Tax Compliance and Public Goods Provision An Agent-based Econophysics Approach

Sascha Hokamp[*], Götz Seibold[†]

Abstract

We calculate the dynamics of tax evasion within a multi-agent econophysics model which is adopted from the theory of magnetism and previously has been shown to capture the main characteristics from agent-based based models which build on the standard Allingham and Sandmo approach. In particular, we implement a feedback of public goods provision on the decision-making of selfish agents which aim to pursue their self interest. Our results imply that such a feedback enhances the moral attitude of selfish agents thus reducing the percentage of tax evasion. Two parameters govern the behavior of selfish agents, (i) the rate of adaption to changes in public goods provision and (ii) the threshold of perception of public goods provision. Furtheron we analyze the tax evasion dynamics for different agent compositions and under the feedback of public goods provision. We conclude that policymakers may enhance tax compliance behavior via the threshold of perception by means of targeted public relations.

Keywords: tax compliance, econophysics, multi-agent model

JEL Classification: C15, H16, H30

[*]University of Hamburg; e-mail: sascha.hokamp@uni-hamburg.de
[†]BTU Cottbus – Senftenberg; e-mail: goetz@physik.tu-cottbus.de

1   Introduction

Theoretical approaches to account for tax compliance are often based on the seminal work of Allingham and Sandmo (1972) which incorporates potential penalties, tax rates and audit probabilities as basic parameters in order to evaluate the expected utility of taxpayers. In the most basic version of this neoclassical standard scheme the behavior of *all* taxpayers enters via the *average* degree of risk aversion, related to the structure of the underlying utility function. However, the reasons and motivations for tax evasion in a society are manifold and it would obviously be desirable to relate the behavior of individuals, embedded in their social network, to global parameters such as risk aversion. In some sense this hierarchy of modelling has a close match in physics where phenomenological laws of thermodynamics can be derived within the concepts of statistical mechanics which are applied to the individual constituents of a macroscopic system. For example, thermodynamic parameters such as the temperature of an ideal gas can be traced back to the average energy which is distributed among the various degrees of freedom of the individual gas molecules.

Within the economics domain, in particular tax evasion, the approach by Allingham and Sandmo (1972) would correspond to the phenomenological level of description. On the other hand so-called agent-based models have been set up as a comparatively new tool for analyzing tax compliance issues on a more individual (i.e. 'microscopic') level. An essential feature of any agent-based model is the direct interaction between agents, which is combined with some process that allows for changes in individual behavior patterns.

According to Hokamp and Pickhardt (2010) and Pickhardt and Seibold (2014) agent-based tax evasion models may be categorized into an economics and econophysics domain. The latter has been initiated by Lima and Zaklan (2008); Zaklan *et al.* (2008, 2009) and Lima (2010) by analyzing a suitable modification of the Ising model (Ising, 1925), originally known from the theory of magnetism. In contrast, if the interaction process is driven by parameter changes that induce behavioral changes via utility functions and/or by stochastic processes that do not have physical roots, these models belong to the economics domain. Examples include Mittone and Patelli (2000); Davis *et al.* (2003); Bloomquist (2004, 2008); Antunes *et al.* (2007); Korobow *et al.* (2007); Szabó *et al.* (2009, 2010); Hokamp and Pickhardt (2010); Méder *et al.* (2012); Nordblom and Žamac (2012); Andrei *et al.* (2014); Hokamp (2014); Pellizzari and Rizzi (2014) of which some are summarized by Bloomquist (2006), Pickhardt and Prinz (2014), and Pickhardt and Seibold (2014).

In previous work Seibold and Pickhardt (2013); Pickhardt and Seibold (2014) and more recently Hokamp and Seibold (2014) have extended the Ising-based econophysics approach to tax evasion toward the implementation of different agent types. This theory is able to reproduce results from agent-based economics models (Hokamp and Pickhardt, 2010) and therefore should be appropriate for a quantitative analysis of tax compliance. Moreover, the econophysics route to tax evasion may provide the formal framework to construct a phenomenological 'global' theory starting from a microscopic

'agent-based' description. However, before tackling this ambitious task it is of course necessary to provide an econophysics description of tax evasion which comprises the main ingredients inherent in contemporary neoclassical approaches based on the work by Allingham and Sandmo (1972).

In this regard, one crucial aspect which we aim to improve in the present paper concerns the time evolution of social norms within the network of agents. In fact, all previous econophysics works mentioned above classified the individual agents by two parameters, (i) a local field describing the moral attitude of agents toward tax evasion and (ii) a local temperature which governs the susceptibility of agents with regard to behavioral changes. Both parameters compete with the interaction between agents which aims to conform the agent's behavior within their social network.

However, these parameters were fixed and therefore behavioral changes only occured as a result of the statistical evaluation of the dynamics incorporating the aforementioned competition between interaction of agents and their moral attitude and/or local temperature, respectively.

Here we aim to go beyond this static description by incorporating social norm updating into the agent-based econophysics approach to tax evasion. In particular, we allow for dynamical changes in the moral attitude of agents due to public goods provision. Within standard agent-based models this issue has been studied by Mittone and Patelli (2000); Antunes *et al.* (2007); Szabó *et al.* (2009, 2010); Méder *et al.* (2012); Hokamp (2014); Pellizzari and Rizzi (2014) with partially contradictory results. For example, Szabó *et al.* (2009, 2010) showed that increasing the level of governmental services (e.g. health care) leads to less tax evasion. Note that the authors consider more than 20 employment types and four agent types, namely (i) elected administration, (ii) tax authority, (iii) workers, and (iv) entrepreneurs. In contrast, Hokamp (2014) finds the counterintuitive result that income tax compliance may decrease with raising marginal per capita returns. The latter analysis is based on a model with back auditing and four agent types which are also implemented in the present paper (cf. Sec. 3).

The paper is organized as follows. In order to put our investigations into the appropriate economic context we present in Sec. 2 a brief literature review regarding public goods provision and tax compliance. In Sec. 3 we outline the basic ingredients of our econophysics model [cf. Seibold and Pickhardt (2013); Hokamp and Seibold (2014); Pickhardt and Seibold (2014)] and exemplify the approach for a society with homogeneous agents which sets the stage for the following discussion. The procedure how the provision of public goods influences on selfish agents within our model is presented in Sec. 4 and corresponding results are shown in Sec. 5. We finally conclude our discussion in Sec. 6.

2 Public goods provision and tax compliance: a literature review

The modern root of public goods theory dates back to the seminal work of Samuelson (1954) introducing a condition for Pareto-optimal allocations: the sum of individual's marginal rate of substitution has to equal the marginal rate of transformation. Pickhardt (2003, 2006) surveys modern public goods theory and the relevant literature therein, e.g. Musgrave (1939); Samuelson (1954, 1955); Musgrave (1999); Samuelson and Nordhaus (1998). Among other things, he observes a large diversity in defining and using the terminus 'public good' that is to some extent conflicting or even contradicting. In our paper we adopt the consumption-act-approach of Pickhardt (2003), i.e. properties of goods are totally atomized into acts of consumption at any point in time and then 'rival' and 'non-rival' consumption acts are used to determine private and public goods characteristics, respectively. Note that the terminus 'non-rival in consumption' was introduced to the literature by Musgrave (1969, pg. 126) meaning '[. . .] that the same physical output (the fruits of the same factor input) is enjoyed by both [individuals,] A and B. This does not mean that the same subjective benefit must be derived, or even that precisely the same product quality is available to both.' However, with respect to income tax evasion and public goods provision we briefly review below theoretical works of Cowell and Gordon (1988); Falkinger (1988, 1991, 1995); Cowell (1992) and Bordignon (1993), as well as experiments by Alm *et al.* (1992a,b); Mittone (2006); Alm (2010) and Bazart and Pickhardt (2011). Note that the literature review orginates from Hokamp (2013).

Cowell and Gordon (1988) postulate a two-dimensional continuum of goods; on the one hand bounded by rivalness and non-rivalness and on the other hand limited by 'excludability' and 'non-excludability'. Note that in the course of our paper excludability in consumption is assumed if and only if options are feasible and enforceable to ban individuals from consumption of these goods. Cowell and Gordon (1988) examine the influence of public goods provision on individual's and average population's tax evasion behavior. The authors investigate (i) large populations, where a single individual decision to commit tax evasion has no observable impact on public goods provision and (ii) decreasing absolute risk aversion when increasing income. In addition, they assume under-provision (over-provision) of public goods to deduce their counterintuitive key findings: raising a flat tax ceteris paribus increases (decreases) tax evasion.

Falkinger (1988) adds the notion of tax as a price for public services to the basic flat tax setting of Allingham and Sandmo (1972). Note that these governmental provided public goods are non-rival – i.e. each agent may consume the same amount – but individual's utility of consumption may differ. Falkinger (1988) then examines equity of government-taxpayer relationships under presence of tax non-compliance and public goods provision. He finds that tax evasion ceteris paribus is reduced if four assumptions are simultaneously fulfilled: (i) public goods provision only depends

on expected tax revenues, (ii) behaviors of other agents do not change (the 'Nash-assumption', cf. ibid., pg. 390), (iii) tax enforcement variables have to allow rational for a positive tax evasion, and (iv) an additive utility function prevails. Eventually, his main insight is '[. . .] that the equity argument is [rather] an ex post rationalization of otherwise motivated tax evasion' than an ex ante driving force (see Falkinger, 1988, pp. 392-393).

Falkinger (1991) addresses a reverse question related to Cowell and Gordon (1988), that is: How might tax evasion influence the optimal level of public goods provision? Using additive and strictly concave utility functions Falkinger (1991) shows that the impact of flat tax evasion on the optimal supply of public goods depends on the magnitude of decreasing individual's absolute risk aversion. To analyze inequity Cowell (1992) enlarges the publicly-supplied-goods-approach of Cowell and Gordon (1988) while maintaining their notion of public goods (cf. above). As pointed out by Cowell (1992, pg. 541), the essential problem of economic, psychological and social effects – which simultaneously influence individual's flat tax evasion behavior and public goods provision – concentrates to one single crucial question: 'Can the person affect inequity directly by his own actions?'.

Bordignon (1993) extends the setting of Cowell and Gordon (1988) via introducing taxpayers' heterogeneity that refers to income, magnitude of private consumption and audit probability. In particular, Bordignon (1993, pg. 359) examines '[. . .] the fairness of the fiscal system [. . .]' depending on tax structure, public goods provision and subjects' beliefs or individuals' estimates of other peoples' tax evasion. He then finds that poor taxpayers ceteris paribus evade more (less) in absolute terms than rich subjects, if the whole population is faced with low (high) tax rates. To allow for a comparison with Falkinger (1988) and Cowell (1992) the author drops the assumption of agents' heterogeneity and derives Pareto-optimal provision levels of public goods. Then – within his homogeneous fairness-approach assuming under-provision (over-provision) of public goods – Bordignon (1993) shows that raising the tax rate leads to zero (positive) tax evasion; where positive tax evasion ceteris paribus increases (decreases) with low (high) tax rates. Falkinger (1995) takes an economic and a psychological point of view on flat tax evasion and equity that both base on Falkinger (1988, 1991) and Cowell (1992). Yet, Falkinger (1995) explains counterintuitive results of Cowell (1992) through a decline of absolute risk aversion while increasing equity.

Alm *et al.* (1992a) present a tax compliance experiment and find that providing a surplus from tax payments – i.e. a public good – may work to increase tax compliance. Alm *et al.* (1992b) provide an experimental contribution to taxpayer's mystery, that is, 'Why do people pay taxes?' Regarding this voluntary contribution puzzle Alm *et al.* (1992b) argue that (i) taxpayers may value public goods provision, (ii) individuals probably err on the real extent of tax enforcement variables – in particular, they might overweight audit probabilities – and (iii) subjects may hold an extreme risk aversion. Further, the authors obtain that flat tax evasion even occurs if expected returns of a tax gamble are negative. Mittone (2006) conducts a dynamic tax evasion

experiment under risk and uncertainty. Among other things, the author finds that '[...] the production of a public good reduces tax evasion but to a lesser extent than tax yield distribution' (Mittone, 2006, pg. 830). Alm *et al.* (1992b) conclude that new theories are quite necessary to explain taxpayers' responses frequently observed in laboratory experiments with respect to taxes, evasion, public goods provision, and the like. An interdisciplinary approach, e.g. behavioral economics, may fill in this gap. See economist (and chemist) Alm (2010, pg. 636) for a history of the terminus 'behavioral economics', meaning '[...] an approach that uses methods and evidence from other social sciences (especially psychology) to inform the analysis of individual and group decision making'. For a psychologist's point of view see Kirchler (2007). Furtheron Alm (2010) briefly surveys behavioral (public) economics; an interdisciplinary research field that deviates from several standard neoclassical assumptions, for instance, subjects may not always act rational and selfish. In particular, the author distinguishes three experimental branches that examine (i) 'public goods', (ii) 'tax compliance' and (iii) 'behavioral responses to taxes' (cf. ibid., pg. 635). Furthermore, Alm (2010, pg. 650) presents a brief overview of open tasks and promising ideas for future experiments in the domain of behavioral public economics, which concern (i) 'intertemporal decisions', (ii) 'social insurance programs' and (iii) 'behavior under uncertainty'. Finally, note that the author quotes the agent-based approach of Bloomquist (2011) to illustrate a fruitful combination of laboratory experiments to computational simulations for investigating the evolution of tax compliance dynamics over time.

Bazart and Pickhardt (2011) conduct a flat tax compliance experiment that examines the relationship of tax evasion and positive incentives under public goods provision and lottery effects. Note that the seminal work of Falkinger and Walter(1991) has added welfare improving positive rewards for audited individuals to the standard model of tax evasion, that is Allingham and Sandmo (1972); i.e. taxpayers get a monetary payback for their voluntarily paid taxes regardless of their actual extent of tax cheating. On contrary, Bazart and Pickhardt (2011) employ non-rival public goods and install an excludable flat tax gamble, where only fully compliant taxpayers can get the extra profit. In each round of the laboratory experiment the public good as well as the permissible extra gain is financed by voluntary contributions – i.e. taxes – forced reimbursements and paid penalties, so that each individual influences marginally the extent of public goods provision. Bazart and Pickhardt (2011) find experimental evidence for gender effects: (i) females respond stronger to an increase of penalties than males, (ii) females evade less than males, and (iii) males react stronger than females to positive lottery rewards – ceteris paribus yielding higher tax compliance rates.

Finally, to sum up reviewed neoclassical theoretical and experimental applications present a wide range of conclusions, effects, findings and results that may appear to some extent ambiguous, counterintuitive, or actually conflicting and contradicting with respect to income tax evasion, public goods provision and their inter

dependencies. Analogous discrepancies also appear in agent-based models as already outlined in Sec. 1. In the next section we proceed by introducing our econophysics agent-based model for which we implement the influence of public goods in Sec. 4.

3   The agent-based econophysics approach

Our considerations are based on the Ising model hamiltonian

$$H = -J \sum_{\langle ij \rangle} S_i S_j - \sum_i B_i S_i \tag{1}$$

where J describes the coupling of Ising variables (spins) $S_i = +1, -1$ between adjacent lattice sites denoted by $\langle ij \rangle$. In the present context $S_i = +1(-1)$ is interpreted as a compliant (non-compliant) taxpayer. Calculations have been done for a two-dimensional square lattice with dimension $1,000 \times 1,000$ and periodic boundary conditions, i.e. a torus structure. We note that the results are not sensitive to the specific lattice geometry. Within a similar econophysics model this issue has been analyzed by Lima and Zaklan (2008) and Zaklan *et al.* (2009, 2008) who consider, in addition, alternative lattice structures like the scale-free Barabási-Albert network or the Voronoi-Delaunay network. In addition Lima (2010) considers Erdös-Rényi random graphs and finds that the results for these alternative lattices do not differ significantly from those obtained with a square lattice.

Eq. (1) contains also the coupling of the spins to a local magnetic field B_i which can be associated with the morale attitude of the agents and corresponds to the parameter γ_i in the theory of Nordblom and Žamac (2012) which have investigated social norm updating within an agent-based model. In addition, our model contains a local temperature T_i which measures the susceptibility of agents to external perturbations (either influence of neighbors or magnetic field). We then use the heat-bath algorithm [cf. Krauth (2006)] in order to evaluate statistical averages of the model. The corresponding program code is written in FORTRAN and is available upon request. The probability for a spin at lattice site i to take the values $S_i = \pm 1$ is given by

$$p_i(S_i) = \frac{1}{1 + \exp\{-[E(-S_i) - E(S_i)]/T_i\}} \tag{2}$$

and $E(-S_i) - E(S_i)$ is the energy change for a spin-flip at site i. Upon picking a random number $0 \leq r \leq 1$ the spin takes the value $S_i = 1$ when $r < p_i(S_i = 1)$ and $S_i = -1$ otherwise. One time step then corresponds to a complete sweep through the lattice.

Following Davis *et al.* (2003) and Hokamp and Pickhardt (2010) we consider societies which are composed of the following four types of agents: (i) *selfish a-type agents*, which take advantage from non-compliance ($S_i = -1$) and, thus, are characterized by $B_i/T_i < 0$ and $|B_i| > J$; (ii) *copying b-type agents*, which conform to the norm of

their social network and thus copy the behavior with respect to tax evasion from their neighborhood. This can be modeled by $B_i << J$ and $J_i/T_i \gtrsim 1$; (iii) *ethical c-type agents*, which are practically always compliant ($S_i = +1$) and thus are parametrized by $B_i/T_i > 0$ and $|B_i| > J$; (iv) *random d-type agents*, which act by chance, within a certain range, due to some confusion caused by tax law complexity. We implement this behavior by $B_i << J$ and $J/T_i << 1$. The parameters distinguishing the different agent types are taken from Seibold and Pickhardt (2013) and Pickhardt and Seibold (2014).

In analogy to Zaklan *et al.* (2008, 2009); Lima (2010) and Pickhardt and Seibold (2014) we further implement the probability p_a of an audit. If tax evasion is detected the agent is forced to stay compliant over h time steps (penalty period). Such a procedure has also been implemented in a randomized variant in Lima and Zaklan (2008). We also note that it is possible to incorporate lapse of time effects, i.e. the situation where a detected agent is also screened over several years in the past by the tax authorities (i.e. backaudit). This variant has been studied within an agent-based econophysics tax compliance model in Seibold and Pickhardt (2013).
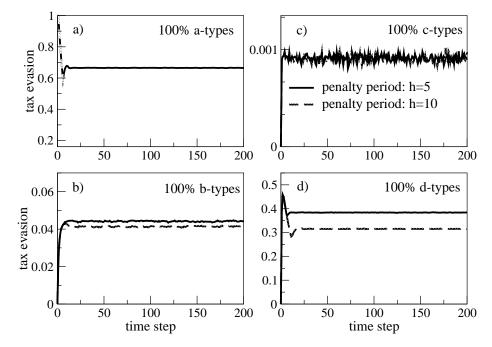
Since in the following we implement the influence of public goods provision which affects selfish a-types only, it is instructive to consider the case of a society with all agents being of the same type. The resulting percentage of tax evasion as a function of time is shown in Fig. 1 for two penalty periods ($h = 5, 10$).

The first case of endogenous non-compliant selfish agents is shown in Fig. 1a where at time step zero all agents are set to non-compliance, i.e. the fraction of non-compliant taxpayers is $p_{non-cp} = 1$. Due to the enforcement mechanism, p_{non-cp} is significantly reduced because at each time step a certain percentage of the remaining non-compliant agents are forced to become compliant. Before reaching a stationary value small oscillations are observed since after h time steps the first detected agents can become non-compliant again. Fig. 1b reports the result for copying b-type agents. As initial condition all agents are set to 'compliant'. Since b-types tend to copy the behavior of their social network only few of them change their behavior and the equilibrium value for tax evasion approaches a rather small value between 4% and 5%. It should be noted that the equilibrium value is independent of the initial condition. If we would have set all agents to non-compliant at time step zero, the audits would have reduced the percentage of tax evasion to the same equilibrium value than shown in Fig. 1b.

The time evolution for ethical c-type agents is reported in Fig. 1c. Here the initial fraction of non-compliant taxpayers is set to $p_{non-cp} = 0$ and there is only a very small probability that one of the agents becomes non-compliant. Since ethical agents avoid tax evasion the results are also almost independent of the audit probability. Hence, any positive audit probability would be inefficient in this case.

Finally, Fig. 1d shows the percentage of tax evasion for random d-type agents. Since these agents act by chance their probability for changing their behavior from compliant to non-compliant or vice versa would be of the order of $\sim 50\%$ without any audit. The enforcement mechanism then leads to a further reduction for the

equilibrium value of $p_{non-cp} = 0$ where larger penalty periods naturally lead to a stronger reduction.

Figure 1: Time evolution of tax evasion corresponding to the fraction of non-compliant taxpayers p_{non-cp}. The panels report results for a society consisting of 100% a-type (panel a), 100% b-type (panel b), 100% c-type (panel c), and 100% d-type (panel d) agents. Solid lines display the tax evasion dynamics for penalty period $h = 5$ whereas dashed lines are for penalty period $h = 10$. Note that in panel c) both cases are not distinguishable. Audit probability is $p_a = 10\%$ in each case.

4   The influence of public goods provision

In our previous considerations the local fields for the ethical (c-type) and selfish (a-type) agents where initially fixed and independent of the time evolution. For ethical agents this means a deep seated moral attitude inculcated by education, religious convictions [cf. Heinemann and Schneider (2011)] etc. However, selfish agents are expected to maximize their personal benefit by deliberating about whether their profit from public goods is supported, independent or hindered by the tendency of tax evasion. We therefore set up the following instructions for selfish a-types:

a) Compare the provision of public goods between the current and previous time step. Within our approach this provision is proportional to the fraction of compliant tax payers p_{cp} (with $p_{cp} + p_{non-cp} = 1$) and we therefore evaluate $\Delta p_{cp} = p_{cp}(t_n) - p_{cp}(t_{n-1})$. Moreover, only a substantial improvement or decline $|\Delta p_{cp}| > \Delta p_{min}$ in the provision can be perceived by an individual, where the threshold of perception Δp_{min} enters as an additional parameter.

b) Each a-type agent compares its behavior between the current and previous time step, i.e. $\Delta S_i^a = S_i^a(t_n) - S_i^a(t_{n-1})$ where i denotes the lattice site. The three possible values $\Delta S_i^a = -2, 0, 2$ indicate a change from compliance to non-compliance, no behavioral change and a change from non-compliance to compliance, respectively.

c) Change the attitude of a-types towards non-compliance (i.e. the local field B_i of a-types) by a step $\Delta B > 0$ depending on Δp_{cp} and ΔS_i^a.

Instruction (c) corresponds to the following possibilities which are in line with the analysis of Falkinger (1995) and summarized in Tab. 1.

The new parameters entering our approach are thus (i) Δp_{min} (threshold of perception) reflecting the lower bound in the provision of public goods which can be perceived by an agent, and (ii) the rate of adaption ΔB with which the a-type agent adjusts its behavior to a change in the provision of public goods. In our simulations we specify Δp_{min} by a certain percentage of compliant agents to which it is obviously proportional. The parameter ΔB is chosen for each a-type agent i as a random number $0 < \Delta B_i < \Delta B_{max}$.

Table 2 summarizes the parameters of our model.

5   Results

In Fig. 2 we investigate the influence of public goods provision on a society consisting of 100% selfish a-type agents for various rates of adaption ΔB_{max} and two thresholds of perception $\Delta p_{min} = 1\%$ (upper panel) and $\Delta p_{min} = 5\%$ (lower panel). This feedback influences on the local field distribution $P(B_i)$ which is now determined self-consistently as described above. At time step $t_n = 0$ selfish a-types are given a flat distribution of local fields confined to the range $-10 < B_i < -20$ and which is shown by the black curve in the main panels of Fig. 2. Note that this is a generalization of Lima and Zaklan (2008) which have used a single field value. However, the feedback of public goods leads to a redistribution of $P(B_i)$ which for large times becomes stationary and for different ΔB_{max} and Δp_{min} is shown in the main panel of Fig. 2. Interestingly one observes two regimes in the distribution of local fields. For small ΔB_{max} the distribution transforms into a Gaussian and shifts to larger mean values B_{mean} which are still in the range $B_{mean} < 0$, i.e. the majority of agents is still characterized as non-compliant taxpayers. From the insets to Fig. 2 we observe that

Table 1: Summary of how changes in public goods provision and behavioral change of a-types influence on the (im)moral attitude change of these agents

Change of Public Goods Provision	Change of a-type Behavior	Resulting Moral Attitude Change
$|\Delta p_{cp}| < \Delta p_{min}$	$\Delta S_i^a = -2, 0, 2$	0
$|\Delta p_{cp}| > \Delta p_{min}$	$\Delta S_i^a = 0$	0
$\Delta p_{cp} > \Delta p_{min}$	$\Delta S_i^a = 2$	$+\Delta B_i$
$\Delta p_{cp} < -\Delta p_{min}$	$\Delta S_i^a = 2$	$-\Delta B_i$
$\Delta p_{cp} > \Delta p_{min}$	$\Delta S_i^a = -2$	$-\Delta B_i$
$\Delta p_{cp} < -\Delta p_{min}$	$\Delta S_i^a = -2$	$+\Delta B_i$

$|\Delta p_{cp}| < \Delta p_{min}$, $\Delta S_i^a = -2, 0, 2$: The provision of public goods does not change significantly, i.e. it is below the threshold of perception Δp_{min} of selfish agents. The agent therefore keeps its moral attitude and the field remains unaltered.

$|\Delta p_{cp}| > \Delta p_{min}$, $\Delta S_i^a = 0$: The provision of public goods has changed significantly (i.e. it is above the threshold of perception Δp_{min} of selfish agents) while the agent did not change its behavior. It is not perceivable for the rational agent whether a change of its behavioral norm would lead to any advantage. Therefore also in this case the agent tends to keep its moral attitude and the field remains unaltered.

$\Delta p_{cp} > \Delta p_{min}$, $\Delta S_i^a = 2$: The agent has changed its behavior from non-compliant to compliant. Simultaneously the provision of public goods has significantly increased. The agent therefore recognizes a confirmation for its behavioral change and thus becomes less 'unethical'. This is implemented by a change of its local field $B_i \to B_i + \Delta B$ which thus becomes less negative.

$\Delta p_{cp} < -\Delta p_{min}$, $\Delta S_i^a = 2$: The agent has changed its behavior from non-compliant to compliant but the provision of public goods has significantly decreased. The agent therefore concludes that its behavioral change is not honored and thus becomes more 'unethical'. The corresponding change in the local field is $B_i \to B_i - \Delta B$.

$\Delta p_{cp} > \Delta p_{min}$, $\Delta S_i^a = -2$: The agent has changed its behavior from compliant to non-compliant while the provision of public goods has significantly increased. The agent is therefore encouraged in its unethical behavior which is implemented by a change in the local field $B_i \to B_i - \Delta B$.

$\Delta p_{cp} < -\Delta p_{min}$, $\Delta S_i^a = -2$: The agent correlates its change from compliant to non-compliant behavior with the decrease of the provision of public goods suggesting a more ethical behavior. Therefore the local field changes as $B_i \to B_i + \Delta B$.

the dynamics in this case is similar to the one without feedback of public goods. Due to the shift of $P(B)$ to larger mean values a slight reduction in the stationary value of tax evasion becomes visible, especially for larger thresholds of perception.

Above a critical value of the adaption rate ($\Delta B_{max}^{crit} \approx 2.5$ for $\Delta p_{min} = 1$ and $\Delta B_{max}^{crit} \approx 4.5$ for $\Delta p_{min} = 5\%$) one observes a broad distribution with a mean value which is now deep in the positive B-regime, i.e. most of the selfish a-type agents show a dominant compliant behavior. As a consequence the stationary value of tax evasion (insets to Fig. 2) is now strongly reduced and the remaining tax evaders are solely due to the 'negative B' agents in the tail of $P(B)$.

We emphasize that the distributions shown in the main panels of Fig. 2 are stationary distributions, i.e. obtained after $t = 200$ time steps. We have checked that the same distributions are obtained after $t = 5,000$ time steps in both regimes. Clearly, the crossover between the two regimes is due to a self-sustaining effect when the agents

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Table 2: Compendium of parameters determining the present multi-agent model. Note that temperature and magnetic field are measured in units of the exchange coupling $J \equiv 1$
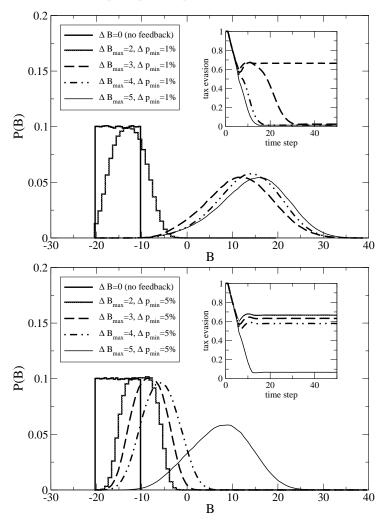
Parameter	Interpretation	Value
Temperature T	Susceptibility to External Perturbations	$T = 5$ for a- and c-types. $1 < T < 3$ (b-types); $10 < T < 30$ (d-types)
Magnetic Field B	(Moral) Attitude towards Tax Evasion	$B = 0$ (b-, d-types); $10 < B < 20$ (c-types) B is self-consistently determined for a-types.
Audit Probability p_a		$p_a = 0.1$
Penalty Period h		$h = 5, 10$ time steps
ΔB_{max}	Maximum Rate of Adaption to Changes in Public Goods Provision	$\Delta B_{max} = 1 \dots 5$
Δp_{min}	Threshold of Perception for Changes in Public Goods Provision	$\Delta p_{min} = 1\%, 5\%$

start to become compliant and thus enhance the provision of public goods, combined with the audit which enforces the detected agent to change its behavior from $S_i = -1$ to $S_i = +1$ over the following h periods. According to the mechanism summarized in the third line of Tab. 1 a regime where the provision of public goods has significantly increased, induces a behavioral change of non-compliant agents upon auditing. This in turn enhances the provision of public goods and is responsible for the self-sustaining effect leading to the crossover between the two regimes.

An interesting (but also pathological) situation may arise when the distribution of local fields is centered around $B = 0$. In this case there may appear long term oscillations between both regimes, i.e. between substantial and small fractions of tax evaders. For the present parameters such situation occurs for threshold value $\Delta p_{min} = 0$ and a critical adaption rate of $\Delta B_{max} = 1$. For values $\Delta B_{max} > 1$ the resulting distributions are similar to the case $\Delta p_{min} = 1\%$ but shifted to larger (positive) mean values.

How can the above results generalized within a multi-agent model? To answer this question we calculate the fraction of tax evasion for a society consisting of 35% b-types, 15% d-types and the residual share of a- and c-types is varied. Note that the percentage of 15% d-types can be motivated from estimates for U.S. households taken from Andreoni *et al.* (1998). Similar agent shares have also been investigated by Hokamp and Pickhardt (2010) and Pickhardt and Seibold (2014). We choose an adaption rate of $\Delta B_{max} = 4$ so that for small threshold of perception $p_{min} = 1\%$ (panel a) the majority of a-type agents undergoes a transition towards compliant behavior (cf. Fig. 2). As a consequence the fraction of evaders for societies consisting of 50%, 40%, and 30% a-types is drastically reduced as is apparent from Fig. 3a. This

Figure 2: Main panel: The distribution of local fields for various rates of adaption ΔB_{max}. The case $\Delta B_{max} = 0$ corresponds to the case with no feedback of public goods which is identical to the distribution used in Fig. 1a. Results are reported for a societcy consisting of 100% a-types, enforcement with fixed compliance period $h = 5$ and audit probability $p_a = 10\%$. The initial distribution of local fields is confined to the range $-20 < B_i < -10$. Inset: Time evolution of tax evasion (fraction of non-compliant agents p_{non-cp}). Top panel: Threshold of perception $\Delta p_{min} = 1\%$, Lower panel: Threshold of perception $\Delta p_{min} = 5\%$

reduction becomes less pronounced for smaller a-type shares (e.g. 20% a-types). In fact, the transition of a-types from non-compliance to compliance observed in Fig. 2 is driven by a significant change (enhancement) in the public goods provision. Now a large share of c-types obviously constitutes the fraction of compliant agents and therefore to the public goods provision but this contribution is constant over time. On the other hand, the behavioral change of a-types is induced by a concomitant change in the public goods provision and this change is naturally reduced when the fraction of c-types becomes to large. Therefore we observe only moderate to small reduction in tax evasion for a-type shares of 20%, 10%, and 0%.
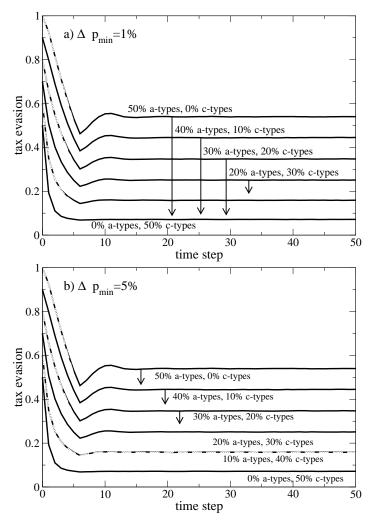
Fig. 3b reports analogous results for a larger threshold of perception $p_{min} = 5\%$. From the previous results shown in Fig. 2 we know that for this parameter the mean value of the field distribution for a-types is still in the negative range so that non-compliance is predominant. As a consequence also the effective share of non-compliant agents for the composed society is only moderately reduced upon allowing for the feedback from public goods provision.

6   Discussion and outlook

We have investigated the influence of public goods provision on tax evasion within an econophysics model which describes the interaction of various behavioral types of agents. Within our framework only selfish (a-types) agents are susceptible to the provision of public goods and we have shown that their behavioral attitude towards tax evasion is determined by a dynamic local 'field-parameter' B_i. As a result we find that an initially flat distribution $P(B_i)$ is altered self-consistently due to the feedback of public goods provision leading to a generic shift towards larger field value (increased 'morality' of agents). This in turn can reduce tax evasion in agreement with previous investigations within the economics domain Szabó *et al.* (2010). Also experimental data in the context of tax evasion Alm *et al.* (1992a,b) support the finding that compliance increases when taxpayers receive a public good in return for their payment. On the other hand Hokamp (2014) implements public goods provision via an utility function which is the crucial factor for the counterintuitive result that income tax evasion may increase providing a higher level of public goods. With regard to our present approach the different result can be traced back to a different behavior upon the increase of public goods provision (cf. third line of Tab. 1) where the selfish agent in Hokamp (2014) does not increase its moral attitude.

Two parameters govern the feedback of public goods provision to the behavioral change of selfish agents: (i) the rate of adaption ΔB_{max} and (ii) the threshold of perception Δp_{min} towards changes in public goods provision. ΔB_{max} is an endogenous parameter of the agents which may depend on age but otherwise is not susceptible to external influence. On the other hand, policymakers can to a certain extend influence on the threshold of perception Δp_{min} by means of targeted public relations. In practice this may be achieved by announcements via public media also

Figure 3: Time evolution of tax evasion (fraction of non-compliant agents p_{non-cp}) for a society consisting of 35% b-types and 15% d-types. The fraction of a- and c-types is indicated in the panels. Solid lines report the result without and dashed lines the result including the feedback of public goods provision. Results are obtained for enforcement with fixed compliance period $h = 5$ and audit probability $p_a = 10\%$. The initial distribution of local fields is confined to the range $-20 < B_i < -10$ and the rate of adaption is $\Delta B_{max} = 4$. Panel (a): Threshold of perception $\Delta p_{min} = 1\%$, Panel (b): Threshold of perception $\Delta p_{min} = 5\%$

about minor state projects.

Since often public goods provision is more important for the elder generation (as e.g. in case of health care) it may also partially account for their stronger tax morale as found by Nordblom and Žamac (2012). However, in our econophysics model we have assumed a constant (over time) distribution of agent types. On the other hand, Nordblom and Žamac (2012) have set up an economic model which describes how social beings update their personal norms. Implementing these mechanisms in our model would allow for the transformation between different agent types and is an interesting perspective for future research.

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References

[1] Allingham, M. G. and A. Sandmo (1972). Income Tax Evasion: A Theoretical Analysis. *Journal of Public Economics* **1**, 323–338.

[2] Alm, J., B. Jackson and M. McKee (1992a). Estimating the Determinants of Taxpayer Compliance with Experimental Data. *National Tax Journal* **45**(1), 107 – 114.

[3] Alm, J., G. H. McClelland and W. D. Schulze (1992b). Why do people pay taxes? *Journal of Public Economics* **48**, 21 – 38.

[4] Alm, J. (2010). Testing Behavioral Public Economic Theories in the Laboratory. *National Tax Journal* **63** (4), 635 – 658.

[5] Andrei, A., K. Comer and M. Koehler (2014). An agent-based model of network effects on tax compliance and evasion. *Journal of Economic Psychology* **40**, 119 – 133.

[6] Andreoni, J., B. Erard and J. Feinstein (1998). Tax Compliance. *Journal of Economic Literature* **36**(2), 818–860.

[7] Antunes, L., J. Balsa, A. Respicio and H. Coelho (2007). Tactical Exploration of Tax Compliance Decisions in Multi-agent Based Simulation, in: Luis Antunes

and Keiki Takadama (Eds.), *Multi-Agent-Based Simulation VII*, International Workshop MABS 2006, LNAI 4442, Heidelberg: Springer, 80–95.

[8] Bazart, C. and M. Pickhardt (2011). Fighting Income Tax Evasion with Positive Rewards. *Public Finance Review* **39**(1), 124–149.

[9] Bloomquist, K. M. (2004). Modeling Taxpayers' Response to Compliance Improvement Alternatives. Paper presented at the Annual Conference of the North American Association for Computational Social and Organizational Sciences, Pittsburgh, PA.

[10] Bloomquist, K. M. (2006). A Comparison of Agent-based Models of Income Tax Evasion. *Social Science Computer Review* **24**(4), 411–425.

[11] Bloomquist, K. M. (2008). Taxpayer Compliance Simulation: A Multi-Agent Based Approach, In: Edmonds, B., Hernández, C. and Troitzsch, K. G. (eds.) *Social Simulation: Technologies, Advances and New Discoveries.* Premier References Series, Chap. 2, 13–25.

[12] Bloomquist, K. M. (2011). Tax Compliance as an Evolutionary Coordination Game: An Agentbased Approach. *Public Finance Review* **39** (1), 25 – 49.

[13] Bordignon, M. (1993). A Fairness Approach to Income Tax Evasion. *Journal of Public Economics* **52**, 345 – 362.

[14] Cowell, F. A. and J. P. F. Gordon(1988). Unwillingness to Pay - Tax Evasion and Public Goods Provision. *Journal of Public Economics* **36**, 305 – 321.

[15] Cowell, F. A. (1992). Tax Evasion and Inequity. *Journal of Economic Psychology* **13**, 521 – 543.

[16] Davis, J. S., G. Hecht and J. D. Perkins (2003). Social Behaviors, Enforcement, and Tax Compliance Dynamics. *The Accounting Review* **78**(1), 39–69.

[17] Falkinger, J. (1988) Tax Evasion and Equity: A Theoretical Analysis. *Public Finance/Finances Publiques* **43**, 388 – 395.

[18] Falkinger, J. (1991) On Optimal Public Good Provision with Tax Evasion. *Journal of Public Economics* **45**, 127 – 133.

[19] Falkinger, J. and H. Walther (1991). Rewards versus Penalties: On a New Policy against Tax Evasion. *Public Finance Quarterly* **19**, 67 – 79.

[20] Falkinger, J. (1995). Tax Evasion, Consumption of Public Goods and Fairness. *Journal of Economic Psychology* **16**, 63–72.

[21] Heinemann, F. and F. Schneider (2011). Religion and the Shadow Economy. *ZEW Discussion Papers* **11-038**.

[22] Hokamp, S. and M. Pickhardt (2010). Income Tax Evasion in a Society of Heterogeneous Agents - Evidence from anAgent-based Model. *International Economic Journal* **24**(4), 541–553.

[23] Hokamp, S. (2013). Income Tax Evasion and Public Goods Provision – Theoretical Aspects and Agent-based Simulations *Brandenburg University of Technology Cottbus, Ph.D. Thesis.*

[24] Hokamp, S. (2014). Dynamics of tax evasion with back auditing, social norm updating and public goods provision - An agent-based simulation. *Journal of Economic Psychology* **40**, 187 – 199.

[25] Hokamp, S. and G. Seibold (2014). How much Rationality tolerates the Shadow Economy? - An Agent-based Econophysics Approach. *Advances in Social Simulation; Conference Series: Advances in Intelligent Systems and Computing* **229**, 119–128.

[26] Ising, E. (1925). Beitrag zur Theorie des Ferromagnetismus. *Zeitschrift für Physik* **31**(1), 253–258.

[27] Kirchler, E. (2007). The Economic Psychology of Tax Behavior. *Cambridge University Press, Cambridge.*

[28] Korobow, A., C. Johnson and R. Axtell (2007). An Agent-based Model of Tax Compliance with Social Networks. *National Tax Journal* **60**(3), 589-610.

[29] Krauth, W. (2006). *Statistical Mechanics; Algorithms and Computations.* Oxford University Press.

[30] Lima, F. W. S. and G. Zaklan (2008). A Multi-agent-based Approach to Tax Morale. *International Journal of Modern Physics C: Computational Physics and Physical Computation* **19**(12), 1797–1808.

[31] Lima, F. W. S. (2010). Analysing and Controlling the Tax Evasion Dynamics via Majority-Vote Model. *Journal of Physics: Conference Series* **246**, 1–12.

[32] Méder, Z. Z., A. Simonovits and J. Vincze (2012). Tax Morale and Tax Evasion: Social Preferences and Bounded Rationality. *Economic Analysis and Policy* **42**(2), 171 –188.

[33] Mittone, L. and P. Patelli (2000). Imitative Behaviour in Tax Evasion, in: Francisco Luna and Benedikt Stefansson (eds.), *Economic Simulations in Swarm: Agent-based Modelling and Object Oriented Programming*, Dordrecht, Boston, London: Kluwer Academic Publishers, 133–158.

[34] Mittone, L. (2006). Dynamic Behaviour in Tax Evasion: An Experimental Approach. *The Journal of Socio-Economics* **35**, 813 – 835.

[35] Musgrave, R. A. (1939). The Voluntary Exchange Theory of Public Economy. *The Quarterly Journal of Economics* **53**, 213 – 237.

[36] Musgrave, R. A. (1969). Provision for Social Goods. *Public Economics, McMillan, London*, 124 – 144.

[37] Musgrave, R. A. (1999). The Nature of the Fiscal State: The Roots of My Thinking. *Public Finance and Public Choice, Cambridge: MIT Press*, 29 – 49.

[38] Nordblom, K. and J. Žamac (2012). Endogenous Norm Formation Over the Life Cycle - The Case of Tax Morale. *Economic Analyis & Policy* **42**(2), 153–170.

[39] Pellizzari, P. and D. Rizzi (2014). Citizenship and power in an agent-based model of tax compliance with public expenditure. *Journal of Economic Psychology* **40**, 35 – 48.

[40] Pickhardt, M. (2003). Studien zur Theorie öffentlicher Güter. *Metropolis, Marburg, Germany.*

[41] Pickhardt, M. (2006). Fifty Years after Samuelson's 'The Pure Theory of Public Expenditure': What Are We Left with? *Journal of the History of Economic Thought* **28** (4) 439 – 460.

[42] Pickhardt, M. and A. Prinz (2014). Behavioral Dynamics of Tax Evasion - A Survey. *Journal of Economic Psychology* **40**, 1 – 19.

[43] Pickhardt, M. and G. Seibold (2014). Income Tax Evasion Dynamcis: Evidence from an Agent-based Econophysics Model. *Journal of Economic Psychology* **40**, 147 – 160.

[44] Samuelson, P. A.(1954). The Pure Theory of Public Expenditure. *The Review of Economics and Statistics* **36**, 387 – 389.

[45] Samuelson, P. A.(1955). Diagrammatic Exposition of a Theory of Public Expenditure. *Review of Economics and Statistics* **37**, 350 – 356.

[46] Samuelson, P. A.(1969). Pure Theory of Public Expenditure. *Public Economics, McMillan, London*, 98 – 123.

[47] Samuelson, P. A. and W. D. Nordhaus (1998). Economics. *McGraw-Hill, New York, 16th edition.*

[48] Seibold, G. and M. Pickhardt (2013). Lapse of time effects on tax evasion in an agent-based Econophysics model. *Physica A* **392**(9), 2079–2087.

[49] Szabó, A., L. Gulyás and I. J. Tóth (2009). Sensitivity Analysis of a Tax Evasion Model Applying Automated Design of Experiments, in: Luis S. Lopes, Nuno Lau, Pedro Mariano, and Luis M. Rocha (Eds.), *Progress in Artificial Intelligence*, 14th Portuguese Conference on Artificial Intelligence, EPIA 2009, Aveiro, Portugal, LNAI 5816, Berlin Heidelberg: Springer, 572-583.

[50] Szabó, A. and L. Gulyás and I. J. Tóth (2010). Simulating Tax Evasion with Utilitarian Agents and Social Feedback. *International Journal of Agent Technologies and Systems* **2**(1), 16 – 30.

[51] Zaklan, G., F. W.S. Lima and F. Westerhoff (2008). Controlling Tax Evasion Fluctuations. *Physica A: Statistical Mechanics and its Applications* **387**(23), 5857–5861.

[52] Zaklan, G., F. Westerhoff and D. Stauffer (2009). Analysing Tax Evasion Dynamics via the Ising Model. *Journal of Economic Interaction and Coordination* **4**, 1–14.