

The effect of sampling interval on the predictions of an asperity contact model of two-process surfaces

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Abstract. Contact of random machined two-process steel textures with a smooth, flat steel surface is discussed in this paper. Two-process surfaces were machined by vapour blasting followed by lapping. An elastic-plastic contact model was applied, assuming distributed radius of asperities. Calculation procedures allowed the mean surface separation, contact pressure, and area fraction to be computed as functions of sampling intervals. Parameters characterizing the summits important in contact mechanics were calculated for different sampling intervals. Plasticity index of two-process textures was calculated using the modified procedure. It was found that the influence of sampling interval on normal contact depended on the rough surface ability to plastic deformation. The use of a traditional method of calculation overestimated the plasticity index. Peaks from plateau surface region governed contact characteristics of two-process surfaces.

Key words: two-process surface, contact, sampling interval.

1. Introduction

All surfaces are rough and consist of asperities of different shapes and sizes. Surface finish plays a huge part in determining the proportion between elastic and plastic deformations. Study of a deformation of contacting asperities is important for understanding the mechanisms of different phenomena like friction, lubrication, wear, and thermal and electrical contact resistance. Understanding the relationship between surface topography and contact characteristics can lead to specification of parameters of manufacturing processes considering functional properties of surfaces. Therefore, contact of rough surfaces was studied by many researchers. The pioneering contribution was made by Greenwood and Williamson [1] who developed the elastic contact model (GW model), in which a rough surface was represented by many spherical summits of the same radius of curvature and Gaussian distribution of the surface height. This model was extended by many researchers. Carbone and Bottiglione [2] slightly improved the GW model in which all the spheres did not have the same radius. They found that multi-asperity contact theories, considering the effect of the summit curvature variation as a function of summit height, gave similar results. Abbott and Firestone [3] developed a plastic contact model in which the rough surfaces were simply truncated. To connect these two approaches (the elastic and plastic models) Chang et al. (CEB model) [4] and Zhao et al. (ZMC model) [5] developed elastic-plastic contact models, based on theoretical assumptions. In subsequent works, the elastic-plastic contact models were established on the basis of finite element analysis (FEA) of a deformable sphere and a rigid flat. These models were de-

veloped by Kogut and Etsion (KE model) [6, 7], Jackson and Green (JG model) [8, 9] as well as Shankar and Mayuram (SM model) [10, 11]. The analysis of the normal contact is helpful in the study of a tangential contact [12, 13].

Asperities of different scales of sizes exist on the surface. Therefore, the results of application of statistical contact models depend on the sampling interval. Sampling interval affects spatial and hybrid parameters of surface texture [14, 15]. It has considerable effects on surface topography parameters important in contact mechanics, like summit density and summit curvature [16–18]. McCool [19] determined a frequency range for which the elastic contact models were applicable. Thomas and Rosen [20] proposed a method of sampling interval selection, on the basis of the assumption that repetitive contact should be elastic. Vallet et al. [21] studied the effect of the optimum sampling interval on the roughness and contact parameters. Sampling interval choice in contact problems can be solved using the analysis of multi-scale rough surfaces [22–24]. Wilson et al. [23] used sinusoids to represent the asperities in contact at each surface scale. A model of surface separation as a function of load was developed by summing the distance between two surfaces at all scales. In order to describe the contact of rough surfaces which occurs at different length scales, Wriggers and Reinert [24] developed a multi-scale approach using the finite element analysis as the numerical simulation tool.

Pawlus and Żelasko [25] analysed the effect of the sampling interval on the predictions of the elastic GW [1] and elastic-plastic CEB [4] contact models. They found that an increase in the sampling interval caused a decrease in the plasticity index. The separation between contacting rough surfaces decreased with higher sampling interval applied for a given load. The predicted contact area was larger for a bigger sampling interval and the same load. Differences among separations and contact

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areas computed with different sampling intervals after using the elastic-plastic model were smaller than after applying the elastic model. These observations were confirmed after the analysis of the elastic-plastic contact of rough random one-process surface with a rigid flat after the application of KE [6, 7] and JG [8, 9] elastic contact models. For high tendency to plastic deformation the relation of contact load to contact area was almost independent of the sampling interval. For surfaces inclined to elastic-plastic or elastic deformation due to an increase of sampling interval, the predicted contact area grew for a given load [26].

Kucharski and Starzyński [27] selected the sampling interval using proposals of Greenwood and Wu [28, 29].

The mode of surface deformation is of considerable interest when understanding contact mechanics. The well-known version of plasticity index Ψ was developed by Greenwood and Williamson [1]. The contact mode should be elastic for plasticity index Ψ smaller than 0.6 and plastic for Ψ higher than 1.0, while for Ψ in range 0.6–1, the mode of deformation was in doubt. The plasticity index can be used as a qualitative wear measure, it is believed that when it is higher, material loss is also larger. Vermeules and Hobleke [30] found that surfaces of steel sheets of smaller plasticity indices were characterized by smaller inclination to seizure. Hirst and Hollander [28] believed that under boundary lubrication regime, the risk of adhesion was related to plasticity index. Rosen and Thomas [32] studied the relationship of the plasticity index to machining parameters in cylinder honing. The problem is that the plasticity index depends on the sampling interval [25, 26, 31].

Most models of rough surface contact used a simple statistical assumption such as the Gaussian distribution of heights. But surfaces of tribological interest seldom display such features in service, because the wear process usually removes peaks, resulting in asymmetric ordinate distribution. These surfaces are examples of two-process textures, and they belong to textured surfaces [33]. Such topographies are also formed in operating process. The plateau honed cylinder surface is a typical example of machined stratified random surface. Two-process surfaces are said to have advantages over conventional one-process textures, but evidence for that is lacking. Only the authors of a few works [34–36] tried to isolate the influence of parameters characterizing two-process surfaces. Greenwood and Williamson [1] thought that, for the purposes of contact analysis, non-Gaussian worn surfaces were characterized by a Gaussian distribution of summit height from the plateau surface part. Leefe [37] also believed that contact of two-process surfaces was governed by the Gaussian distribution of peak heights from the plateau surface portion. Only a few works were concerned with the contact of two-process surfaces [37–39].

In this paper, the effect of the sampling interval on the predictions of an asperity contact model of two-process machined surfaces will be studied.

2. Materials and methods

Discs made of 42CrMo4 steel of 4 GPa hardness obtained after heat treatment were machined. Two-process surfaces were fin-

ished by vapour blasting followed by lapping. In the vapour blasting process, aloxite 95-A-30-J was used as the abrasive material. The pressure of the feeding system was 0.6 MPa, while the flux angle was 60°. Different disc textures after one-process resulted from a change of the distance between the disc and nozzle, as well as the machining time. Lapping was performed using a metallographic polishing lathe. The size of grains was lower than 12 μm . This was an attempt to obtain isotropic surfaces. Machined surfaces were measured by a Talysurf CCI Lite white light interferometer with 0.01 nm height resolution. Its measuring area is 3.3 \times 3.3 mm, 1024 \times 1024 points.

Contact of steel-on-steel flat surfaces with composite Young modulus $E' = 113.115$ GPa was considered. It was assumed that the machined disc contacted over a smooth, flat surface.

In contact mechanics of rough surfaces, statistical methods are widely used. They are sensitive to the sampling interval. Therefore, studying the effect of the sampling interval on the predictions of asperity contact of two-process surfaces using an approach based on a statistical model is a task of vital importance. In contact calculation, the fundamental assumptions that the asperities are spherical near their summits, that there is no interaction between asperities, and that only the asperities deform during contact [1], were adopted. The surface point was chosen to be a summit if its ordinate was higher than those of its eight neighbors. This criterion was assumed based on the works of Greenwood [40], Sayles and Thomas [41], and a previous research of one of the authors of this paper [42]. The radius of each summit was calculated as reciprocal of its mean average curvature in perpendicular directions. The summit curvature was computed using a three-point formula [43]. Other surface topography parameters important in contact mechanics, like areal density of summits S_{ds} , standard deviation of their heights σ_s , and the distance between the mean of asperity heights and that of surface ordinates y_s , were calculated. For each summit in contact, interference and then the contact area and load were determined using the JG [8, 9] model. The total contact area A and total contact load P were calculated for various separations h , based on surface heights [4]. The vertical range of summit ordinates was divided into 200 intervals. The total contact area A and total contact load P were obtained by summing the contributions of all contacting asperities located above a given separation. The computations of the contact area and load were repeated for various sampling intervals. Similar procedures were used in [25] for CEB and GW models, in [26] for KE and JG models, and in [39] for the ZMC model.

For one-process surfaces, the plasticity index is calculated, based on the work of Greenwood and Williamson [1], using the following formula:

$$\Psi = \frac{E'}{H} \left(\frac{\sigma_s}{R} \right)^2, \quad (1)$$

where: H – hardness of softer material, E' – Young's elastic modulus, σ_s – standard deviation of asperity height, R – mean asperity radius of curvature.

For two-process surface, a modified method of plasticity index computations was used. First, the standard deviation of summit heights of plateau surface portion σ_{2ps} was calculated. To achieve this goal, the heights of all summits were saved. Then, the special profile was obtained from asperity heights. This profile was studied using the probability approach [44]. The obtained curve is similar to the material probability plot of a two-process texture, with clearly visible plateau (peak) and valley regions. The lowest summit belonging to the plateau part of this special profile is the lower limit of the plateau portion. The standard deviation of the plateau part was assumed to be σ_{2ps} . It was calculated in a way similar to the calculation of standard deviation of plateau height of the material probability curve [44]. Then, the proposed average radius of curvature of two-process surface R_{2p} was determined only for summits positioned above the lower limit of the plateau portion. Finally, the plasticity index of two-process surfaces is equal to:

$$\Psi_{2p} = \frac{E'}{H} \left(\frac{\sigma_{2ps}}{R_{2p}} \right)^2 \quad (2)$$

The procedure of obtaining the plasticity index of two-process textures is described in detail in [45].

The normal contact of eight two-process surfaces was considered. However, the behavior of three representative two-process textures will be analyzed in detail.

3. Results and discussion

Fig. 1. presents isometric views of details of three two-process surfaces.

Table 1 shows a list of selected parameters (important in contact mechanics) of two-process surfaces calculated using the procedure described above [45]. In addition, the parameters characterising two-process surfaces are shown. They are: standard deviation of plateau height Spq, standard deviation of valley height Svq, and material ratio at plateau-to-valley transition Smq [44]. Parameters calculated in a traditional manner are listed in Table 2.

Parameters characterising the height of surface IIA were the highest. The Spq parameter of surface IIB was much smaller

Table 1

Surface topography parameters and plasticity indices of two-process surfaces for various sampling intervals SI; Parameters σ_{2ps} , R_{2p} , and Ψ_{2p} were calculated according to [44]

Surface	SI μm	Sq μm	Spq μm	Svq μm	Smq %	σ_{2ps} μm	R_{2p} μm	Ψ_{2p}
IIA	6.4	3.8	1.44	6.4	58.3	1.15	34.2	5.19
	12.8					1.1	64.7	3.69
	19.2					1.06	95.2	2.99
	25.6					0.97	137.2	2.38
IIB	6.4	3.0	0.65	5.7	62.4	0.512	52.7	2.79
	12.8					0.473	113.7	1.82
	19.2					0.454	198.7	1.35
	25.6					0.447	288.4	1.11
IIC	6.4	1.26	0.24	5.9	91	0.165	141.8	0.97
	12.8					0.132	384.3	0.52
	19.2					0.13	727.6	0.38
	25.6					0.12	1187.4	0.28

Table 2

Surface topography parameters important in contact mechanics and plasticity indices of two-process surfaces; Parameters σ_s , R, Sds, and Ψ were calculated in the traditional manner

Surface	SI μm	σ_s μm	Sds $(\mu\text{m})^{-2}$	R μm	$\sigma_s \text{SdsR}$	ys μm	Ψ
IIA	6.4	2.38	0.00092	29.7	0.065	2.28	8.0
	12.8	1.82	0.00035	60.2	0.038	2.88	4.92
	19.2	1.5	0.00019	92.6	0.026	3.32	3.6
	25.6	1.35	0.00012	135.2	0.022	3.35	2.82
IIB	6.4	1.3	0.0011	48.1	0.069	1.27	4.65
	12.8	0.88	0.00039	114.5	0.039	1.38	2.48
	19.2	0.68	0.0002	191.1	0.026	1.48	1.69
	25.6	0.6	0.00013	284.3	0.022	1.52	1.3
IIC	6.4	0.23	0.0014	140.6	0.045	0.28	1.14
	12.8	0.18	0.00038	380	0.026	0.34	0.62
	19.2	0.15	0.00023	721.6	0.025	0.34	0.41
	25.6	0.14	0.00014	1184.3	0.023	0.35	0.31

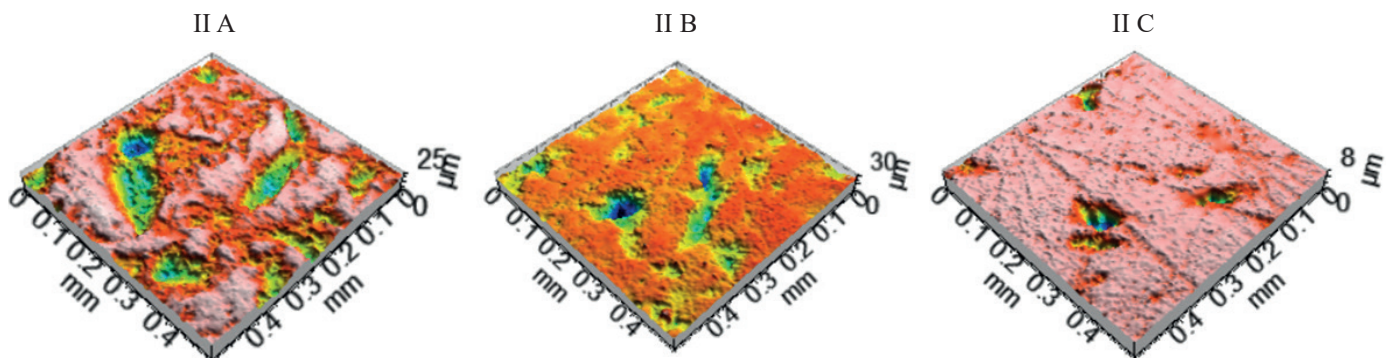


Fig. 1. Isometric views of details of two-process surfaces

than that of texture IIA. Amplitude parameters of surface IIC were considerably smaller than those of the remaining two-process surfaces, while the Smq parameter was much higher. The statistical parameters Spq, Svq, and Smq were insensitive to sampling interval changes. It is important that the Smq parameter is higher than 50% (Table 1). It means that using the traditional formula (1), bulk deformation should be considered, and therefore, the modified formula (2) must be used.

Various plasticity indices calculated using two methods were caused by differences between standard deviations of surface heights calculated by traditional and modified procedures. Mean radii of summit curvature R_{2p} and R were similar. Deviations between parameters σ_{2ps} and σ_s were smaller when the sampling interval increased.

Behavior of parameters characterising two-process surfaces with an increase of the sampling interval were similar to those of one-process textures [25, 26] – the standard deviation of summit height σ_{2ps} decreased, while the mean radius of summits curvature R_{2p} increased and the plasticity index Ψ_{2p} decreased. The σ_{2ps} parameter was smaller than the Spq parameter. Leeffe [37] thought that small errors can be made by making an assumption that the standard deviation of asperity heights is equal to the Spq parameter of two-process surfaces. However, this assumption is true only for small sampling interval corresponding to high correlation between ordinates of neighboring points.

Application of the traditional method had overestimated the plasticity index of surface IIA from 1.54 to 1.33, 1.2 and 1.18 times, of surface IIB from 1.67 to 1.44, 1.25 and 1.17 times, but of surface IIC – from 1.21 to 1.19, 1.08 and 1.11 times, when the sampling interval increased. One can see that the smallest deviations occurred for surface IIC. For all surfaces the sampling interval increase caused a decrease of the relative deviation of

plasticity indices calculated using two methods. The mentioned tendency can be explained with the analysis of cumulative distribution of summit heights (Figs. 2–4). The increase of the sampling intervals caused an increase of the transition points between summits in both plateau and valley surface portions. Therefore, owing to an increase of the sampling interval, the effect of summit heights from the valley surface part on the standard deviation of summit heights σ_s became smaller, and therefore the relative difference between σ_{2ps} and σ_s decreased. Abscissa of the transition point of cumulative distribution of summit heights is larger than the Smq parameter. This finding was obtained from the analysis of other machined or simulated [45] two-process surfaces. When this abscissa is larger, the effect of the valley part on the standard deviation of summit height σ_s is smaller. Therefore, the relative differences between plasticity indices σ_{2ps} and σ_s of surface IIC are much smaller than those of surfaces IIA and IIB – especially for comparatively small sampling intervals 6.4 μm and 12.8 μm . For larger sampling intervals 19.2 μm and 25.6 μm , the cumulative distribution of summits heights became symmetric. In this case, one can compute the plasticity index of surface IIC on the basis of the traditional procedure. Greenwood and Williamson [1] and Leeffe [37] thought that on two-process surface the summits originated from the plateau part – however, this supposition is true only for comparatively high sampling intervals and Smq parameter.

Due to an increase of the sampling interval, the correlation between ordinates of neighboring points decreased, as Whitehouse and Archard [46] predicted. It was determined [45] that for a small correlation, the summit heights distribution had a Gaussian shape.

One can conclude from this analysis that the plasticity index of two-process textures is affected only by the plateau portion of

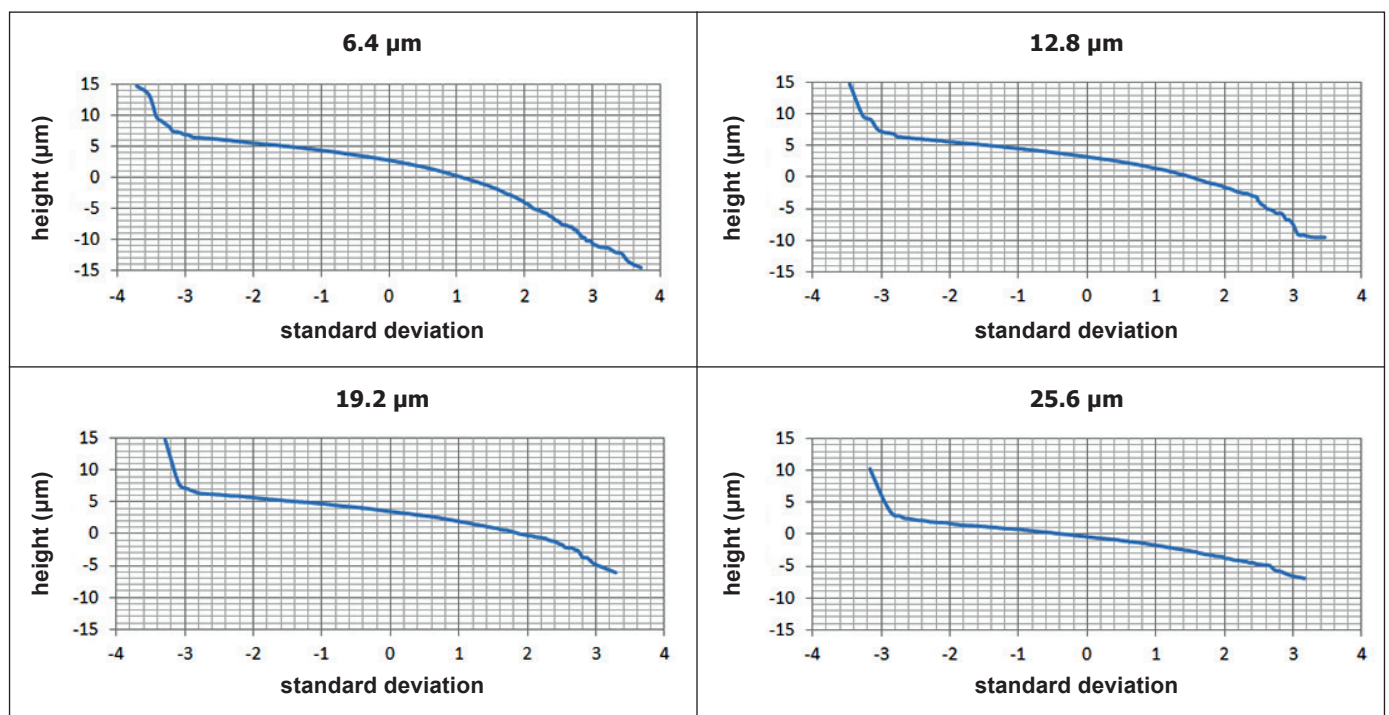


Fig. 2. Cumulative distribution of summit heights of surface IIA for different sampling intervals

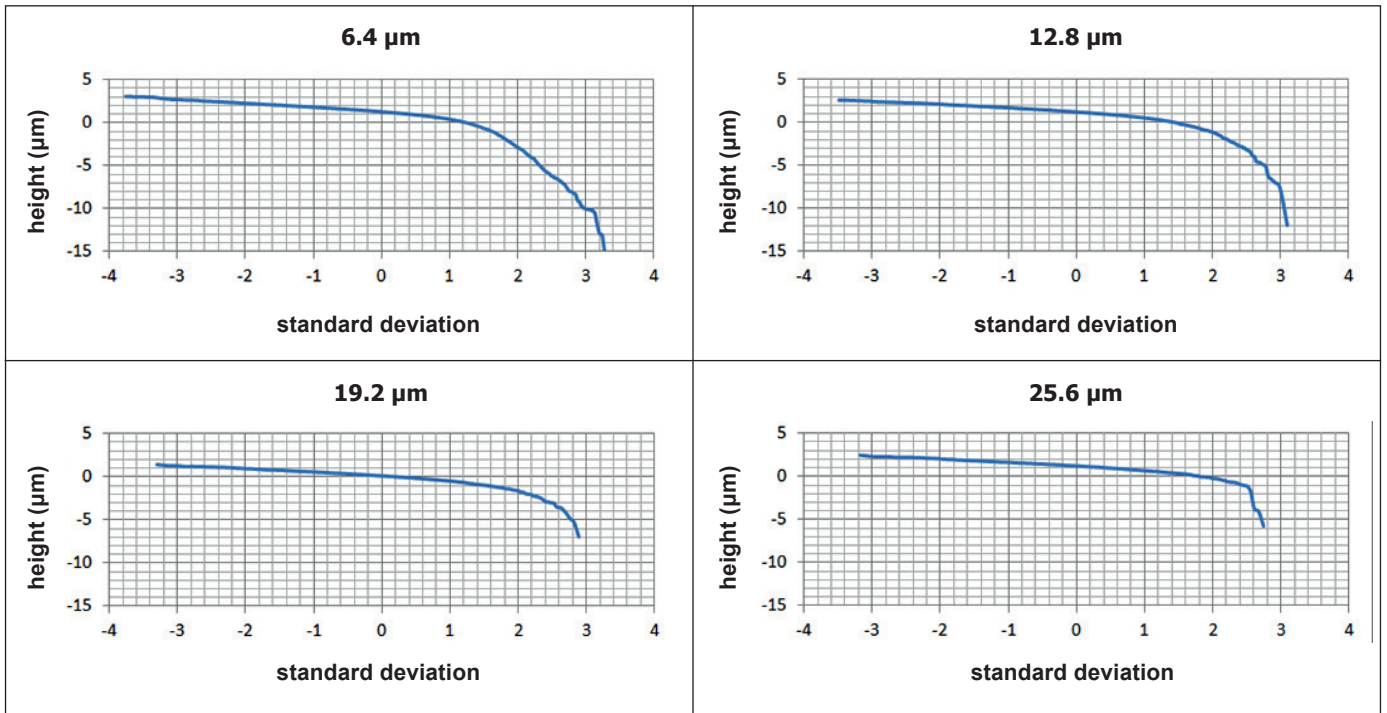


Fig. 3. Cumulative distribution of summit heights of surface IIB for different sampling intervals

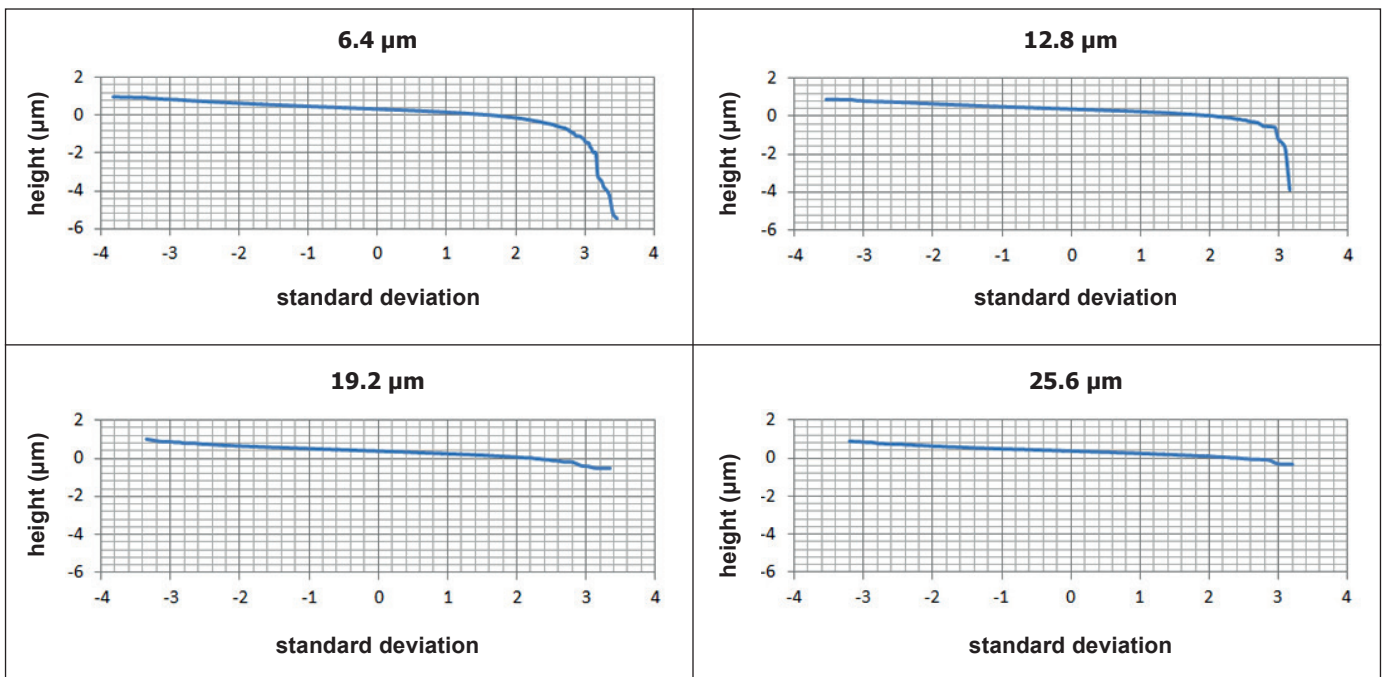


Fig. 4. Cumulative distribution of summit heights from surface IIC for different sampling intervals

surfaces. However, the valley surface part influences the mean radius of summit curvature. A summit consists of the highest central point and eight neighboring points. In the calculations, only the summits whose highest point belonged to the plateau part were considered. This method of summit definition seems to be correct because the standard deviation of asperity heights identified by this method was very similar to the σ_{2ps} value.

However, other summit points, except for the highest one, could have originated from the valley part. This is the reason why R_{2p} was similar to R . When all surface points (the highest and its neighbors) used for summit identification originated from the plateau portion, the obtained mean radius of curvature was higher than R_{2p} . One can conclude from this that the valley surface part leads to an increase of plasticity index.

Similar result was obtained in [44], where the mean radius of curvature of simulated two-process surfaces was smaller than that of a one-process plateau texture. Two-process textures were created by imposition of two surfaces of a Gaussian ordinate distribution, therefore the mean radius of curvature of plateau surface was known a priori.

Superiority of the modified method of plasticity index calculation over the traditional method was confirmed in experimental research [47].

Surfaces IIA and IIB should be plastically deformed, however the predicted behavior of surface IIC should be elastic-plastic or elastic, depending on the sampling interval. Contact of two-process surfaces IIA, IIB, and IIC was studied using the JG model. The results are presented in Figs. 5–7. Top graphs (a) show separation h as a function of contact pressure p and the sampling interval, middle graphs (b) show separation h versus dimensionless contact area (area fraction) A/A_n , and bottom graphs (c) show the real area of contact A/A_n as a function of contact pressure for different sampling intervals.

For two-process surfaces separation h was counted from height corresponding to a material ratio of 50%, because the mean plane of two-process surfaces situated typically under this height seems to be unrealistic.

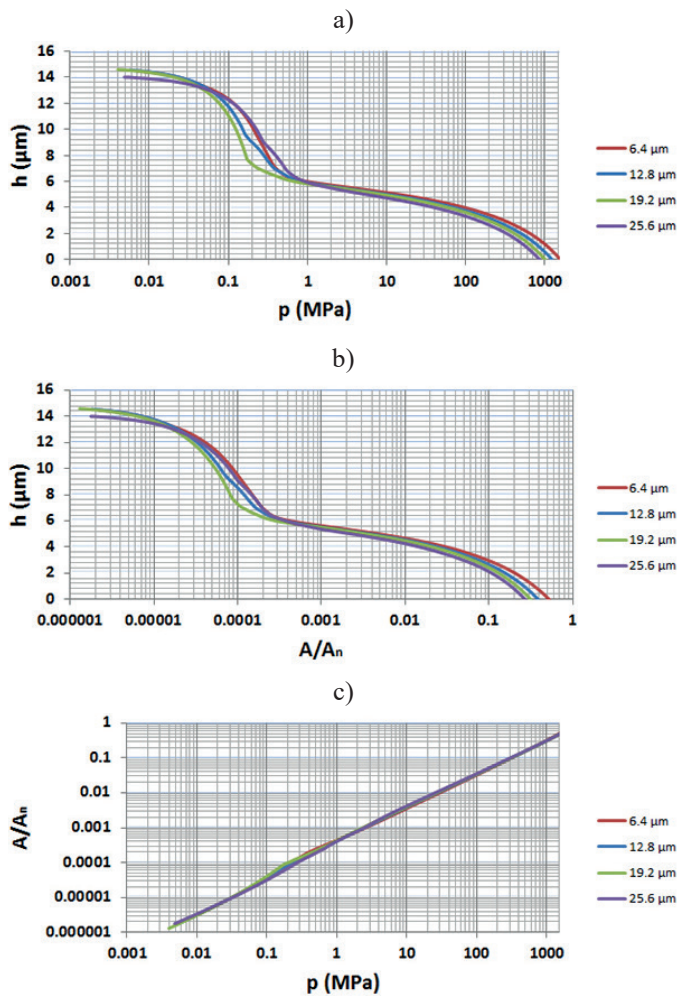


Fig. 5. Contact characteristics of surface IIA

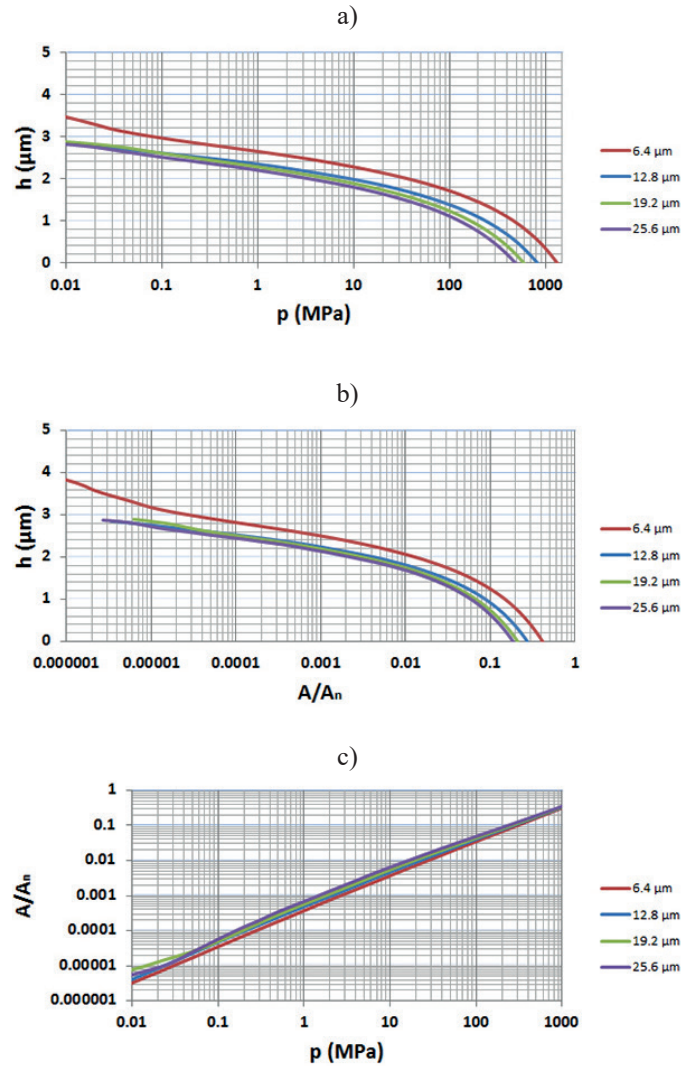


Fig. 6. Contact characteristics of surface IIB

For surface IIA up to $4Spq$, an increase of the sampling interval caused a decrease of contact pressure p and contact area A/A_n for a given separation h (Figs. 5a and 5b); contact behavior is governed by individual summits for larger separations (Fig. 2).

As a result, the dependence between the load and the contact area is almost independent of the sampling interval (Fig. 5b).

For surface IIB, an increase of the sampling interval also caused (similarly to surface IIA) a decrease of the contact pressure p and the dimensionless contact area A/A_n for a given separation h . However, the ratios of the largest and smallest loads were larger than similar ratios of the contact areas for the same separations, and therefore an increase of the sampling interval caused an increase of contact area for the same load.

The increase of sampling interval caused a decrease of the contact pressure p for the same separation h of surface IIC, however no similar changes were observed in relation to the area fraction A/A_n for separation h up to $4Spq$ (Fig. 7). As a result, for a given load the contact area increased with an increase of the sampling interval. A similar situation was

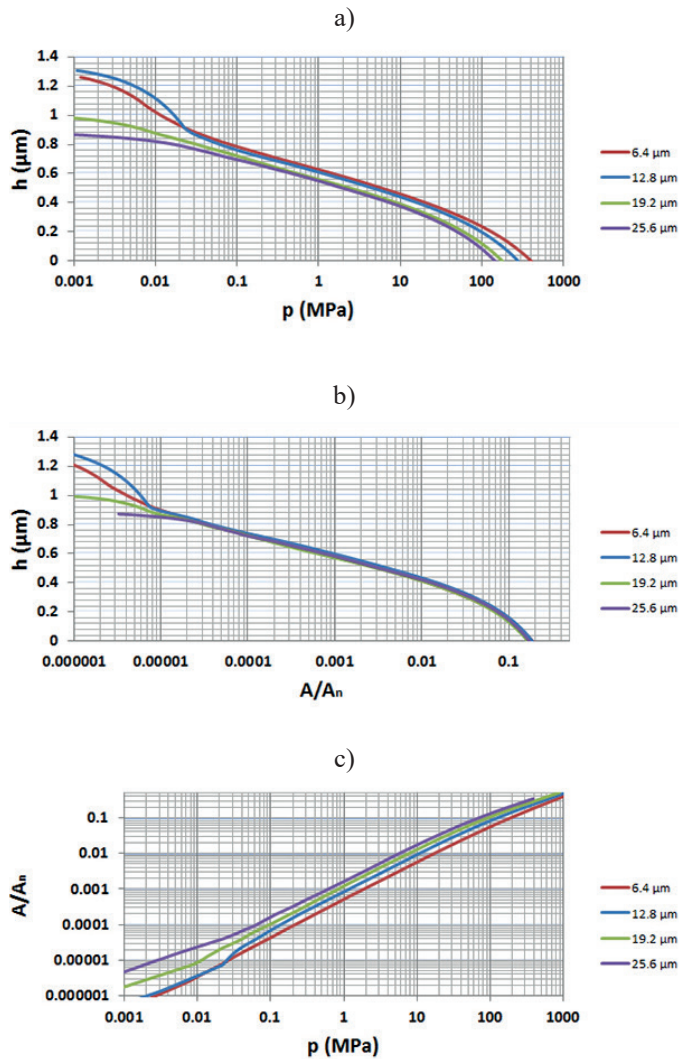


Fig. 7. Contact characteristics of surface IIC

observed for one-process surfaces inclined to elastic deformation [25, 26].

One can see from the analysis of Figs. 5–7 that contact characteristics of two-process textures are governed by summits of plateau portions.

4. Conclusions

The elastic-plastic contact of random isotropic machined two-process steel surfaces with a smooth steel flat was analysed. Two-process surfaces were machined by vapour blasting followed by lapping. Predictions of contact mechanics of random two-process textures surfaces depend on the sampling interval. Dependences among contact characteristics of one-process textures, obtained previously by the present authors, were confirmed for two-process surfaces. As predicted by the model, increasing the sampling interval causes smaller surface tendency to plastic deformation, described by the plasticity index. For a high ability to plastic deformation, the dependence be-

tween contact load and contact area is nearly independent of the sampling interval used.

Summits from the plateau surface portion determine the contact characteristics of two-process surfaces. For this surface type, only those asperities should be taken into consideration whose highest points belong to the plateau surface part in the plasticity index computation. Calculation of the plasticity index using the traditional method overestimated the surface inclination to plastic deformation. Relative errors caused by improper calculation of asperity height standard deviation are smaller for higher sampling intervals.

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