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A new mutual coupling compensation method for receiving antenna array-based DOA estimation

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Abstract: Most receiving antenna arrays suffer from the mutual coupling problem between antenna elements, which can critically influence the performance of the array. In this work, a novel and accurate form of compensation matrix is applied to compensate the mutual coupling in a uniform linear array (ULA). This is achieved by applying a new method based on solving a boundary value problem for the whole ULA. In this method, both self and mutual impedances are exploited in an accurate characterization of mutual impedance matrix which results in a perfect mutual coupling compensation method, and hence a very accurate direction of arrival (DOA) estimation. In the new scheme, the compensation matrix is obtained by using the relationship between measured voltage and theoretical coupled voltage based on the MOM. Numerical results show that using DOA estimation algorithms to the decoupled voltage obtained by using this method leads to an excellent performance of DOA estimation with higher accuracy and resolution.

Key words: antenna array, beamforming, compensation matrix, DOA estimation, mutual coupling, MUSIC algorithm

1. Introduction

The mutual coupling as an electromagnetic event is very severe in compact antenna arrays [1–3]. In most cases, the mutual coupling has a destructive impact on the antenna's performance. Mutual coupling effect was developed in different areas, both in conventional antenna applications and new applications such as multiple input-multiple output (MIMO) systems, diversity systems, radar, and sonar systems. Over the past few years, many methods for mutual coupling have been studied and many solutions have been proposed. Application of DOA of incident signals is very significant. The independence of the array elements is critical to the performance of the signal processing algorithms [4]. One of the common ways to deal with the mutual coupling between the array elements is to increase the distance between them. It leads to an increase in the physical level of the array, which is contradicted by modern telecommunication because of the tendency to use smaller devices [5]. Other methods of reducing the mutual coupling in the flat plate antennas are the use of the photonic band-gap (PBG) structure between the elements and the neutral lines [6], which both make the design more complicated and are not applicable in the dipole and monopole array antennas.

Therefore, the use of the compensator matrix in the processing section of a receiver can be a good option for removing the mutual coupling in the array antennas. Several mutual coupling compensation methods have been proposed and evaluated. The most significant contributions are the open-circuit voltage method [4], the S-parameter method [1], the full-wave electromagnetic method of moments [7–9], calibration method [10], using the mathematical tool of the genetic algorithm (GA) [11], impedance matching technique [12] and finally the receiving mutual impedance methods [13–14]. The method of moments (MOM) [8] is used to find a solution for the boundary value problem of the antenna array. By using this method, mutual coupling effect is characterized by computing the current distribution on the array elements and through the definition of mutual impedance matrix (MIM). However, this requires the knowledge (or assumption) of a precise angle of incidence, which is practically unknown. In general, the current distribution on the elements of an antenna array, and hence the mutual coupling varies with the direction of the incoming signals. Thus, the MIM depends on the direction of the incident waves.

Gupta and Ksienski [4] used a circuit theory method to analyze the mutual coupling effect. In this method, the mutual coupling of an N-element antenna array is modeled as $(N+1)$-port linear time-invariant network. The compensated open circuit voltages are derived from the measured voltages, using the MIM matrix. These voltages are utilized as input signals to the array signal processing algorithms, such as DOA estimation. Later on, Yeh *et al.* [15] used the MIM to overcome the undesirable effects of mutual coupling on the measured voltage output of the antenna in receiver array antenna to estimate the DOA of signals.

A widely accepted definition of receiving mutual coupling has been presented in [13, 14]. An antenna array, whose elements are connected to an impedance Z_L, is taken into consideration. It is assumed that an externally excited incident plane wave illuminates the antenna array. A method for computing the mutual impedance matrix is presented in [14]. The current distribution on the antenna arrays with monopole and dipole elements remains unchanged without considering the azimuth angle of arrival. This is correct when the signal incident angle is fairly perpendicular to the antenna axis [13]. So, the receiving mutual impedance matrix is invariant and do not depend on the azimuth angle of arrival. Voltages and currents of antenna terminals at $N-1$ distinct arrival angles are measured according to [14]. The data is utilized to calculate the mutual impedance between each pair of elements. Recently, new methods for compensating mutual coupling have been investigated by many researchers.

As pointed out in [2, 17] the receiving mutual coupling compensation methods introduced by Hui *et al.* [13] and Lui *et al*. [14] have not been considered some self-impedance term for the coupling effect compensation, therefore, the results of these methods are not exactly correct. In the new scheme, extreme care has been taken into account for both self and mutual impedance, relating to the mutual coupling effects. According to [2, 17], these new MIM can more efficiently eliminate the mutual coupling effects in a ULA with the application of DOA estimation.

In this paper, a new mutual coupling compensation matrix along with a decoupling method is introduced by using the MIM that is obtained in [2, 16, 17]. The compensation matrix is obtained using the relationship between the measured voltages (coupling voltage) and theoretical coupled voltages (method of MOM).

The commonly available software packages such as FEKO or NEC do not give us the parameters of the mutual coupling as default. On the other hand, in this paper, the effect of mutual coupling is investigated in a signal processing context.

The rest of the paper is organized as follows: in Section 2, an algorithm is presented to calculate the compensation matrix based on the relationship between the actual and theoretical coupled voltages in the presence of mutual coupling. A theoretical method to calculate voltage at the antenna terminals in the presence of mutual coupling, and the relation between this voltage and the actual voltage, is introduced in Section 3. In Section 4, numerical simulation of the DOA estimation, using the multiple signal classification (MUSIC) algorithm is demonstrated. Conclusions are made in Section 5.

2. Calculation of compensation matrix

DOA estimation algorithms, such as the MUSIC algorithm, Root-MUSIC algorithm and signal parameter estimation via rotation invariance techniques (ESPRIT) can separate closely spaced sources [18–22]. These are model based on parameter estimation algorithms and are as valid as the model [9]. Practically, these algorithms are often severely affected by mutual coupling between array elements so they cannot provide high-resolution DOA estimation of the signal. The steering vector does not satisfy the model when the array is influenced by mutual coupling [10]. The mutual coupling can severely decline the performance of DOA estimation techniques and signal processing algorithms [1–12].

Mutual coupling effect must be considered in calculating the antenna array surface current distribution to obtain compensation matrix. By utilizing standard MOM methods and applying Galerkin's procedure the antenna array surface current distribution is calculated in presence of mutual coupling effect. The MOM ascribes a push factor current and a basic function to each segment by parsing each antenna to N segments. Finally, the equation of the antenna surface current distribution is obtained by assuming that the antenna is placed along the z-axis.

$$I(z) = \sum_{n=1}^{MN} a_n f_n(z). \tag{1}$$

M is the number of array elements, a_n is the coefficient push current, and $f_n(z)$ is the basic function for each segment [24].

To achieve the compensator factor for mutual coupling effect, a basic model must be designed for the system. Theoretically, the received voltage from the antenna is expected to be ideal with no distortion however, it is not what happens in practice. The following equation indicates the relationship between the practical measured voltage and the ideal voltage.

$$\boldsymbol{u}^t = \boldsymbol{C}_{\text{com}} \boldsymbol{v}^t_{\text{meas}} \,. \tag{2}$$

In Equation (2), vector $\boldsymbol{v}^t_{\text{meas}}$ as the known columned vector of size $M*1$ contains the measured (actual) output voltage from the antenna terminals and vector $\boldsymbol{u}^t$ as the unknown columned vector of size $M*1$ contains the ideal voltage of antenna terminals. The superscript "t" is to signify that these referred to the antenna terminals. Matrix $\boldsymbol{C}_{\text{com}}$ with size $M*M$ indicates the compensation matrix [24].

The problem can be considered as a uniform linear array which consists of M dipole elements ocated along the x-axis with an arbitrary separation d (Figure 1). Surface current distribution of each dipole in the antenna array must satisfy the Hallen's integral equation, in the receiving mode. It can be solved by MOM method. So, a matrix can be obtained according to Equation (3).

$$\boldsymbol{V}_m = \boldsymbol{Z}\boldsymbol{I}_m\,. \tag{3}$$

In this equation, $\boldsymbol{v}_m$ designates for the $M*N$ voltage segments matrix in the presence of mutual coupling. Matrix $\boldsymbol{Z}$ is the $M*M$ mutual impedance matrix of the antenna array, and $\boldsymbol{I}_m$, is the $M*N$ surface currents matrix, in which each line contains the current segments of each array element.

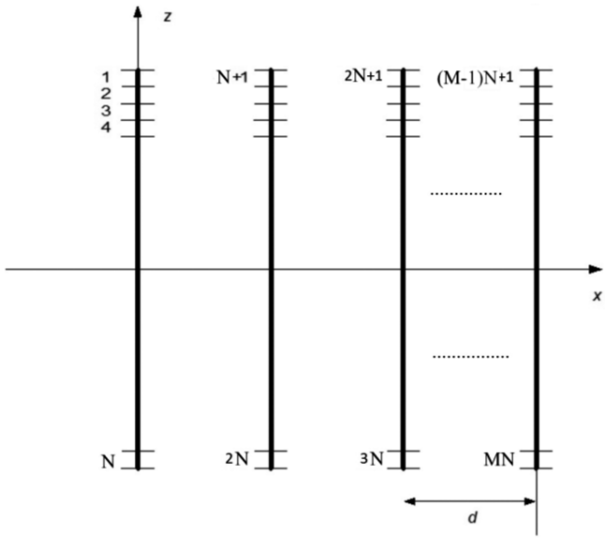

Fig. 1. Illustration of dipole antennas array

In theory, the output voltage of the array elements $\boldsymbol{v}_m$, is obtained by considering the mutual coupling effect in computing the surface current distribution elements and calculating the impedance matrix among array elements $\boldsymbol{Z}$, according to Equation (3).

The relationship between the output voltage of the antenna terminals in theory $\boldsymbol{v}^t_m$, and an ideal voltage $\boldsymbol{u}^t$ is as follows:

$$\boldsymbol{u}^t = \boldsymbol{C}\boldsymbol{v}^t_m. \tag{4}$$

$\boldsymbol{C}$ denotes the conversion matrix. The main purpose of this research is to calculate matrix $\boldsymbol{C}_{\text{com}}$. By establishing a relationship between the measured voltage (actual), $\boldsymbol{v}^t_{\text{meas}}$, and the output voltage of the antenna terminals in theory, $\boldsymbol{v}_m^t$, in the presence of mutual coupling, the compensation matrix $\boldsymbol{C}_{\text{com}}$, is obtained. By calculating the compensation matrix $\boldsymbol{C}_{\text{com}}$, the compensated voltage ($\boldsymbol{v}_{\text{decouple}}^t$) is calculated according to the following equation:

$$\boldsymbol{v}_{\text{decouple}}^t = \boldsymbol{C}_{\text{com}} \boldsymbol{v}_{\text{meas}}^t . \tag{5}$$

The next sections explain how to calculate $\boldsymbol{C}_{\text{com}}$.

3. Theoretical method to calculate voltage at the antenna terminals

To calculate the output voltage of an antenna array in the presence of the mutual coupling effect, the corresponding mutual impedance matrix must be obtained and the array voltage segments could be calculated using Equation (3). Regarding the first and the second-order characteristics of basic function, the elements of the mutual impedance matrix of an array element can be obtained as follows [23]:

$$Z^{ij} = I_m^{j^H} M^{ij} I_m^j; \quad i,\ j = 1,\ 2,\ \ldots,\ M. \tag{6}$$

In Equation (6) I_m^j is the $N{*}1$ current vector of j-th element of the array in presence of the mutual coupling effect, and M_{ij} is the $N{*}N$ moment impedance matrix of the i-th element to the j-th element, the elements of which are calculated in the following way [2]:

$$M_{mn}^{ij} = \int_{f_m} f_m(z) \int_{f_n} f_n(z') \frac{e^{-jkr}}{4\pi r} \,\mathrm{d}z'\,\mathrm{d}z; \quad \begin{cases} m,\ n = 1,2,\ldots,N \\ i,\ j = 1,2,\ldots,N \end{cases}, \tag{7}$$

where $f_n(z')$ and $f_m(z)$ are black pulse basic functions, r is the antenna radius, and k is the propagation constant. Thus $\mathbf{Z}_{\text{moment}}$ is created.

$$\boldsymbol{Z}_{\text{moment}} = \begin{bmatrix} [\boldsymbol{M}^{11}]_{N*N} & [\boldsymbol{M}^{12}]_{N*N} & \ldots & [\boldsymbol{M}^{1M}]_{N*N} \\ [\boldsymbol{M}^{21}]_{N*N} & [\boldsymbol{M}^{22}]_{N*N} & \ldots & [\boldsymbol{M}^{2M}]_{N*N} \\ \vdots & \vdots & \ddots & \vdots \\ [\boldsymbol{M}^{M1}]_{N*N} & [\boldsymbol{M}^{M2}]_{N*N} & \ldots & [\boldsymbol{M}^{MM}]_{N*N} \end{bmatrix}_{(M*N)(M*N)} . \tag{8}$$

Since the mutual impedance matrix $\mathbf{Z}$ is $M{*}M$, and the number of unknown parameters, Z^{ij}, is M times more than the number of equations, therefore it is necessary to increment the number of equations to M times with no correlation between rows or columns of the matrix $\mathbf{Z}$. So, for any M desired entry angles of the signal to the array $\varphi_1, \varphi_2, \ldots, \varphi_M$, the array elements currents are calculated.

$$\begin{gathered}
\varphi_1 \to \boldsymbol{I}_m^{\varphi_1} = \begin{bmatrix} I_{m1}^{1\varphi_1} & I_{m2}^{1\varphi_1} & \dots & I_{mN}^{1\varphi_1} \\ I_{m1}^{2\varphi_1} & I_{m2}^{2\varphi_1} & \dots & I_{mN}^{2\varphi_1} \\ \vdots & \vdots & \ddots & \vdots \\ I_{m1}^{M\varphi_1} & I_{m2}^{M\varphi_1} & \dots & I_{mN}^{M\varphi_1} \end{bmatrix}_{M*M} \\
\varphi_2 \to \boldsymbol{I}_m^{\varphi_2} = \begin{bmatrix} I_{m1}^{1\varphi_2} & I_{m2}^{1\varphi_2} & \dots & I_{mN}^{1\varphi_2} \\ I_{m1}^{2\varphi_2} & I_{m2}^{2\varphi_2} & \dots & I_{mN}^{2\varphi_2} \\ \vdots & \vdots & \ddots & \vdots \\ I_{m1}^{M\varphi_2} & I_{m2}^{M\varphi_2} & \dots & I_{mN}^{M\varphi_2} \end{bmatrix}_{M*M} \\
\vdots \\
\vdots \\
\varphi_M \to \boldsymbol{I}_m^{\varphi_M} = \begin{bmatrix} I_{m1}^{1\varphi_M} & I_{m2}^{1\varphi_M} & \dots & I_{mN}^{1\varphi_M} \\ I_{m1}^{2\varphi_M} & I_{m2}^{2\varphi_M} & \dots & I_{mN}^{2\varphi_M} \\ \vdots & \vdots & \ddots & \vdots \\ I_{m1}^{M\varphi_M} & I_{m2}^{M\varphi_M} & \dots & I_{mN}^{M\varphi_M} \end{bmatrix}_{M*M}
\end{gathered} . \tag{9}$$

Now, using Equation (6) Z^{ij} would be calculated and for the mutual impedance matrix $\boldsymbol{Z}$ it is obtained as follows:

$$\boldsymbol{Z} = \begin{bmatrix} Z^{11} & Z^{12} & \dots & Z^{1M} \\ Z^{21} & Z^{22} & \dots & Z^{2M} \\ \vdots & \vdots & \ddots & \vdots \\ Z^{M1} & Z^{M2} & \dots & Z^{MM} \end{bmatrix}_{M*M} . \tag{10}$$

From Equation (4), one can get $\boldsymbol{u}^t = \boldsymbol{C}\,\boldsymbol{v}_m^t$ in which $\boldsymbol{v}_m^t$ and $\boldsymbol{u}^t$ dimensions are $M{*}1$, and $\boldsymbol{C}$ dimension is $M{*}M$. In order to calculate $\boldsymbol{C}$, the number of unknown parameters and equations should be equal. So for M angle, φ_1, φ_2, …, φ_M, voltage segments matrix, $\boldsymbol{V}_m^{\varphi_i}$, is calculated in the presence of mutual coupling. Using Equation (10) along with Equation (3), the voltage for each angle is obtained.

$$\boldsymbol{V}_m^{\varphi_i} = \begin{bmatrix} Z^{11} & Z^{12} & \dots & Z^{1M} \\ Z^{21} & Z^{22} & \dots & Z^{2M} \\ \vdots & \vdots & \ddots & \vdots \\ Z^{M1} & Z^{M2} & \dots & Z^{MM} \end{bmatrix}_{M*M} \begin{bmatrix} I_{m1}^{1\varphi_i} & I_{m2}^{1\varphi_i} & \dots & I_{mN}^{1\varphi_i} \\ I_{m1}^{2\varphi_i} & I_{m2}^{2\varphi_i} & \dots & I_{mN}^{2\varphi_i} \\ \vdots & \vdots & \ddots & \vdots \\ I_{m1}^{M\varphi_i} & I_{m2}^{M\varphi_i} & \dots & I_{mN}^{M\varphi_i} \end{bmatrix}_{M*N} . \tag{11}$$

The result will be:

$$\boldsymbol{V}_m^{\varphi_i} = \begin{bmatrix} V_{m1}^{1\varphi_i} & V_{m2}^{1\varphi_i} & \cdots & V_{mN}^{1\varphi_i} \\ V_{m1}^{2\varphi_i} & V_{m2}^{2\varphi_i} & \cdots & V_{mN}^{2\varphi_i} \\ \vdots & \vdots & \ddots & \vdots \\ V_{m1}^{M\varphi_i} & V_{m2}^{M\varphi_i} & \cdots & V_{mN}^{M\varphi_i} \end{bmatrix}_{M*N} . \tag{12}$$

To calculate the transformation matrix $\boldsymbol{C}$, the output voltage of each element of the array V_{mt}^i, $i = 1, 2, \ldots, M$ is required, where $\boldsymbol{V}_m^{\varphi_i}$ designates the segment voltage of the i-th element, which is considered as output. The voltage matrix of the output segment in the presence of mutual coupling for M angles is as follows:

$$\boldsymbol{V}_m^{\varphi} = \begin{bmatrix} V_{mt}^{1\varphi_1} & V_{mt}^{1\varphi_2} & \cdots & V_{mt}^{1\varphi_M} \\ V_{mt}^{2\varphi_1} & V_{mt}^{2\varphi_2} & \cdots & V_{mt}^{2\varphi_M} \\ \vdots & \vdots & \ddots & \vdots \\ V_{mt}^{M\varphi_1} & V_{mt}^{M\varphi_2} & \cdots & V_{mt}^{M\varphi_M} \end{bmatrix}_{M*M} . \tag{13}$$

In the ideal case the output voltage matrix $\boldsymbol{U}^{\varphi}$ is calculated for M angles, $\varphi_1, \varphi_2, \ldots, \varphi_M$.

$$\boldsymbol{U}^{\varphi} = \begin{bmatrix} U_t^{1\varphi_1} & U_t^{1\varphi_2} & \cdots & U_t^{1\varphi_M} \\ U_t^{2\varphi_1} & U_t^{2\varphi_2} & \cdots & U_t^{2\varphi_M} \\ \vdots & \vdots & \ddots & \vdots \\ U_t^{M\varphi_1} & U_t^{M\varphi_2} & \cdots & U_t^{M\varphi_M} \end{bmatrix}_{M*M} . \tag{14}$$

where $U_t^{i\varphi_k}$; $i,\ k = 1, 2, \ldots, M$ is obtained from the following equation:

$$U_t^{i\varphi_k} = e^{j\frac{2\pi(i-1)d}{\lambda}\sin(\varphi_k)}; \quad i,\ k = 1, 2, \ldots, M. \tag{15}$$

In Equation (15),d is the distance between array elements. By calculating $\boldsymbol{V}_m^{\varphi}$ and $\boldsymbol{U}^{\varphi}$, the conversion matrix $\boldsymbol{C}$ can then be obtained using Equation (4).

$$\boldsymbol{C} = \boldsymbol{V}_m^{\varphi}(\boldsymbol{U}^{\varphi})^{-1}. \tag{16}$$

According to the explanation given at the end of Section 2, by using the conversion matrix $\boldsymbol{C}$ and the mutual impedance matrix $\boldsymbol{Z}$, the compensation matrix $\boldsymbol{C}_{\mathrm{com}}$, is obtained as follows:

$$\boldsymbol{u}^t = C_{\mathrm{com}}\boldsymbol{v}^t_{\mathrm{meas}} \Rightarrow C_{\mathrm{com}} = \boldsymbol{U}^{\varphi}(\boldsymbol{V}^{\varphi}_{\mathrm{meas}})^{-1}, \tag{17}$$

$$\boldsymbol{u}^t = C\boldsymbol{v}^t_m \Rightarrow C = \boldsymbol{U}^{\varphi}(\boldsymbol{V}^{\varphi}_m)^{-1}. \tag{18}$$

$\boldsymbol{V}_{\text{meas}}^{\varphi}$ is the practical voltage matrix of the array output segment for M angles. Now the relationship between practical and theoretical voltages is achieved in the presence of mutual coupling:

$$\begin{cases} \boldsymbol{V}_{\text{meas}}^{\varphi} = \boldsymbol{Z}_L \boldsymbol{I}_m^{\varphi} \\ \boldsymbol{V}_{\text{meas}}^{\varphi} = \boldsymbol{Z} \boldsymbol{I}_m^{\varphi} \end{cases} \Rightarrow \boldsymbol{V}_m^{\varphi} = \boldsymbol{Z}(\boldsymbol{Z}_L)^{-1} \boldsymbol{V}_{\text{meas}}^{\varphi}, \tag{19}$$

where $\boldsymbol{Z}_L$ is a diagonal matrix with a dimension of $M{*}M$ with the 50 Ω resistor on the main diagonal. $\boldsymbol{I}_m^{\varphi}$ is the matrix of the array output segment for M angles.

Using Equation (19) along with Equation (18) one gets:

$$\boldsymbol{C} = \boldsymbol{U}^{\varphi} \left(\boldsymbol{Z}(\boldsymbol{Z}_L)^{-1} \boldsymbol{V}_{\text{meas}}^{\varphi} \right)^{-1} = \boldsymbol{U}^{\varphi} \left(\boldsymbol{V}_{\text{meas}}^{\varphi} \right)^{-1} \boldsymbol{Z}_L \boldsymbol{Z}^{-1}. \tag{20}$$

So, we have:

$$\boldsymbol{U}^{\varphi} \left(\boldsymbol{V}_{\text{meas}}^{\varphi} \right)^{-1} = \boldsymbol{C} \boldsymbol{Z} \boldsymbol{Z}_L^{-1}. \tag{21}$$

Finally, by replacing (21) in (17), the compensation matrix $\boldsymbol{C}_{\text{com}}$, is obtained:

$$\boldsymbol{C}_{\text{com}} = \boldsymbol{C} \boldsymbol{Z} \boldsymbol{Z}_L^{-1}. \tag{22}$$

Once the compensation matrix has been calculated, it can be applied to the array output voltage vector in the presence of mutual coupling to reduce the destructive effects of mutual coupling on the DOA and beamforming.

$$\boldsymbol{v}_{\text{decoup}}^{t} = C_{\text{com}} \boldsymbol{v}_{\text{meas}}^{t}, \tag{23}$$

where $\boldsymbol{v}_{\text{decoup}}^{t}$ is the output compensation voltage vector of the array elements that is used in DOA and beamforming and other processing algorithms.

4. Simulation

In this section, for the evaluation of the mutual coupling compensation capability of the proposed scheme, two scenarios are performed. In the first experiment, a uniform linear array with 4 elements along the x-axis is considered. The distance between each two elements is $d = \lambda/2$ and they are terminated to the load impedance of $Z_L = 50$ Ω. In this case, two incident plane waves from azimuth angles of $\varphi_1 = 30°$ and $\varphi_2 = 45°$, with an elevation angle of $\theta = 90°$, are taken into consideration. The incoming signals are contaminated with an additive white Gaussian noise (AWGN), with a signal to noise ratio (SNR) of 30 dB. The input signals have a wavelength of $\lambda = 0.125$ m and the radius of the cross-section of any antenna is $a = \lambda/200$. For calculation of the compensation matrix, the full-wave electromagnetic MOM algorithm is implemented using MATLAB computer programming language to take all the mutual coupling effects in to consideration. By applying a MUSIC algorithm, the estimation of the DOA of the incident wave is done after determination of the compensation matrix [19]. The incoming signal parameters are stated in Table 1.

Two different voltages as the MUSIC algorithm input are considered. They are the terminal voltages without decoupling and the decoupled terminal voltages. The decoupled terminal voltages are considered by using the proposed method. The terminal voltages are computed using

Table 1. Incoming signal parameters

Parameters	First signal	Second signal
Wavelength	$\Lambda = 0.125$ m	$\Lambda = 0.125$ m
Polarization	Vertical	Vertical
Signal to noise ratio	30 dB	30 dB
Direction of arrival (DOA)	$\varphi_1 = 30°$	$\varphi_2 = 45°$

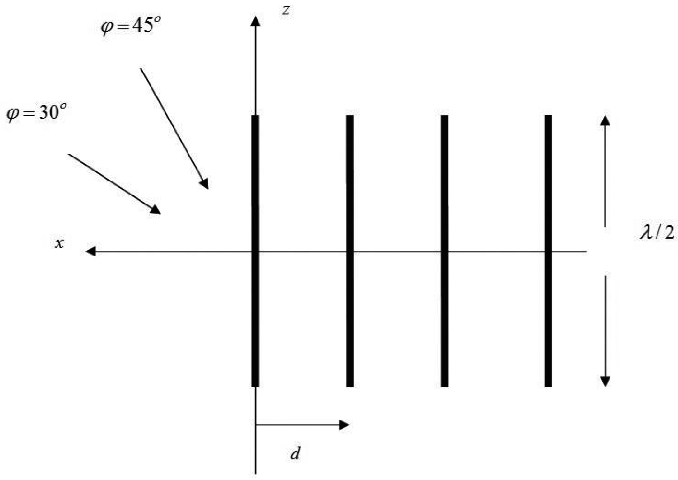

Fig. 2. The DOA estimation with a 4-dipole antenna array, separation ($\varphi_1 = 30°$ and $\varphi_2 = 45°$, and $\theta = 90°$)

MOM algorithm (rectangular basis functions and discretization rate of 16 segments per half-wavelength, with $L = \lambda/2$, and $M = 16$).

The DOA of incoming signals to array from azimuth angles $\varphi_1 = 30°$ and $\varphi_2 = 45°$ and an elevation angle of $\theta = 90°$ in the presence of mutual coupling without compensation and with compensation is presented in Figure 3 and Figure 4, respectively.

As shown in Figure 3, using the terminal voltages without decoupling, the spectral function detects two signals, one peak at $\varphi_1 = 24.3°$, and other at $\varphi_2 = 47.1°$ with biases more than $2.1°$. As clearly observed in Figure 4, by using the proposed method, if the terminal voltages are decoupled, sharper peaks will be formed for both angles of arrivals, $\varphi_1 = 30°$ and $\varphi_2 = 45°$.

Figure 5 shows the results of DOA estimation by considering two kinds of voltages as the inputs for the MUSIC algorithm. The better accuracy of the proposed method can be understood by considering the locations of the detected peaks in this figure. Summary of the estimation results in Figure 5 are presented in Table 2.

In the second experiment the FEKO software was utilized to verify the results obtained from numerical calculations. In this simulation three signals impinge the array from horizontal direction ($\theta = 90°$) and azimuth angles $\varphi_1 = 37°$, $\varphi_2 = 50°$ and $\varphi_3 = 67°$. In this case the dipole

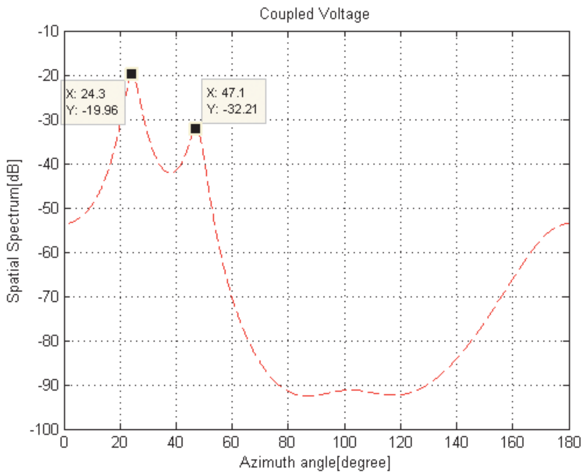

Fig. 3. The DOA of incoming signals to array from $\varphi_1 = 30°$ and $\varphi_2 = 45°$ and $\theta = 90°$ in the presence of mutual coupling without compensation

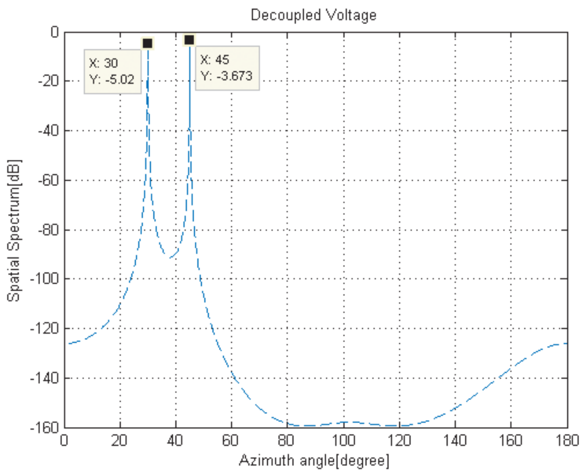

Fig. 4. The DOA of incoming signals to array from $\varphi_1 = 30°$ and $\varphi_2 = 45°$ and $\theta = 90°$ in the presence of mutual coupling with compensation

antenna terminals are connected to the $Z_L = 50\ \Omega$ load while the distance between antennas is $d = \lambda/2$. To demonstrate the statistical performance of the proposed method, the Monte Carlo experiments i.e., 30 trials for the simulation is performed. In addition, the arrival signal is corrupted by AWGN with 10 dB. In each trial 256 snapshots of data were captured and processed by the MUSIC algorithm. To evaluate the performance of our proposed method, three different

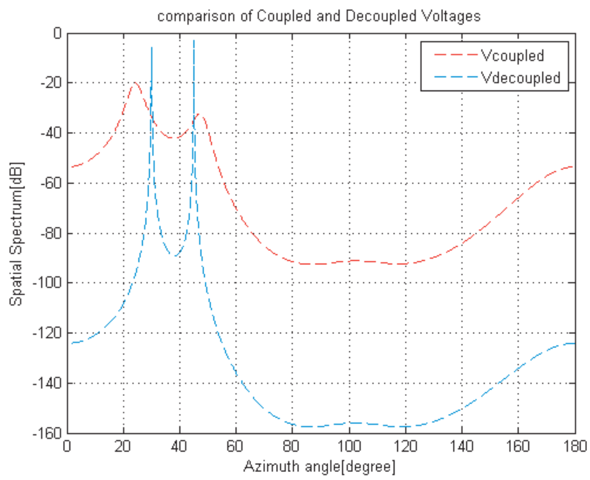

Fig. 5. The DOA estimation of incoming signals to array from $\varphi_1 = 30°$ and $\varphi_2 = 45°$, and $\theta = 90°$ in the presence of mutual coupling with compensation and without compensation

Table 2. The estimation results from Figure 5

Signal/Direction		**Voltages without decoupling**	**Voltages decoupled by using the proposed method**
1	$\varphi_1,\ \theta_1$	30°, 90°	
	$E[\hat{\varphi}_1]$	24.1°	30°
	$\lvert\varphi_1 - E[\hat{\varphi}_1]\rvert$	5.9°	0.00°
2	$\varphi_2,\ \theta_2$	45°, 90°	
	$E[\hat{\varphi}_2]$	47.1°	45°
	$\lvert\varphi_2 - E[\hat{\varphi}_2]\rvert$	2.1°	0.00°

types of voltages are applied as input to the MUSIC algorithm. The first type is the ideal terminal voltages without mutual coupling effect. The second type is the terminal voltages obtained by the FEKO software. This way, all mutual coupling effects are taken into account in the calculation of the terminal voltages. And finally, the third one is terminal voltage obtained by the FEKO software, modified by our proposed method. The spatial spectrums of the MUSIC algorithm for detecting three signals are illustrated in Figure 6.

As shown in Figure 6, using the terminal voltages obtained by the FEKO software, the spectral function detects only one peak at $\varphi_3 = 67°$, while the signal is completely missed at $\varphi_1 = 37°$ and $\varphi_2 = 50°$. As clearly observed, if the terminal voltages are decoupled using our proposed method, there will be sharper peaks in comparison to the data obtained by the FEKO software for three angles of arrivals $\varphi_1 = 37°$ and $\varphi_2 = 50°$, and $\varphi_3 = 67°$.

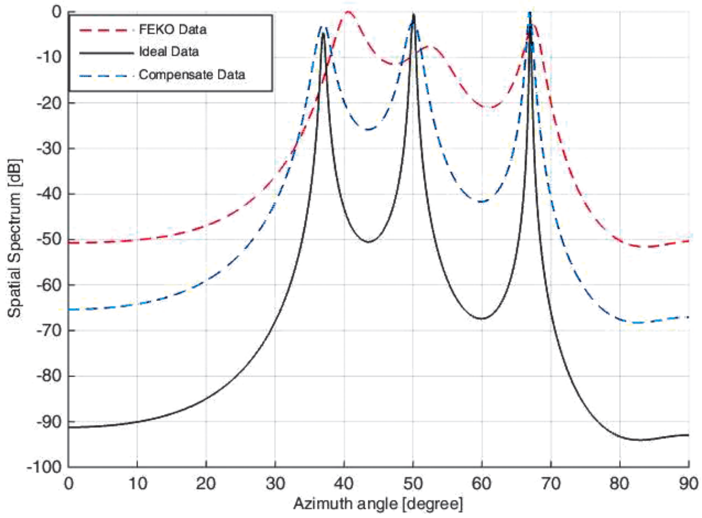

Fig. 6. The DOA estimation of incoming signals to array from $\varphi_1 = 37°$ and $\varphi_2 = 50°$, and $\varphi_3 = 67°$ for different types of voltages as input in the MUSIC algorithm

5. Conclusions

In this paper, a novel mutual coupling compensation matrix and decoupling methodology, in order to compensate the mutual coupling in a uniform linear array (ULA) has been proposed. The aim of this method is to relate the measured voltage and the theoretical coupled voltage based on the MOM to achieve a compensation matrix. Perfect decoupling voltage is obtained by applying the proposed compensation matrix. Simulation results show that applying high-resolution direction of arrival estimation algorithms to the decoupled voltage causes a perfect performance of DOA estimation with more accuracy and resolution.

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