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Evaluation of temperature oscillation experiment for the determination of heat transfer coefficient and dispersive Peclet number

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Abstract An evaluation method is developed for temperature oscillation experiments in heat exchangers. The unity Mach number dispersion model is applied. For the consideration of lateral wall heat conduction an effective wall thickness is introduced together with a wall heat transfer coefficient. The evaluation method may also be applied to single blow experiments with pulse signals. A sensitivity analysis describes and discusses the accuracy of different evaluation procedures.

Keywords: Heat exchanger; Temperature oscillation experiment; Evaluation method; Dispersion model

Nomenclature

A	–	area, m^2
a	–	exponent (complex)
a_w	–	thermal diffusivity of wall, m^2/s
B	–	capacity ratio, $B = V\rho c_p/V_w\rho_w c_w = C/C_w$
C	–	capacity, J/K; dummy variable; propagation velocity, m/s
c	–	specific heat capacity, J/(kg K)

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D	–	dummy variable
F	–	transfer function
f	–	function
i	–	imaginary unit $i = \sqrt{-1}$
k	–	counter
L	–	length of flow path, m
Ma	–	dispersive thermal Mach number, $\mathrm{Ma} = w/C$
M	–	coefficient matrix
N	–	number of transfer units, $N = \alpha A \big/ \dot{W}$
Nu	–	Nusselt number
n_t	–	number of temperature measuring points
Pe	–	dispersive Peclet number, $\mathrm{Pe} = \frac{wL\rho c_p}{\lambda_d} = \frac{\dot{W}L}{A_c\lambda_d}$
$\dot{q}$	–	lateral wall heat flux, W/m^2
$\dot{q}_x$	–	axial dispersive heat flux, W/m^2
Re	–	Reynolds number
S	–	dummy variable
s	–	Laplace variable
T	–	dimensionless fluid temperature outside the heat exchanger, $T = \frac{\theta - \vartheta_0}{\Delta\vartheta^*}$
t	–	dimensionless fluid temperature inside the heat exchanger, with index w wall temperature, $t = \frac{\vartheta - \vartheta_0}{\Delta\vartheta^*}$
U	–	amplitude, K
V	–	volume of fluid inside the flow channel, m^3
V	–	variable, (A3)
$\dot{V}$	–	volumetric flow rate, m^3/s
W	–	dummy variable
$\dot{W}$	–	heat capacity rate, W/K
w	–	mean flow velocity, m/s
X	–	dimensionless wall thickness
x	–	dimensionless flow length, $0 \le x \le 1$
z	–	dimensionless time coordinate, $z = \tau/\tau_R$

Greek symbols

α	–	heat transfer coefficient, W/(m^2 K)
α	–	coefficient, (A1)
β	–	coefficient, (A1)
β	–	sensitivity factor
γ	–	dummy variable
δ	–	wall thickness, m
ϵ	–	sensitivity factor
η	–	dimensionless dispersive heat flux, $\eta = \frac{\dot{q}_x L}{\lambda_d \Delta\vartheta^*}$
θ	–	fluid temperature outside the heat exchanger, K
ϑ	–	fluid temperature inside the heat exchanger, with index w wall temperature, K
ϑ	–	sensitivity factor
$\Delta\vartheta^*$	–	arbitrary temperature difference, K
κ	–	sensitivity factor
λ	–	thermal conductivity, W/(m K)

ξ	–	length coordinate, m
ρ	–	density, m^3/kg
σ	–	sensitivity factor
τ	–	time coordinate, s
τ_R	–	residence time, $\tau_R = V/\dot{V}$, s
φ	–	phase
$\Delta\varphi$	–	phase shift
Ω	–	angular frequency, s^{-1}
ω	–	dimensionless angular frequency, $\omega = \Omega\tau_R$

Subscripts and superscripts

A	–	at the heat transfer surface
c	–	cross-section
d	–	dispersive
p	–	isobaric or period
w	–	wall
0	–	$x = 0$ (inlet) or τ =0 or $D_3 = 0$ or mean value of oscillation
1	–	$x = 1$ (outlet)
–	–	Laplace transform,
$\sim$	–	effective value

1 Introduction

Recently an evaluation method has been proposed for the evaluation of single blow pulse experiments [1]. The method is based on the new unity Mach number dispersion model [2], which is very convenient for design calculations and more appropriate to account for simultaneous maldistribution and axial mixing than the original parabolic dispersion model. The measured temperature pulses are evaluated in the Laplace frequency domain which is only possible if the pulse signals go back to the initial value within the measurement region. The method presumes thermally thin walls, i.e., no axial wall heat conduction and no lateral conduction resistance. If these assumptions are not fulfilled a temperature oscillation technique may be preferable. Several papers have been published on evaluation methods and experiments using temperature oscillations [3–5]. They all apply the original parabolic dispersion model. In this paper the evaluation method is developed which is based on the unity Mach number dispersion model [2]. Axial and lateral wall heat conduction are taken into account.

2 Analysis of the temperature oscillation experiment

2.1 Thermally thin walls

The analysis is based on the previously developed single blow evaluation method [1]. The governing equations are repeated here

$$\frac{\partial t}{\partial z} + \frac{\partial t}{\partial x} + \frac{1}{\mathrm{Pe}}\frac{\partial \eta}{\partial x} + N\left(t - t_w\right) = 0\ , \tag{1}$$

$$\eta + \frac{\mathrm{Ma}^2}{\mathrm{Pe}}\left(\frac{\partial \eta}{\partial z} + \frac{\partial \eta}{\partial x}\right) = -\frac{\partial t}{\partial x}\ , \tag{2}$$

$$\frac{\partial t_w}{\partial z} = NB\left(t - t_w\right)\ , \tag{3}$$

$$\begin{array}{lcl} x = 0,\ x = 1 & : & T = t + \frac{\eta}{\mathrm{Pe}}\ , \\ \tau \le 0,\ z \le 0 & : & T = t = t_w = 0\ , \\ \tau > 0,\ z > 0 & : & T\left(x = 0\right) = T_0 = f\left(z\right)\ . \end{array} \tag{4}$$

Equation (1) is the energy equation for the fluid with consideration of the hyperbolic dispersion model [2]. Equation (2) describes the axial dispersive heat flux for the special case of dispersive Mach number Ma = 1. Equation (3) is the energy equation of the thermally thin wall (no heat conduction in the axial direction, no conductive resistance perpendicular to the heat transfer surface A). This case is first considered. Wall heat conduction is considered later in this paper. Equations (4) contain inlet, outlet and initial conditions. The Laplace transform solution yields the transfer function

$$F\left(s\right) = \frac{\overline{T}_1\left(s\right)}{\overline{T}_0\left(s\right)} = \frac{\int_0^\infty T_1 e^{-sz}\mathrm{d}z}{\int_0^\infty T_0 e^{-sz}\mathrm{d}z} = e^{-a(s)}\ , \tag{5}$$

with

$$\frac{1}{a\left(s\right)} = \frac{1}{s + \frac{1}{\frac{1}{N} + \frac{B}{s}}} + \frac{1}{\mathrm{Pe} + s}\ . \tag{6}$$

The Laplace solution is now applied to the temperature oscillation experiment. The harmonic sinusoidal temperature oscillation

$$\theta\left(x, z\right) = \vartheta_0 + U\left(x\right)\sin\left[\omega z + \varphi\left(x\right)\right] \tag{7}$$

is considered, which is generated at the inlet $x = 0$ with amplitude U_0 and phase φ_0. The temperature ϑ_0 is now the temporal mean value of fluid and wall temperatures. This oscillation can also be extracted from an arbitrarily shaped periodic oscillation with the help of known Fourier series approximation (see appendix A1).

The amplitude attenuation and phase shift from inlet $x = 0$ to outlet $x = 1$ can be calculated using the Laplace solution Eqs. (5), (6) if s is replaced by the imaginary angular frequency

$$s = i\omega \ , \tag{8}$$

resulting in

$$a\,(i\omega) = \ln \frac{U_0}{U_1} + i\,(\varphi_0 - \varphi_1)\,. \tag{9}$$

Rearranging Eq. (6) with $s = i\omega$ yields

$$\ln \frac{U_0}{U_1} = a_r = \frac{D_1 D_3 + D_2 D_4}{D_3^2 + D_4^2} \ , \tag{10}$$

$$\varphi_0 - \varphi_1 = \Delta\varphi = \frac{D_2 D_3 - D_1 D_4}{D_3^2 + D_4^2} \ , \tag{11}$$

$$\begin{aligned} D_1 &= -\omega^2 \left[1 + \tfrac{N}{\mathrm{Pe}}\,(1+B)\right] , \\ D_2 &= \omega \left[N\,(1+B) - \tfrac{\omega^2}{\mathrm{Pe}}\right] , \\ D_3 &= NB - 2\tfrac{\omega^2}{\mathrm{Pe}} \ , \\ D_4 &= \omega \left[1 + \tfrac{N}{\mathrm{Pe}}(1+2B)\right] \ . \end{aligned} \tag{12}$$

2.2 Consideration of wall heat conduction

2.2.1 Axial heat conduction

First axial heat conduction is investigated. For this purpose an extended Laplace solution is developed, presuming zero lateral conduction resistance ($t_w = t_w(x,z)$). All necessary equations for the calculation of the transfer function $F(s) = e^{-a(s)}$ are given in the appendix, Eqs. (A2). Substituting $s = i\omega$ into the Eqs. (A2) yields $a(i\omega) = -\ln F(i\omega)$ and Eq. (9) the required values of $a_r = \ln(U_0/U_1)$ and $\Delta\varphi$. Calculations show that in most cases particularly with liquids the axial wall heat conduction can be neglected.

2.2.2 Lateral heat conduction

The differential equation for the one-dimensional transient heat conduction in a plain wall, cylinder or sphere is

$$\frac{1}{a_w}\frac{\partial \vartheta_w}{\partial \tau} = \frac{\partial^2 \vartheta_w}{\partial \xi^2} + \frac{m}{\xi}\frac{\partial \vartheta_w}{\partial \xi},$$
$$m = 0 : \text{plain wall},\ m = 1 : \text{cylinder}\ \ (\xi = r)\,,\ m = 2 : \text{sphere}\ \ (\xi = r)\ \ . \tag{13}$$

A plain wall of surface, A, and wall thickness, δ, is considered. The rear surface of the wall is adiabatic. At the front surface a sinusoidal temperature oscillation is generated

$$\vartheta_{wA} = U_A \sin\left(\Omega\tau\right)\ , \tag{14}$$

which causes the conductive heat flux at the surface

$$\dot{q}_A = -\,\lambda_w \frac{\partial \vartheta_w}{\partial \xi}\bigg|_{\xi=0}\ . \tag{15}$$

This periodic heat flux is calculated from the solution of Eq. (13). The wall of thickness δ and thermal diffusivity a_w is now replaced by a wall with infinite thermal diffusivity $a_w = \infty$ and thickness $\widetilde{\delta} \le \delta$. The oscillating uniform only time dependent wall temperature is denoted with $\widetilde{\vartheta}_w$. The periodic heat flux $\dot{q}_A$ is now expressed as

$$\dot{q}_A = \alpha_w\left(\vartheta_{wA} - \widetilde{\vartheta}_w\right) = \widetilde{\delta}\rho_w c_w \frac{\mathrm{d}\widetilde{\vartheta}_w}{\mathrm{d}\tau}\ . \tag{16}$$

This equation defines an internal conductive heat transfer coefficient, α_w, and an effective wall thickness, $\widetilde{\delta}$, which are determined such that the heat flux, $\dot{q}_A$, of Eq. (16) is equal to the real heat flux according to Eq. (15). The derivations show that with constant values of α_w and $\widetilde{\delta}$ the heat fluxes are equal, independent of time.

With the notations

$$\delta = \frac{V_w}{A},\ \widetilde{\delta} = \frac{\widetilde{V}_w}{A}, X = \frac{V_w}{A}\sqrt{\frac{\Omega}{2a_w}},\ \widetilde{X} = \frac{\widetilde{V}_w}{A}\sqrt{\frac{\Omega}{2a_w}}, \widetilde{\mathrm{N}}\mathrm{u}_w = \frac{\alpha_w \widetilde{V}_w}{\lambda_w A} \tag{17}$$

the effective wall thickness and internal heat transfer coefficient can be expressed in the dimensionless form as

$$\widetilde{X} = \frac{\left(\sinh^2 2X + \sin^2 2X\right)}{2\left(\sinh^2 X + \cos^2 X\right)\left(\sinh 2X + \sin 2X\right)}\ ,$$
$$\lim_{X\to\infty}\widetilde{X} = 1,\qquad \lim_{X\to 0}\widetilde{X} = X\ , \tag{18}$$

and

$$\widetilde{\mathrm{Nu}}_w = \frac{\left(\sinh^2 2X + \sin^2 2X\right)^2}{2\left(\sinh^2 2X - \sin^2 2X\right)\left(\sinh^2 X + \cos^2 X\right)^2} \, . \tag{19}$$

$$\lim_{X\to\infty} \widetilde{\mathrm{Nu}}_w = 2, \qquad \lim_{X\to 0} \widetilde{\mathrm{Nu}}_w = 3$$

For the full cylinder and full sphere corresponding equations are derived. They are given in the appendix, Eqs. (A3) and (A4).

Figure 1 shows the dimensionless effective wall thickness $\widetilde{X}$ and the wall Nusselt number, $\widetilde{\mathrm{Nu}}_w$, as function of X as well as $\widetilde{\mathrm{Nu}}_w$ as function of $\widetilde{X}$.

The previous solution, Eqs. (6)–(12), can further be applied as the effective capacity ratio

$$\widetilde{B} = \frac{V\rho c_p}{\widetilde{V}_w \rho_w c_w} = B\left(\omega = 0\right)\frac{X}{\widetilde{X}} \, , \tag{20}$$

is introduced together with the effective number of transfer units, which is formed with the overall heat transfer coefficient

$$\frac{1}{\widetilde{N}} = \left(\frac{1}{\alpha} + \frac{1}{\alpha_w}\right)\frac{\dot{W}}{A} = \frac{1}{N}\left(1 + \frac{\alpha}{\alpha_w}\right) . \tag{21}$$

The effective B depends on the frequency ω of the harmonic under consideration. The internal conductive wall resistance $1/\alpha_w$ can be expressed as a function of the effective B

$$\frac{1}{\alpha_w} = \frac{1}{\widetilde{B}\,\widetilde{\mathrm{Nu}}_w\,A}\frac{V\rho c_p}{\rho_w c_w \lambda_w} \, . \tag{22}$$

The value of $\widetilde{\mathrm{Nu}}_w$ depends on the geometry of the walls or storage material and $\widetilde{X}$. Usually the resistance $1/\alpha_w \ll 1/\alpha$, and a rough estimation of $\widetilde{\mathrm{Nu}}_w$, e.g., $\widetilde{\mathrm{Nu}}_w = 1/3$, will be sufficient. Separate iterative calculations would provide the correct value.

Both one-dimensional methods for axial and lateral heat conduction can approximately be used for real combined cases, if axial conduction is calculated with the effective values $\widetilde{B}$ and $\widetilde{N}$. Inaccuracies of this approach may be compensated by slight corrections of $\widetilde{B}$ and $\widetilde{N}$.

2.3 Evaluation methods

Two evaluation methods are suggested and discussed in the following.

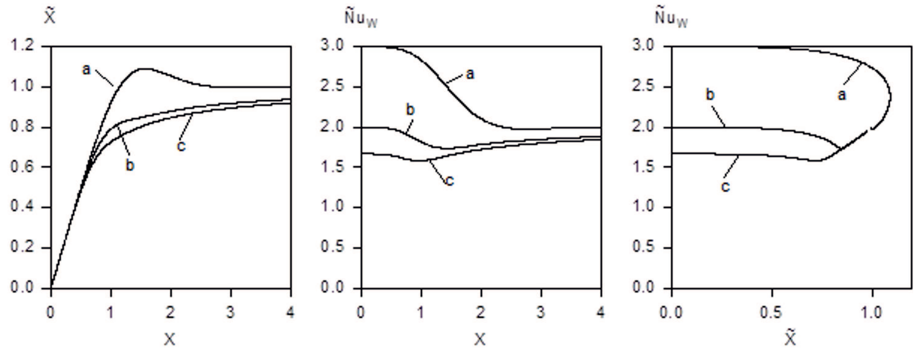

Figure 1: Left: Effective dimensionless wall thickness $\widetilde{X}$ as function of real dimensionless wall thickness X, Eqs. (18), (A3), and (A4). Middle: Wall Nusselt number $\widetilde{\mathrm{Nu}}_w$ as function of X, Eqs. (19), (A3), and (A4). Right: $\widetilde{\mathrm{Nu}}_w$ as function of $\widetilde{X}$, Eqs. (18), (19), (A3), and (A4). a: plain wall, b: full cylinder, c: full sphere. For $X \to \infty$: $\widetilde{X} \to 1$ and $\widetilde{\mathrm{Nu}}_w \to 2$ for a, b, and c.

2.3.1 Method 1

Method 1 is the standard method in which one oscillation is evaluated. Equating measured, (A1), and calculated (Eqs. (9)–(12), (17)–(22), (A3) and (A4)) values of a_r and $\Delta\varphi$ yields two equations for the two unknowns N and Pe. The residence time $\tau_R = V \big/ \dot{V}$ has to be measured separately, it is required for the calculation of the dimensionless angular frequency $\omega = \Omega\tau_R$. Also the effective capacity ratio $\widetilde{B}$ has to be determined separately from geometrical and thermophysical data and frequency. The obtained experimental N and Pe are affected by inaccuracies of a_r, $\Delta\varphi$, ω, and $\widetilde{B}$.

2.3.2 Method 2

The separate determination of τ_R and $\widetilde{B}$ is avoided with method 2 in which two oscillations of frequencies ω_1 and $\omega_2 = k\omega_1$, $k = 1, 2, 3, ...$, are commonly evaluated. The two harmonic oscillations can be extracted from one non-harmonic periodic oscillation, generated, e.g., by periodic on-off heating. One can also generate two oscillations (ω_1, ω_2) successively at constant fluid flow rate and mean temperature. Equating measured and calculated values of a_{r1}, $\Delta\varphi_1$, a_{r2}, and $\Delta\varphi_2$ yields 4 equations for the unknowns N, Pe, $\widetilde{B}_1$, and ω_1. The second capacity ratio $\widetilde{B}_2 = C\widetilde{B}_1$ with $C = 1$ for thin walls and $C > 1$ for thick walls. In the latter case C can be calculated (iteratively) as well as α_w from Eqs. (17)–(22), (A3), and (A4).

Both methods can also be applied to experiments with pulse signals, if the pulses are regarded as one half of a symmetrical periodic oscillation as shown in Fig. 2.

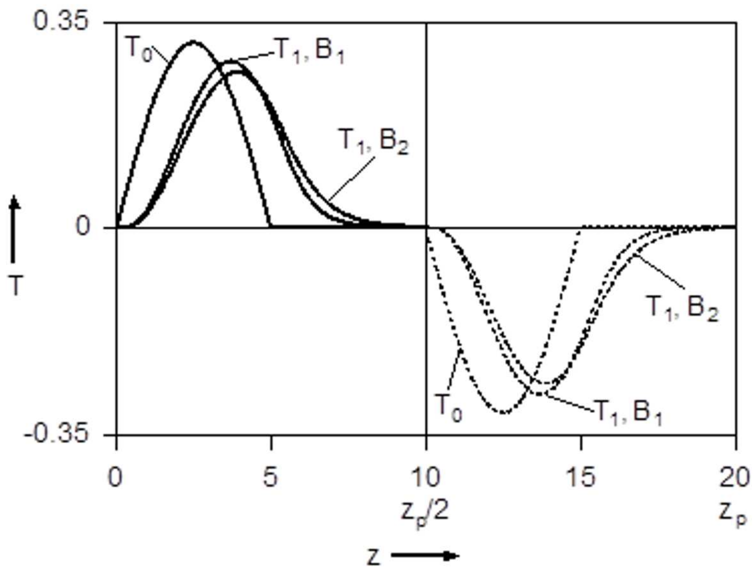

Figure 2: Pulse signals (left side) of example of ref [1], extended to the hypothetical periodic oscillation with $z_p = 20$. $N = 2.4$, $\mathrm{Pe} = 6$, $B_1 = 4$, $B_2 = 1.892$, $\omega = \pi/10$.

3 Sensitivity analysis

For the estimation of error propagation during the evaluation procedure the following sensitivity factors are defined

$$y = N,\ \mathrm{Pe},\ B,\ \omega\ ,$$
$$\varepsilon_y = \frac{\Delta y}{y}\frac{\omega}{\Delta\omega},\ \beta_y = \frac{\Delta y}{y}\frac{B}{\Delta B},\ \sigma_y = \frac{\Delta y}{y}\frac{1}{\Delta a_r},\ \kappa_y = \frac{\Delta y}{y}\frac{1}{\Delta\left(\Delta\varphi\right)}\ . \tag{23}$$

Their calculation is based on the series development (total differential) of the complex function $a = (a_r, \Delta\varphi)$ according to

$$\frac{1}{a} = \frac{1}{(a_r, \Delta\varphi)} = \frac{1}{(0,\omega) + \frac{1}{\frac{1}{N} + \frac{B}{(0,\omega)}}} + \frac{1}{(\mathrm{Pe},\omega)}\ , \tag{24}$$

$$\frac{\partial a}{\partial N}N = \frac{N a^2}{(N(1+B),\omega)^2} = (A_N, \phi_N) \ , \tag{25}$$

$$\frac{\partial a}{\partial \mathrm{Pe}}\mathrm{Pe} = \frac{\mathrm{Pe}\, a^2}{(\mathrm{Pe},\omega)^2} = (A_{\mathrm{Pe}}, \phi_{\mathrm{Pe}}) \ , \tag{26}$$

$$\frac{\partial a}{\partial B}B = \frac{-N^2 B a^2}{(0,\omega)(N(1+B),\omega)^2} = (A_B, \phi_B) \ , \tag{27}$$

$$\frac{\partial a}{\partial \omega}\omega = a^2(0,\omega)\left[\frac{1}{(\mathrm{Pe},\omega)^2} + \frac{(\omega,-NB)^2 - N^2 B}{\omega^2 (N(1+B),\omega)^2}\right] = (A_\omega, \phi_\omega) \ , \tag{28}$$

$$\Delta a_r = A_N \frac{\Delta N}{N} + A_{\mathrm{Pe}}\frac{\Delta \mathrm{Pe}}{\mathrm{Pe}} + A_B \frac{\Delta B}{B} + A_\omega \frac{\Delta\omega}{\omega} \ , \tag{29}$$

$$\Delta(\Delta\varphi) = \phi_N \frac{\Delta N}{N} + \phi_{\mathrm{Pe}}\frac{\Delta \mathrm{Pe}}{\mathrm{Pe}} + \phi_B \frac{\Delta B}{B} + \phi_\omega \frac{\Delta\omega}{\omega} \ . \tag{30}$$

The Eqs. (29) and (30) are applied to each frequency considered for the evaluation. The superscripts ($\tilde{}$) for effective values are omitted in this analysis.

3.1 Method 1

For inaccurate residence time but precise values of a_r, $\Delta\varphi$, and B the Eqs. (29) and (30) are divided by $\Delta\omega/\omega$ leading to two equations for ϵ_N and ϵ_{Pe}:

$$\begin{gathered} \tfrac{\Delta\omega}{\omega} = \tfrac{\Delta\tau_R}{\tau_R} \neq 0 \ , \quad \Delta a_r = \Delta(\Delta\varphi) = \Delta B/B = 0 \ , \\ M = \begin{bmatrix} A_N & A_{\mathrm{Pe}} \\ \phi_N & \phi_{\mathrm{Pe}} \end{bmatrix} , \\ \begin{bmatrix} \varepsilon_N & \varepsilon_{\mathrm{Pe}} \end{bmatrix} = \begin{bmatrix} -A_\omega & -\phi_\omega \end{bmatrix} \tfrac{1}{M} \ . \end{gathered} \tag{31}$$

For inaccurate B, a_r, $\Delta\varphi$ one has to divide Eqs. (29) and (30) by $\Delta B/B$, Δa_r, and $\Delta\varphi$, respectively, leading to the sensitivity factors β, σ, and κ:

$$\begin{gathered} \tfrac{\Delta B}{B} \neq 0 \ , \quad \Delta a_r = \Delta(\Delta\varphi) = \Delta\omega/\omega = 0 \ , \\ \begin{bmatrix} \beta_N & \beta_{\mathrm{Pe}} \end{bmatrix} = \begin{bmatrix} -A_B & -\phi_B \end{bmatrix} \tfrac{1}{M} \ , \end{gathered} \tag{32}$$

$$\begin{gathered} \Delta a_r \neq 0 \ , \quad \Delta(\Delta\varphi) = \Delta B/B = \Delta\omega/\omega = 0 \ , \\ \begin{bmatrix} \sigma_N & \sigma_{\mathrm{Pe}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \end{bmatrix} \tfrac{1}{M} \ , \end{gathered} \tag{33}$$

$$\begin{gathered}\Delta\left(\Delta\varphi\right)\neq 0\ , \quad \Delta a_r = \Delta B/B = \Delta\omega/\omega = 0\ , \\ \left[\begin{array}{cc}\kappa_N & \kappa_{\mathrm{Pe}}\end{array}\right] = \left[\begin{array}{cc}0 & 1\end{array}\right]\tfrac{1}{M}\ .\end{gathered} \tag{34}$$

The final mean relative error can be calculated from

$$\begin{gathered} y = N, \mathrm{Pe}\ , \\ \left(\tfrac{\Delta y}{y}\right)^2 = \left(\varepsilon_y \tfrac{\Delta\omega}{\omega}\right)^2 + \left(\beta_y \tfrac{\Delta B}{B}\right)^2 + \left(\sigma_y \Delta a_r\right)^2 + \left(\kappa_y \Delta\left(\Delta\varphi\right)\right)^2\ ,\end{gathered} \tag{35}$$

if the relative errors $\Delta\omega/\omega$, $\Delta B/B$, and absolute errors Δa_r and $\Delta(\Delta\varphi)$ are known. The error $\Delta\omega/\omega$ depends mainly on the measurement error for the flow rate. The inaccuracy $\Delta B/B$ depends on the construction, wall material, fluid properties and frequency for thick walls. The errors Δa_r and $\Delta(\Delta\varphi)$ depend on the temperature measurement errors $\Delta\theta$, the amplitudes of oscillations and the numbers of measuring points. The following formula has been derived:

$$(\Delta a_r)^2 = \Big(\Delta(\Delta\varphi)\Big)^2 = \frac{2\Delta\theta_0^2}{n_{t_0} U_0^2} + \frac{2\Delta\theta_1^2}{n_{t_1} U_1^2}\ . \tag{36}$$

Since $(\Delta a_r)^2 = \Big(\Delta(\Delta\varphi)\Big)^2$ it makes sense to introduce the combined sensitivity coefficient ϑ,

$$\vartheta_y^2 = \sigma_y^2 + \kappa_y^2\ , \tag{37}$$

which simplifies the former Eq. (35) to

$$\left(\frac{\Delta y}{y}\right)^2 = \left(\varepsilon_y \frac{\Delta\omega}{\omega}\right)^2 + \left(\beta_y \frac{\Delta B}{B}\right)^2 + \left(\vartheta_y \Delta a_r\right)^2\ . \tag{38}$$

For design purposes (steady state) the effective apparent heat transfer coefficient α_d (index d for dispersive) defined by

$$\frac{1}{N_d} = \frac{1}{N} + \frac{1}{\mathrm{Pe}} \tag{39}$$

is of interest [1,2]. So it is useful also to define sensitivity coefficients for N_d. For small differences $\Delta y/y \approx \mathrm{d}y/y$ one can derive:

$$\begin{gathered}\gamma = \sigma,\ \kappa,\ \vartheta,\ \varepsilon\ , \\ \gamma_{N_d} = \frac{\gamma_N}{1+\frac{N}{\mathrm{Pe}}} + \frac{\gamma_{\mathrm{Pe}}}{1+\frac{\mathrm{Pe}}{N}}\ .\end{gathered} \tag{40}$$

3.2 Method 2

Applying Eqs. (29) and (30) to both frequencies and neglecting possible frequency induced changes in N leads to:

$$\begin{gathered} y = N,\ \mathrm{Pe},\ N_d,\ B_1,\ \omega_1\ , \\ \omega_2 = k\omega_1, \quad k = 1,\,2,\,3,\,\dots\ , \\ B_2 = CB_1, \quad C \geq 1\ , \end{gathered} \tag{41}$$

$$M = \begin{bmatrix} A_{N_1} & A_{\mathrm{Pe}_1} & A_{B_1} & A_{\omega_1} \\ \phi_{N_1} & \phi_{\mathrm{Pe}_1} & \phi_{B_1} & \phi_{\omega_1} \\ A_{N_2} & A_{\mathrm{Pe}_2} & A_{B_2} & A_{\omega_2} \\ \phi_{N_2} & \phi_{\mathrm{Pe}_2} & \phi_{B_2} & \phi_{\omega_2} \end{bmatrix}, \tag{42}$$

$$\Delta a_{r1} \neq 0: \begin{bmatrix} \sigma_{N_1} & \sigma_{\mathrm{Pe}_1} & \sigma_B & \sigma_\omega \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \frac{1}{M}\ , \tag{43}$$

$$\Delta\left(\Delta\varphi\right)_1 \neq 0: \begin{bmatrix} \kappa_{N_1} & \kappa_{\mathrm{Pe}_1} & \kappa_B & \kappa_\omega \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \frac{1}{M}\ , \tag{44}$$

$$\Delta a_{r2} \neq 0: \begin{bmatrix} \sigma_{N_2} & \sigma_{\mathrm{Pe}_2} & \sigma_B & \sigma_\omega \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \frac{1}{M}\ , \tag{45}$$

$$\Delta\left(\Delta\varphi\right)_2 \neq 0: \begin{bmatrix} \kappa_{N_2} & \kappa_{\mathrm{Pe}_2} & \kappa_B & \kappa_\omega \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \frac{1}{M}\ , \tag{46}$$

$$\left(\frac{\Delta y}{y}\right)^2 = \left(\vartheta_{y_1}\Delta a_{r1}\right)^2 + \left(\vartheta_{y_2}\Delta a_{r2}\right)^2\ . \tag{47}$$

The errors $(\Delta a_{r1})^2 \neq (\Delta a_{r2})^2$, if the amplitudes are different for the frequencies ω_1 and ω_2.

4 Examples

4.1 Experiments with liquids

In Tab. 1 two separate oscillation experiments with liquids are considered which are evaluated according to method 1. For experiment 1 $B_1 = 2.5$ and $\omega_1 = 2$, for experiment 2 $B_2 = 4.0$ and $\omega_2 = 6$. The sensitivity factors σ, κ, ϑ, ε, and β are calculated with which the relative errors of N, Pe, and N_d can be estimated if the uncertainties $\Delta\omega/\omega$, $\Delta B/B$ and Δa_r are known.

Using Eq. (36) $(\Delta a_r)^2 = (\Delta(\Delta\varphi))^2$ can be calculated from the temperature measurement error. With a high number of measuring points along numerous oscillation periods a high accuracy can be attained and Δa_r may be one order of magnitude smaller than $\Delta\omega/\omega$ and $\Delta B/B$. For the comparison of the examples presented with Tabs. 1–6 the following uncertainties are assumed: $\Delta\omega/\omega = \Delta B/B = 2\%$ and $\Delta a_r = 0.2\%$. With these uncertainties and the sensitivity factors of Tab. 1 one receives from Eq. (38) for experiment 1: $\Delta N/N = \pm 88$ %, $\Delta\mathrm{Pe}/\mathrm{Pe} = \pm 21.3\%$, and $\Delta N_d/N_d = 56.8\%$. The errors are definitely too high. The main reason for the extreme inaccuracies is the inaccurate separate measurement of residence time (see ε). The experiment 2 is slightly better. Equation (38) yields $\Delta N/N = 22.6\%$, $\Delta\mathrm{Pe}/\mathrm{Pe} = 0.9\%$, and $\Delta N_d/N_d = 16.2\%$.

Table 1: Evaluation method 1. Calculated sensitivity factors σ, κ, ϑ, ε, and β (Eqs. (24)–(28), (31)–(34), (37), and (40)) for two separate experiments with liquids: $\omega_1 = 2$, $B_1 = 2.5$, $\omega_2 = 6$, $B_2 = 4$, $N = 2.4$, Pe = 6.

B	ω	y	σ_y	κ_y	ϑ_y	ε_y	β_y
2.5	2.0	N	–36.24	–3.62	±36.42	40.48	–16.87
		Pe	7.52	2.13	±7.81	–9.97	3.68
		N_d	–23.73	–1.98	±23.82	26.07	–11.00
4.0	6.0	N	–5.07	5.40	±7.40	–11.03	–2.13
		Pe	1.17	–0.20	±1.18	–0.13	0.43
		N_d	–3.29	3.80	±5.02	–7.95	–1.40

Table 2: Evaluation method 2. Calculated sensitivity factors σ_1, κ_1, ϑ_1, σ_2, κ_2, and ϑ_2 (Eqs. (24)–(28), (37), and (40)–(46)) for one experiment with a liquid and thermally thin walls: $B_1 = B_2 = 4$, $\omega_1 = 2$, $\omega_2 = 6$, $N = 2.4$, Pe = 6.

	$\omega_1 = 2$, $B_1 = 4.0$			$\omega_2 = 6$, $B_2 = 4.0$		
y	σ_{y1}	κ_{y1}	ϑ_{y1}	σ_{y2}	κ_{y2}	ϑ_{y2}
N	14.40	–58.63	±60.37	19.96	19.16	±27.67
Pe	–2.16	5.27	±5.69	–1.07	–1.58	±1.91
N_d	12.47	–40.37	±42.25	13.95	13.23	±19.23
B	–5.15	13.20	±14.17	–5.61	–3.41	±6.56
ω	–0.31	2.75	±2.77	–1.18	–0.59	±1.32

In the following examples the evaluation method 2 is applied, avoiding the separate determination of residence time. Table 2 considers one experiment with evaluation of two oscillations. Thermally thin walls are assumed,

$B_1 = B_2$, the frequencies $\omega_1 = 2$, $\omega_2 = 6$. Now only Δa_{r1} and Δa_{r2} affect the results. Assuming equal amplitudes of both frequencies $\Delta a_{r1} = \Delta a_{r2} = 0.002$ leads to $\Delta N/N = \pm 13.3\%$, $\Delta \mathrm{Pe}/\mathrm{Pe} = \pm 1.2\%$ and $\Delta N_d/N_d = \pm 9.3\%$. The evaluation method 2 provides also values for B and ω with accuracy $\Delta B/B = \pm 3.1\%$ and $\Delta\omega/\omega = \pm 0.61\%$.

Further improvement can be attained with thermally thick walls as shown with Tab. 3. In this example it is assumed that the effective capacity ratios increase by the factor $\widetilde{B}_2 \big/ \widetilde{B}_1 = 1.6$ due to the frequency ratio $\omega_2/\omega_1 = 3$. For thick walls with $\widetilde{X} = 1$ one would reach $\widetilde{B}_2 \big/ \widetilde{B}_1 = \sqrt{\omega_2/\omega_1} = \sqrt{3} = 1.732$, see also Eq. (17) and Fig. 1. Applying the sensitivity factors of Tab. 3 yields $\Delta N/N = \pm 3.74\%$, $\Delta \mathrm{Pe}/\mathrm{Pe} = \pm 0.41\%$ and $\Delta N_d/N_d = \pm 2.57\%$, $\Delta B/B = \pm 0.81\%$, $\Delta\omega/\omega = 0.22\%$. These results show that the frequency induced change of B can improve the accuracy when applying evaluation method 2.

Table 3: Evaluation method 2. Calculated sensitivity factors σ_1, κ_1, ϑ_1, σ_2, κ_2, and ϑ_2 (Eqs. (24)–(28), (37), and (40)–(46)) for one experiment with a liquid and thermally thick walls: $B_1 = 2.5$, $\omega_1 = 2$, $B_2 = 4.0$, $\omega_2 = 6$, $N = 2.4$, $\mathrm{Pe} = 6$.

	$\omega_1 = 2,\ B_1 = 2.5$			$\omega_2 = 6,\ B_2 = 4.0$		
y	σ_{y1}	κ_{y1}	ϑ_{y1}	σ_{y2}	κ_{y2}	ϑ_{y2}
N	–0.47	16.58	±16.59	–6.39	–5.74	±8.60
Pe	–0.63	–1.28	±1.43	1.36	0.51	±1.45
N_d	–0.83	11.48	±11.51	–4.18	–3.96	±5.75
B	–1.38	–3.28	±3.55	0.46	1.88	±1.94
ω	0.31	0.87	±0.92	0.03	0.64	±0.64

The best possible ratio $C = B_2/B_1 = \infty$ can be reached with a combined heat transfer and tracer experiment. Such one experiment could be realized by mixing of hot salt water of constant concentration and temperature with cold pure water of constant temperature. Changing periodically the mixing ratio generates a periodic concentration ($B_2 = \infty$) and temperature (B_1) oscillation of same frequency.

Evaluation according to method 2 yields N, Pe, B_1, and ω_1. Table 4 presents the sensitivity factors for this experiment. Assuming the same error Δa_{r1} for temperature and Δa_{r2} for concentration yields the final errors $\Delta N/N = \pm 1.25\%$, $\Delta \mathrm{Pe}/\mathrm{Pe} = \pm 0.37\%$ and $\Delta N_d/N_d = \pm 0.89\%$,

Table 4: Evaluation method 2. Calculated sensitivity factors σ_1, κ_1, ϑ_1, σ_2, κ_2, ϑ_2 (Eqs. (24)–(28), (37), (40)–(46)) for one combined heat transfer and tracer experiment with a liquid: $B_1 = 4$, $B_2 = \infty$, $\omega_1 = \omega_2 = 6$, $N = 2.4$, Pe = 6.

	$\omega_1 = 6$, $B_1 = 4$			$\omega_2 = 6$, $B_2 = \infty$		
y	σ_{y1}	κ_{y1}	ϑ_{y1}	σ_{y2}	κ_{y2}	ϑ_{y2}
N	0.77	4.38	±4.44	–1.33	–4.21	±4.41
Pe	0	0	0	1.83	–0.33	±1.86
N_d	0.55	3.13	±3.17	–0.43	–3.10	±3.13
B	–2.73	0.48	±2.78	4.09	–0.63	±4.14
ω	0	0	0	–0.67	0.50	±0.83

$\Delta B/B = \pm 1.00\%$, $\Delta\omega/\omega = 0.17\%$. Obviously this combined experiment yields the highest accuracy.

This would also allow to apply the pulse technique with its lower number of temperature measuring points n_t.

4.2 Experiments with gases

The evaluation method 2 with evaluation of two temperature oscillations is now applied to an experiment with a gas and thermally thin walls: $B_1 = B_2 = 0.002$, $\omega_1 = 1$, $\omega_2 = 3$, $N = 3$, and Pe = 12. With sensitivity factors of Tab. 5 and the previous values $\Delta a_{r1} = \Delta a_{r2} = 0.002$ one receives $\Delta N/N = \pm 2.87\%$, $\Delta \mathrm{Pe}/\mathrm{Pe} = \pm 11.26\%$ and $\Delta N_d/N_d = \pm 0.076\%$, $\Delta B/B = \pm 29.52\%$, $\Delta\omega/\omega = \pm 3.28\%$. Typical for gas experiments (low values of B) is the very precise determination of the effective value of N_d. The relative error of Pe is usually higher than that of N. If mainly the effective value of N_d is of interest, the high inaccuracy of Pe in this example could be tolerated.

An improvement by frequency induced change of B is impossible in most cases or less pronounced than with liquids. So the method 2 may not be advantageous compared with the standard method 1. This is shown in Table 6. Separate experiments with $N = 3$, Pe = 12, $N_d = 2.4$ and $B = 0.002$ are considered with various frequencies $\omega = 0.10 - 3.00$. With the previous assumptions $\Delta a_r = 0.002$, $\Delta B/B = \Delta\omega/\omega = 0.02$ one receives for the lowest frequency $\omega = 0.1$: $\Delta N/N = \pm 3.34\%$, $\Delta \mathrm{Pe}/\mathrm{Pe} = \pm 13.31\%$ and $\Delta N_d/N_d = \pm 0.10\%$. For the highest frequency $\omega = 3$: $\Delta N/N = \pm 1.34\%$,

Table 5: Evaluation method 2. Calculated sensitivity factors σ_1, κ_1, ϑ_1, σ_2, κ_2, and ϑ_2 (Eqs. (24)–(28), (37), and (40)–(46)) for one experiment with a gas and thermally thin walls: $B_1 = B_2 = 0.002$, $\omega_1 = 1$, $\omega_2 = 3$, $N = 3$, Pe = 12.

	$\omega_1 = 1,\ B_1 = 0.002$			$\omega_2 = 3,\ B_2 = 0.002$		
y	σ_{y1}	κ_{y1}	ϑ_{y1}	σ_{y2}	κ_{y2}	ϑ_{y2}
y	σ_{y1}	κ_{y1}	ϑ_{y1}	σ_{y2}	κ_{y2}	ϑ_{y2}
N	10.00	–0.58	$\pm$10.02	–10.15	1.73	$\pm$10.30
Pe	–38.10	2.14	$\pm$38.16	40.82	–6.95	$\pm$41.40
N_d	0.38	–0.04	$\pm$0.38	0.04	–0.00	$\pm$0.04
B	–64.92	102.37	$\pm$121.22	69.65	–47.35	$\pm$84.22
ω	15.15	–1.04	$\pm$15.19	–16.09	3.35	$\pm$16.43

Table 6: Evaluation method 1. Calculated sensitivity factors σ, κ, ϑ, ϵ, β (Eqs. (24) – (28), (31)–(34), (37) and (40)) for five separate experiments with gases. All experiments $N = 3$, Pe = 12, $B = 0.002$. Frequencies $\omega = 0.1$, 0.19, 0.30, 1.0, 3.0.

$N = 3$, Pe = 12, $N_d = 2.4$, $B = 0.002$					
ω	0.100	0.190	0.300	1.000	3.000
a_r	8.3959	2.4005	2.4027	2.4247	2.5871
$\Delta\varphi$	0.1830	0.1898	0.2423	0.6882	1.9689
σ_N	1.419	0.695	0.529	0.434	0.468
σ_{Pe}	–3.580	–0.691	–0.028	3.370	0.362
σ_{Nd}	0.419	0.417	0.417	0.421	0.447
κ_N	–10.386	–5.479	–3.475	–1.064	–0.444
κ_{Pe}	41.565	21.908	13.875	4.134	1.320
κ_{Nd}	0.004	–0.001	–0.005	–0.025	–0.092
ϑ_N	$\pm$10.482	$\pm$5.522	$\pm$3.515	$\pm$1.149	$\pm$0.645
ϑ_{Pe}	$\pm$41.719	$\pm$21.919	$\pm$13.875	$\pm$4.151	$\pm$1.369
ϑ_{Nd}	$\pm$0.419	$\pm$0.417	$\pm$0.417	$\pm$0.421	$\pm$0.456
ε_N	–0.499	0.372	0.572	0.681	0.665
$\varepsilon_{\mathrm{Pe}}$	1.973	–1.499	–2.293	–2.742	–2.535
ε_{Nd}	–0.005	–0.002	–0.001	–0.003	0.025
β_N	1.203	0.333	0.133	0.012	0.001
β_{Pe}	–4.793	–1.327	–0.532	–0.047	–0.005
β_{Nd}	0.004	0.001	0.000	–0.000	–0.000

$\Delta \mathrm{Pe}/\mathrm{Pe} = \pm 5.08\%$ and $\Delta N_d/N_d = \pm 0.10\%$. The errors decrease with increasing frequency. With sufficiently high frequency a better accuracy can be attained with method 1 than with method 2.

4.3 Simple approximation for gases

For the frequency

$$\omega_0^2 = \frac{1}{2} \mathrm{Pe}\, N\, B \tag{48}$$

the Eqs. (10), (11), and (12) give $D_3 = 0$, $a_{r0} = (D_2/D_4)_0$, and $\Delta\varphi_0 = -(D_1/D_4)_0$. Substituting ω_0 according to Eq. (48) and rearranging leads to

$$\frac{1}{a_{r0}} = \frac{1}{N} + \frac{1}{\mathrm{Pe}} + \underbrace{\frac{B}{2+B}\left(\frac{3}{\mathrm{Pe}} - \frac{1}{N}\right)}_{\approx 0} \approx \frac{1}{N_d} \tag{49}$$

and

$$(\Delta\varphi_0)^2 = \omega_0^2 \underbrace{\left[\frac{1+\frac{N}{\mathrm{Pe}}(1+B)}{1+\frac{N}{\mathrm{Pe}}(1+2B)}\right]^2}_{\approx 1} \approx \frac{1}{2}\mathrm{Pe} N B\ . \tag{50}$$

The approximation of Eq. (49) is exact for $B = 0$ or $\mathrm{Pe} = 3\ N$, that of Eq. (50) for $B = 0$ or $N/\mathrm{Pe} = 0$. The simple approximation rule can be given: If $\Delta\varphi = \omega$ then $a_r = N_d$.

The rule is confirmed with the data for $\omega = 0.19$ of Tab. 6. The exact values would be: $\omega_0 = \sqrt{0.036} = 0.1897367$, $\Delta\varphi_0 = 0.1896608$, and $a_{r0} = 2.4004796$. The data of Tab. 6 show that even for remarkable deviations of $\Delta\varphi$ from ω the equation $a_r = N_d$ is a good approximation.

If the frequency $\omega_0 = \Delta\varphi_0$ has been found the approximations of Eqs. (49) and (50) give

$$\begin{aligned} \frac{1}{N} &= \frac{1}{2a_{r0}}\left(1 + \sqrt{1 - 2B\left(\frac{a_{r0}}{\Delta\varphi_0}\right)^2}\right)\ , \\ \frac{1}{\mathrm{Pe}} &= \frac{1}{2a_{r0}}\left(1 - \sqrt{1 - 2B\left(\frac{a_{r0}}{\Delta\varphi_0}\right)^2}\right)\ . \end{aligned} \tag{51}$$

This equation is valid for $\mathrm{Pe} \geq N$. For cases $N \geq \mathrm{Pe}$ N and Pe have to be exchanged. The sensitivity factors for $\omega = 0.19$ lead to the errors $\Delta N/N = \pm 1.49\%$, $\Delta \mathrm{Pe}/\mathrm{Pe} = \pm 5.94\%$ and $\Delta N_d/N_d = \pm 0.10\%$. Additional errors may arise from the approximations in Eqs. (49) and (50).

5 Conclusions

1. Periodic temperature oscillation experiments on heat exchangers can be evaluated with consideration of axial fluid dispersion according to the unity Mach number dispersion model as well as approximate consideration of axial and lateral wall heat conduction. Two evaluation procedures are proposed.
2. Evaluation method 1 is the standard procedure in which only one harmonic oscillation is evaluated. Amplitude ratio and phase shift yield the unknown number of transfer units N and the dispersive Peclet number Pe. The residence time and the effective fluid to wall capacity ratio B have to be determined separately. This method should preferably be applied to experiments with gases.
3. Evaluation method 2 avoids the separate determination of residence time and effective capacity ratio by evaluating commonly two oscillations of different frequencies, extracted from the complete non-harmonic periodic oscillation. A frequency induced change of effective wall thickness and consequently capacity ratio B helps to improve the accuracy of the results. This method should preferably be applied to liquids. The highest accuracy could be attained with a combined heat transfer and tracer experiment.
4. For planning and performing temperature oscillation experiments a sensitivity and error analysis should be carried out. All necessary equations for this purpose are given in this paper.

Appendix

A. 1 Concerning Eg. (7)

Extraction of amplitude U and phase φ from arbitrary periodic oscillation

$$\begin{aligned} &U = \sqrt{\alpha^2 + \beta^2}, \quad \varphi = \arctan\left(\frac{\alpha}{\beta}\right), \quad \omega = \frac{2\pi}{z_p}, \quad \nu = 1, 2, 3, \ldots, \\ &\alpha = \frac{2}{z_p}\int_0^{z_p} \theta \cos\left(\nu 2\pi \frac{z}{z_p}\right) \mathrm{d}z, \quad \beta = \frac{2}{z_p}\int_0^{z_p} \theta \sin\left(\nu 2\pi \frac{z}{z_p}\right) \mathrm{d}z\,. \end{aligned} \tag{A1}$$

Simplification for pulse signals (Fig. 2): Integration from $z = 0$ to $z_p/2$ and multiplying integral by 2. Only $\nu = 1, 3, 5, \ldots$, as integrals with even ν vanish.

A. 2 Axial heat conduction

$$\frac{\partial t_w}{\partial z} = NB\,(t - t_w) + \frac{1}{\mathrm{Pe}_w}\frac{\partial^2 t_w}{\partial x^2}, \quad x = 0, x = 1 : \frac{\partial t_w}{\partial x} = 0$$

$$\mathrm{Pe}_w = \frac{wL}{a_w} = \frac{wL\rho_w c_w}{\lambda_w}\ , \quad C_1 = \left(\frac{1}{N+s} + \frac{1}{\mathrm{Pe}+s}\right)^{-1}$$

$$C_2 = -\mathrm{Pe}_w\left[\frac{NB\,(\mathrm{Pe}+2s)}{N+\mathrm{Pe}+2s} + s\right], C_3 = -\frac{s\mathrm{Pe}_w\,(\mathrm{Pe}+s)\,[N\,(1+B)+s]}{N+\mathrm{Pe}+2s}$$

$$V^3 + C_1V^2 + C_2V + C_3 = 0 \to V_1, V_2, V_3 \tag{A2}$$

$$E_j = V_j + \frac{1}{C_1}V_j^2 \quad j = 1,2,3$$

$$F\,(s) = \frac{\frac{1}{E_1}\left(e^{V_3} - e^{V_2}\right)e^{V_1} + \frac{1}{E_2}\left(e^{V_1} - e^{V_3}\right)e^{V_2} + \frac{1}{E_3}\left(e^{V_2} - e^{V_1}\right)e^{V_3}}{\frac{1}{E_1}\left(e^{V_3} - e^{V_2}\right) + \frac{1}{E_2}\left(e^{V_1} - e^{V_3}\right) + \frac{1}{E_3}\left(e^{V_2} - e^{V_1}\right)}\ .$$

A. 3 Full cylinder

$$S_1 = 1 + \sum_{k=1}^{\infty}\frac{2^{2k}X^{4k}\,(-1)^k}{[(2k)!]^2}\ , \quad S_2 = \sum_{k=1}^{\infty}\frac{k2^{2k+1}X^{4k-1}\,(-1)^k}{[(2k)!]^2}\ ,$$

$$S_3 = \sum_{k=1}^{\infty}\frac{2^{2k-1}X^{4k-2}\,(-1)^{k+1}}{[(2k-1)!]^2}\ , \quad S_4 = \sum_{k=1}^{\infty}\frac{(2k-1)\,2^{2k-1}X^{4k-3}\,(-1)^{k+1}}{[(2k-1)!]^2}\ ,$$

$$\widetilde{X} = \frac{S_2^2 + S_4^2}{2\,(S_1S_4 - S_2S_3)}\ , \quad \lim_{X\to\infty}\widetilde{X} = 1\ , \quad \lim_{X\to 0}\widetilde{X} = X\ , \tag{A3}$$

$$\widetilde{\mathrm{Nu}}_w = \frac{(S_2^2 + S_4^2)^2}{2\,(S_1S_4 - S_2S_3)\,(S_1S_2 + S_3S_4)}\ , \quad \lim_{X\to\infty}\widetilde{\mathrm{Nu}}_w = 2\ , \quad \lim_{X\to 0}\widetilde{\mathrm{Nu}}_w = 2\ .$$

A. 4 Full sphere

$$f_1 = \frac{\sinh 6X + \sin 6X}{2\left(\sinh^2 3X + \sin^2 3X\right)} - \frac{1}{3X}, \quad f_2 = \frac{\sinh 6X - \sin 6X}{2\left(\sinh^2 3X + \sin^2 3X\right)}\ ,$$

$$\widetilde{X} = \tfrac{1}{2}f_2\left[1 + \left(\tfrac{f_1}{f_2}\right)^2\right]\ , \quad \lim_{X\to\infty}\widetilde{X} = 1\ , \quad \lim_{X\to 0}\widetilde{X} = X\ , \tag{A4}$$

$$\widetilde{\mathrm{Nu}}_w = 2\frac{f_2}{f_1}\left(\widetilde{X}\right)^2\ , \quad \lim_{X\to\infty}\widetilde{\mathrm{Nu}}_w = 2\ , \quad \lim_{X\to 0}\widetilde{\mathrm{Nu}}_w = \frac{5}{3}\ .$$

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