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THERMOELASTIC INTERACTIONS IN A ROTATING INFINITE ORTHOTROPIC ELASTIC BODY WITH A CYLINDRICAL HOLE AND VARIABLE THERMAL CONDUCTIVITY

In the present article, we introduced a new model of the equations of generalized thermoelasticity for unbounded orthotropic body containing a cylindrical cavity. We applied this model in the context of generalized thermoelasticity with phase-lags under the effect of rotation. In this case, the thermal conductivity of the material is considered to be variable. In addition, the cylinder surface is traction free and subjected to a uniform unit step temperature. Using the Laplace transform technique, the distributions of the temperature, displacement, radial stress and hoop stress are determined. A detailed analysis of the effects of rotation, phase-lags and the variability thermal conductivity parameters on the studied fields is discussed. Numerical results for the studied fields are illustrated graphically in the presence and absence of rotation.

1. Introduction

In material science, building insulation, electronics, research, and associated fields, particularly where high operational temperatures are achieved, variability of thermal conductivity is very important. The effect of temperature on thermal conductivity is distinctive for metals and nonmetals. Thermal conductivity of metals is around relative to the absolute temperature (in Kelvin) times the electrical conductivity. It is to be noted that the electrical conductivity in pure metals dimin-

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ishes with expanding temperature, the thermal conductivity remains approximately constant. In amalgams the change in electrical conductivity is normally smaller and in this way thermal conductivity increments with temperature, regularly relatively to temperature [1].

The generalized theories of thermoelasticity have created to defeat unbounded proliferation speed of thermal signals, which was predicted in classical dynamical coupled theory of thermoelasticity. Biot [2] built up the coupled model of thermoelasticity which tackles the first shortcoming of the uncoupled theory, however, it shares the second shortcoming of the uncoupled theory. Lord and Shulman [3] presented generalized theory thermoelasticity with one relaxation time by proposing another law of heat transfer to change traditional Fourier's law. This law includes heat flux vector and also its time derivative. Green and Lindsay [4] developed a theory that contains two constants times that act as relaxation times and modifies not the heat conduction equation only but also all the equations of the coupled theory. Zenkour [5] presented a comparison between various generalized thermoelasticity theories to discuss three-dimensional thermal shock plate problem. Zenkour [6–8] restricts his attention to the theory of Green and Naghdi of type III to deal with nano-machined beam resonators subjected to various boundary conditions and those resting on visco-Pasternak's foundations. Tzou [9] suggested a dual phase delay model of the heat conduction (DPL) to combine the effects of infinitesimal interactions in the fast-transient procedure of heat transportation mechanism in a macroscopic design. Two diverse phase-lags have been presented in the constitutive relation among the vector of heat flux and the gradient of the temperature [10].

Abouelregal [11] studied a piezo-thermo-electric half-space medium with dependent properties of temperature under a ramp-type changing heating using the generalized theory with fractional order. Zenkour and Abouelregal [12] introduced a generalized new model of the nonlocal generalized thermoelasticity for nanobeams due to a harmonically changing in heat taking into consideration the variability of thermal conductivity. Zenkour et al. [13] discussed the generalized theory of the nonlocal thermoelasticity presented by Lord and Shulman [3] to study the Euler-Bernoulli nanobeams vibrations assuming the temperature-dependent thermal conductivity. Ezzat and El-Bary [14] studied the fractional derivative effects in a perfect conducting infinite hollow cylinder and considering that the thermal conductivity is variable.

It has been shown up there that rotation causes thermoelastic generalized medium to be diffusive and having a physical property that has different values when measured in different directions. This treatise joined some examination on the phenomenon of the free surface in a rotating medium. It gives the idea that little consideration has been paid to examination of propagation of plane thermoelastic waves in a rotating medium.

Many investigations have been devoted to studies on the rotation effect of thermoelastic propagation of waves in an isotropic infinite cylinder materials. Abouelregal and Abo-Dahab [15, 16] introduced the effect of dual phase lag model on a non-homogeneous magneto-thermoelastic infinite solid having a spherical cavity and studied the diffusion effect and Thomson's phenomenon on magneto-thermoelastic solid cylinder. Abouelregal and Zenkour [17] concerned the effect of rotation on an isotropic homogeneous thermoelastic half-space due to a crack of Mode-I using the generalized theory of thermoelasticity. Singh and Kumar [18] explained the rotation effect on micropolar magneto-thermoelastic body. Xiong and Guo [19] investigated the effect of heat source moving with constant speed, and variable properties of a magneto-thermoelastic medium, under the fractional order theory of thermoelasticity. Kumar et al. [20] analyzed the interactions subjected to the effect of rotation and hall current in a magneto- thermoelastic micropolar half-space under the fractional order thermoelasticity. Sherief and Hamza [21] considered the variability of thermal conductivity effect on a magneto-thermoelastic infinitely hollow cylinder.

In the current article, a problem of an infiite homogeneous orthotropic thermally conducting body containing a cylindrical cavity affected by the angular velocity is discussed in the context of Tzou model. The thermal conductivity of the body is thought to be changing with the temperature [22–24]. The surface of the cylinder is exposed to thermal shock that depends on the time and surface of the body drop free. The outcomes for generalized and classical theories of thermoelasticity have been obtained from the resultant model as special cases. To explain and compare the theoretical results, the numerical solution is done by means of Laplace transform technique. The variability effects of thermal conductivity, rotation, phase lags on distributions of displacements, temperature and stresses are displayed graphically.

2. The basic equations

We consider an infinite orthotropic, homogeneous, isotropic conducting thermoelastic rotating body with density ρ at initial constant temperature T_0. The field equations in generalized thermoelasticity with dual-phase-lags in the absence of the body force are [25, 26]:
equations of motion:

$$C_{ijmn}\varepsilon_{mn,j} - \beta_{ij}\theta_{,j} + F_i = \rho\ddot{u}_i + \rho\left[\vec{\Omega}\times(\vec{\Omega}\times\vec{u})\right]_i + 2\rho\left(\vec{\Omega}\times\dot{\vec{u}}\right)_i . \tag{1}$$

Constitutive relations are

$$\sigma_{ij} = C_{ijmn}\varepsilon_{mn} - \beta_{ij}\theta\delta_{ij}, \qquad i, j = 1, 2, 3, \tag{2}$$

where

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \tag{3}$$

The generalized heat conduction equation in the context of dual-phase-lag theory (DPL) suggested by Tzou is given by

$$\left(1 + \tau_\theta \frac{\partial}{\partial t}\right)\left(K_{ij}\theta_{,j}\right)_{,i} = \left(\delta + \tau_q \frac{\partial}{\partial t}\right)\left(\rho C_E \frac{\partial \theta}{\partial t} + \beta_{ij} T_0 \frac{\partial u_{m,m}}{\partial t} - \rho Q\right). \tag{4}$$

In all the above governing equations (1)–(4), an over dot indicates the partial derivative as for the time variable, $\theta = T - T_0$ represents the thermodynamical temperature, δ_{ij} is Kronecker's delta, β_{ij} are the thermoelastic components of the coupling, C_{ijmn} are isothermal elastic constants, σ_{ij} are the stress tensor components, F_i are the body force components, $\vec{\Omega} = \Omega\vec{n}$ is the angular velocity, $\vec{n}$ is the direction of the axis of rotation, $\vec{\Omega}\times(\vec{\Omega}\times\vec{u})$ is the centripetal acceleration, $2(\vec{\Omega}\times\dot{\vec{u}})$ is the Coriolis acceleration and u_i are the displacement vector components. In addition, ε_{ij} symbolizes the strain tensor, $\varepsilon_{kk} = e$ is cubical dilatation, K_{ij} is the thermal conductivity tensor which is considered to be temperature-dependent, C_E represents the specific heat of the body at constant strain, Q is the heat source, τ_q and τ_θ are the phase-lags of the heat flux and gradient of temperature, respectively, such that $0 \leqslant \tau_\theta < \tau_q$.

The coupled and generalized theories thermoelasticity can be obtained as limited cases depending on the values of δ, τ_q and τ_θ. Putting $\tau_\theta = 0$, $\delta = 1$, and $\tau_q = \tau_0$ (the first relaxation time), gives the basic equations of governing Lord and Shulman's theory (LS). Also, when $\tau_q = \tau_\theta = 0$, governing equation for a classical thermoelastic body (CTE) are obtained.

3. The problem formulation

In this section, we study an infinite thermoelastic orthotropic body with cylindrical cavity. The surface of cavity is exposed to a time dependent varying heat and traction free. We take the cylindrical coordinates (r, ξ, z) with the z-axis considering all the functions depending on the radial space r and the variable time t according to the symmetry about z-axis.

For axially symmetric problem, the displacement vector has its components

$$u_r = u(r,t), \qquad u_\xi(r,t) = u_z(r,t) = 0. \tag{5}$$

Consequently, the radial and hoop strains ε_{rr} and $\varepsilon_{\xi\xi}$ are given by:

$$\varepsilon_{rr} = \frac{\partial u}{\partial r}, \qquad \varepsilon_{\xi\xi} = \frac{u}{r}. \tag{6}$$

The sum of normal strains gives the cubic dilatation

$$e = \varepsilon_{rr} + \varepsilon_{\xi\xi} = \frac{\partial u}{\partial r} + \frac{u}{r}. \tag{7}$$

The stress-displacement relations (2) can be obtained after using Eq. (6) as

$$\begin{Bmatrix} \sigma_{rr} \\ \sigma_{\xi\xi} \\ \sigma_{zz} \end{Bmatrix} = \begin{bmatrix} c_{11} & c_{12} & -\beta_{11} \\ c_{12} & c_{22} & -\beta_{22} \\ c_{13} & c_{23} & -\beta_{33} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial u}{\partial r} \\ \dfrac{u}{r} \\ \theta \end{Bmatrix}. \tag{8}$$

Taking the rotation term about the z-axis as a body force only, Eq. (1) in the r direction will be in the following form

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\sigma_{rr} - \sigma_{\xi\xi}}{r} = \rho \frac{\partial^2 u}{\partial t^2} - \rho \Omega^2 u, \tag{9}$$

where Ω is the uniform angular velocity. Introducing Eqs. (8) into equation of motion (9), we get

$$c_{11}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) - c_{22}\frac{u}{r^2} = \rho \frac{\partial^2 u}{\partial t^2} + \beta_{11}\frac{\partial \theta}{\partial r} + (\beta_{11} - \beta_{22})\frac{\theta}{r} - \rho \Omega^2 u. \tag{10}$$

Also, the one-dimensional generalized heat equation (4) can be obtained as

$$\left(1 + \tau_\theta \frac{\partial}{\partial t}\right)(K_r \theta_{,r})_{,r} = \left(\delta + \tau_q \frac{\partial}{\partial t}\right)\left[\rho C_E \frac{\partial \theta}{\partial t} + T_0 \frac{\partial}{\partial t}\left(\beta_{11}\frac{\partial u}{\partial r} + \beta_{22}\frac{u}{r}\right)\right], \tag{11}$$

where the thermal conductivity K_r is temperature dependence, the above heat Eq. (11) is nonlinear of temperature and possibly of specific heat C_E.

4. Temperature-dependent thermal conductivity

Experimentally, the thermal properties of the material differ with temperature and this translates into a nonlinear heat equation and nonlinear thermoelastic problem, so that the temperature dependence of material properties must be contemplated in the thermal stress investigation of these components [27–30]. The solution of the nonlinear problems can be observed and obtained by supposing the material to be temperature dependent implying that the specific heat C_E and the r direction thermal conductivity K_r depend on the distribution of temperature [31]. Additionally, the experimental information demonstrates that variations of Poisson's ratio and the thermal expansion coefficient, because of the high temperature, can be dismissed [31].

Assuming that the thermal conductivity K_r is a linear function of temperature θ [32]:

$$K_r = K_r(\theta) = k_0(1 + k_1\theta), \tag{12}$$

where k_0 is the thermal conductivity at initial temperature T_0 and k_1 is a factor characterizing the variety of thermal conductivity.

To solve the problem, we will take the following transformation [32]

$$\psi = \frac{1}{k_0} \int_0^\theta K_r(\theta) \mathrm{d}\theta, \tag{13}$$

where the new function ψ expressing the heat conduction. Substituting from Eq. (12) into Eq. (13) and integrating, we obtain [32]

$$\psi = \theta \left(1 + \frac{1}{2} k_1 \theta \right). \tag{14}$$

From Eqs. (13) and (14), we deduce the following two relations

$$\nabla \psi = \frac{K_r(\theta)}{k_0} \nabla \theta, \tag{15}$$

$$\frac{\partial \psi}{\partial t} = \frac{K_r(\theta)}{k_0} \frac{\partial \theta}{\partial t}. \tag{16}$$

The generalized heat conduction equation (11) of thermally conducting orthotropic, homogeneous solids with variable thermal conductivity after using Eqs. (15) and (16) reduce to

$$\left(1 + \tau_\theta \frac{\partial}{\partial t} \right) \nabla^2 \psi = \left(1 + \tau_q \frac{\partial}{\partial t} \right) \left[\rho C_E \frac{\partial \psi}{\partial t} + \frac{T_0}{K_0} \frac{\partial}{\partial t} \left(\beta_{11} \frac{\partial u}{\partial r} + \beta_{22} \frac{u}{r} \right) \right], \tag{17}$$

where $\rho C_E = K_r / k$ and

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r}. \tag{18}$$

Substituting from Eqs. (15) and (16) into motion equation (10), we have

$$c_{11} \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) - c_{22} \frac{u}{r^2} = \rho \frac{\partial^2 u}{\partial t^2} + \frac{\beta_{11}}{1 + 2k_1 \theta} \frac{\partial \psi}{\partial r} - \rho \Omega^2 u \\ + \frac{\beta_{11} - \beta_{22}}{k_1 r} \left[\sqrt{1 + 2k_1 \psi} - 1 \right]. \tag{19}$$

Assuming that $\theta = T - T_0$ is small and does not cause insignificant variations of elastic and thermal coefficients, these will be regarded as independent of T. In addition to the assumption $|\theta / T_0| \ll 1$, one can assume that second powers and products of the components of strain may be neglected in comparison with the strains ε_{ij}.

For linearity and by means of the binomial theorem for fractional powers and the assumption $|\theta / T_0| \ll 1$, the radial equation of motion (19) and constitutive relations (8) will be in the forms

$$c_{11} \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) - c_{22} \frac{u}{r^2} = \rho \frac{\partial^2 u}{\partial t^2} + \beta_{11} \frac{\partial \psi}{\partial r} + (\beta_{11} - \beta_{22}) \frac{\psi}{r} - \rho \Omega^2 u, \tag{20}$$

$$\begin{Bmatrix} \sigma_{rr} \\ \sigma_{\xi\xi} \\ \sigma_{zz} \end{Bmatrix} = \begin{bmatrix} c_{11} & c_{12} & -\beta_{11} \\ c_{12} & c_{22} & -\beta_{22} \\ c_{13} & c_{23} & -\beta_{33} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial u}{\partial r} \\ \dfrac{u}{r} \\ \psi \end{Bmatrix}. \tag{21}$$

Presenting the following dimensionless variables,

$$\begin{gathered} \{r', u', R'\} = \frac{c_0}{k}\{r, u, R\}, \qquad \{t', \tau_q', \tau_\theta'\} = \frac{c_0^2}{k}\{t, \tau_q, \tau_\theta\}, \\ \psi' = \frac{\psi}{T_0}, \qquad \Omega' = \frac{\Omega}{\eta c_0^2}, \qquad \sigma_{ij}' = \frac{\sigma_{ij}}{c_{11}}, \qquad k_1' = T_0 k_1, \qquad c_0^2 = \frac{c_{11}}{\rho}, \end{gathered} \tag{22}$$

the dimensionless basic equations are given by

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - c_2\frac{u}{r^2} = \frac{\partial^2 u}{\partial t^2} + \varepsilon_1\frac{\partial \psi}{\partial r} + \varepsilon_3\frac{\psi}{r} - \Omega^2 u, \tag{23}$$

$$\left(1 + \tau_\theta\frac{\partial}{\partial t}\right)\nabla^2\psi = \left(1 + \tau_q\frac{\partial}{\partial t}\right)\left[\frac{\partial \psi}{\partial t} + \frac{\partial}{\partial t}\left((\varepsilon_4\frac{\partial u}{\partial r} + \varepsilon_5\frac{u}{r}\right)\right], \tag{24}$$

$$\begin{Bmatrix} \sigma_{rr} \\ \sigma_{\xi\xi} \\ \sigma_{zz} \end{Bmatrix} = \begin{bmatrix} 1 & c_1 & -\varepsilon_1 \\ c_1 & c_2 & -\varepsilon_2 \\ c_3 & c_4 & -\varepsilon_6 \end{bmatrix} \begin{Bmatrix} \dfrac{\partial u}{\partial r} \\ \dfrac{u}{r} \\ \psi \end{Bmatrix}, \tag{25}$$

where

$$\begin{gathered} \{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_6\} = \frac{T_0}{c_{11}}\{\beta_{11}, \beta_{22}, (\beta_{11} - \beta_{22}), \beta_{33}\}, \\ \{\varepsilon_4, \varepsilon_5\} = \frac{1}{\rho C_E}\{\beta_{11}, \beta_{22}\}, \qquad \{c_1, c_2, c_3, c_4\} = \frac{1}{c_{11}}\{c_{12}, c_{22}, c_{13}, c_{23}\}. \end{gathered} \tag{26}$$

5. Application and boundary conditions

We take the conditions that initially occurred in the problem as

$$u = \left.\frac{\partial u}{\partial t}\right|_{t=0} = 0, \qquad \psi = \left.\frac{\partial \psi}{\partial t}\right|_{t=0} = 0. \tag{27}$$

Also, one can consider that the medium is quiet and the cylindrical surface is subjected to the following boundary conditions:

• Thermal shock varying heat

$$\theta(r, t) = \theta_0 H(t), \qquad r = R, \qquad \theta_0 = \text{const.}, \qquad t > 0. \tag{28}$$

From Eq. (16), then boundary condition (28) is given by

$$\psi(R,t) = \theta_0 H(t) + \frac{1}{2} k_1 [\theta_0 H(t)]^2. \tag{29}$$

- The cylindrical surface is traction free, i.e.,

$$\sigma_{rr}(r,t) = 0, \quad r = R. \tag{30}$$

6. Solution in Laplace's transform domain

Laplace's transform is utilized to change governing equations from the time domain into Laplace and space field. Assume that the underlying conditions for dimensionless temperature and displacement are zero. Applying Laplace's transformation

$$\bar{f}(r,s) = \int_0^\infty \mathrm{e}^{-st} f(r,t)\mathrm{d}t, \tag{31}$$

under the initial conditions (27) to Eqs. (23)–(25) and assuming that $\beta_{11} = \beta_{22}$ (i.e., $\varepsilon_4 = \varepsilon_5 = \varepsilon$) and $c_{11} = c_{22}$. That is

$$\frac{\mathrm{d}^2\bar{u}}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}\bar{u}}{\mathrm{d}r} - \frac{\bar{u}}{r^2} - (s^2 - \Omega^2)\bar{u} = \varepsilon_1 \frac{\mathrm{d}\bar{\psi}}{\mathrm{d}r}, \tag{32}$$

$$\frac{\mathrm{d}^2\bar{\psi}}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}\bar{\psi}}{\mathrm{d}r} = \frac{s(1+\tau_q s)}{1+\tau_\theta s}\left[\bar{\psi} + \varepsilon\left(\frac{\mathrm{d}\bar{u}}{\mathrm{d}r} + \frac{\bar{u}}{r}\right)\right], \tag{33}$$

$$\begin{Bmatrix} \bar{\sigma}_{rr} \\ \bar{\sigma}_{\xi\xi} \\ \bar{\sigma}_{zz} \end{Bmatrix} = \begin{bmatrix} 1 & c_1 & -\varepsilon_1 \\ c_1 & c_2 & -\varepsilon_2 \\ c_3 & c_4 & -\varepsilon_6 \end{bmatrix} \begin{Bmatrix} \dfrac{\mathrm{d}\bar{u}}{\mathrm{d}r} \\ \dfrac{\bar{u}}{r} \\ \bar{\psi} \end{Bmatrix}. \tag{34}$$

Equations (32) and (33) can be reduces to

$$(DD_1 - s^2 + \Omega^2)\bar{u} = \varepsilon_1 D\bar{\psi}, \tag{35}$$

$$\varepsilon q D_1 \bar{u} = (D_1 D - q)\bar{\psi}, \tag{36}$$

where

$$D = \frac{\mathrm{d}}{\mathrm{d}r}, \quad D_1 = \frac{\mathrm{d}}{\mathrm{d}r} + \frac{1}{r}, \quad q = \frac{s(1+\tau_q s)}{1+\tau_\theta s}. \tag{37}$$

Presenting the potential function $\phi(r)$, defined by

$$u = \frac{\mathrm{d}\phi}{\mathrm{d}r}, \tag{38}$$

into reduced equations (35) and (36), thus we have

$$(DD_1 - s^2 + \Omega^2)\bar{\phi} = \varepsilon_1 \bar{\psi}, \tag{39}$$

$$\varepsilon q D_1 D\bar{\phi} = (D_1 D - q)\bar{\psi}. \tag{40}$$

Combining Eqs. (39) and (40) one obtains the equation

$$\{\nabla^4 - [s^2 - \Omega^2 + q(1 + \varepsilon_1\varepsilon)]\nabla^2 + q(s^2 - \Omega^2)\}\bar{\phi} = 0. \tag{41}$$

If m_1^2 and m_2^2 are roots of the specific equation

$$m^4 - [s^2 - \Omega^2 + q(1 + \varepsilon_1\varepsilon)]m^2 + q(s^2 - \Omega^2) = 0, \tag{42}$$

we can factorize Eq. (41) as

$$(\nabla^2 - m_1^2)(\nabla^2 - m_2^2)\bar{\phi} = 0. \tag{43}$$

The roots m_1^2 and m_2^2 are obtained as

$$m_1^2 = \frac{1}{2}\left(2A + \sqrt{A^2 - 4B}\right), \quad m_2^2 = \frac{1}{2}\left(2A - \sqrt{A^2 - 4B}\right), \tag{44}$$

where

$$A = s^2 - \Omega^2 + q(1 + \varepsilon_1\varepsilon), \quad B = q(s^2 - \Omega^2). \tag{45}$$

Equation (43) can be written as the modified Bessel equation form of order zero as

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r} - m_1^2\right)\left(\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r} - m_2^2\right)\bar{\phi} = 0. \tag{46}$$

The general solution of Eq. (46), which is bounded as $r \to \infty$, is given by

$$\bar{\phi} = \sum_{i=1}^{2} A_i K_0(m_i r), \tag{47}$$

where A_1 and A_2 represent arbitrary integration parameters determined from the boundary conditions and $K_0(\cdot)$ are modified Bessel's functions of order zero of the first kind. Accordingly, the solution of the function $\bar{\psi}$ is derived as

$$\bar{\psi} = \frac{1}{\varepsilon_1}\sum_{i=1}^{2}(m_i^2 - s^2)A_i K_0(m_i r). \tag{48}$$

Furthermore, using the Bessel function relation

$$zK_n'(z) = -zK_{n\pm1}(z) \pm nK_n(z), \tag{49}$$

leads to

$$\bar{u} = -\sum_{i=1}^{2} m_i A_i K_1(m_i r). \tag{50}$$

After some mathematical manipulations, the thermal stress can be obtained as

$$\left\{\bar{\sigma}_{rr}, \bar{\sigma}_{\xi\xi}, \bar{\sigma}_{zz}\right\} = \frac{1}{2r}\sum_{i=1}^{2}\{\Psi_i, \Gamma_i, \Phi_i\}A_i, \tag{51}$$

where

$$\begin{aligned}
\Psi_i &= r(2s^2 - m_i^2)K_0(m_i r) - 2c_1 m_i K_1(m_i r) + m_i^2 r K_2(m_i r),\\
\Gamma_i &= r\left[m_i^2(2+c_1) - 2s^2\right]K_0(m_i r) - 2m_i K_1(m_i r) + c_1 m_i^2 r K_2(m_i r),\\
\Phi_i &= r\left[m_i^2\left(c_3 - \frac{2\varepsilon_6}{\varepsilon_3}\right) + \frac{2\varepsilon_6}{\varepsilon_3}s^2\right]K_0(m_i r) - 2c_4 m_i K_1(m_i r) + c_3 m_i^2 r K_2(m_i r).
\end{aligned} \tag{52}$$

From Eq. (17), we get the solution of temperature $\bar{\theta}$ as

$$\bar{\theta}(r,s) = \frac{-1 + \sqrt{1 + 2k_1\bar{\psi}}}{k_1}. \tag{53}$$

Also, Laplace's transform of the boundary conditions (29) and (31) gives

$$\bar{\psi}(R,s) = \theta_0\left(\frac{1}{s} + \frac{k_1}{2s}\right) = \bar{G}(s), \tag{54}$$

$$\bar{\sigma}_{rr}(R,s) = 0. \tag{55}$$

Using Eqs. (48) and (51) of the functions $\bar{\psi}$ and $\bar{\sigma}_{rr}$ into the boundary conditions (54) and (55), we obtain

$$\sum_{i=1}^{2}(m_i^2 - s^2)A_i K_0(m_i R) = \varepsilon_1 \bar{G}(s), \tag{56}$$

$$\sum_{i=1}^{2}\left[R(2s^2 - m_i^2)K_0(m_i R) - 2c_1 m_i K_1(m_i R) + m_i^2 R K_2(m_i R)\right]A_i = 0. \tag{57}$$

Solution of the system of Eqs. (56) and (57) gives the values of the constants A_1 and A_2 completing the solution in the domain of Laplace's transform.

7. Numerical inversion of Laplace's transforms

Keeping in mind the end goal to get the solutions in the physical domain, we convert Laplace's transform into the governing functions. We adopt a numerical reversal strategy that depends on a Fourier's series expansion [33]. In this technique, any function $\bar{g}(s)$ in Laplace domain can be modified to the time area $g(t)$ as

$$g(t) = \frac{\mathrm{e}^{ct}}{t}\left\{\frac{1}{2}\bar{g}(c) + \mathrm{Re}\left[\sum_{n=1}^{N}(-1)^n\bar{g}\left(c + \frac{\mathrm{i}n\pi}{t}\right)\right]\right\}, \quad \mathrm{i} = \sqrt{-1}. \tag{58}$$

For speedier convergence, various numerical trials have demonstrated that the estimation of c fulfills the relation $ct \approx 4.7$ [34].

8. Discussions of numerical results

In this section, to explain the general solution behavior of the obtained theoretic results, we display some discussions and numerical results. For the purpose of numerical evaluations, we take cobalt as an orthotropic material. Values of the appropriate parameters of the material are [35]

$c_{11} = c_{22} = 3.071 \cdot 10^{11}\ \mathrm{Nm}^{-1}$, $c_{12} = c_{13} = 1.650 \cdot 10^{11}\ \mathrm{Nm}^{-1}$, $c_{23} = \dfrac{1}{2}c_{12}$,
$\rho = 8836\ \mathrm{kg/m}^3$, $\beta_{11} = \beta_{22} = 7.04 \cdot 10^6\ \mathrm{N/m^2K}$, $\beta_{33} = 6.90 \cdot 10^6\ \mathrm{N/m^2K}$,
$C_E = 427$ J/kg K, $K_r = 69$ W/m K, $T_0 = 298$ K.

Using the above material parameters, the distributions of dimensionless physical quantities; displacement u, temperature θ and thermal stressess σ_{rr} and $\sigma_{\xi\xi}$ have been calculated mathematically and presented graphically in Figs. 1–12. Comparisons of numerical calculations were carried out for three cases.

8.1. Effect of phase-lags

Let us discuss how the non-dimensional temperature, displacement and thermal stresses vary with the phase-lag of the heat flux τ_q and the phase-lag of temperature gradient τ_θ when the variability thermal conductivity parameter k_1 remains constant. In this case (Figs. 1–4), we take different values of the parameters τ_q and τ_θ. To get the governing equations of CTE theory from the introduced new model, we take ($\tau_\theta = \tau_q = 0$). To obtain the basic equations of LS model, we take ($\tau_\theta = 0$, $\tau_q = 0.1$). Finally, in the case of generalized theory of thermoelasticity suggested by Tzou (DPL), we put ($\tau_q > 0$, $\tau_\theta > 0$). We can conclude the following points from the demonstrated figures:

- The heat wave front moves forward with a finite speed in the medium with the passage of time.

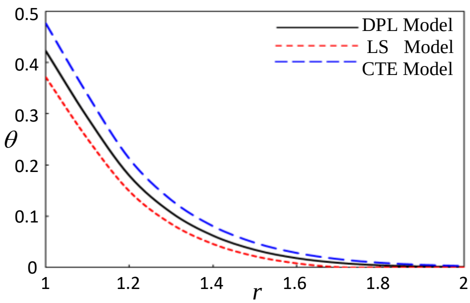

Fig. 1. Temperature distribution θ for different values of phase-lags τ_q and τ_θ

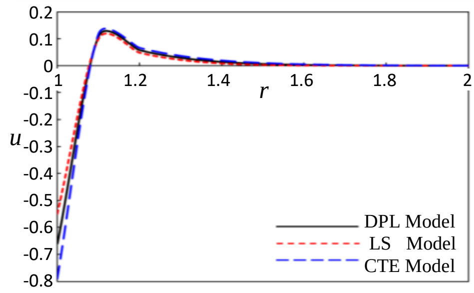

Fig. 2. Displacement distribution u for different values of phase-lags τ_q and τ_θ

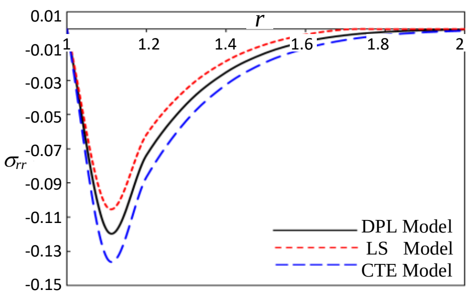

Fig. 3. Thermal stress distribution σ_{rr} for different values of phase-lags τ_q and τ_θ

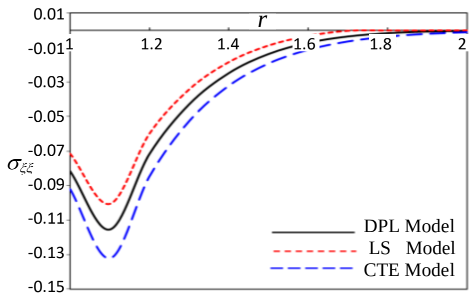

Fig. 4. Thermal stress distribution $\sigma_{\xi\xi}$ for different values of phase-lags τ_q and τ_θ

- The temperature distribution decreases as the space variable r increases (Fig. 1).
- From Fig. 2, it is seen that value of displacement u increases as r decreases in the interval $1.1 \geqslant r \geqslant 0$ and decreases in the interval $1.6 \geqslant r \geqslant 1.1$, then it reaches to steady state when $r \geqslant 1.6$.
- From Figs. 3 and 4, it can be found that thermal stresses σ_{rr} and $\sigma_{\xi\xi}$ increase as the distance r increases. Also, it is clear that the most extreme estimations happen close to the surface of the hole and it decreases when r increases.
- Near the surface of the cylinder, where the boundary conditions dominate, the coupled and the generalized theories give very close results. Inside the cylinder, the solution is markedly different. This is due to the fact that thermal waves in the coupled theory travel with an infinite speed of propagation as opposed to a finite speed in the generalized case.
- In all Figs. 1–4, it is observed that the phase-lag of the heat flux τ_q and the phase-lag of temperature gradient τ_θ have significant effects on all fields.
- It is observed that all the waves reach the steady state depending on the values of the phase-lags τ_q and τ_θ.
- The variations of temperature for DPL theory is small in comparison to CTE theory.

- The difference between the three curves at any fixed point as well as at fixed parameter k_1 for the three theories is clearly visible from these figures.
- The values in classical theory of thermoelasticity (CTE model) are different compared to those of other theories. The fact that in generalized thermoelasticity theories (DPL and LS), the waves propagate with finite speeds is evident in all figures. This validates clearly the difference between the modified theories of thermoelasticity and the classical coupled model.
- The behavior of three theories is generally quite similar. With the increase in distance, the results are quite close to each other, which is in agreement with the generalized theories of thermoelasticity.

8.2. Effect of thermal conductivity parameter

Investigating the variability thermal conductivity parameter k_1 effect on the non-dimensional temperature, displacement and thermal stresses when phase-lag of the heat flux τ_q and the phase-lag of temperature gradient τ_θ remain constants. Here, we consider three distinct quantities of the parameter of variability thermal conductivity k_1 to examine the effect of temperature on thermal conductivity. We take $k_1 = -1, -0.5$ when the thermal conductivity is dependent of temperature and $k_1 = 0$ for fixed thermal conductivity.

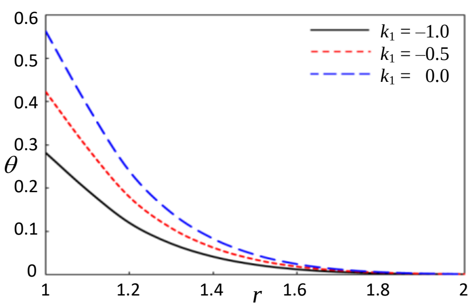

Fig. 5. Temperature distribution θ for different values of thermal conductivity parameter k_1

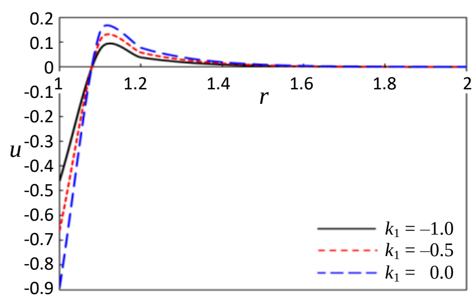

Fig. 6. Displacement distribution u for different values of thermal conductivity parameter k_1

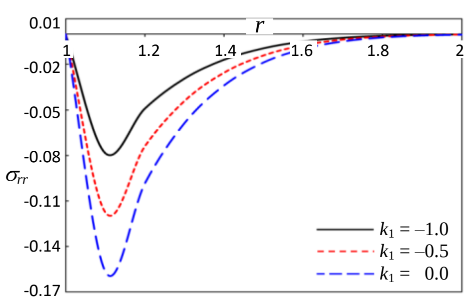

Fig. 7. Thermal stress distribution σ_{rr} for different values of thermal conductivity parameter k_1

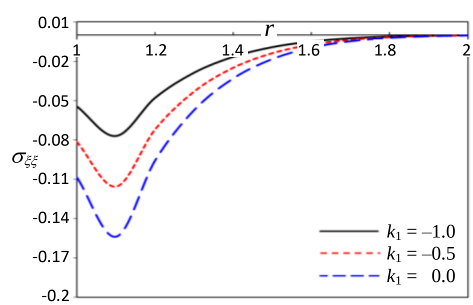

Fig. 8. Thermal stress distribution $\sigma_{\xi\xi}$ for different values of thermal conductivity parameter k_1

We selected the rotation parameter as $\Omega = 5$ and lags $\tau_q = 0.02$ and $\tau_\theta = 0.01$ in this case. We also observe the following important facts from Figs. 5–8:

- The variability parameter k_1 has pronounced effects on all the studied fields.
- From Fig. 5, it is easily seen that the value of temperature increases with the increase of k_1 in all contexts of all theories of thermoelasticity and satisfied the considered boundary conditions.
- From Fig. 6, it is shown that the amplitude of distribution of the displacement u rises with the increase of k_1.
- From Figs. 7 and 8, it can be found that the absolute values of thermal stresses σ_{rr} and $\sigma_{\xi\xi}$ increase as the parameter k_1 decreases.
- We have noticed from these figures that the variability thermal conductivity parameter k_1 has a significant effect on all the fields which add an importance to our consideration about the thermal conductivity to be variable.
- From these figures, we find that the field quantities depend not only on the state and space variables t and r, but also on the variability thermal conductivity parameter and phase-lags parameters. The phenomenon of finite speeds of propagation is manifested in all figures.

8.3. Effect of rotation

Studying the effect of rotation on dimensionless physical quantities when phase-lag of the heat flux τ_q and the phase-lag of temperature gradient τ_θ and the variability thermal conductivity parameter k_1 remain constants. In this case, Figs. 9–12 show the variations of temperature, displacement and thermal stresses along the radial direction at various values of rotation parameter Ω. From these figures it is observed that:

- The amplitude of the temperature has a slight increase for the rotating case in comparison with the non-rotating case due to the presence of rotation term (see Fig. 9).
- It is clear from the graph in Fig. 10 that the variations of displacement with the varied values of the rotation parameter Ω in the context of phase-lag model is close in both rotating and non-rotating case.
- Also, a significant difference in thermal stresses is noticed for different values of rotation parameter Ω (see Figs. 11 and 12).
- Consequently, the idea of rotation in a thermoelastic orthotropic medium with the parameters of phase-lags will yield more destruction as contrast with non-rotating type.
- Therefore, the presence of rotation in current model is of significance.
- Rotation will play its role in the wave propagations that has been appeared by looking at the distributions of various waves in a rotating and non-rotating thermoelastic orthotropic medium.
- As detected from the introduced graphical results, the parameter of rotation plays a significant role on the deformation variants in the body.

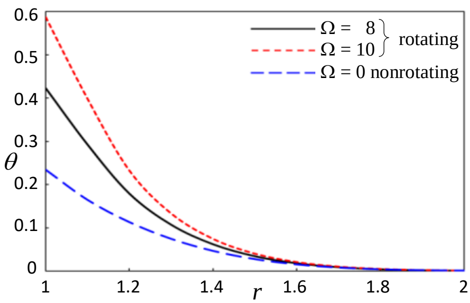

Fig. 9. Temperature distribution θ for different values of angular velocity (rotation) Ω

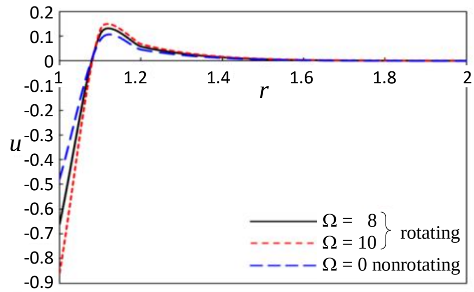

Fig. 10. Displacement distribution u for different values ofangular velocity (rotation) Ω

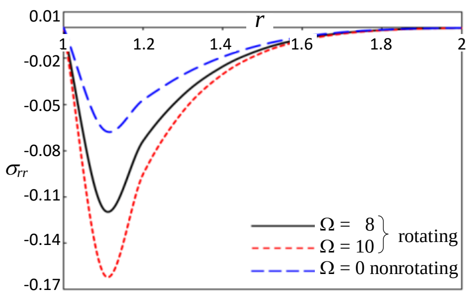

Fig. 11. Thermal stress distribution σ_{rr} for different values of angular velocity (rotation) Ω

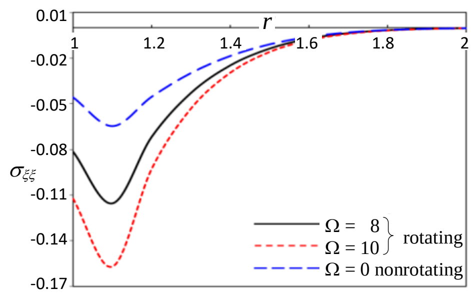

Fig. 12. Thermal stress distribution $\sigma_{\xi\xi}$ for different values ofangular velocity (rotation) Ω

9. Conclusions

In this work, we have investigated a one-dimensional problem for an infinite homogeneous orthotropic thermally conducting body containing a cylindrical cavity affected by the angular velocity under thermal shock surface heating using the method of Laplace's transform. From the obtained numerical results, some main conclusions are given by:

- The studied field quantities depend on time t and space r and on the variability of thermal conductivity, rotation and phase-lags parameters.
- The dependence of thermal conductivity on the temperature has significant effects on the velocity of propagation of waves and mechanical interactions.
- In the introduced model, the presence of rotation terms is essential.
- When the considered medium rotates with some angular velocity, the values of temperature distribution are much smaller in magnitude.
- The idea of rotation in a thermoelastic orthotropic medium will yield more pulverization as contrast with non-rotating sort.
- The theories of Biot and generalized thermoelasticity proposed by Lord and Shulman can be obtained as special cases from the current model.

- The results displayed in this work should prove to be valuable for researchers in scientific and designing, in addition, for those working on the improvement of mechanics of materials.

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