a natural, and intuitive interpretation. It shows how good the auction mechanism is in what is its main goal: providing the auctioneer with the best possible price. In case of reverse auctions under study, the auction is the more price efficient, the lower is the price reached due to the auction mechanism.
In order to test the price efficiency, two measures were used. First one is standard, as it is simply the value of the final price reached (denoted as *PR*). But as this measure might be to some extent affected by the cost realizations, yet another measure was introduced. The μ measure was calculated in case of each auction in the following way:

$$\mu^j = p^j - c^j_{1:n}, \tag{11}$$

where: j is the auction index, p^j is the final price reached in the auction, and $c^j_{1:n}$ is the lowest cost among n bidders participating in the j-th auction.
The measure defined in (11) has a very convenient interpretation. It shows the distance between the final price actually reached, and the lowest potential price, i.e. the value of the lowest cost among participating bidders. The more price efficient is the given auction design, the closer should those two values be to each other, and therefore the closer to 0 should be the value of μ.
Price efficiency of the reverse auction is relied to the buyer's surplus. Assuming a constant utility of the object traded, the lower is the price, the higher is the surplus reached by the buyer. But buyer's surplus is not the only aspect that interests the auction theorists. A crucial element of auction's efficiency is also the extent to which it manages to maximize the total surplus, resulting from the transactions. This value would be maximized, when the transaction is made with the lowest cost bidder (the final price does not affect the total surplus, and only its division between a buyer and a seller). In order to measure the allocative efficiency, two standard measures were used, i.e. the percentage of auctions won by the lowest-cost participant (denoted as *AL*), and the percentage of the total surplus reached, typically referred to as Pareto efficiency, but here denoted as *KH* in order to refer to the Kaldor-Hicks efficiency, which is a generalization of Pareto efficiency. The latter in case of each auction was

calculated in the following way:

$$KH^j = \frac{\beta - c^j}{\beta - c^j_{1:n}} \cdot 100\%. \tag{12}$$

Table 2 presents the average values of the efficiency measures.

Table 2: Average values of the efficiency measures

Auction design	Price efficiency		Allocative efficiency	
	PR	μ	AL	KH
ENG	280 a	40 a	95.1% b	99.86% b
FPS	255.01	15.58	90%	99.12%
FSR	258.87	17.07	97%	99.63%
FSPR	255.02	15.12	96.9%	99.88%

a the theoretical prediction
b values estimated on the basis of English stages in FSR and FSPR experiments

Unfortunately none of the variables under study has a normal distribution; the normality hypotheses were rejected using a Durbin-Watson test in R (R Core Team, 2013). The distribution of PR and μ are actually right asymmetric. In order to test the significance of differences in values of these measures a one-tailed Wilcoxon-Mann-Whitney U test was applied. The null hypothesis in the test says that the efficiency measures equal, with an alternative which was changed depending on which efficiency measure had a higher value. Table 3 provides the p-values.

Table 3: P-values in price efficiency tests

Auction design	Efficiency	Tested against			
		ENG	FPS	FSR	FSPR
FPS	PR	2.2e-16 **	-		
	μ	2.2e-16 **	-		
FSR	PR	2.2e-16 **	0.0058 **	-	
	μ	2.2e-16 **	0.1474	-	
FSPR	PR	1.1e-15 **	0.1957	0.126	-
	μ	2.2e-16 **	0.364	0.128	-

All tests using Wilcox test in R (R Core Team, 2013). The alternative hypotheses were changed, depending on which auction had a bigger value of the efficiency measure, i.e. if experimentally it was found that $PR(\mathcal{M}_1) > PR(\mathcal{M}_2)$, then it was used as an alternative hypothesis.

As we can see all mechanisms turn out to be more price efficient than the English auction, which is in accordance with the theoretical predictions. In case of ENG there were no experiments conducted, and instead for the test purposes the theoretical values were used. The optimal strategy in the English auction, which consists in bidding one's opponent's price down by the minimal increments, as long as it stays profitable, is simple enough to feel safe that the theoretical values are a good approximation of the values that would have been reached, had the actual experiments been started. In order to confirm that assumption, the English stages of FSR and FSPR were analyzed in detail. The expected final price in English auction is the value of the cost realization of the second-best bidder, and so in order to check if auction participants followed the optimal strategies, the differences between the final prices and the best opponents' costs were studied. The median value of them turned out to be exactly 0, and the average was 0.21, therefore using the theoretical values for the test purposes seems to be well-justified.

Unfortunately the weak power of the U test makes it impossible to draw more unambiguous conclusions. The only exception is the comparative test of efficiencies of FPS and FSR, which showed that prices reached in the first-price sealed-bid auction are significantly lower than those reached in case of the first-price sealed-bid auction with a dynamic run-off ($p = 0.0058$). One has to be careful with the interpretation of this result, though. First of all, the final prices reached in the auctions are affected by the cost realizations, and the test of μ measures, that did not have this flaw, provided no grounds for rejection of the null hypothesis that price efficiencies of FPS and FSR are equal ($p = 0.1474$). Secondly, we have to bear in mind, that the U test allows us to draw conclusions concerning the median values, whereas the efficiency measures presented in Table 2, that seem to matter the most, are the average values. Nevertheless, the order of the values of the price efficiencies is generally in accordance with the predictions made in Section 2.

Let us now take a look at the values of the allocative efficiency measures, that are also provided in Table 2. In order to test the significance of differences in case of KH measures again the one-tailed Wilcoxon-Mann-Whitney test was applied, and in case of AL measures a one-tailed binomial test for two proportions was used. Table 4 presents the p-values.

Table 4: P-values in allocative efficiency tests

Auction design	Efficiency	Tested against			
		ENG	FPS	FSR	FSPR
FPS	KH	0.03345 (*)	-		
	AL	0.0591	-		
FSR	KH	0.8258	0.0001 (**)	-	
	AL	0.2449	0.00035 (**)	-	
FSPR	KH	0.2442	0.0067 (**)	0.5235	-
	AL	0.3493	0.0121 (*)	0.5	-

All tests in R (R Core Team, 2013). In case of KH a Wilcox test was used, and in case of AL a binomial test of equal proportions was applied. The alternative hypotheses were changed, depending on which auction had a bigger efficiency measure. If experimentally it was found that $KH(M_1) > KH(M_2)$, then it was used as an alternative hypothesis.

Again, the values of the allocative efficiencies generally follow the theoretical predictions. The mechanism of the lowest allocative efficiency is the first-price sealed bid auction, that is significantly less efficient than the other mechanisms in case of both efficiency measures applied. A bit surprisingly the AL value in case of the English auction, contrary to expectations, is lower than in case FSR and FSPR mechanisms, but this is most likely a chance result - actually both tests are inconclusive when comparing the allocative efficiency of ENG, FSR, and FSPR.
The idea that the hybrid auction, designed on the basis of two standard mechanisms, might take the best of both of them, seems to find a confirmation in the experimental results. The FSR and FSPR rules have a price efficiency close to that of the first-price sealed-bid auction, accompanied by the allocative efficiency similar to that of the English auction. The Wilcox test is not strong enough to differentiate between the two newly defined mechanisms, nevertheless the non-transparency built within the FSPR rule seems to have a positive effect on the price efficiency, without any negative consequences for the allocative efficiency, which gives this rule a marginal advantage over FSR.
Let us now take a closer look at the bidding strategies of the auction participants.

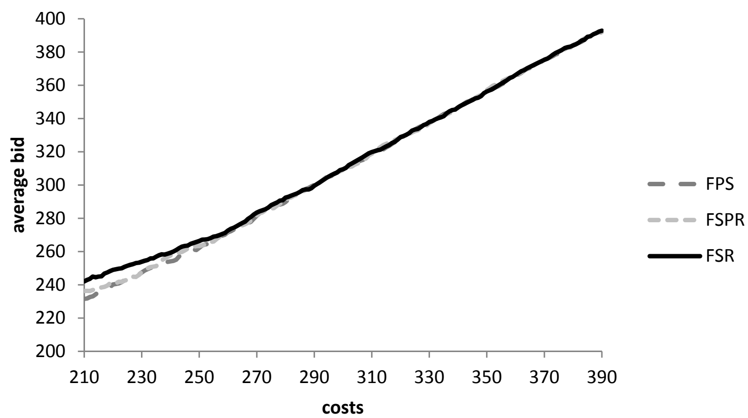
5 Results: bidding strategies

According to Proposition 1 the optimal strategy in the first stage of the first-price sealed-bid auction with a dynamic run-off consists in bidding the price equal (or very close) to the maximal price. If the bidders followed it, then the second stage should start with a high price, and the final winner would be selected in the English phase. The analysis of the auctions run in the experiment shows, that it did not happen. Out of 335 FSR auctions in the experiment only 100 (that is 30%) had any bidding in the second stage. In case of 70% of auctions the best price asked in the first stage was lower than the second best bidder's cost, and therefore there was no potential for any run-off. This has to be regarded as bidders' mistake, as they missed a potential for much higher profits.
In order to compare the prices bid in the first stages of FSR and FSPR auctions with that of the FPS auction, the moving average of bids (MAB) for subsequent values of cost realizations has been calculated:

$$MAB\,(c) = \frac{1}{m\,(c)} \cdot \sum_{t=c-10}^{t=c+10} b(t) \tag{13}$$

where: $b\,(t)$ is the price bid by the participant with a cost t, and $m\,(c)$ is the number of cost realizations from the interval $[c-10, c+10]$.
Figure 2 compares the bids in FPS, FSR, and FSPR.
As we can see, the prices bid in the first stages of FSR are significantly lower than the optimal bid of 400, and quite close to the prices offered in FPS, and first stages

Figure 2: Moving average of prices bid in FPS, and first stages of FSR and FSPR, as a function of cost realizations



of FSPR. The moving average of bids in case of those 3 mechanisms is almost indistinguishable for costs over 250. Most likely, the bidders with high cost values did not put much effort in determining the optimal bids, knowing that their chances of winning, and the potential profit are relatively small. But the differences in the strategic behavior of bidders with low cost realizations can be noticed with the naked eye. Those participants were aware of the differences in the auction rules, and adapted to them (semi)rationally: the highest prices, as expected, were bid in case of the first-price sealed-bid auction with a dynamic run-off, and the lowest in case of the first-price sealed-bid auction (see Table 5).

Table 5: Bidding in case of FPS and first stages of FSR, and FSPR in case of bidders with $c \leq 250$

Auction design	Average price	Median value of profit margin asked	Percentage of bids with profit margin ≥ 20
FPS	243.73	15	36.5%
FSR	251.55	22	58.6%
FSPR	246.39	15.5	43.1%

Moreover, the differences in bidding strategies of the low cost sellers go in line with the expectations, that one could have on the basis of Figure 1 (even though it only presents the optimal strategies for risk-neutral bidders in the auction with 2 participants). The differences in bids in case of FPS and first stage of FSPR are relatively small: the median value of profit margin asked (understood as a difference between the price asked by the seller, and his cost value) was 15 in case of FPS, and just 15.5 (+0.5) in case of FSPR, and the rest of the variables also do not differ that much: the average price asked in FPS is 243.73 compared to 246.39 in case of FSPR (+2.66), the percentage of bidders who dared to ask the price giving them the profit of at least 20 was 36.5% in case of FPS, and 43.1% in FSPR (+6.6%). On the other hand the low cost participants of FSR were much bolder. The median value of profit margin asked by them was 22 (+7, compared to FPS), and as many as 58.6% (+22.1%, compared to FPS) asked a price, giving them a potential profit of at least 20. These participants were aware of the rules of FSR, and knew that in order to qualify for the second stage they just need to have a bid in the top two. The probability of being one of the two lowest cost bidders, when your cost equals 250, was as high as 0.844 (and of course increasing as the cost decreases), which explains why the bidders felt more free to ask higher prices. As a consequence the average price asked by those bidders was 251.55 (+7.82, compared to FPS).

The optimal strategy in the English auction stages of FSR, and FSPR was straightforward: one should bid the price down by the minimal increments, as long as it stays above his cost level, and quit afterwards. In case of FSR, as has already been mentioned, only in case of 100 auctions (30% of all) the second stage brought any competition (in other cases the price from the first stage was too low for the second-best bidders to go for the run-off). In case of FSPR the number of run-offs was even smaller, as the second-stage was optional. Actually, on the basis of (10), the second stage was initiated in 51% of auctions, but just few of them involved any real competition (for the same reason as in case of FSR); in total that happened in case of 23 auctions (18% of all). Table 6 summarizes the most important information concerning the second stages of FSR and FSPR.

Table 6: Bidding in case of the second stages of FSR, and FSPR

Variable	FSR	FSPR
% of auctions with bidding in the second stage	30%	18%
average distance between the final price and the cost of the 2nd best bidder	0.26	0
median distance between the final price and the cost of the 2nd best bidder	0	0
% of auctions with final price higher than the cost of the 2nd best bidder	21%	22%
% of auctions with final price equal to the cost of the 2nd best bidder	41%	26%
% of auctions with final price lower than the cost of the 2nd best bidder	38%	52%

As we can see the participants behaved in accordance with the theoretical predictions. The median difference between the final price and the cost of the second-best bidder is 0 in both cases, and the average of it is only marginally above 0 in case of FSR. A more detailed analysis shows that the discrepancies between those values are of both types. When the second-best bidder asks the price that equals exactly his cost, then the winning bidder must go minimally below that cost. On the other hand, when the best bidder asks a price, higher than the second-best bidder's cost value by exactly 1, then he is indifferent between bidding the price down, or quitting, and so the final price can be marginally higher than his cost. But, as we can see from Table 6, those

cases balance out, and so the second stage's results match the theoretical predictions almost perfectly.
The bidding of participants in the dynamic run-offs of FSR and FSPR went according to the optimal strategies, therefore the main factors that altered the final price level were the bidding strategies in the sealed-bid stage. As has been demonstrated, the prices asked in the first stages of FSR were much lower than the theoretically predicted value of the maximal price β, which must have had a positive (from the buyer's perspective) impact on the final price level, lowering it below the expected value. Nevertheless p_{FSR} did not turn out to be lower than p_{FPS}, which is caused by the fact that bidders deviated from the theoretically optimal strategies in all sealed-bid auctions, including FPS.

6 Conclusions

Private entrepreneurs are not obliged to apply the standard auction mechanisms, when they try to sell their products, or when they search for the cheapest supplies. In fact, especially in the latter case, they keep inventing new rules or apply modifications to the existing ones on a daily basis. Those innovations can take a form of additional stages, incorporation of non-transparencies, data manipulations, asymmetries of rights, and many others. Many of them would work only once, with the unexpecting participants, but some can survive in a long run, if they continue to provide the auctioneers with the efficient outcomes.
This paper studied two hybrid mechanisms, that seem to bode well for the future. Both were designed as a hybrid of the first-price sealed-bid auction and the English auction, with the first-price sealed-bid auction with a dynamic run-off (FSR) being a two-stage mechanism, and the first-price sealed-bid auction with a possible dynamic run-off (FSPR) being a non-transparent variant of the former. The experiments demonstrated, that both mechanisms under study are characterized by a high price efficiency, that matches the one of the first-price sealed-bid auction, but at the same time provide a high allocative efficiency, that is on the level observed only in case of the English auction. As such, the FSR and FSPR mechanisms come close to an ideal of auction theorists: the rules that maximize the buyers' surplus and the total surplus at the same time.
The experimental research described in this paper is only a first step. Experiments concerning FSR, and FSPR should be repeated in different environments and circumstances to confirm that high values of efficiency reached in this study were not a coincidence. Out of two rules under study the slightly better values of efficiency measures were observed in case of the non-transparent FSPR mechanism. A deepened study could give answers to the questions concerning the role of the uncertainty in case of this auction rule. Does it indeed work in favor of both efficiency measures, or could we just focus our attention on FSR, which is a much more classical, and therefore "elegant" auction design? Above all, we cannot forget, that experiments work only as

a pre-test of the efficiency of the institutional innovations, and the ultimate answer to the questions, concerning their efficiency, would only be reached, once we test those innovations in the real life. That is actually a path that auction theory has followed in the past, and in which it has succeeded (see e.g. Klemperer, 2002), and so could be tried in case of FSR, and FSPR, as well. Nevertheless it might yet be too soon to discuss it know.

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