

Bayesian Estimation and Prediction for ACD Models in the Analysis of Trade Durations from the Polish Stock Market

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Abstract

In recent years, autoregressive conditional duration models (ACD models) introduced by Engle and Russell in 1998 have become very popular in modelling of the durations between selected events of the transaction process (trade durations or price durations) and modelling of financial market microstructure effects. The aim of the paper is to develop Bayesian inference for the ACD models. Different specifications of ACD models will be considered and compared with particular emphasis on the linear ACD model, Box-Cox ACD model, augmented Box-Cox ACD model and augmented (Hentschel) ACD model. The analysis will consider models with the Burr distribution and the generalized Gamma distribution for the innovation term. Bayesian inference will be presented and practically used in estimation of and prediction within ACD models describing trade durations. The MCMC methods including Metropolis-Hastings algorithm are suitably adopted to obtain samples from the posterior densities of interest. The empirical part of the work includes modelling of trade durations of selected equities from the Polish stock market.

Keywords: autoregressive conditional duration model (ACD model), trade durations, financial market microstructure, Bayesian inference

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1 Introduction

The last thirty years have witnessed dynamic development of financial econometrics spurred by the use of new time series modelling methods, simulation-based methods, enhanced computing power and relatively easy access to large databases. Prior to the mid-1990s, empirical research in financial econometrics drew mainly on daily data and mostly involved modelling and forecasting of daily rates of return on financial instruments or their volatility and relied chiefly on ARCH and GARCH models put forward by R. Engle (1982) and T. Bollerslev (1986). The automation of stock exchange trading systems has led to the availability of accurate and complete data sets describing the transaction process. The accessibility of information about each and every individual transaction and its characteristics means that now tests based on intraday data have become standard. Therefore, data describing individual financial events i.e. financial ultra-high frequency data have come to be widely used in empirical analyses. The concept of *ultra-high frequency data (UHF data)* was first used in financial literature by Engle (2000). These are time series developed from the characteristics of the events of the transaction process with exactly assigned time of occurrence. In practice, this simply means an analysis of transaction data otherwise known as tick-by-tick data. These transaction data consist of information about the transaction's timing, price, volume, best bid and ask prices and orders. The availability of such detailed information about the transaction process opens up many new possibilities and directions to researchers. The accessibility of data on individual transactions, above all, allows research insights into the so-called market microstructure (see Madhavan 2000). Analysis of the transaction process offers an added value in explaining the behaviour of prices, investors and markets (see O'Hara 1995). Theoretical microstructure models attempt to explain the behaviour of the transaction prices, trading volumes and spreads between the bid and offer prices. They explain the reasons for volume and volatility grouping at certain times of the day and the underlying relationship. A review of theoretical market microstructure models and their wider analysis can be found in the following works: O'Hara (1995), Madhavan (2000), Vives (2008), De Jong and Rindi (2009).

Testing some hypotheses on market microstructure and empirical analysis of intraday seasonality patterns in the financial markets are therefore possible only when data representing individual transactions can be used. This in turn requires new modelling tools incorporating the specific nature of the processes generating this type of data. The random distribution of observations on the time axis is a characteristic feature of ultra-high frequency financial data. One way of the transaction process analysis involves modelling the time intervals between successive points of this process, i.e. modelling the process of durations. This approach proved to lend itself to modelling the dynamics of the transaction process. Moreover, the analysis of time intervals between successive events of the transaction process can allow for a more detailed insight into the various types of dependencies prevailing on the market. Transaction durations play an important role in the market microstructure theory where they are

used as a proxy variable indicative of the presence of new information on the market. The importance of the time intervals between transactions in the transaction process is discussed in works pertaining to the market microstructure theory, e.g. in Glosten and Milgrom (1985), Diamond and Verrecchia (1987), Easley and O'Hara (1987), Admati and Pfleiderer (1988).

The primary tool used for econometric modelling of transaction durations, analysis of the transactional intensity of assets and research into the effects of market microstructure is currently constituted of Autoregressive Conditional Duration models (ACD models) introduced by R. Engle and J. Russell in 1998 (cf. Engle and Russell 1998). The dynamic development of financial markets requires reaching out for new data analysis tools and techniques. There is a strong interest around the world in financial transaction data and ACD models and their use for modelling financial time series. In the case of ACD models, inference about the parameters is usually based on the Maximum Likelihood (ML) method, or on the Quasi-Maximum Likelihood (QML) method. Because of the not so-well known properties of maximum likelihood estimators for ACD models with conditional distributions other than the exponential distribution the Bayesian approach relying on the Monte Carlo methods seems to provide a valuable, interesting and theoretically consistent estimation method. Hence, there is a need for broadening the scope of research based on financial data from individual transactions with the use of Bayesian ACD models. Moreover, this research can add new merits to the published literature on ACD models.

The main objective of this work is to develop and apply the Bayesian approach to estimation of and prediction within ACD models, as well as a practical use of Bayesian ACD models in the analysis of the dynamics of transaction durations of selected companies listed on the Polish stock exchange. The paper summarizes the key aspects of a more extensive study from the author's Ph.D. dissertation. The remainder of the paper is organized as follows. In Section 2 of the paper we introduce and discuss the basic definition of the ACD process and present selected extensions of the basic linear ACD model. Section 3 provides a short discussion about properties of maximum likelihood estimators for ACD models. Next, in Section 4 we discuss Bayesian estimation and prediction of ACD processes. Then in Section 5 the approach presented is used to model transaction durations of selected equities listed in the main index of Polish stock exchange – the WIG20 index. Section 6 of the paper contains concluding remarks.

2 ACD models in duration analysis

ACD processes are one of the primary tools used in modelling the time intervals between events of the transaction process, analysing trading intensity of companies and examining the effects of financial markets microstructure. This paper illustrates in particular the potential of the family of ACD processes in explaining the dynamics of the durations ascertained in the financial market. In literature ACD processes

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already enjoy a well-established position (cf. e.g. Engle and Russell 1998, Lunde 1999, Grammig and Maurer 2000, Bauwens and Giot 2000, 2001, 2003, Dufour and Engle 2000, Zhang, Russell and Tsay 2001, Hautsch 2002, 2004, 2012, Bauwens, Giot, Grammig and Veredas 2004, Bauwens and Veredas 2004, Drost and Werker 2004, Ghysels, Gouriéroux and Jasiak 2004, Fernandes and Grammig 2006, Strickland, Forbes and Martin 2006). As regards the Bayesian approach, the only known work on this approach to duration modelling in the financial markets by means of ACD models is a paper by Brownlees and Vannucci (2013). Thus, the research presented in this paper constitutes one of the first Bayesian analyses of ACD models.

Let us consider a sequence of moments $t_1, t_2, \dots, t_n, \dots$ in which events of the transaction process occur. An event of the transaction process shall mean the conclusion of a transaction, a change in the transaction price or trading volumes etc. The time interval between successive events of the transaction process that occur at the moments t_i and t_{i-1} is denoted as $x_i = t_i - t_{i-1}$; x_i will be called the duration. Let Ψ_i ($i = 1, 2, \dots$) represent the expected durations conditional on information available at time t_{i-1} , i.e.:

$$\Psi_i = E(x_i | \mathfrak{S}_{i-1}, \theta),$$

where \mathfrak{S}_{i-1} denotes a set of information available prior to and inclusive moment t_{i-1} and θ denotes a vector of parameters. Today the most common approach to the description of transaction, price and volume durations involves the ACD model.

The Autoregressive Conditional Duration model was proposed for modelling of the dynamics of financial durations by Engle and Russell (1997, 1998). The main idea behind the ACD model involves a dynamic parameterisation of the conditional expected duration Ψ_i :

$$\Psi_i = E(x_i | \mathfrak{S}_{i-1}, \theta) = E(x_i | x_{i-1}, \dots, x_1; \theta) = \Psi_i(x_{i-1}, \dots, x_1; \theta).$$

In the ACD model, duration x_i is expressed as the following product:

$$x_i = \Psi_i \cdot \varepsilon_i,$$

where ε_i follows i.i.d. process defined on positive support with density function $f_\varepsilon(\varepsilon_i)$ and expected value $E(\varepsilon_i) = 1$. Different types of ACD models may result either from different functional forms for the conditional mean function Ψ_i , or the selection of different probability distributions for the random variable ε_i . Obtaining an ACD process with a certain type of conditional distribution of durations x_i (given the entire past of the process) consists in the introduction of various probability distributions for the variable ε_i . As far as the distribution of innovations ε_i is concerned, ACD models can only use for ε_i probability distributions defined on the set of positive real numbers (cf. Engle and Russell 1998, Lunde 1999, Grammig and Maurer 2000, Bauwens and Giot 2001, 2003, Hautsch 2002, Bauwens, Giot, Grammig and Veredas 2004, De Luca and Gallo 2004, Fernandes and Grammig 2006, Luca and Zuccolotto 2006, Allen, Chan, McAleer and Peiris 2008). The most common and simplest distribution for the

variable ε_i is the exponential distribution. In the context of conditional distributions in the analysis of ACD processes one may also consider e.g.: Weibull distribution, gamma distributions, generalized gamma distributions and Burr distributions. It is worth emphasising that the families of generalized gamma distributions and Burr distributions are disjoint.

The basic ACD model specification, due to the conditional mean equation, as proposed by Engle and Russell (1998), is based on linear parameterisation of the dynamics of expected duration Ψ_i :

$$\Psi_i = \omega + \sum_{j=1}^p \alpha_j x_{i-j} + \sum_{j=1}^q \beta_j \Psi_{i-j}, \quad (1)$$

where $\omega > 0$, $\alpha_j \geq 0$, $\beta_j \geq 0$. It is worth noting that these constraints are sufficient, although not necessary, for the non-negativity of the duration process. This is the so-called linear ACD process of p and q order – ACD(p, q), where p and q determine delay orders of past durations x_i and past expected durations Ψ_i respectively. The conditional mean of duration in a linear ACD process by definition equals $\Psi_i = E(x_i | \mathcal{S}_{i-1}, \theta)$.

The simplest ACD process which we will consider in empirical research is the ACD(1,1) linear process:

$$\Psi_i = \omega + \alpha \cdot x_{i-1} + \beta \cdot \Psi_{i-1}, \quad (2)$$

where $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$, $\alpha + \beta < 1$ (the condition imposed in order to ensure existence of the conditional mean of duration). The basic specification of the ACD process may however be too restrictive to even lend itself to a correct description of financial duration processes. Numerous empirical studies conducted e.g. by Dufour and Engle (2000), Zhang, Russell and Tsay (2001) or Fernandes and Grammig (2006) showed that news impact curves (cf. Engle and Ng 1993), depicting the impact of the disturbance ε_{i-1} on the expected duration Ψ_i , are too inelastic to adjust the process of conditional durations to observed durations. Let us notice that the news impact curve in the case of the basic ACD process is linear and has an $\alpha \cdot \Psi_{i-1}$ slope parameter. Moreover innovations are introduced into the equation in a multiplicative impact with Ψ_{i-1} . It is also worth emphasising that it was shown as early as in Engle and Russell's work (1998) that standard linear ACD model tends to overpredict after very short or very long durations. This gave rise to numerous augmented ACD models.

Extensions of the basic functional form of the conditional expected duration are two-pronged. As regards the first direction, pertinent literature proposes specifications that allow both additive and multiplicative introduction into the model of a stochastic component, the so-called delayed innovations. On the other hand, there are specifications that take into account news impact curves that are non-linear, i.e. much more flexible.

With respect to specifications underlying the description of the functional form of

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the conditional expected duration, below we present only selected augmented ACD models which were used in the empirical research described in the subsequent part of the paper. These include:

1. logarithmic ACD model – LACD(1,1) model, cf. Bauwens and Giot (2000), Lunde (1999):

$$\ln \Psi_i = \omega + \alpha \cdot \ln \varepsilon_{i-1} + \beta \cdot \ln \Psi_{i-1} = \omega + \alpha \cdot \ln x_{i-1} + (\beta - \alpha) \cdot \ln \Psi_{i-1}, \quad (3)$$

where $|\beta| < 1$ (the condition of non-explosiveness and strict stationarity of the process),

2. Box-Cox ACD model – BCACD(1,1) model cf. Hautsch (2002):

$$\Psi_i^{\delta_1} = \omega + \alpha \cdot \varepsilon_{i-1}^{\delta_2} + \beta \cdot \Psi_{i-1}^{\delta_1}, \quad (4)$$

where $\omega > 0$, $\alpha > 0$, $0 < \beta < 1$ (the condition of non-explosiveness and strict stationarity of the process), $\delta_1 > 0$, $\delta_2 > 0$,

3. asymmetric logarithmic ACD model – AsLACD(1,1) model, cf. Fernandes and Grammig (2006):

$$\ln \Psi_i = \omega + \alpha \cdot [|\varepsilon_{i-1} - b| + c \cdot (\varepsilon_{i-1} - b)] + \beta \cdot \ln \Psi_{i-1}, \quad (5)$$

where $b > 0$ and $|\beta| < 1$ (the condition of non-explosiveness and strict stationarity of the process),

4. augmented Box-Cox ACD model – ABCACD(1,1) model cf. Hautsch (2004, 2012):

$$\Psi_i^{\delta_1} = \omega + \alpha \cdot [|\varepsilon_{i-1} - b| + c \cdot (\varepsilon_{i-1} - b)]^{\delta_2} + \beta \cdot \Psi_{i-1}^{\delta_1}, \quad (6)$$

where $\omega > 0$, $\alpha > 0$, $0 < \beta < 1$ (the condition of non-explosiveness and strict stationarity of the process), $\delta_1 > 0$, $\delta_2 > 0$, $b > 0$, $|c| \leq 1$ (this restriction has to be imposed in order to circumvent complex values whenever $\delta_2 \neq 1$),

5. augmented ACD model – AACD(1,1) model, cf. Fernandes and Grammig (2006):

$$\Psi_i^{\delta_1} = \omega + \alpha \cdot \Psi_{i-1}^{\delta_1} \cdot [|\varepsilon_{i-1} - b| + c \cdot (\varepsilon_{i-1} - b)]^{\delta_2} + \beta \cdot \Psi_{i-1}^{\delta_1}, \quad (7)$$

where $\omega > 0$, $\alpha > 0$, $0 < \beta < 1$ (the condition of non-explosiveness of the process), $\delta_1 > 0$, $\delta_2 > 0$, $b > 0$, $|c| \leq 1$ (this restriction has to be imposed in order to circumvent complex values whenever $\delta_2 \neq 1$).

The above-described functional forms for the conditional duration Ψ_i do not represent an exhaustive set of all possible directions of generalizations of the basic ACD process. They illustrate, however, the specification process of an adequate functional form of Ψ_i . The presentation of these generalizations draws primarily on Fernandes and Grammig (2006) as well as Hautsch (2004, 2012).

3 Maximum Likelihood method versus Bayesian approach for ACD models estimation

In the case of ACD models, inference about the parameters is usually based on the classical approach, i.e. on the Maximum Likelihood (ML) method, or on the Quasi-Maximum Likelihood (QML) method. In the case of ACD model estimation the density of the exponential distribution is the simplest choice for the density function of the error term. It should be emphasised, however, that the results of empirical modelling of durations between events in the transaction process showed that the assumption of the exponential distribution for innovations may be, unfortunately, too restrictive. For this reason, the literature on ACD models offers other probability distributions for error term, such as the Burr distribution or the generalized gamma distribution. Maximum likelihood estimators are consistent and asymptotically efficient provided the right distribution of innovations in the model is chosen. Otherwise the parameter estimators may not even be consistent. An alternative way to estimate the parameters in the ACD models involves the use of the Quasi-Maximum Likelihood method. Engle and Russell (1998) showed that for the ACD (1,1) model assuming exponential distribution for innovation term consistent and asymptotically normal estimators of model parameters are obtained by the use of the QML method. However, a crucial assumption is that the conditional expected duration equation is correctly specified (cf. e.g. Hautsch 2004, 2012). It should also be noted that the above properties hold for the basic linear ACD model and need not necessarily hold true for more general specifications of ACD models. In the case of the QML method the actual implementation of estimation procedures is admittedly straightforward, but is achieved at the expense of estimator efficiency. Neither does the QML estimation method warrant that the resulting parameter estimators will be unbiased in finite samples. Grammig and Maurer (2000) showed that QML estimators of ACD model parameters can be biased and inefficient even for samples of over 15,000 observations. In practice therefore the more efficient ML estimates are preferred.

The situation described above suggests the use of the Bayesian approach relying on the Monte Carlo methods to estimate ACD models. Given the above problems surrounding the use of ML and QML methods and furthermore the not so-well known properties of maximum likelihood estimators for ACD models with conditional distributions other than the exponential distribution, Bayesian inference can constitute a comfortable, attractive and theoretically consistent estimation method, even if it is numerically demanding and time consuming.

4 Bayesian inference for ACD models

We will estimate proposed ACD models in the Bayesian framework; see, e.g. Zellner (1971), Osiewalski (2001, in Polish), Koop (2003), Geweke (2005), Pajor (2010, in Polish). Let $\{x_i : i = 0, 1, 2, \dots, T\}$ denote an observed time series of transaction

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durations. The following general ACD model is assumed:

$$\begin{aligned}x_i &= \Psi_i \cdot \varepsilon_i \\ \Psi_i &= E(x_i | \mathfrak{S}_{i-1}, \theta) = \Psi_i(x_{i-1}, \dots, x_1; \theta)\end{aligned}$$

where ε_i follows a Burr type distribution or a generalized gamma distribution and Ψ_i may be one of six specifications of the conditional expected duration: linear ACD model (2), a type I logarithmic ACD model (3), Box-Cox ACD model (4), logarithmic asymmetric ACD model (5), augmented Box-Cox ACD model (6) and augmented ACD model (7). Thus 12 Bayesian ACD models were specified.

Let $x = (x_1, \dots, x_T) \in X \subset \mathbb{R}^T$ denote the vector of observed transaction durations (in the empirical part it is a vector of deseasonalised transaction durations) and $x_f \in X_f \subset \mathbb{R}^k$ denotes a vector of forecasted durations. The vector of unknown parameters in the model M_j will be marked $\theta_{(j)}$, where $j = 1, \dots, 12$. The Bayesian ACD model M_j , describing the dynamics of durations x_i , is uniquely determined by the density of the joint distribution of the vector of observations (durations), the vector of forecasted durations and the vector of parameters:

$$p(x, x_f, \theta_{(j)} | M_j) = p(x_f | x, \theta_{(j)}; M_j) \cdot p(x | \theta_{(j)}; M_j) \cdot p(\theta_{(j)} | M_j),$$

where $p(x | \theta_{(j)}; M_j)$ is the density of the conditional distribution of durations with fixed parameters, i.e. sampling distribution and $p(\theta_{(j)} | M_j)$ stands for prior density. The density $p(x_f | x, \theta_{(j)}; M_j)$ is the so-called sampling predictive distribution. Estimation of the vector of model parameters involves determining the conditional distribution for this vector of parameters given the vector of observations x , i.e. a posterior distribution with a density defined as follows:

$$p(\theta_{(j)} | x; M_j) = \frac{p(x | \theta_{(j)}; M_j) \cdot p(\theta_{(j)} | M_j)}{p(x | M_j)},$$

where $p(x | M_j) = p_j(x) = \int_{\Theta_{(j)}} p_j(x | \theta_{(j)}) \cdot p(\theta_{(j)} | M_j) d\theta_{(j)}$ denotes marginal data density. Bayesian prediction, in turn, involves determining the density of the conditional distribution of the vector x_f given the observed vector x , i.e. (post-sample) predictive distribution density:

$$p(x_f | x; M_j) = \int_{\Theta} p(x_f | x, \theta_{(j)}; M_j) \cdot p(\theta_{(j)} | x; M_j) d\theta_{(j)}.$$

Bayesian inference provides also a tool which allows to compare models, but formal Bayesian comparison of competing models is not subject of this paper.

It should also be acknowledged that Bayesian inference is carried out conditionally given the vector $x_{(0)} = (x_0, \Psi_0)$ – the vector of initial conditions which was hitherto omitted in the notation.

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The sample density and sample predictive density, respectively, are the products of appropriate conditional densities of durations:

$$p(x|x_{(0)}, \theta_{(j)}, M_j) = \prod_{i=1}^T f(x_i|\mathfrak{S}_{i-1}, x_{(0)}; \theta_{(j)}),$$

$$p(x_f|x, x_{(0)}, \theta_{(j)}, M_j) = \prod_{i=T+1}^{T+k} f(x_i|\mathfrak{S}_{i-1}, x_{(0)}; \theta_{(j)}),$$

where the conditional duration density $f(x_i|\mathfrak{S}_{i-1}, x_{(0)}; \theta_{(j)})$ is expressed by the following formulas:

1. for an ACD model with a Burr distribution:

$$f(x_i|\mathfrak{S}_{i-1}, x_{(0)}; \theta_{(j)}) = \frac{\kappa}{x_i} \cdot \left(\frac{x_i}{\Psi_i} \cdot \frac{\Gamma(1 + \frac{1}{\kappa}) \cdot \Gamma(\frac{1}{\eta} - \frac{1}{\kappa})}{\eta^{1+\frac{1}{\kappa}} \cdot \Gamma(1 + \frac{1}{\eta})} \right)^\kappa \cdot \left[1 + \eta \cdot \left(\frac{x_i}{\Psi_i} \cdot \frac{\Gamma(1 + \frac{1}{\kappa}) \cdot \Gamma(\frac{1}{\eta} - \frac{1}{\kappa})}{\eta^{1+\frac{1}{\kappa}} \cdot \Gamma(1 + \frac{1}{\eta})} \right)^\kappa \right]^{-1 - \frac{1}{\eta}}$$

where $\kappa > \eta > 0$,

2. for an ACD model with a generalized gamma distribution:

$$f(x_i|\mathfrak{S}_{i-1}, x_{(0)}; \theta_{(j)}) = \frac{\gamma}{x_i \cdot \Gamma(\frac{\nu}{\gamma})} \cdot \left(\frac{x_i}{\Psi_i} \cdot \frac{\Gamma(\frac{1+\nu}{\gamma})}{\Gamma(\frac{\nu}{\gamma})} \right)^\nu \cdot \exp \left[- \left(\frac{x_i}{\Psi_i} \cdot \frac{\Gamma(\frac{1+\nu}{\gamma})}{\Gamma(\frac{\nu}{\gamma})} \right)^\gamma \right]$$

where $\gamma, \nu > 0$.

In both above cases Ψ_i is determined by one of these specifications: (2), (3), (4), (5), (6) or (7).

For Bayesian models to be fully specified, one must determine prior distributions of parameters, $p(\theta_{(j)}|M_j)$. It should be emphasised that a specification of the prior distribution in Bayesian ACD models cannot adhere to the general rules of construction of reference priors recommended in Bayesian literature because these models are too complex to analytically determine the reference priors in the vein

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of Jeffreys and Bernardo. Therefore we assumed prior distributions reflecting subjectively weak preliminary knowledge about the parameters.

The vector $\theta_{(j)}$ of all parameters in M_j was decomposed into two prior independent random subvectors: $\vartheta_{(j)}$ – the subvector of parameters characteristic for a given specification Ψ_i , and $\nu_{(j)}$ – the subvector of parameters characteristic for ε_i . The following joint prior was obtained:

$$p(\theta_{(j)}|M_j) = p(\vartheta_{(j)}, \nu_{(j)}|M_j) = p(\vartheta_{(j)}|M_j) p(\nu_{(j)}|M_j) \quad j = 1, \dots, 12.$$

In ACD models with the Burr distribution (these will be Bayesian models M_j for $j = 1, \dots, 6$) the vector of parameters of the distribution of ε_i is $\nu_{(j)} = (\kappa, \eta)$, where $\kappa > \eta > 0$. In models with this distribution it was assumed that:

$$p(\nu_{(j)}|M_j) = p(\kappa, \eta) \propto f_N(\kappa|\mu_\kappa, \sigma_\kappa^2) \cdot f_N(\eta|\mu_\eta, \sigma_\eta^2) \cdot I_{(0,\infty)}(\kappa) \cdot I_{(0,\infty)}(\eta) \cdot I_{(\kappa>\eta)}(\kappa, \eta),$$

where $j = 1, \dots, 6$, $\mu_\kappa = 0$, $\sigma_\kappa = 5$, $\mu_\eta = 0$, $\sigma_\eta = 5$ and $f_N(\cdot|\mu_0, \sigma_0^2)$ denotes the density of a normal distribution with an expected value of μ_0 and variance of σ_0^2 . The initial knowledge of the vector of the parameters (κ, η) is reflected in truncated normal distributions.

In turn, in the ACD models with a generalized gamma distribution (i.e. Bayesian models M_j for $j = 7, \dots, 12$) the vector of parameters of ε_i is $\nu_{(j)} = (\gamma, \nu)$, where $\gamma > 0$, $\nu > 0$. Then the prior density is:

$$p(\nu_{(j)}|M_j) = p(\gamma, \nu) = p(\gamma)p(\nu) \propto f_N(\gamma|\mu_\gamma, \sigma_\gamma^2) \cdot I_{(0,\infty)}(\gamma) \cdot f_N(\nu|\mu_\nu, \sigma_\nu^2) \cdot I_{(0,\infty)}(\nu),$$

where $j = 7, \dots, 12$, $\mu_\gamma = 0$, $\sigma_\gamma = 5$, $\mu_\nu = 0$, $\sigma_\nu = 30$. The initial knowledge of the vector of parameters (γ, ν) is embodied in the above appropriate truncated normal distributions, similarly as in the Burr distribution.

The specification of the prior density $p(\theta_{(j)}|M_j)$ in the model M_j must be supplemented with prior density of the vector of parameters $\vartheta_{(j)}$ characteristic for a given specification of the conditional expected duration Ψ_i . Depending on the specification of conditional expected duration Ψ_i , the parameters vector $\vartheta_{(j)}$ will be: $\vartheta_{(j)} = (\omega, \alpha, \beta)$ in the linear ACD model and logarithmic ACD model, $\vartheta_{(j)} = (\omega, \alpha, \beta, \delta_1, \delta_2)$ in the Box-Cox ACD model, $\vartheta_{(j)} = (\omega, \alpha, \beta, b, c)$ in the asymmetric logarithmic ACD model and $\vartheta_{(j)} = (\omega, \alpha, \beta, \delta_1, \delta_2, b, c)$ in the augmented Box-Cox ACD model and augmented ACD model. We assume prior independence of parameters. Then the joint prior density of the vector $\vartheta_{(j)}$ is a product of the prior densities of its coordinates. The exception here is the linear ACD model with additional restrictions imposed on selected parameters.

We assume that prior distributions for the parameters with values spanning the entire set of real numbers are normal with zero mean and standard deviation of five. For the remaining parameters of the models considered we assume normal distributions with zero mean and standard deviation of five, adequately truncated, due to the restrictions imposed on the parameters by the individual models. The exception is the parameter

b in the augmented Box-Cox ACD model and the augmented ACD model for which the prior distribution is an inverted gamma distribution with density:

$$p(b) = f_{IG}(b|\nu, s) = \frac{s^\nu}{\Gamma(\nu)} \cdot \left(\frac{1}{b}\right)^{\nu+1} \cdot \exp\left\{-\frac{s}{b}\right\} \cdot I_{(0,\infty)}(b), \quad \nu = 1, s = 0, 3.$$

The above prior distributions reflect the subjectively weak preliminary knowledge of the parameters.

5 Empirical study

5.1 Data sets

The empirical study was carried out based on data from the Warsaw Stock Exchange. The Warsaw Stock Exchange is the stock market with the highest capitalization in Eastern and Central Europe. Thus, it is noticed as one of the most important and the best developed market in that region. However, there are limited works focusing on ACD models and their empirical applications to Polish stock data; see e.g. Doman (2005), Bień (2006, 2006a, in Polish), Doman (2008), Doman and Doman (2010), Doman (2011, in Polish). This study can fill that gap to some extent. Moreover it can also develop the existing knowledge about the Polish stock market microstructure.

The methods presented above were empirically verified on the basis of transaction data on equities of three companies listed in the Warsaw Stock Exchange's WIG 20 index: Polish Telecom (TPSA), the media company Agora SA (AGORA) and the PKOBP SA bank (PKOBP). The present study is based on tick-by-tick quotations between 23 March 2009 and 19 June 2009. The companies were selected to reveal differences in liquidity (trading intensity) during the study period. The analysis covers only transactions carried out in the continuous trading phase, i.e. in the case of the Warsaw Stock Exchange between 10:00 and 16:10. Transaction data are derived from the Stooq.pl website. Transaction data sheets contain information on the date and time of the transaction with the accuracy of one second and the transaction's closing price and volume. When several transactions were recorded in the same second, data were partially aggregated and such transactions were deemed to constitute a single transaction with the price being a volume-weighted average. It is worth noting that in the case of the companies surveyed, the share of transactions registered in the same second is very high and ranges from 37% for AGORA to 50% for PKOBP. Based on the aggregate transactional data, durations between transactions i.e. transaction durations were determined. In addition, the time intervals between the close of the session and the beginning of the next day's session were removed. In this case, it was used the most common convention adopted in the foreign literature.

The basic descriptive statistics of transaction durations for the companies surveyed are shown in Table 1. We are dealing with three companies revealing divergent trading activity patterns. The majority of the transactions involved PKOBP equities, for

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which the average duration between transactions is approximately 13 seconds. The company belongs with a group of WSE's most liquid entities. The fewest transactions were reported for AGORA, for which the average transaction duration is almost 80 seconds. With an average duration between transactions of approximately 24 seconds, TPSA represents an averagely liquid company. In addition, the medians of the series are markedly smaller than the average values, which of course means that duration distributions are characterized by a strong right-sided asymmetry. Analysis of the descriptive statistics of empirical duration distributions reveals their strikingly overdispersion. The dispersion indices are generally very high, which may be indicative of high dynamics of the surveyed series. The values of the variation coefficients range from 1.6 to 1.8. With its dispersion index ascertained at a high 2.13 the company AGORA is an exception. It is worth noting that the higher the transaction intensity, the lower the dispersion index. Similar results were reported in Bień (2006) only for the 2003 Polish data (also cf. Bień 2006a). The dynamics of transaction durations can be seen in Figure 1 showing graphs for series of the first 10,000 observations. In addition, the graphs clearly indicate clustering of short and long transaction waiting times. This suggests the presence of a strong autocorrelation in the tested series. Polish durations are characterized by properties analogous to data recorded at mature foreign markets, i.e. a strong right-sided asymmetry, overdispersion or clustering of shorter and longer durations (cf. Hautsch 2004, Hautsch 2012, Bauwens and Giot 2001 and numerous other works).

The values of Q Ljung and Box statistics in Table 1 allow formally (but in non-Bayesian approach) to perform test of the null hypothesis such that there is no autocorrelation of durations respectively from the first to the twentieth order delay. The critical values of χ^2 distribution for the significance level of 0.05 are $\chi^2(5) = 11,070$ and $\chi^2(20) = 31,410$ respectively. Of course, on the basis of the values of test statistics the null hypothesis whereby there is no autocorrelation of durations is easily rejected for all three companies. The autocorrelation of transaction durations is therefore very strong.

The presence of a very strong autocorrelation of durations may be due to intraday seasonality patterns of trading activity. Seasonality is one of the most characteristic properties of financial time series for ultra-high frequency data. Therefore we estimated the intraday seasonality patterns. With a view to that we used the Nadaraya-Watson estimator of regression of the duration on the time of the day, determined separately for each day of the week (cf. Bauwens and Veredas 2004, Veredas, Rodriguez-Poo and Espasa 2001, Huptas 2009):

$$\phi(t) = \frac{\sum_{i=1}^n x_i K\left(\frac{t-t_i}{h_n}\right)}{\sum_{i=1}^n K\left(\frac{t-t_i}{h_n}\right)}, \quad (8)$$

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Table 1: Descriptive statistics of transaction durations, autocorrelation coefficients and the Ljung-Box statistics for the analysed companies

	plain durations			adjusted (deseasonalised) durations		
	AGORA	TPSA	PKOBP	AGORA	TPSA	PKOBP
Number of observations	19627	64616	118480	19627	64616	118480
Mean	79.97	24.34	13.28	0.98	0.99	0.99
Standard deviation (SD)	170.44	43.09	21.23	1.98	1.67	1.48
Dispersion index (=Mean/SD)	2.13	1.77	1.59	2.02	1.69	1.49
Median [s]	20	9	6	0.263	0.396	0.486
Minimum [s]	1	1	1	0.006	0.023	0.042
Maximum [s]	4003	833	653	44.06	30.96	31.90
ACF(1)	0.218	0.216	0.230	0.208	0.198	0.206
Q(5)	2931.74	9352.83	23259.5	2575.94	7596.65	17492.8
Q(20)	6256.26	20744.45	60290.1	5106.05	15600.7	41943.0

ACF(k) – the value of the k -th order autocorrelation coefficient; Q(k) – the value of the Ljung-Box Q-statistic of k -th order; descriptive statistics in seconds.

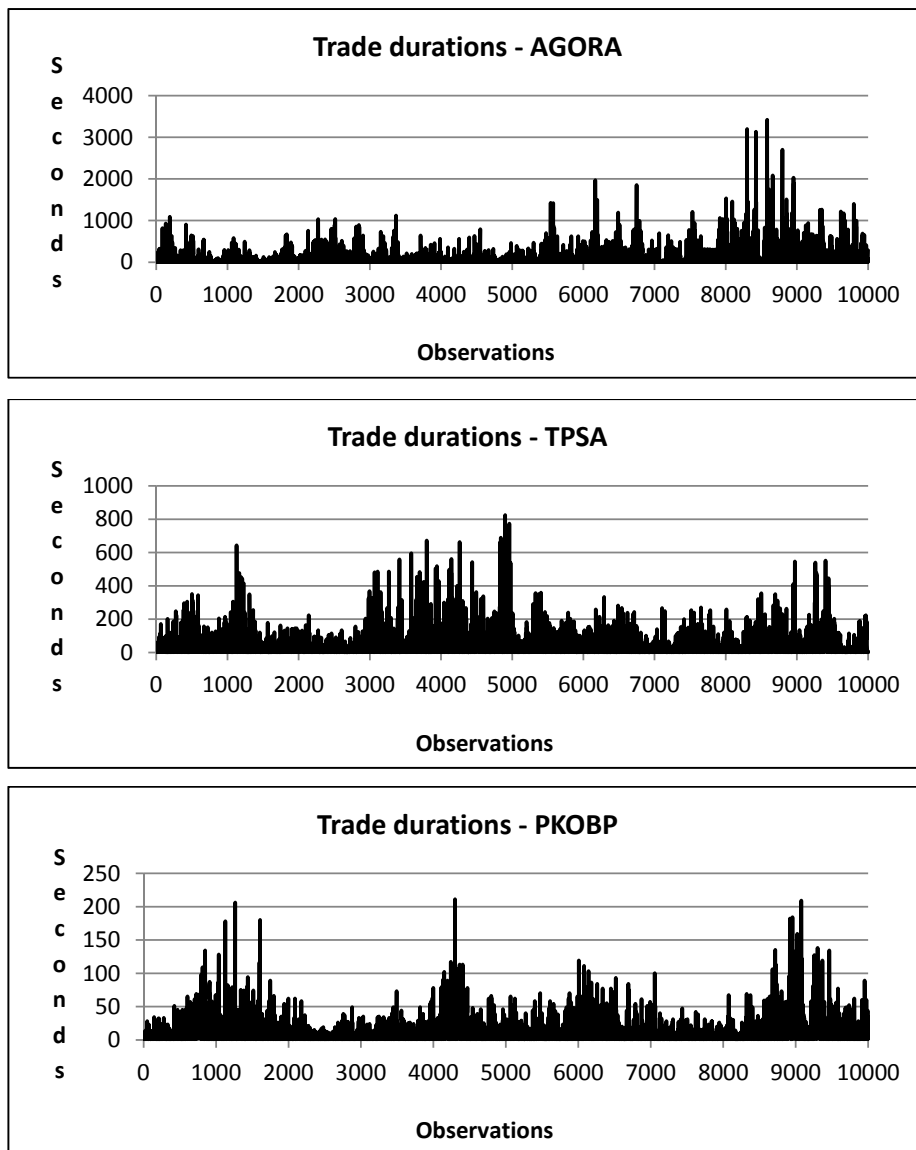
where: t – number of seconds from the midnight of every day (or alternatively from the beginning of the session), x_i – durations corresponding to time t_i (x_i is a dependent variable), t_i – number of seconds from the midnight of every day (or from the beginning of the session) until the time of a given transaction, K - kernel function, h_n – bandwidth, s -standard deviation of sample t_i , n - number of observations. In the case of the kernel estimators use was made of the *quartic* kernel with an optimal bandwidth of $h = 2,78sn^{-1/5}$. It is worth noting that the kernel method applied is a non-Bayesian one. Figure 2 illustrates the intraday seasonality patterns for the three companies surveyed taking into account the day of the week effect.

Graphs of the estimated function of intraday seasonality have the distinct shape of an inverted U and show unequivocally that the transaction durations are subject to daily seasonality. Durations between transactions are much shorter at the beginning and at the end of a session than at noontime. The busy transaction activity shortly after the opening of the market reflects response to overnight news (information incoming from foreign stock exchanges, macroeconomic data etc). With the assimilation of information incoming from other markets the transaction activity becomes subdued. Between 12:00 and 14:00 it is significantly lower due to the so-called lunch-break effect. The intervals between successive transactions are then definitely the longest. Then, as the close of the session approaches, transaction activity gradually increases, which is justified by some traders closing or adjusting their positions. It is worth noting that the intraday seasonality varies depending on the day of the week. It appears that regardless of the type of company, transaction activity is higher on Tuesdays and Wednesdays than on any other day of the week.

Analysis of seasonal patterns also indicates that in the case of a liquid company – PKOBP in our analysis – investors' trading activity at the end of the session is a little

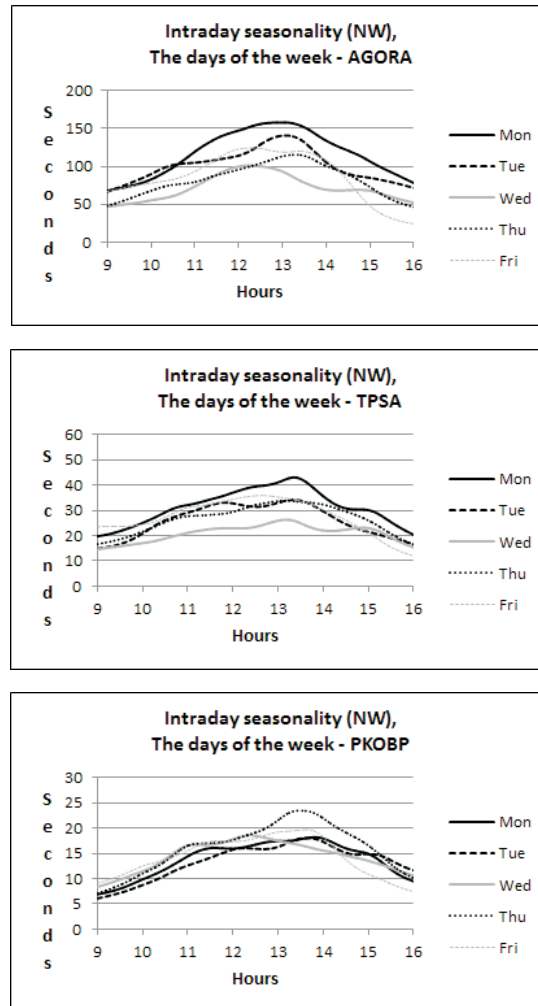
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Figure 1: Trade durations for the analysed companies – first 10000 observations



smaller, which translates into longer durations than at the beginning of the day. On the other hand, we can note the reverse situation in the case of AGORA company – traders are more active before the close of the market than just after its opening.

Figure 2: Intraday seasonality patterns for the days of the week for the analysed companies



To be able to further analyse the series surveyed on the basis of Bayesian ACD models, we should eliminate the seasonality effect from the series, which will help reduce or even eliminate the detected autocorrelation. Guided by pertinent literature on the subject (e.g. Engle and Russell 1998), after estimating the intraday seasonality factor we determined the transaction durations devoid of the seasonality effect in accordance

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with the formula

$$\hat{x}_i = \frac{x_i}{\phi(t_{i-1})}, \quad (9)$$

where: $x_i = t_i - t_{i-1}$ - duration between transactions at time t_i and time t_{i-1} , \hat{x}_i -duration devoid of the seasonal effect, $\phi(t_i)$ -multiplicative factor of intraday seasonality at time t_i . The $\phi(t_i)$ seasonality factor (determined on the basis of formula (8)) is construed as the average duration corresponding to each unit of time in which data was observed (generally, average duration corresponding to each second).

Descriptive statistics of durations after the elimination of the intraday seasonality effect are shown in Table 1. The elimination of the seasonal factor from the data resulted in partial reduction in transaction durations autocorrelation. The values of the Ljung and Box test statistic declined for the companies surveyed by approx. 15% – 25%, but continue to remain very high. The rather high values of low-order autocorrelation coefficients suggest that the rather strong clustering of short and long durations will still continue. The null hypothesis of absence of autocorrelation is still rejected at any reasonable level of significance. Of course, this indicates that the dynamics of transaction durations are influenced by factors other than a merely deterministic seasonality effect resulting from the structure of the stock market.

5.2 Main posterior results

This section presents the results of Bayesian inference conducted for series of transaction durations derived from data on the Polish stock market. We present the results of the Bayesian estimation of ACD model parameters as defined in section 3 of the paper. We also consider the predictive quality of models. It should be emphasised here that the question of Bayesian estimation of the ACD class of models and Bayesian prediction have not so far been considered in the literature.

In order to calculate the ultimate characteristics of the posterior distribution of parameters and the predictive distribution, in each model the Monte Carlo methods based on Markov chains (MCMC) are used. It is used the Metropolis and Hastings algorithm with a symmetric proposal density (see e.g. Hastings 1970, O'Hagan 1994). For a candidate generating distribution we use the multivariate Student's t distribution with three degrees of freedom for which the expected value is equal to previous state of the Markov chain and the covariance matrix is obtained based on a numerical strategy using the Monte Carlo – Importance Sampling method. The length of the generated Markov chain and the number of burnt-in states depend on the speed of convergence of the algorithm within the model framework. For the more general models e.g. the augmented Box-Cox ACD model and the augmented ACD model we made 6 000 000 draws, including 1 000 000 burnt-in states. On the other hand, in simpler, less parameterized models we made 2 200 000 draws, including 200 000 burnt-in states. To assess the convergence of the Metropolis and Hastings algorithm we used CUMSUM plots (cf. Yu and Mykland 1994). All the empirical results presented were obtained using author's own codes implemented in the GAUSS

13.0 econometric environment.

The object of Bayesian modelling is the dynamics of transaction durations calculated for three companies listed on the Warsaw Stock Exchange, namely the Polish Telecom company (TPSA), the media company Agora SA (AGORA) and the PKOBP SA bank (PKOBP). The initial time series of transaction durations under analysis were truncated and consist of 10,000 latest observations culled from the original series. The procedure involving the truncating of data series was performed due to the fact that in the case of longer time series (let it be remembered that the AGORA sample consisted of approximately 20,000 observations, PKOBP sample consisted of almost 120,000 observations) modelling would be extremely demanding from the numerical point of view. Long time series in conjunction with Monte Carlo simulations used in Bayesian estimation made the estimation process extremely cumbersome and time-consuming. It seems however that the arbitrary truncation of data series to 10,000 observations was not highly detrimental to the empirical results. Finally, ACD processes were used to model the dynamics of transaction durations for the company AGORA from 5 May 2009 to 19 June 2009, the company TPSA from 6 June 2009 to 19 June 2009 and the company PKOBP from 8 June 2009 to 19 June 2009. The initial observations were used as the initial conditions $x_{(0)}$.

The course of modelled transaction durations devoid of the seasonality effect for all companies surveyed is plotted in Figure 3. Although the analysed durations are devoid of the seasonality effect, all cases reveal a rather considerable variability and areas of duration clustering, i.e. long periods of waiting times for transactions followed by periods of shorter durations. In addition, there are outliers.

Table 2: Descriptive statistics of deseasonalized transaction durations for the analysed datasets

	AGORA	TPSA	PKOBP
Number of observations	10000	10000	10000
Mean	1.029	0.927	1.230
Standard deviation (SD)	2.097	1.449	1.791
Dispersion index ($=\text{Mean}/\text{SD}$)	2.038	1.563	1.456
Median	0.267	0.420	0.564
Skewness	5.258	4.093	3.510
Minimum	0.006	0.023	0.043
Maximum	44.065	22.281	24.439

The basic descriptive characteristics of the examined deseasonalized time series are shown in Table 2. Analysis of the descriptive statistics of empirical duration distributions reveals their overdispersion despite the fact that transaction durations have been purged of their seasonality patterns. The dispersion indices (variation coefficients) are generally very high, ranging between 1.45 and 1.56. AGORA is an exception – its dispersion index climbed to the high mark of 2.04. High skewness

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values indicate that duration distributions are characterized by a strong right-sided asymmetry. The very high values of skewness, overdispersion and modal values on the far side of zero indicate that the empirical distributions in these cases are very far from exponential distribution. These assumptions are undoubtedly corroborated by nonparametric density and hazard functions graphs for transaction durations of the companies surveyed which are presented in Figure 4.

In order to model transaction durations of AGORA, TPSA and PKOBP we used twelve Bayesian ACD models defined and presented in section 3: Burr-ACD, Burr-LACD, Burr-BCACD, Burr-As-LACD, Burr-ABCACD, Burr-AAACD, GGam-ACD, GGam-LACD, GGam-BCACD, GGam-As-LACD, GGam-ABCACD and GGam-AAACD. In the class of $\{M_1, \dots, M_{12}\}$ models, we consider six models with the Burr distribution for the random term and the six models with the generalized gamma distribution for innovations.

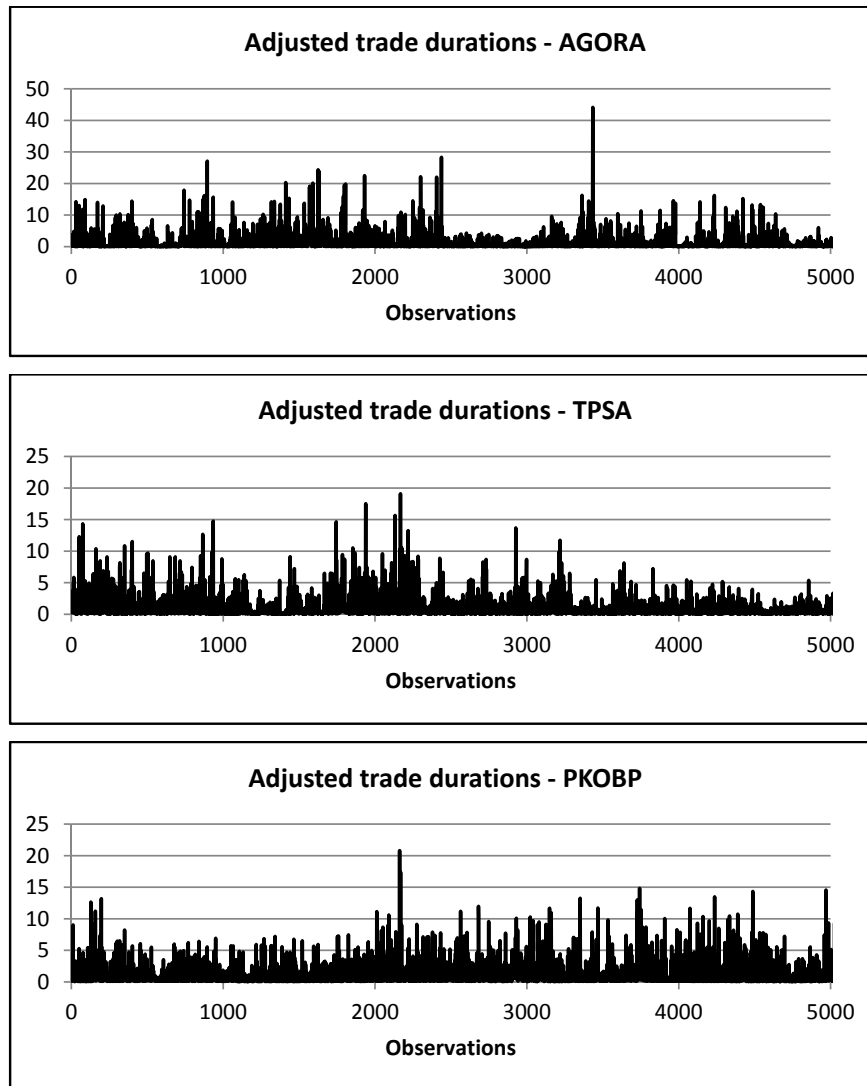
5.2.1 Bayesian estimation results

The Bayesian estimation was carried out for all twelve considered models. Bayesian estimation results for TPSA, AGORA and PKOBP are presented in Tables 3, 4 and 5 respectively. These set out the posterior means and standard deviations of the posterior marginal distributions of the parameters of the ACD models under consideration. The characteristics presented show that in the construction of the posterior marginal distributions an essential role was played by the information contained in the observed data and not by the prior distributions initially accepted. As a matter of fact, posterior distributions are characterised by a different location and a markedly smaller dispersion than prior distributions. Figure 5 presents the marginal prior distributions (bold solid lines) and posterior ones (bars) of the GGam-BCACD model parameters fitting the TPSA data.

The marginal posterior distributions for the parameters of the conditional Burr distribution and conditional generalized gamma distribution clearly indicate that data rejects the conditional exponential distribution of transaction durations (in the case of the conditional Burr distribution this corresponds to the assumption $\kappa = 1, \eta \rightarrow 0$, and in the case of the conditional generalized gamma distribution to the assumption $\gamma = 1, \nu = 1$) for all three companies. It is worth noting that the results also exclude conditional Weibull distributions (in the case of the conditional Burr distribution this corresponds to the assumption $\eta \rightarrow 0$, and in the case of the conditional generalized gamma distribution to the assumption $\gamma = \nu$).

In the case of TPSA, posterior distributions of the parameters κ and η are located far from $\kappa = 1$ and $\eta = 0$, respectively, with the distributions being heavily concentrated. The posterior means of the parameter κ oscillate around 1.26, and a standard deviation around 0.02, and for the parameter η posterior means oscillate around 0.6 with a standard deviations hovering around 0.03. On the other hand, in the case

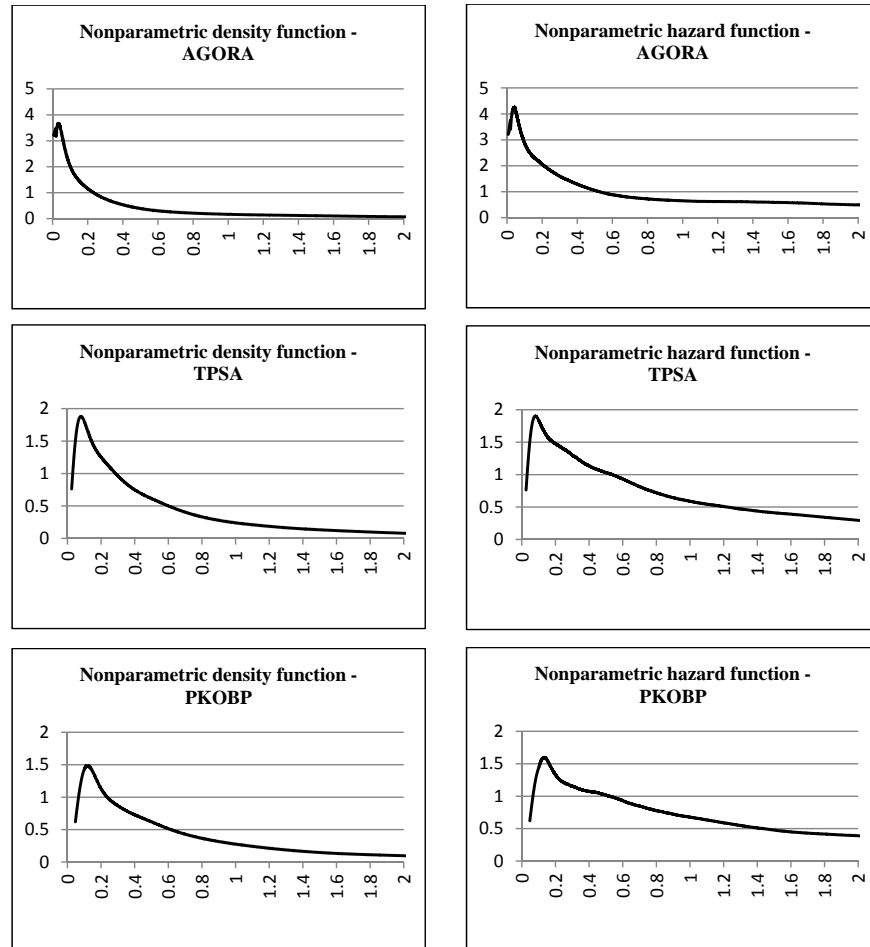
Figure 3: Trade durations devoid of the seasonality effect for the analysed companies – first 5000 observations



of models with the conditional generalized gamma distribution, the characteristics clearly indicate that the posterior distributions of the parameters γ are located on the far left of the value $\gamma = 1$ and reveal a small dispersion. The posterior distributions of the parameters ν , however, are on the far right of the value $\nu = 1$ and that despite the

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Figure 4: Nonparametric density functions and nonparametric hazard functions for trade durations devoid of the seasonality effect for the analysed companies



relatively high level of standard deviations equalling 1, with the posterior means equal to about 5.2 (for the GGam-LACD model, the mean and the standard deviation reach even 7.85 and 1.95) the values $\nu = 1$ are practically improbable. This is confirmed by the histograms of marginal posterior distributions of the parameters γ and ν in the GGam-BCACD model presented in Figure 5. In the case of AGORA we observe that the characteristics of the central tendency of the parameter κ are lower and, at the same time, the posterior means of the parameter η are higher in comparison to the models for TPSA. Hence, the value of the dispersion index of the conditional

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Table 3: Posterior means and standard deviations (in parentheses) of parameters in all ACD models for TPSA company

	M_1 Burr-ACD	M_2 Burr - LACD	M_3 Burr - BCACD	M_4 Burr - As-LACD	M_5 Burr - ABCACD	M_6 Burr - AACD
ω	0.0260 (0.0036)	0.0852 (0.0066)	0.0086 (0.0066)	0.0499 (0.0514)	0.0142 (0.0089)	0.0448 (0.0055)
α	0.1474 (0.0113)	0.1167 (0.0076)	0.0432 (0.0099)	-0.0971 (0.0275)	0.0373 (0.0111)	0.0211 (0.0123)
β	0.8381 (0.0119)	0.9754 (0.0046)	0.9543 (0.0057)	0.9622 (0.0051)	0.9497 (0.0058)	0.9361 (0.0115)
δ_1	-	-	0.1762 (0.0461)	-	0.2202 (0.0644)	0.1056 (0.0644)
δ_2	-	-	0.6090 (0.0592)	-	0.5912 (0.0540)	0.5477 (0.0525)
b	-	-	-	0.8799 (0.3593)	0.1783 (0.0253)	0.1809 (0.0233)
c	-	-	-	-1.9389 (0.2580)	0.7119 (0.2329)	0.5952 (0.2502)
κ	1.2492 (0.0199)	1.2908 (0.0211)	1.2646 (0.0204)	1.2633 (0.0204)	1.2621 (0.0203)	1.2633 (0.0202)
η	0.5951 (0.0334)	0.6836 (0.0358)	0.6161 (0.0341)	0.6139 (0.0338)	0.6097 (0.0339)	0.6125 (0.0338)
	M_7 GGam-ACD	M_8 GGam - LACD	M_9 GGam - BCACD	M_{10} GGam - As-LACD	M_{11} GGam - ABCACD	M_{12} GGam - AACD
ω	0.0299 (0.0040)	0.0692 (0.0051)	0.0077 (0.0064)	0.0566 (0.0614)	0.0117 (0.0083)	0.0504 (0.0061)
α	0.1288 (0.0100)	0.1097 (0.0076)	0.0512 (0.0100)	-0.0912 (0.0302)	0.0460 (0.0111)	0.0195 (0.0115)
β	0.8417 (0.0124)	0.9668 (0.0054)	0.9462 (0.0065)	0.9555 (0.0061)	0.9412 (0.0067)	0.9306 (0.0118)
δ_1	-	-	0.2267 (0.0536)	-	0.2975 (0.0707)	0.1064 (0.0602)
δ_2	-	-	0.5971 (0.0639)	-	0.5882 (0.0581)	0.5284 (0.0535)
b	-	-	-	0.9981 (0.4764)	0.1737 (0.0246)	0.1813 (0.0222)
c	-	-	-	-1.8690 (0.2631)	0.6994 (0.2433)	0.6872 (0.2389)
γ	0.1542 (0.0232)	0.0942 (0.0229)	0.1385 (0.0228)	0.1418 (0.0229)	0.1407 (0.0237)	0.1401 (0.0235)
ν	4.6792 (0.7503)	7.8460 (1.9533)	5.2716 (0.9172)	5.1357 (0.8537)	5.2066 (0.9624)	5.2209 (0.9552)

distribution is higher (compared to TPSA) while distribution’s modal value is still located in the proximity of zero. This is not surprising, given that the dispersion index for the AGORA data series exceeded 2. On the other hand, in the case of models with the conditional generalized gamma distribution, the posterior distributions of the parameters γ are now located slightly to the left of $\gamma = 0.1$. The posterior means of the parameters γ are approximately 0.05-0.09 (except for the linear ACD model). The posterior distributions of the parameters ν in most models tend to be centrally located around 5-5.2, with standard deviations of 1-1.15. Lower values of the posterior means of the parameter γ compared to the TPSA models with unchanged posterior means of the parameter ν allow for greater dispersion of the conditional distributions.

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Table 4: Posterior means and standard deviations (in parentheses) of parameters in all ACD models for AGORA company

	M_1 Burr-ACD	M_2 Burr-LACD	M_3 Burr-BCACD	M_4 Burr-As-LACD	M_5 Burr-ABCACD	M_6 Burr-AACD
ω	0.0552 (0.0054)	0.3590 (0.0283)	0.0115 (0.0099)	0.4329 (0.0315)	0.0142 (0.0116)	0.0949 (0.0090)
α	0.3628 (0.0168)	0.2503 (0.0099)	0.0968 (0.0162)	-0.5704 (0.0628)	0.1054 (0.0166)	0.0472 (0.0248)
β	0.6347 (0.0167)	0.9241 (0.0078)	0.9108 (0.0083)	0.8863 (0.0092)	0.8998 (0.0086)	0.8666 (0.0217)
δ_1	-	-	0.0649 (0.0132)	-	0.1435 (0.0235)	0.0627 (0.0339)
δ_2	-	-	0.2158 (0.0237)	-	0.3001 (0.0193)	0.2994 (0.0234)
b	-	-	-	0.7589 (0.0781)	0.0741 (0.0066)	0.0739 (0.0064)
c	-	-	-	-1.0730 (0.0117)	0.9812 (0.0231)	0.9675 (0.0371)
κ	0.9356 (0.0134)	1.0571 (0.0173)	1.0253 (0.0167)	1.0114 (0.0166)	1.0196 (0.0162)	1.0231 (0.0166)
η	0.4678 (0.0225)	0.7735 (0.0371)	0.6841 (0.0356)	0.6739 (0.0356)	0.6645 (0.0335)	0.6760 (0.0344)
	M_7 GGam-ACD	M_8 GGam-LACD	M_9 GGam-BCACD	M_{10} GGam-As-LACD	M_{11} GGam-ABCACD	M_{12} GGam-AACD
ω	0.0683 (0.0066)	0.2429 (0.0128)	0.0133 (0.0119)	0.3670 (0.0341)	0.0190 (0.0146)	0.1175 (0.0095)
α	0.3778 (0.0198)	0.2326 (0.0095)	0.1164 (0.0100)	-0.4743 (0.0520)	0.1196 (0.0185)	0.0427 (0.0230)
β	0.6141 (0.0189)	0.9002 (0.0091)	0.8874 (0.0096)	0.8644 (0.0104)	0.8761 (0.0092)	0.8449 (0.0213)
δ_1	-	-	0.0848 (0.0171)	-	0.1844 (0.0310)	0.0639 (0.0352)
δ_2	-	-	0.2073 (0.0258)	-	0.2911 (0.0203)	0.2941 (0.0221)
b	-	-	-	0.8514 (0.0920)	0.0866 (0.0069)	0.0889 (0.0065)
c	-	-	-	-1.0776 (0.0123)	0.9841 (0.0211)	0.9781 (0.0297)
γ	0.1562 (0.0175)	0.0543 (0.0153)	0.0869 (0.0171)	0.0865 (0.0176)	0.0908 (0.0171)	0.0873 (0.0170)
ν	2.7732 (0.3154)	8.6511 (2.7272)	5.2181 (1.1372)	5.1822 (1.1644)	4.9882 (1.0316)	5.1963 (1.1458)

As regards PKOBP, descriptive characteristics of posterior marginal distributions of the parameters κ and η are slightly bigger than those obtained for TPSA. Marked differences compared to the previous companies were ascertained for posterior marginal distributions of the parameters of the conditional generalized gamma distribution. The posterior distributions of the parameters γ are now located in the very close vicinity of zero. The posterior means of the parameters γ range from 0.022 to 0.045. The posterior distributions of the parameters ν , however, are located on the far to the right of the $\nu = 1$ and reveal overdispersion. The posterior means of the parameters ν range from 19.8 for the GGam-ACD model to 37.8 for the GGam-LACD model. Posterior standard deviations range from 9.86 in the GGam-ACD model to

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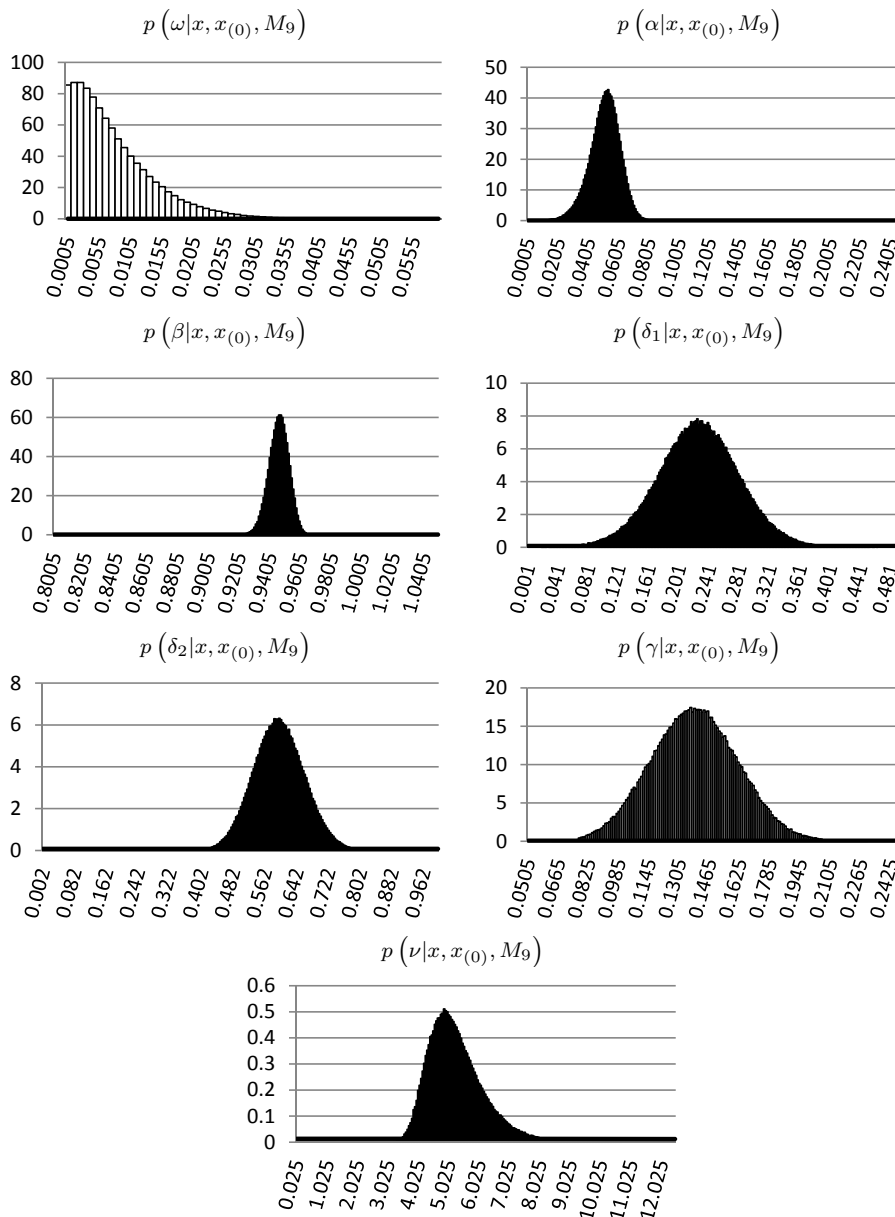
Table 5: Posterior means and standard deviations (in parentheses) of parameters in all ACD models for PKOBP company

	M_1 Burr-ACD	M_2 Burr - LACD	M_3 Burr - BCACD	M_4 Burr - As-LACD	M_5 Burr - ABCACD	M_6 Burr - AACD
ω	0.0424 (0.0055)	0.0850 (0.0068)	0.0091 (0.0068)	0.1049 (0.0283)	0.0154 (0.0093)	0.0440 (0.0055)
α	0.1267 (0.0093)	0.1024 (0.0065)	0.0362 (0.0102)	- 0.0999 (0.0166)	0.0366 (0.0126)	0.0514 (0.0285)
β	0.8533 (0.0104)	0.9748 (0.0044)	0.9619 (0.0051)	0.9629 (0.0051)	0.9557 (0.0054)	0.9159 (0.0235)
δ_1	-	-	0.1193 (0.0375)	-	0.1712 (0.0615)	0.2187 (0.1186)
δ_2	-	-	0.4332 (0.0555)	-	0.4094 (0.0553)	0.3860 (0.0570)
b	-	-	-	1.0314 (0.2048)	0.2112 (0.0218)	0.2020 (0.0145)
c	-	-	-	- 1.5268 (0.1089)	0.7122 (0.1844)	0.5625 (0.2003)
κ	1.2991 (0.0219)	1.3550 (0.0233)	1.3243 (0.0228)	1.3247 (0.0232)	1.3196 (0.0229)	1.3208 (0.0227)
η	0.6665 (0.0373)	0.7724 (0.0398)	0.7053 (0.0387)	0.7061 (0.0392)	0.6932 (0.0383)	0.6958 (0.0385)
	M_7 GGam-ACD	M_8 GGam - LACD	M_9 GGam - BCACD	M_{10} GGam - As-LACD	M_{11} GGam - ABCACD	M_{12} GGam - AACD
ω	0.0511 (0.0060)	0.0679 (0.0051)	0.0093 (0.0076)	0.0836 (0.0236)	0.0157 (0.0103)	0.0525 (0.0065)
α	0.1061 (0.0078)	0.0935 (0.0063)	0.0468 (0.0115)	- 0.0983 (0.0180)	0.0493 (0.0149)	0.0453 (0.0257)
β	0.8556 (0.0105)	0.9651 (0.0052)	0.9516 (0.0061)	0.9546 (0.0056)	0.9446 (0.0066)	0.9107 (0.0224)
δ_1	-	-	0.1626 (0.0480)	-	0.2650 (0.0833)	0.2223 (0.1199)
δ_2	-	-	0.4052 (0.0614)	-	0.4175 (0.0552)	0.3791 (0.0561)
b	-	-	-	0.9688 (0.1855)	0.2276 (0.0366)	0.2191 (0.0239)
c	-	-	-	- 1.4626 (0.0990)	0.7845 (0.1982)	0.7217 (0.1928)
γ	0.0453 (0.0198)	0.0225 (0.0085)	0.0339 (0.0146)	0.0354 (0.0143)	0.0373 (0.0189)	0.0346 (0.0164)
ν	19.8357 (9.8651)	37.2879 (13.9249)	26.2235 (12.2436)	24.3536 (9.8601)	25.5310 (13.4491)	26.7087 (13.0379)

13.92 in GGam-LACD model. In the best GGam-BCACD model, the posterior mean and standard deviation stand at 26.22 and 12.24 respectively. The very high posterior standard deviations of the parameters ν leave a lot of doubt about the values of these parameters, and thus the shape of the conditional density function. It should also be emphasised that the marginal posterior distributions $p(\nu|x, x_{(0)}, M_i)$ reveal right-sided asymmetry and in the case of the GGam-BCACD model the modal value of the distribution equals approximately 18. Little does it change the fact that in the case of PKOBP the most probable values of the parameter ν are much higher than they are for TPSA and AGORA. In addition, the conditional distributions parameters are always on a similar level regardless of the specification of the conditional duration

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Figure 5: Marginal posteriors (bars) and priors (solid lines) of parameters of the GGam-BCACD model for TPSA company



equation. Furthermore, in such configurations of conditional distributions parameters estimates, the data favour non-monotonic conditional hazard functions of durations. The results clearly indicate that the effect of constant conditional duration in the considered ACD models corresponding to the assumption $\alpha = 0$, $\beta = 0$ is strongly rejected by the modelled series. The posterior marginal distributions of the parameters α or β are located far from zero. Above observations also indicate that durations are not generated by a Poisson process. In order to describe the dynamics of the analysed transaction durations the properties of probability distributions alone used in the models as conditional distributions do not suffice.

In the case of TPSA posterior marginal distributions of the parameters β are alike in all models. This parameter serves to measure the dependence of the expected duration at time t_i on Ψ_{i-1} . The expected transaction duration is characterized by a relatively strong persistence. The posterior means of the parameters β range in fact from 0.93 to 0.96, except for the linear ACD models whose posterior means stand at about 0.84. Note, however, that in the case of the linear ACD model it is the sum of the parameters $\alpha + \beta$ that is responsible for the persistence of the process. Posterior marginal distributions $p(\beta|x, x_{(0)}, M_i)$ reveal a very small dispersion. This is evidenced by posterior standard deviations of 0.01. Although the persistence of transaction durations is high for TPSA, the coefficients β do not border on 1. From the point of view of market microstructure, this may indicate that the market is dominated by well-informed investors. Liquidity investors' share of the market is small or these investors are not able to properly grasp the changing market conditions or are simply risk-averse and while away times of uncertainty associated with new information. Hence this weaker persistence. Should the coefficient β be very close to 1, it would mean that in the market uninformed traders mimic the moves of well-informed players, and hence we would see strong dependences in transaction intensity. Posterior marginal distributions of the parameters α are located close to zero and also seem to be centred on the posterior means, as evidenced by the low posterior standard deviations.

Analysis of the estimation results for AGORA company suggests that the expected trade duration reveals less persistence than does TPSA's. At the same time it reacts more strongly to new observations or new random disturbances. The posterior marginal distributions of the parameters β remain markedly on the right side of zero, and are heavily concentrated around the posterior means, as evidenced by the relatively small posterior standard deviations. The posterior means of the parameters β for AGORA range from 0.84 in the GGam-AAACD to 0.92 in the Burr-LACD model (except for the linear ACD models). On the other hand, for all the specification considered, the posterior marginal distributions of the parameters α are located farther away from 0 than they are in the case of TPSA and reveal very a small dispersion measured in terms of standard deviations. The smaller persistence of expected duration may be due to the small share of liquidity traders in the transaction process or to their strong risk aversion. A stronger reaction to new observations is

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probably due to the presence of more individual investors on the market. On the other hand, PKOBP's expected transaction duration reveals a much stronger persistence than in the case of AGORA and remains at a level similar to that for TPSA. At the same time, we see a weaker response to new random disturbances. The posterior marginal distributions of the parameters β are located near zero, and the posterior means range between 0.94 and 0.96. The distributions are highly concentrated around the posterior means, as evidenced by very low posterior standard deviations. The somewhat stronger duration persistence in the case of TPSA suggests that the market is dominated by well-informed players, but the number of liquidity traders is also growing. The latter are trying to emulate the decision of well-informed investors, but to confirm these suppositions deeper studies are required in order to take into account the volatility and volume. The posterior marginal distributions of the parameters α are located in the vicinity of zero and also seem to be centred around the posterior means, as evidenced by their low posterior standard deviations. We will now proceed to analyse the parameters of the Box and Cox transformation in the GGam-BCACD models. The posterior distributions of the parameters δ_1 are located on the far right of zero and to the left of 1 for each of three companies. The posterior mean for the parameter δ_1 for TPSA stands at about 0.22 and the posterior standard deviation is at about 0.05. The posterior mean for this parameter for AGORA equals 0.08 with posterior standard deviation not exceeding 0.02. In the case PKOBP the posterior mean of the parameter δ_1 is at the level of 0.16 and the standard deviation equals 0.048. Both the location and dispersion of the posterior marginal distributions $p(\delta_1|x, x_{(0)}, M_9)$ unambiguously indicate that the data reject linear and logarithmic specifications. The dispersion of distributions $p(\delta_1|x, x_{(0)}, M_9)$ measured in terms of posterior standard deviations indicates that the $\delta_1 = 0$ and $\delta_1 = 1$ values have practically no prior probability. It should also be admitted that the distributions edge closer toward zero, suggesting that logarithmic specifications could be more probable than linear model transformations. The results therefore corroborate legitimacy of the use of the Box and Cox transformation in the specifications considered.

Analysing, in turn, the characteristics of the posterior distributions of the parameter δ_2 of Box and Cox transformation in GGam-BCACD models we should note the location of the marginal distributions between $\delta_2 = 0$ and $\delta_2 = 1$?. Both the location and dispersion of the posterior distributions of the parameter δ_2 clearly indicate that the data for the companies surveyed favour concave news impact curves. Figure 6 displays the empirical news impact curves for selected ACD models with the conditional generalized gamma distribution for TPSA company, which represent conditional duration Ψ_i depending on the observed disturbance ε_{i-1} at the time t_{i-1} . In addition, plotting curves we established that the conditional duration Ψ_{i-1} at the time t_{i-1} equals 1 (see Fernandes and Grammig 2006), and substituted the posterior means for the parameters. It is very interesting that Ψ_i in each case reacts to new shock ε_{i-1} in a similar manner. The news impact curves of the nonlinear models are

concave. The concavity of the curves allows reducing the problem of overprediction after very short and very long durations ascertained in the case of the simplest linear model. In the case of concave shocks impact curves, the difference in duration of reaction to disturbances is stronger in the case of small disturbances than in large ones. It seems, therefore, that the concavity of the news impact curves is one of the most important properties that should be taken into account in the analysis of transaction durations for companies on the Polish stock market.

The concavity of functions of response to the random shocks is also confirmed by the characteristics of posterior distribution of parameters in the ABCACD and As-LACD models. For GGam-ABCACD models, posterior marginal distributions of the δ_2 parameter are located on the far side of the $\delta_2 = 1$. As far as the GGam-As-LACD models (and Burr-As-LACD models) are concerned, the posterior distributions of the α and c parameters tend to the left of zero, which ensures concavity of the news impact curves.

Figure 6: The estimated news impact curves for selected ACD models with the conditional generalized gamma distribution for TPSA company

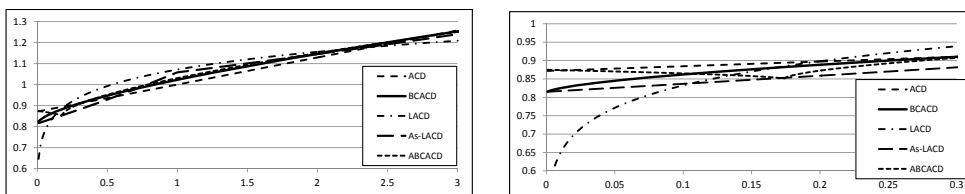
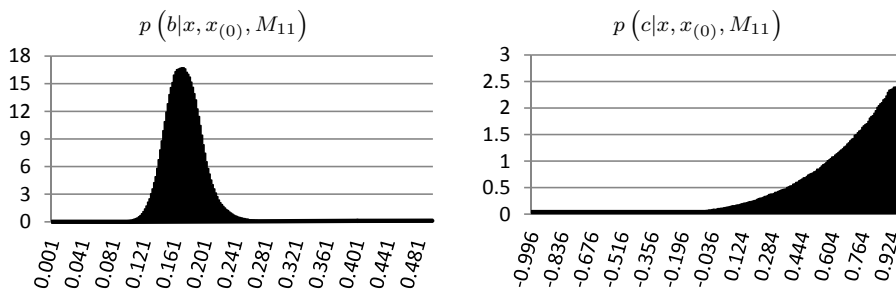


Figure 7: Marginal posteriors (bars) and priors (solid lines) of b and c parameters of the GGam-ABCACD model for TPSA company



Let us look at the characteristics of the posterior distributions of the parameters b and c in the GGam-ABCACD models. The marginal posterior distributions of the

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parameters b and c in the GGam-ABCACD models are displayed in Figure 7. In the case of TPSA the posterior mean of the distribution $p(b|x, x_{(0)}, M_{11})$ is approximately 0.17 and the standard deviation at about 0.025 is relatively small. The parameter b is therefore statistically significant. The posterior distribution $p(c|x, x_{(0)}, M_{11})$, in turn, is located around 0.7 and heavily dispersed. A similar situation is revealed for PKOBP. The posterior marginal distribution $p(b|x, x_{(0)}, M_{11})$ of the GGam-ABCACD model is located centrally at around 0.227, a value that is the farthest from zero of all three companies surveyed. The posterior distribution $p(c|x, x_{(0)}, M_{11})$ has posterior mean of 0.78 and reveals a large dispersion measured in terms of a standard deviation of about 0.2. Thus the large posterior standard deviations of the parameters c leave a lot of uncertainty about the values of the rotation parameters. On the other hand, an analysis of the marginal posterior distributions of the parameters b and c in the GGam-ABCACD model for AGORA can bring us to the conclusion that the parameter b is statistically significant, the same as in the case of TPSA, but the marginal posterior distribution $p(b|x, x_{(0)}, M_{11})$ this time edges much closer to zero. The posterior mean stands at 0.08 and the standard deviation of about 0.01 is very low. In turn, the posterior distribution $p(c|x, x_{(0)}, M_{11})$ is now highly concentrated around the posterior mean of about 0.98 due to the small standard deviation of 0.02. In addition, it is worth noting that the marginal posterior distributions of the parameter c reveal a strong left-sided asymmetry and mode values in very close proximity of the marginal value of $c = 1$. The empirical results also lead to the conclusion that in the case of the ABCACD model, which combines power transformation with the asymmetry of response to small and large shocks, the power transformation can play the dominant role in explaining the nature of duration dynamics and causes a problem with unambiguous identification of the rotation parameter by data. Allowing an asymmetric response to a disturbance does not have to increase the explanatory power of the model.

An important issue for the use of Monte Carlo methods based on Markov chains involves analysis of the convergence speed of the simulated chains. Figure 8 shows the performance of the CUMSUM statistics after rejection of burnt-in states in the case of the GGam-BCACD model for TPSA. In the case of TPSA, the values of CUMSUM statistics were calculated after a prior rejection of 200 000 burnt-in states and relied on 2 million Gibbs' states. A very fast convergence of the cumulative sums for the parameters of the equation defining the expected duration is manifest. CUMSUM statistics range between (-0.05, 0.05) already after about 60,000 Gibbs' states. In the case of the parameters γ and ν we see a slower convergence in the chain. This may result from a strong posterior correlation between these parameters. Assuming a relative error of $\varepsilon = 0.05$, convergence of CUMSUM statistics can be noted after about 160 000 Gibbs' states. Similar results were reported for the GGam-BCACD model for AGORA. In the case of PKOBP convergence of the chains was significantly slower than for the less liquid companies – TPSA and AGORA. To achieve satisfactory estimation results the values of CUMSUM statistics were calculated based on 4 million

Gibbs' draws generated after the rejection of as many as 500 000 burnt-in states i.e. twice more than in previous empirical illustrations. The γ and ν parameters revealed the slowest convergence of chains. This results of course from a strong posterior correlation between these parameters, but it can be gathered that also the high degree of uncertainty as to the value of the parameter ν is partly to blame. Assuming $\varepsilon = 0.05$, the convergence of CUMSUM statistics was determined after about 1.2 million Gibbs' draws. Assuming, in turn, a relative error $\varepsilon = 0.1$ the convergence of CUMSUM statistics was reported earlier – after about 500 000 Gibbs' states. There was a faster convergence of the cumulative sums for the parameters of the equation defining the expected duration, although the CUMSUM statistics ranged between $(-0.05, 0.05)$ only after about 400 000 Gibbs' states. It is also worth mentioning that in the case of all three companies, simpler models i.e. those that were less parameterized revealed faster convergence. Augmented models i.e. the ABCACD and AACD models revealed slower convergence and required more burnt-in states.

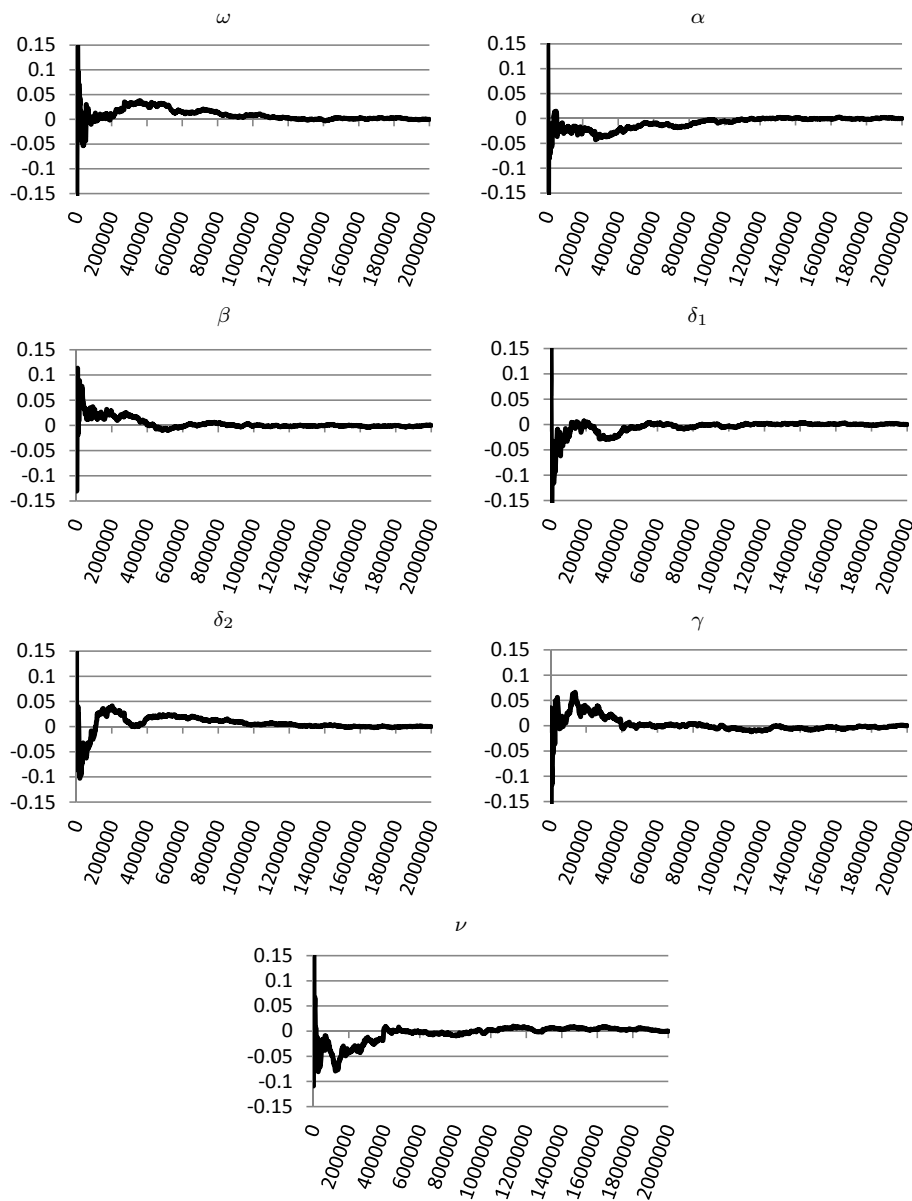
5.2.2 Bayesian prediction

We will now present the forecasting properties of Bayesian ACD models. In the light of the results obtained predictive distributions of durations were determined solely on the basis of the GGam-BCACD models. We forewent the construction of predictive distributions based on the Bayesian technique of knowledge combination. Instead of presenting forecasts of waiting times between successive transactions i.e. x_{T+k} we decided to designate and present forecasts (predictive distributions) of times of occurrence of subsequent k transactions (which are rather intuitive in these analyses). Let it be remembered again that pursuant to the literature on the subject, we applied modelling using ACD specifications to transaction durations purged of intraday seasonality patterns. We therefore predicted the times of occurrence of subsequent k transactions taking into account the seasonality component, i.e. forecasts defined by the formula $\tilde{y}_{T+k} = \sum_{i=1}^k \tilde{x}_{T+i} = \sum_{i=1}^k x_{T+i} \cdot \phi(t_{x_{T+i}})$, where $k = 1, 2, 3, 4, 5, 10, 20, 100$ (forecast horizons), and x_{T+i} stands for the forecast of trade duration between transaction $T+i-1$ and transaction $T+i$, ignoring the seasonality effect resulting from the time of day, and $\phi(t_{x_{T+i}})$ is the value of the function describing the seasonal component at time $t_{x_{T+i}}$. Table 6 presents quantiles of predictive distributions. Figure 9 shows histograms of marginal predictive distributions for the trade durations \tilde{y}_{T+k} ($k = 1, 2, 3, 4, 5, 10, 20, 100$) using TPSA company as an example. In the graphs, realised durations \tilde{y}_{T+k} are marked as black triangles. The histograms of marginal predictive distributions for durations for AGORA and PKOBP companies look the same, hence they are not included in the diagrams.

In general, we observe a very wide dispersion of predictive distributions of transaction durations. Their dispersion measured in terms of their inter-quartile range increases with the forecast horizon, which is due to the nature of the forecast and the consequent accumulation of prediction errors. Marginal predictive distributions are

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Figure 8: The CUMSUM statistics after rejection of burnt-in states for parameters in the GGam-BCACD model for TPSA company



generalized gamma type distributions wherein an increase in the forecast horizon sees an inclination towards normal distribution.

In the case of TPSA actual durations y_{T+k} range between the median and the 0.75-quantile of the predictive distribution for the y_{T+1} , the 0.25-quantile and the median for y_{T+2} and y_{T+3} , between the 0.1- and 0.25-quantiles for y_{T+4} and y_{T+5} . In the case of "long-term" forecasts, the observed times stand, in turn, between the 0.75- and 0.9-quantiles for y_{T+10} , slightly below the 0.75-quantile for y_{T+20} and above the 0.9-quantile for y_{T+100} . Assuming a point forecast at the median value, the relative posterior errors would be very high and would thus suggest an inconsistency between the forecasts and reality. Similar conclusions can be drawn for AGORA and PKOBP. It should be emphasised, however, that the significant dispersion and heavy right-sided tails of the predictive distributions cause the point forecasts at the level of the median to be obviously burdened with *ex ante* errors, which implies serious uncertainty about durations' future values (the outlying bars in the histograms, although not to be seen in the diagram, are located at the very end of the horizontal axis). Due to the strong asymmetry of predictive distributions, in the case of short-term forecasts we would report a much better accuracy if we considered the point forecasts at their modal value.

TPSA's posterior analysis shows that the observed values of the cumulative transaction duration occupy areas of high predictive density, even for long forecast horizons. It can therefore be concluded that in that sense short-term and long-term forecasts based on the GGam-BCACD model are accurate. AGORA's and PKOBP's situation is different, though. As far as "long-term" forecasts for $k = 4, 5, 10, 20, 100$ horizons are concerned, in the case PKOBP the probabilities of realisation above the observed values stand at 0.016, 0.003, 0.003, 0.013 and 0.004 respectively, and in the case of AGORA at below 0.06. Thus the posterior analyses show that the observed values of the cumulative transaction duration for long horizons are not located in areas of high predictive density and in that sense "long-term" forecasts are not accurate. Predictive distributions are, however, characterized by very fat right tails and overdispersion. Therefore, forecasting the precise occurrence of k consecutive transactions on the basis of the process's past seems to be an extremely challenging task.

6 Concluding remarks

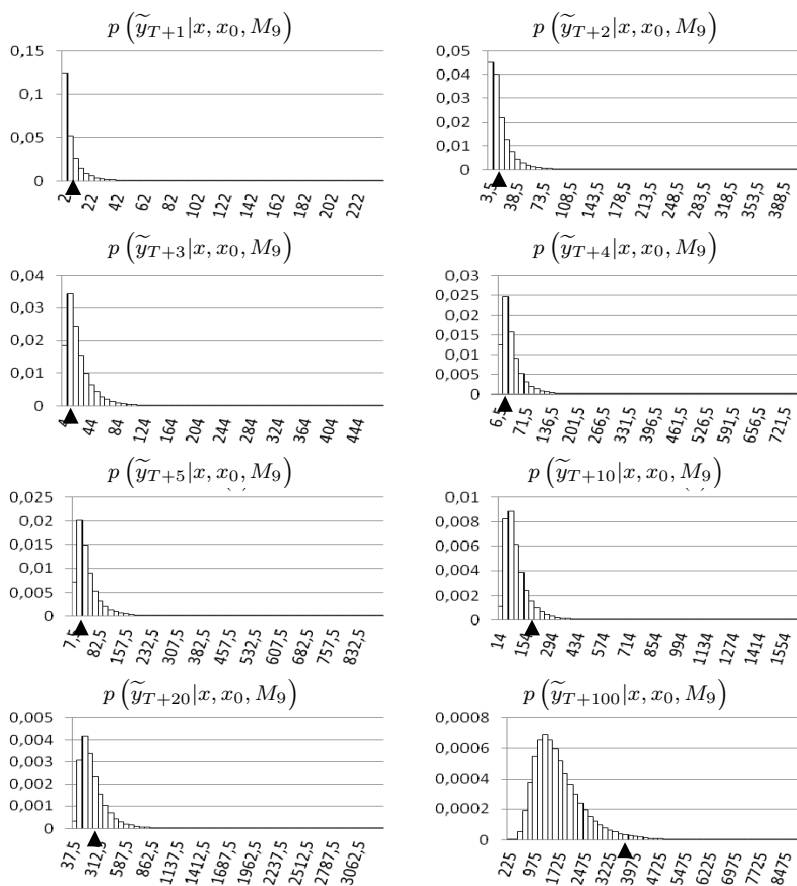
The main aim of this paper was to develop and apply the Bayesian approach to the estimation and testing of predictive capabilities of autoregressive conditional duration (ACD) models, as well as to practically use Bayesian ACD models to analyse transaction duration dynamics of selected companies listed on the Polish stock exchange. It is worth mentioning again that the Bayesian methodology was not used thus far as an alternative method for the estimation of and prediction within

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Table 6: Quantiles of predictive distributions of the cumulative trade durations taking into account the seasonality component for TPSA company

Quantiles	\tilde{y}_{T+1}	\tilde{y}_{T+2}	\tilde{y}_{T+3}	\tilde{y}_{T+4}	\tilde{y}_{T+5}	\tilde{y}_{T+10}	\tilde{y}_{T+20}	\tilde{y}_{T+100}
0.1	1	4	7	11	15	39	98	913
0.25	2	6	11	17	23	55	135	1164
0.5	5	12	19	27	36	83	197	1538
0.75	10	21	33	45	58	128	295	2057
0.9	20	37	55	73	91	193	431	2713
Real values	9	10	11	12	21	166	287	3727

Figure 9: Histograms of marginal predictive distributions of the cumulative trade durations taking into account the seasonality component for TPSA company



competing models in relation to the whole class of the ACD models discussed in the paper. In the context of foreign and Polish literature this application of the proposed approach therefore constitutes a new outlook on the issue of ACD models estimation. The results of the Bayesian inference for ACD models carried out on the basis of time series of transaction durations on the Polish stock exchange proved to be similar for all the companies surveyed. The empirical results allow us to conclude that the conditional distributions of transaction durations are far from being either an exponential distribution or a Weibull distribution. In addition, the generalized gamma distribution can be more flexible in modelling the conditional distribution of transaction durations than the Burr distribution. The results obtained clearly indicate that the effect of constant conditional duration in the models is strongly rejected by all the series under consideration. The posterior means and posterior standard deviations of the parameters α and β in the specifications under consideration are indicative of a significant deviation from this effect. Properties of probability distributions alone used in the models as conditional distributions do not suffice, therefore, to describe the dynamics of the analysed transaction durations. These observations also show that transaction durations are not distributed by the Poisson process. Analysis of the estimation results of ACD models for each of the companies indicate that in the light of the data the Box-Cox ACD model with the conditional generalized gamma distribution seems to be the most appropriate model for determining transaction durations but formal comparison of considered models is required. It occurred that the model's ability to ensure a concave news impact curve is crucial for the modelling of transaction durations on the Polish market. One can say that from the viewpoint of a model's parameter estimation, the dynamics of expected duration for companies with less liquidity are not significantly different from the dynamics ascertained for liquid companies. The nature of the dynamics is similar for all companies. It is also worth noting that what was essential in the construction of marginal posterior distributions was the information embedded the observed data, and not the initially adopted prior distributions themselves. Posterior distributions are characterized by a different location and a markedly smaller dispersion than prior distributions. The results of empirical research required the use of numerical methods. Monte Carlo methods based on Markov chains proved to be an effective tool for the approximation of the characteristics of posterior and predictive distributions. Due to the high variability of transaction durations, forecasts are subject to considerable *ex ante* uncertainty. Marginal predictive distributions are in fact highly dispersed as evidenced by the fat tails revealed by these distributions.

The main finding arising from the empirical research is that ACD models provide an adequate description tool for the dynamics of transaction durations and can be used for modelling. It should be noted, however, that a formal Bayesian comparison of ACD models' explanatory power (based on Bayes factors) is required in order to confirm the markedly higher adequacy of nonlinear ACD models than that of the simplest variant from this class – the linear ACD model. This challenge will be faced

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in the future research. On the basis of the results one can also state that the Bayesian estimation and prediction methods provide a universal and convenient inference tool.

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