Delivery-flow routing and scheduling subject to constraints imposed by vehicle flows in fractal-like networks

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The problems of designing supply networks and traffic flow routing and scheduling are the subject of intensive research. The problems encompass the management of the supply of a variety of goods using multi-modal transportation. This research also takes into account the various constraints related to route topology, the parameters of the available fleet of vehicles, order values, delivery due dates, etc. Assuming that the structure of a supply network, constrained by a transport network topology that determines its behavior, we develop a declarative model which would enable the analysis of the relationships between the structure of a supply network and its potential behavior resulting in a set of desired delivery-flows. The problem in question can be reduced to determining sufficient conditions that ensure smooth flow in a transport network with a fractal structure. The proposed approach, which assumes a recursive, fractal network structure, enables the assessment of alternative delivery routes and associated schedules in polynomial time. An illustrative example showing the quantitative and qualitative relationships between the morphological characteristics of the investigated supply networks and the functional parameters of the assumed delivery-flows is provided.

Key words: transport network, fractal structure, declarative modeling, multimodal process, delivery flow, vehicles flow.

1. Introduction

Assuming that, just as any system, a supply network (SN) has a structure and behavior (Bocewicz and Banaszak 2015), one can distinguish elements of the structure of such a system (including transport roads, a fleet of vehicles, goods transfer facilities, etc.) and the processes which determine its behavior (e.g. the frequency and timeliness of deliveries, downtime, costs, etc.). This distinction is reflected in the researchers perspective take on SNs. Some researchers accentuating the context of analysis of behavior

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(i.e. the execution of processes which guarantee the desired quality of supplies) reachable in networks with arbitrarily given structures, and others focusing on the synthesis of structures which enable processes executed in these structures to be completed in a satisfactory manner.

The approach proposed in this paper assumes the possibility of decomposing the above-discussed problems of synthesis and analysis. Such a decomposition would entail treating the flow of means of transport and flow of materials transported by those means separately. In other words, traffic and delivery flow problems observed within SN can be seen as decomposed into the two separate problems of:

- analysis/synthesis of processes related to the movement of means of transport and

- analysis/synthesis of processes of supply/distribution of the goods transported by those means.

If this type of decomposition is implemented, the above problems can be formulated either as problems of finding an optimum goods flow solution for a given structure of a given SN or as problems of scheduling the fleet (and/or configuring the transport structure) of the SN. The solution of the latter should guarantee the expected quality of goods flow (deliveries).

Assuming the SN structure encompasses a part of a transportation network (TN), the following SN analysis and synthesis problems are usually considered: Does an arbitrarily given SN following TN topology make it possible to carry out transport processes that meet user expectations? Is there a SN network structure which guarantees the execution of transport processes that match the expectations of its users? The distinction made between the above two classes of problems assumes that just as any structure of a system determines its admissible behavior, so too the behavior of a system can be determined by its different structures. The elements that determine the solution to these problems are the relationships between selected structural and functional parameters of the system. This means that declarative models of analysis and synthesis problems should incorporate decision variables specifying the topology of the transport networks, the fleet of vehicles that use it and the stations and loading/unloading stops across the network as well as the transport routes of the objects being moved and transport route schedules. The structure of the constraints which connect the decision variables in this type of models allows the formulation of suitably dedicated constraint satisfaction problems (CSPs) (Banaszak and Bocewicz 2014). These types of problems have the added advantage that they are easily implemented in constraint programming languages such as OzMozart, ILOG, ECLiPSe, (Banaszak and Bocewicz 2014; Dang et al. 2014; Sitek and Wikarek 2015).

In the above context, the class of TN analyzed here is limited to network structures with a regular, recursive morphology typical of tree or mesh (grid) topologies (Fig. 1). This category of topologies, which include urban transport systems, are the subject of intensive research (Bahrehdar and Moghaddam 2014; Buhl et al. 2006; Courtat 2012; Sandkuhl and Kirikova 2011).
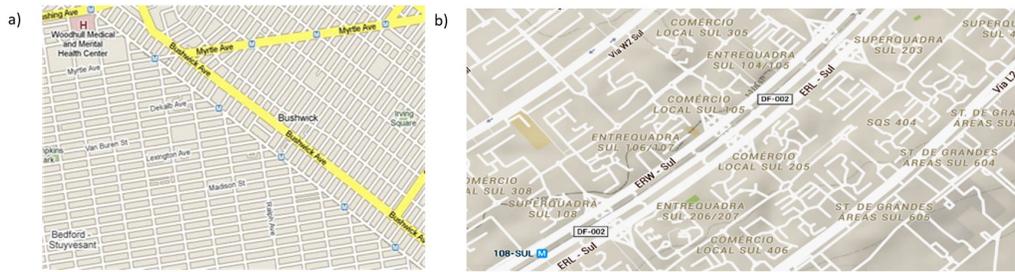
Figure 1: TN structures with a mesh topology (a), and routes with a tree-like structure in the city of Brasilia (b)

For the sake of further considerations let us assume that a SN encompasses all possible branches of transport and transport technologies, including road and rail (surface and underground) transport, e.g. buses, streetcars, subway lines etc.

Multimodal transport is by definition the transportation of freight performed with different, alternative modes of transport along the same transport route, during which goods can be transshipped between different transport modes (Susan 2014). Therefor the concept of a multimodal transport process (MTP) has been introduced (Bocewicz et al. 2015a, 2015b). According to this definition, a MTP involves the movement of objects using different modes of transport in a single, integrated transport chain on a given route. Examples of MTPs include the processes associated with daily commuting (bus – street car – subway), courier services (e.g. DHL), etc. A characteristic feature of such multimodal processes is that their routes are made up of local segments operated by one mode of transport and the objects are moved by suitable local means of transportation. A good illustration of this feature is a passenger travelling by subway who, in the course of his journey, changes from one line to another in accordance with an itinerary.

The considered problem is the simultaneous supply/distribution of different goods from suppliers to recipients, who are all located at various points of the transport network. Knowing the parameters of the local transport companies, one for example wishes to identify delivery routes which would minimize transport time or satisfy the constraints regarding the number of vehicles that can be simultaneously loaded/unloaded at a transshipment point. On the other hand, in solving a synthesis problem subject to the same assumptions and constraints, one builds on knowledge of routes and transport times to find the parameters of local transport companies which would guarantee the delivery of goods within a given time window. Both problem classes belong to the class of computationally hard problems.

In that context the aim of this research is to develop a methodology for the synthesis of regularly structured SNs that follow a fractal-like topology of a TN ensuring fixed cyclic execution of local transport processes. The proposed methodology, which implements sufficient conditions for the synchronization of local cyclic processes allows to develop a method for rapid prototyping of supply/distribution processes which satisfy

the time window constraints given. The adopted simplifying assumptions enable rapid prototyping of admissible solutions in polynomial time. The investigations presented in this paper and the results obtained in the course of this study are a continuation of previous studies collected in (Bocewicz et al. 2015a, 2015b; Bocewicz and Banaszak 2015).

The article is divided into seven sections. Section 2 contains a review of the most important trends in the research into Vehicle Routing Problems. Section 3 presents a declarative modelling driven approach to grid-like SN design while being a part of a TN fractal-like topology. Section 4 provides a problem statement while Section 5 describes a methodology for rapid prototyping of congestion free traffic flows. Computational experiments illustrating the proposed methodology and the scope of future work are presented in Section 6 and Section 7, respectively.

2.   Related work

The Vehicle Routing Problem (VRP) is an optimization problem which encounters in logistics and transport and applies to supply chain management in the physical delivery of goods and services. In a simplest form VRP can be defined as the problem of designing least cost delivery routes from a depot to a set of geographically dispersed locations subject to a set of constraints. The VRP is an NP-hard problem (Kumar and Panneerselvam 2012) and one of the most widely studied topics in Operations Research.

A special instance of the VRP is the Periodic VRP (PVRP), in which delivery routes are constructed over a period of time (Francis and Smilowitz 2006; Coene and Arnout 2010). The objective of both the VRP and the PVRP is to minimize travel costs or the total travel distance required to visit all customers during the planning horizon. The difference is that in PVRPs the frequency of visits to customers within a given time horizon varies from customer to customer. Besides of the assumptions traditionally associate with the VRP, the PVRP has to take into account a time horizon, usually subdivided into regular periods, as well as a customer visit frequency stating how often within a particular period a customer must be visited. A solution to the PVRP consists of sets of routes which jointly satisfy demand constraints and frequency constraints. The objective is to minimize the sum of the costs of all routes over the planning horizon (Coene and Arnout 2010).

There are several variants to the PVRP as there are for the general VRP. Among special instances of the PVRPs the Service Choice (PVRP-SC) plays a pivotal role through its focus on more efficient vehicle tours and/or greater service benefit to customers. The PVRP-SC can be stated as follows: Given is a set of customers with known demand and minimum visit frequency requiring service over the given time horizon as well as a fleet of capacitated vehicles, a set of service schedules with headways and service benefits as well as a network with travel times. The goal is to find an assignment of nodes to service schedules and a set of vehicle routes for each period of the given time horizon that minimizes the total routing cost incurred net of the service benefit accrued (Francis and Smilowitz 2006).

Since the design of an optimal route for a fleet of vehicles to service a set of customers is the main objective of the VRP, most study efforts are focused on the problems of management of different modes of transport services rather than on the routing of delivery processes within a given TN. The majority of research focuses either on scheduling available transport modes so that they can service customers in given time windows or on designing supply networks taking into account the size and capacity of the planned fleet and the topology and traffic capacity of routes. Gąska et al. (2015)'s work synchronization of public transport services and urban network analysis by Sevtsuk and Mekonnen (2012) are classical examples of such research.

Relatively few studies are devoted to delivery-flow routing and scheduling in the existing supply networks; examples of this type of solutions are courier, express and parcel services (CEP), the rail-road freight services (Crisalli et al. 2013), and many other door-to-door multimodal supply chains. The vast majority of studies focus on the supply/distribution of goods. Besides vehicle routing other media routings are considered such as message routing (Socievole et al. 2015), energy routing, data routing (Gurakan 2016), cargo routing (Cargo Movement, Defense Transportation Regulation), and courier routing (Lan et al. 2007).

An alternative approach to traffic flow modelling the specific character of passenger traffic is found in the work of Gao and Wu (2011). Their concept of multimodal transport processes assumes that supply processes are executed in an environment of cyclic transport processes. An example of a serial multimodal process is the movement of passengers travelling by different metro lines along a route of arbitrarily selected stations. A characteristic feature of Gao and Wu's approach is that it offers the possibility of distinguishing levels that determine traffic flows of transportation means and traffic flows of multi-commodity flows.

3. Fractal-like supply networks

The numerous road network patterns deployed in public and/or freight transport systems range from tightly structured fractal networks with perpendicular roads in a regular raster pattern to hierarchical networks with sprawling secondary and tertiary roads feeding into arterial roads in a branch like system (Courtat 2012; Kelly and McCabe 2006; Levinson and 2012). The throughput of passengers and/or freight depends on geometrical and operational characteristics of public transportation and/or cargo supply networks.

Recent research (Courtat 2012; Sun et al. 2015; Zhang 2011) draws attention to the fact that the development of urban agglomerations, and in particular the morphology of urban regions, is subject to the laws of recursion, which are best modeled by fractal structures. The consequences of this fact can be used both in predicting the needs related to the expansion of the existing transport infrastructure, as well as planning new industrial and/or urban agglomerations. The main advantages following from the regular structure of supply network layout are flexibility and robustness which are vital to the improvement of robustness of supply/distribution networks (Haghani and Oh 1996;
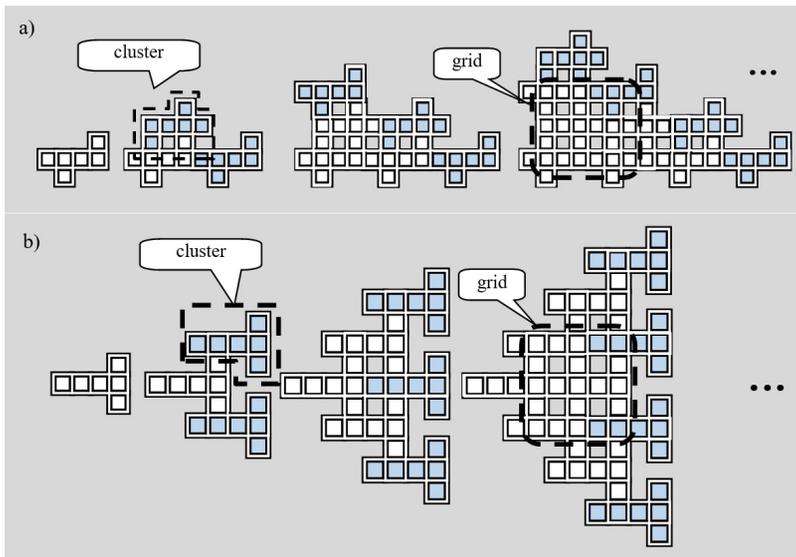
Figure 2: Examples of routes in transport networks with a fractal structure: generated by clusters of shape (a), generated by clusters of shape (b)

Susan 2014). Among numerous reports concerning mesh-like or grid-like as well as fractal-like structures of urban transportation networks the approach proposed in Buhl et al. (2006) should be mentioned. A positive aspect of this the approach is that it allows easy estimation of delivery schedules while taking into account the cyclic behavior of both local transportation modes and the structure of the whole supply network. This is because transportation processes executed by particular lines are usually cyclical and as a consequence multimodal transport processes supported by them also have a periodic character. In other words, these assumptions explicitly constrain the topology of a SN to TNs with fractal structures and implicitly make the efficiency of potential MTPs (e.g. regarding the possible delivery dates) conditional on the admissible flow of traffic (e.g. congestion-free traffic) operating under local transport processes.

For illustration, let us consider two types of TNs with fractal structures as those shown in Fig. 2. A graph theoretical model of SN mirroring the TN structure from Fig. 2 (a) with vertices (which represent network resources, i.e. stations and stops and shared route sections) and edges is shown in Fig. 3 (a). In general case the SN structure can be treated as a part mapped of TN topology. Further discussion, will focus on SNs of grid-like structures, i.e. structures obtained in the course of clusters aggregation following specific rules of fractal growth. This assumption implies that research can be limited to certain elementary structures, the combination of which define the whole transport network. Examples of such structures are marked with a dotted line in Figs. 3 (a), (b).

The considered grid-like structures are composed of clusters (involved in a fractal growth) which are identical repeating substructures, called Elementary Covering Struc-

tures (ECS). The proper example provides ECS from Fig. 3 (b), resources of which are denoted by ^kR_r that means the r-th resource in the k-th elementary transport network. Local transport processes are marked with labeled arcs whose orientations indicate the direction of flow of local traffic (transport modes). For example, the arc labeled P_i^j denotes that the considered traffic flow, in the local transport process, is comprised of "j" units of an i-th transport mode (Figs. 3 (b)).

Dispatching rules for synchronization of access of local transport processes to shared network resources are marked with labels $^k\sigma_r$, e.g. $^k\sigma_{10} = (P_4^1, P_5^1)$ describe the order of access of means of transport, e.g. P_4^1, P_5^1 to a shared resource ^kR_r, e.g. $^kR_{10}$; see Figs. 3 (b).

Models of MTP routes that show the sequences of the resources between which objects are moved are represented graphically with bold symbols of nodes and arcs; see Fig. 4 (a).
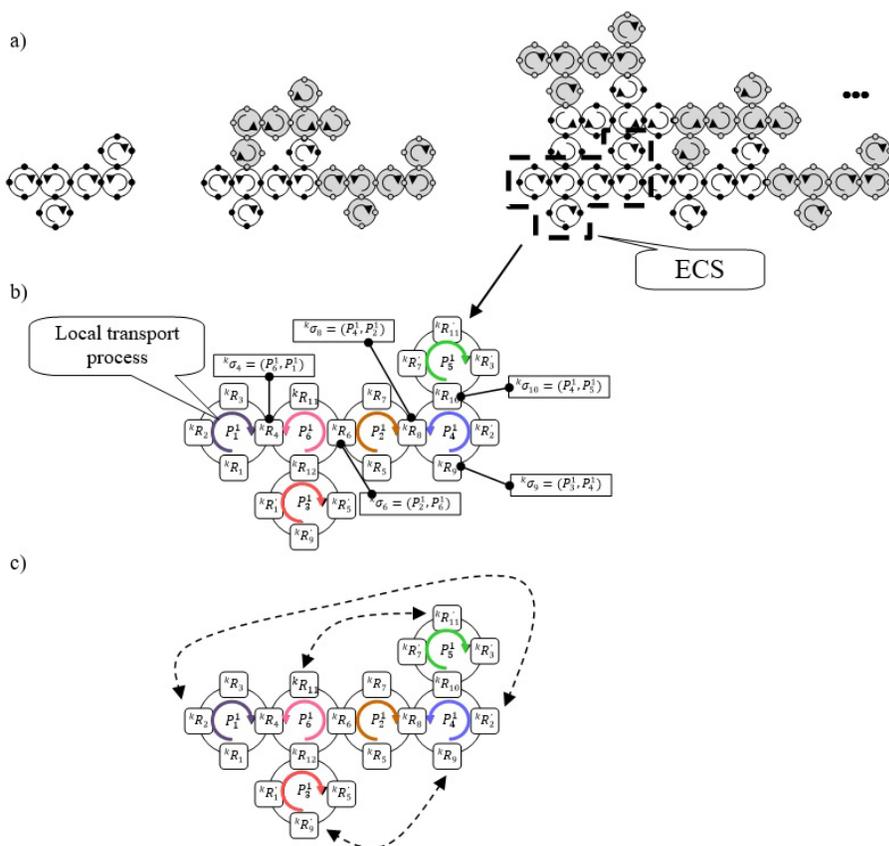
Travel and/or dwelling times of means of transport for various network resources are designated with $t_{i,j}$ which denotes the time of execution of the j-th operation of the i-th transport process. A graphic model which can be used to represent both travel/dwelling times and the waiting times of the means of transport used and the goods moved using those means is a Gantt chart (Fig. 4 (b)). Its graphical representation of timetables and/or supply schedules), allows one to assess waiting times associated with the fact that a means of transport has to wait for access to a requested but currently occupied resource, as well as waiting times of objects transported in MTP chains resulting from unavailability of the scheduled means of transport.

A further assumption is that the repetitive, cyclic behavior of an ECS described in above way implies cyclic, i.e. congestion-free flow of traffic across the network SN (Bocewicz and Banaszak 2015; Bocewicz et al. 2015a).

4. Problem statement

In order to determine the relationships between the structure of TNs along with the means of transport moving in these network and MTPs which determine the routes for the transport of objects, let us consider a reference TN model which integrates the models of a SN structure, local transportation processes and MTP. The reference model of a TN with a fractal structure presented in the previous section makes it possible to develop an appropriate dedicated declarative model allowing the formulation of the aforementioned analysis and synthesis problems as CSPs.

A constraint satisfaction problem: $SC = (X, D, C)$ is usually given by (Sitek and Wikarek 2015) a finite set of decision variables $X = \{x_1, x_2, ..., x_n\}$, a finite family of finite domains of discrete decision variables $D = \{D_i \mid D_i = \{d_{i,1}, d_{i,2}, ..., d_{i,j}..., d_{i,m}\}, i = 1..n\}$, and a finite set of constraints limiting the values of the decision variables $C = \{C_i \mid i = 1..L\}$, where: C_i is a predicate $P[x_k, x_l, ..., x_h]$ defined on a subset of set X. What

Legend:
$^{k}R_r$ − r-th resource in the k-th transport network,
P_i^j − local transportation process performed by j- transport units of the i-th transport mode
$^{k}\sigma_r$ − dispatching priority rule assigned to $^{k}R_r$

Figure 3: Graph theoretical models of fractal structures corresponding to the route patterns from Fig. 2 (a) are shown in (a); uncovered and covered forms of ECS are shown in (b) and (c), respectively

is sought is an admissible solution, i.e. a solution in which the values of all decision variables X satisfy all constraints C.

Accordingly, the declarative TN model comprises:

- Sets of decision variables describing the structures of

  - local transport processes, i.e. the type and number of resources and modes of transport they use, as well as the associated travel/ dwelling times,

  - the MTP, i.e. the type and number of resources in a chain and the type and number of transport modes used, as well as the associated travel/dwelling times,
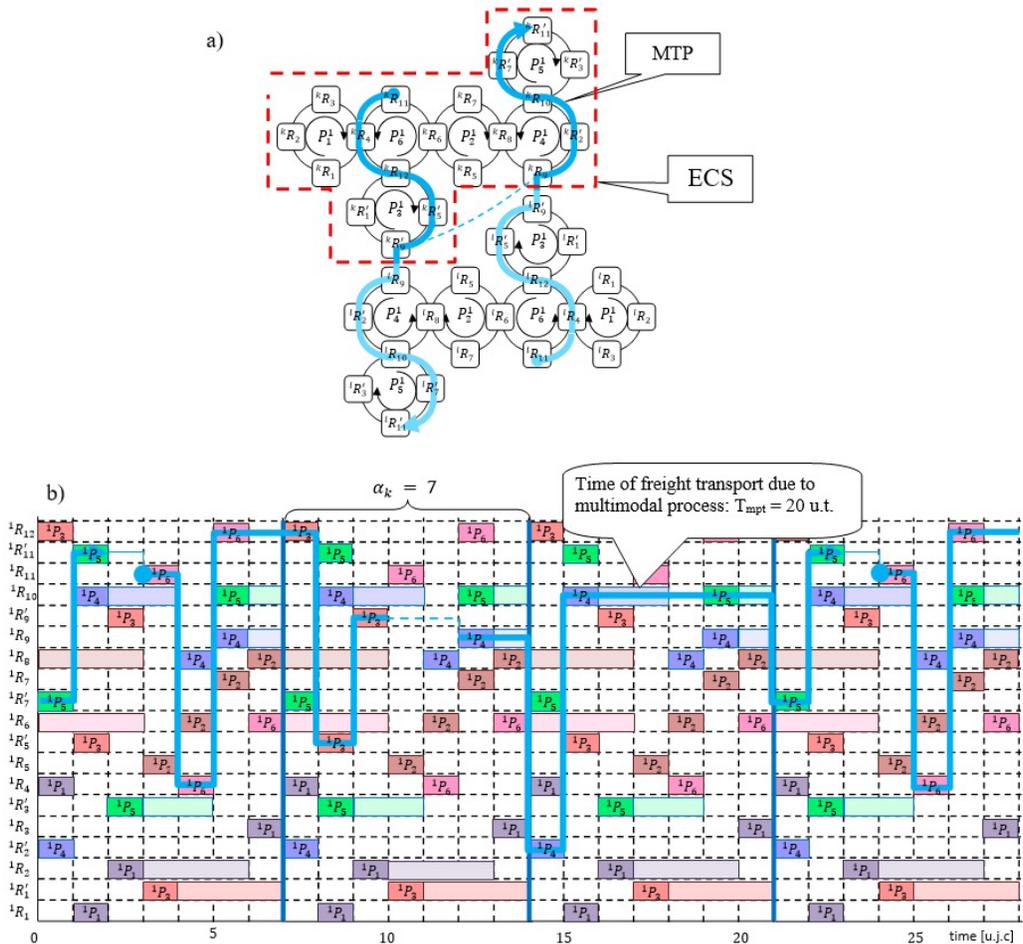
Figure 4: A representation of a MTP performed in a SN given in Fig. 3 (a), Gantt's chart of local transport processes and the MTP (b)

- Domains of decision variables,

- Sets of determining constraints:

    - sets of dispatching rules assigned to shared network resources,

    - transport schedules determining the periods (tacts) and dates of delivery of transported goods.

The set of decision variables found in this model includes:

- a set of resources R found in the structure of the SN and a set of resources $\overline{R}$ determining MTPs,

- a set of local processes occurring in the SN and transportation routes of multi-modal processes,

- a set of dispatching rules θ governing the access of local processes/streams to shared resources of the SN and a set of dispatching rules $\overline{\theta}$ governing the access of multimodal processes/streams to the resources of set $\overline{R}$,

- initial states: S^0 in the SN (corresponding to the initial allocation of local processes) and $\overline{S^0}$ corresponding to the initial allocation of MTPs in the set of resources $\overline{R}$.

Sets of decision domains corresponding to the above-mentioned decision variables determine the integer values of resource availability, integer operation execution times, integer vectors specifying the number of local process streams (vehicles moving along shared routes), permutation vectors of local processes sharing the resources and binary-valued vectors which specify the values of initial states.

Sets of constraints imposing regularity of structure of an SN and the principle of mutual exclusion of local processes as well as constraints enforcing the principle of mutual exclusion of MTPs (Bocewicz et al. 2015) also include constraints which treat as identical the resources of these structures. For example, $\overline{R} \subset R$, and times of goods transport operations with the travel times of the means of transport used for this purpose (Bocewicz et al. 2014). Additionally, it is assumed that there are constraints which relate routes of multimodal processes mP_i with fragments of routes of local processes.

Let us consider the "covered" form of ECS from Fig. 3. (b). The so called "covered" form arises as a result of "gluing together" selected vertices of ECS. Which particular vertices are "glued" together in the "covered" form is determined by the choice of those resources of the elementary transport structure which are shared with the resources of neighboring structures of the TN. For example, a vertex corresponding to resource $^kR_9$ is glued with a vertex corresponding to resource $^kR'_{11}$, because resource $^kR'_9$ is shared with resource $^lR_9$ which is a counterpart of $^kR_9$, see Fig. 4 (a).

It can be shown that if the traffic flow in a given covered form of an ECS is free of congestion, i.e. the schedule which specifies it is a cyclic schedule, the flow of traffic in the entire transport network consisting of uncovered forms of ECSs also has a cyclic nature (Bocewicz and Banaszak 2015). This observation allows one to focus on formulating the following CSP, the solution to which is a structure (a set of dispatching rules) that guarantees a congestion-free flow of traffic. In other words, assuming that the behavior of each i-th ECS is represented by a cyclic schedule $^{(i)}X' = (^{(i)}X_k \mid k = 1, \ldots, h, \ldots, L_i)$, where: $^{(i)}X_h$ is a set of beginning moments of operation of the h-th local process of the i-th ECS and L_i denotes the cardinality of the set of local processes comprising the $i - th$ ECS, the constraint satisfaction problem in question has the following form:

$$PS_i = (\{^{(i)}X', ^{(i)}\theta, ^{(i)}\alpha\}, \{D_X, D_\theta, D_\alpha\}, \{C_L, C_M, C_D\}) \tag{1}$$

where: $^{(i)}X', ^{(i)}\theta, ^{(i)}\alpha$ – decision variables,

- $^{(i)}X'$ – cyclic schedule of the i-th ECS,

- $^{(i)}\theta$ – set of dispatching rules determining the order of operations competing for access to the common resources of the i-th ECS,

- $^{(i)}\alpha$ – set of values of periods of local processes occurring in the i-th ECS,

D_X, D_θ, D_α – domains of admissible values of discrete decision variables C_L, C_M, C_D – finite sets of constraints limiting the values of decision variables

- C_L, C_M – sets of conditions constraining the set of potential behaviors of the i-th ECS (Bocewicz et al. 2015a),

- C_D – a set of sufficient conditions that if satisfied guarantee congestion-free (i.e. deadlock-free and collision-free) flow of traffic in a transport network modeled by the i-th ECS and execution of transport operations and loading/unloading operations (i.e. operations competing for access to common resources).

The sought for solution to problem (1) is schedule $^{(i)}X'$ which satisfies all the constraints of the family of sets $\{C_L, C_M, C_D\}$. Constraints C_L, C_M (Bocewicz et al. 2015a) ensure that local processes in the "uncovered" form of an ECS are executed in a cyclic manner, i.e. the execution of operations is specified by an appropriate cyclic schedule. They do not guarantee, however, the same for the "covered" form of this structure. The additional constraints C_D, which follow from the match-up rule (Bocewicz and Banaszak 2015) that conditions the fit between cyclic schedules, guarantee that the local processes occurring in the structures which satisfy them are executed in a cyclic manner.

5. Methodology for congestion free traffic flows prototyping

The methodology proposed assumes a regular (grid-like or fractal-like) topology of a SN composed of a finite set of identical repeating substructures, called Elementary Covering Structures (ECSs). An ECS is a directed coherence graph comprised of elementary substructures (modelling local cyclic processes) which occur in the regular structure of an SN such that:

- the graph is composed of elementary cyclic digraphs which model local transportation processes,

- it can be used to tessellate a given regular structure.

The patterns do not exhaust all possible cases of ECSs which may differ in shape (i.e. in the way the elementary substructures are put together) and in scale (i.e. in the number of elementary structures they contain). However, their common feature is that they can be tessellated (like a mosaic) to form a pattern that recreates a given regular structure. Each ECS has a corresponding covered-form ECS which is formed by gluing

together the vertices of the ECS which in the regular structure of the SN are joined with the corresponding fragments of the elementary structures of an adjacent ECS.

The behavior of a given supply network can be predicted on the basis of the behavior of its ECSs. It is easy to observe that an initial allocation of local processes in a regular network (which maps the allocation of processes in the ECS), will be followed in each individual ECS by a next allocation of local processes in compliance with the same priority selection rules. That is because if one replicates the same initial process allocation in all remaining ECSs, the structure will be free of collisions between processes which use the same resources (both shared resources and those integrated/merged in the covered representation). The new allocation in the regular structure concerns the processes and resources which are "copies" of the processes and resources of the contemplated ECS. This means that the successive process allocations which occur in cycles after each initial allocation in the ECS will have their counterparts in the overall regular structure – allocations of all the local processes of the structure. Thus, period α of the processes executed in the regular structure is equal to the period of processes executed in the ECS. This means that cyclic behavior of the ECS implies cyclic behavior of the regular-structure SN.

In summary, a cyclic behavior of the covered form of the ECS entails a cyclic behavior of the supply network. It means that by solving a small-scale computationally hard problem (associated with an ECS), one can solve, in online mode, a large scale problem associated with a corresponding regular-structure supply network.

Moreover, it can be shown that every cyclic behavior of the supply network with a regular structure can be associated with an appropriate time-driven event system, whose period is determined by the period of the covered form of the ECS. Viewing those systems as time-driven discrete event systems, provides the possibility of quantitative analysis of the behavior of an SN oriented toward estimating traffic flow rates, routing of distribution streams, and scheduling of transport fleet and the material flows it supports.

6. Illustrative example

As an illustration of the approach, consider the grid-like networks shown in Fig. 2. Problem (1) considered for corresponding ECSs from Figs. 4 (a) and 5 (a), respectively was implemented and solved in the constraint programming environment Oz-Mozart (CPU Intel Core 2 Duo 3GHz RAM 4 GB). When the assumption was made that all operation times in local processes are the same and equal to $t_{i,j} = 1$ u.t. (unit of time), the first acceptable solution was obtained in less than one second. The sets of the dispatching rules obtained are collected in the Table 1.

An analysis of cyclic schedules $^{k}X'$ allows an easy deduction in both cases of the same value of the cycle length $\alpha_k = 7$ u.t. However, the time of freight $T_{mpt} = 20$ u.t. in the case from Fig. 4 (a) is significantly shorter, than in the case observed in Fig. 5 (a) i.e. $T_{mpt} = 52$ u.t. Therefore, in the considered case, the grid-like SNs obtained in the course
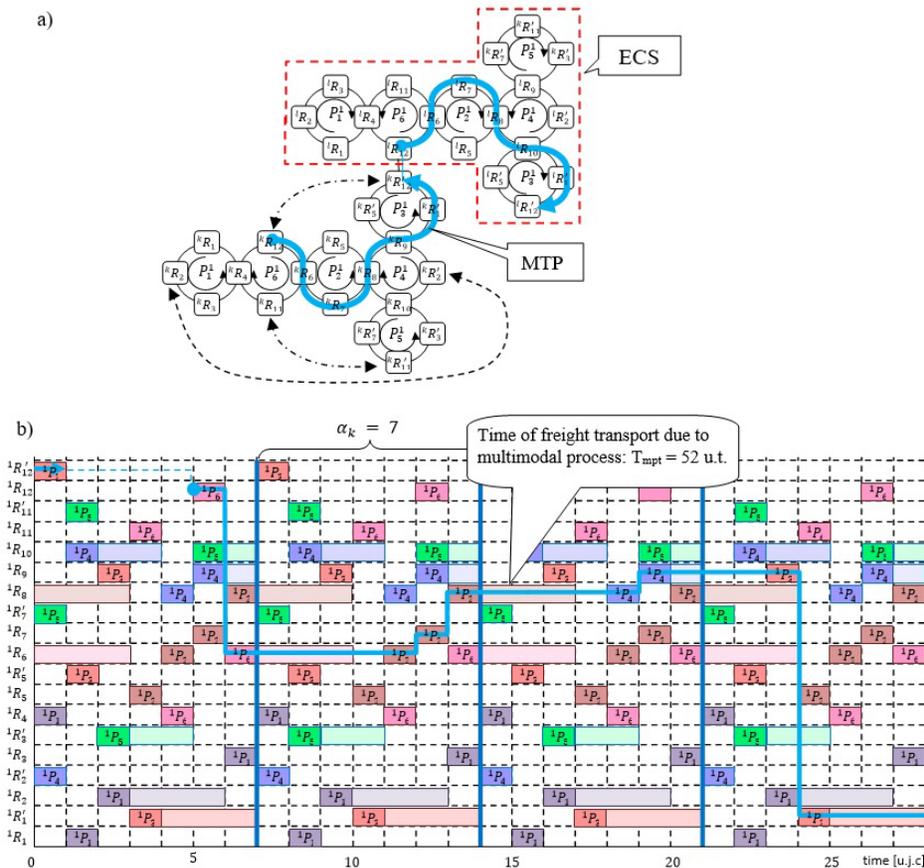
Figure 5: A representation of a MTP performed in a SN following TN from Fig. 2 (b),
Gantt's chart of local transport processes and the MTP (b)

Table 1: The sets of the dispatching rules

ECS	Dispatching rules
ECS following SN from Fig. 4 (a)	${}^k\sigma_{11} = (P_5^1, P_6^1), {}^k\sigma_9 = (P_3^1, P_4^1),$ ${}^k\sigma_2 = (P_4^1, P_1^1), {}^k\sigma_4 = (P_1^1, P_6^1),$ ${}^k\sigma_6 = (P_6^1, P_2^1), {}^k\sigma_8 = (P_2^1, P_4^1)$
ECS following SN from Fig. 5 (a)	${}^k\sigma_{12} = (P_3^1, P_6^1), {}^k\sigma_{11} = (P_5^1, P_6^1),$ ${}^k\sigma_2 = (P_4^1, P_1^1), {}^k\sigma_4 = (P_1^1, P_6^1),$ ${}^k\sigma_6 = (P_6^1, P_2^1), {}^k\sigma_8 = (P_2^1, P_4^1)$

of growth of different fractal structures results in the same cyclic steady state of traffic flow guaranteeing congestion free delivery flows.

7.   Conclusion

The declarative reference model of a transport system presented in this study enables an analysis of the relationships between the structure of the system and its potential behavior, thus allowing formulation and solving of analysis and synthesis problems corresponding to questions such as: Is it possible to make supplies which meet customer demands in a SN with a preset structure? Is there a SN structure that ensures deliveries which meet user expectations? Because this model focuses on TN with a fractal-like and/or grid-like structure, it allows one to formulate a constraint satisfaction problem and determine the constraints of this problem in the form of sufficient conditions, the satisfaction of which guarantees smooth execution of traffic and delivery flows in this type of networks. These conditions, when implemented in commercially available constraint programming platforms, allow rapid prototyping of alternative transport routes and associated schedules in polynomial time.

The issues of planning and/or prototyping of alternative structures and/ or behavior of transport networks with fractal structures presented in this work are part of the broader topic of cyclic scheduling which includes problems occurring in tasks associated with determining timetables, telecommunications transmissions, production planning, etc. In future, while continuing along the line of inquiry related to preventing traffic flow congestion in transport networks, we plan to broaden the scope of our research to include the problems of robust scheduling and the related problem of preventing re-scheduling of timetables in urban transport networks.

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