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Input-output pairing criterion applied in the genetic algorithm for unstable linear systems

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Abstract. In this paper, a new approach towards input-output pairing for an unstable system has been proposed. First, it is demonstrated that the previous method of input-output pairing for unstable plants cannot find appropriate pairs as it only checks necessary conditions for stability and integrity. Then, a new approach using relative error matrix and genetic algorithm for finding appropriate pairs in unstable systems is proposed. As it is shown, this approach takes into consideration both static and dynamic information of plant in measuring interaction. Finally, the accuracy of proposed method is demonstrated by an example and closed loop simulation.

Key words: decentralized control, input-output pairing, unstable plant, relative error matrix, genetic algorithm.

1. Introduction

Multivariable control systems are of great importance in industry. The pairing of input-output variables and the effect of the interactions are problems to be solved at the earlier stages of the design. For that purpose, various approaches have been proposed. Recently, decentralized control has been widely used due to its advantages, such as easy tuning and robustness. In decentralized control, single input-single output (SISO) controllers are designed for SISO loops, thus, the first step is finding loops with minimal interaction. This process is called control input-output pairing which many methods have been introduced for finding appropriate pairs while minimum interaction. The most known approach is relative gain array (RGA) which is based on the interactions among variables [1]. In RGA, for ease of computation, dynamic information and interaction in high frequencies is not evaluated. In order to compensate this flaw, many other methods have been introduced such as dynamic relative gain array (DRGA) that uses transfer function for interaction measuring [2]. Effective relative gain array (ERGA) [3] takes into account the dynamic information on the system, while remaining relatively simple. There are also several methods that utilize state space in order to find suitable pairs [4, 5].

All mentioned approaches can be applied to stable open-loop multivariable plants while most industrial plants are unstable open-loop plants. Hence, it seems necessary to introduce a new approach to find appropriate pairs in an unstable open-loop plant. In [6], Niederlinski index (NI) and RGA are used to perform input-output pairing in unstable systems such that certain conditions for stability and integrity are met. However, the method proposed in [6] uses certain conditions for stability, and this can be problematic. In this paper, it will be shown that the method suggested in [6] cannot find appropriate pairs where both pairs met necessary stability and integrity conditions, while the method proposed in this paper can achieve this goal.

The remainder of this paper is organized as follows. In Section 2, the method suggested in [6] is introduced. Then, the genetic algorithm is explained briefly. Section 3 presents problems described in [6] through an example. Then, our new approach is introduced and its efficiency is shown by an example, followed by a conclusion in Section 4.

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2. Preliminaries

2.1 Input-output pairing approach for an unstable system. The introduced approach for input-output pairing for unstable systems in [6] is explained in this section.

First, consider plant with n-inputs n-outputs as below:

$$G(s)=\begin{bmatrix} g_{11}(s) & g_{12}(s) & \cdots & g_{1n}(s) \\ g_{21}(s) & g_{22}(s) & \cdots & g_{2n}(s) \\ \vdots & \vdots & \vdots & \vdots \\ g_{n1}(s) & g_{n2}(s) & \cdots & g_{nn}(s) \end{bmatrix}. \tag{1}$$

For the above system, Niederlinski Index (NI) is defined as below:

$$NI=|G|/|\tilde{G}|, \tag{2}$$

where $|.|$ is determinant and $\tilde{G}$ is matrix consisting of the diagonal elements of G. RGA elements λ_{ij} in (1) can be computed as below:

$$\lambda_{ij}=g_{ij}\left[G^{-1}\right]_{ji}, \tag{3}$$

where $[G^{-1}]_{ji}$ represents the jith element of G^{-1}.

Theorem 1. Niederlinski Index. Let the number of open-loop unstable poles (excluding poles at s = 0) of G(s) and $\tilde{G} = \text{diag}\{g_{11}, g_{22}, \ldots, g_{nn}\}$ be p and $\tilde{p}$, respectively. Assume that the controller C is such that $\tilde{G}C$ has integral action in all loops and is stable, and that the transfer functions GC is strictly proper. Then if:

$$NI = \begin{cases} < 0 & \tilde{P} - P \quad \text{even} \\ > 0 & \tilde{p} - p \quad \text{odd} \end{cases}, \tag{4}$$

at least one of the following instabilities will occur:

a) the overall system is unstable;

b) at least one of the loops is unstable by itself.

Theorem 2. Relative gain array. Let the number of open-loop unstable poles (excluding poles at s = 0) of G(s) and $G'_{ii}(s) = \text{diag}\{g_{ii}(s), G^{ii}(s)\}$ be p and p'_{ii}, respectively. Assume that the decentralized controller C is such that $G'_{ii}(s)C$ has integral action in all channels and is otherwise stable. Then, if:

$$\lambda_{ii} = \begin{cases} < 0 & P'_{ii} - P \quad \text{even} \\ > 0 & P'_{ii} - P \quad \text{odd} \end{cases}, \tag{5}$$

at least one of the following instabilities will occur:

a) the overall system is unstable;

b) loop *i* is unstable by itself;

c) the system is unstable as loop *i* is removed.

In [6], the proofs of proposed theorems are provided and their accuracy is demonstrated through an example.

2.2. Genetic algorithm. Genetic algorithm differs from other search techniques as it utilizes natural genetics in an optimization procedure. A genetic algorithm works with a population of strings, and each string is called an individual. By exchanging information between individuals, new individuals with more desirable properties are produced, just like in nature. Furthermore, mutation also occurs in individuals and thus the value of one chromosome changes randomly. There are more comprehensive explanations of genetic algorithm parameters available in [7, 8].

The genetic algorithm used in this paper has real value strings. In each population, 8 individuals are used. We used uniform crossover. Thus, given two parents, a random binary vector is created. Uniform crossover then selects the genes in which the vector from the first parent is 1, and the genes in which the vector from the second parent is 0, and combines the genes to form the child. The crossover fraction – the fraction of individuals in the next generation – other than elite children that are created by crossover is 0.8. The elite account, number of current individuals that survive to form the next generation is established at 2. For mutation, the Gaussian function is used, adding a random number to each vector entry of an individual. This number is taken from a Gaussian distribution centered on zero.

3. Introduction of a new criterion

In this section, the deficiency of approach in [6] is illustrated by an example. Then a new approach is proposed.

3.1. Example 1. Consider system as below:

$$G(s) = \frac{1}{s-1}\begin{bmatrix} 0.8 & 0.6 \\ -0.8 & 0.6 \end{bmatrix}, \tag{6}$$

First of all, RGA in (6) is as follows:

$$RGA = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}. \tag{7}$$

The above system has two unstable poles at $s = 1$. Hence, in order to find the appropriate pairs in decentralized control, only the scheme proposed in [6] can be used. There are two possible pairings for plant in Eq. (6), which are diagonal pairing and off-diagonal pairing. Here, conditions in Eq. (4) and Eq. (5) will be checked in the system (6) to discard pairings which do not met those conditions.

First, consider diagonal pairing. In this situation, by using Eq. (2), we get NI = 2, which is positive. As P = 2 and $\tilde{P} = 2$, thus $\tilde{P} - P = 2$ which is even, and based on Theorem 1 and Eq. (4), this pairing cannot be discarded as this pairing met conditions of stability and integrity.

Off-diagonal pairing is another option. Like diagonal pairing, we have NI = 2, which is positive. As P = 2 and $\tilde{P} = 2$, $\tilde{P} - P = 2$, which is even, and based on Theorem 2 and Eq. (5), this pairing is acceptable as it fulfills the requirements.

This example shows that since proposed method in [6] only considers certain stability conditions, it cannot find appropriate pairs; in the above example, both pairs meet conditions and consequently, none of them will be discarded. Therefore, it seems necessary to propose a new criterion for input-output pairing in unstable systems.

3.2. New input-output pairing approach based on genetic algorithm. In our new approach, as genetic algorithm will be used, first we have to define cost function which has to be minimized. We use Relative Error Matrix in [9] as our cost function:

$$E = GC(I + GC)^{-1} - \tilde{G}C(I + \tilde{G}C)^{-1}, \tag{8}$$

where G is gained after putting selected loops on main diagonal and $\tilde{G}$ is the main diagonal of G.

As can be seen in (8), the relative error matrix is the difference between closed loop system while other loops are open and close, respectively. Thus, less magnitude of this matrix means less interaction and vice versa. This matrix has been previously used in [9] for interaction measurement in open-loop stable system. In this paper, we use it as interaction measuring for unstable system. Maximum singular value will be used as a tool for measuring size of matrix. As (8) depends on the controller, first the controller should be designed and genetic algorithm should be used to gain the optimal value of controllers. Through this procedure, a controller is designed such that the magnitude of relative error matrix is minimal, which leads to minimal interaction.

The proposed scheme can be summarized as follows:

1. Check stability and integrity conditions based on [6].
2. For each pairing that fulfills stability and integrity conditions, find optimal coefficients of the controller by using genetic algorithm.
3. Compute relative error matrix in (8) and maximum singular value at each frequency. Then, find the maximum value among maximum singular values in the desired frequency range as follows:

$$m = \max_{\omega}(\bar{\sigma}(E(j\omega))). \tag{9}$$

4. Select pairing with the maximum value of *m*.

By using the above procedure, the final pairing not only met stability and integrity conditions but also showed minimal interaction among selected loops, which led to a better performance in decentralized control. The advantage of the proposed method over the one in [6] is taking interaction among loops into account. Thus, in cases in which certain conditions cannot determine final pairing, the proposed method can be used in order to determine the most suitable pair. In the following subsection, the proposed method will be applied to Example 1, in which method in [6] cannot find an appropriate pair.

3.3. Example 2. Consider plant as below:

$$G(s) = \frac{1}{s-1}\begin{bmatrix} 0.8 & 0.6 \\ -0.8 & 0.6 \end{bmatrix}. \tag{10}$$

We apply the proposed approach to the plant in (10). The steps described in the previous section are applied as follows:

1. Check the stability and integrity of both pairings. As we discussed in Example 1, both pairings met stability and integrity conditions, hence, [6] cannot determine suitable pair.
2. For both pairs, find optimal coefficient of controller such that the relative error matrix is minimal. This procedure is done by genetic algorithm. In this example, proportional integral (PI) controllers are used as below:

$$PI = k_p + k_i/s, \tag{11}$$

since

$$T = \frac{g_{ii}c_{ii}}{1+g_{ii}c_{ii}} = \frac{g_{ii}k_p s + g_{ii}k_i}{s^2 + s(g_{ii}k_p - 1) + g_{ii}k_i}. \tag{12}$$

And as each SISO closed loop system must be stable, all coefficients in the denominator of (12) must be positive. The following constraints must be held in genetic algorithm optimization:

$$g_{ii}k_i > 1 \quad \& \quad g_{ii}k_p - 1 > 1. \tag{13}$$

Genetic algorithm optimization for diagonal pairing loops (y_1, u_1) and (y_2, u_2) can be seen in (14) and (15), respectively:

$$C_{11} = 5.7 + 1.2/s, \tag{14}$$

$$C_{22} = 5.4 + 1.6/s. \tag{15}$$

In frequency range 0–5 rad/sec and by computing maximum singular value of relative error matrix of controllers in (14) and (15), we have:

$$\max_{\omega}(\bar{\sigma}(E(j\omega))) = 1.3 \tag{16}$$

For off-diagonal pairing, the PI controllers optimized by genetic algorithm for loops (y_1, u_1) and (y_2, u_2) are as follows:

$$C_{12} = 20.3 + 18.6/s, \tag{17}$$

$$C_{21} = -17.7 - 12.6/s. \tag{18}$$

And (9) for off-diagonal pairing takes the form:

$$\max_{\omega}(\bar{\sigma}(E(j\omega))) = 0.37. \tag{19}$$

Comparing (16) to (17) leads to the conclusion that interaction in off-diagonal pairing must be lower in order to achieve better performance than in diagonal pairing.

To compare diagonal and off-diagonal pairing, close loop simulation with controllers in (14) and (16) has been carried out, and one criterion is utilized in order to settle time for comparing these configurations.

If set point y_1 for is applied at $t = 0$ (sec) and set point for y_2 is applied at $t = 30$ (sec), then closed loop simulation for y_1 and y_2 with PI controllers in (14) and (16) are shown in Fig. 1 and Fig. 2, respectively.

For comparing performances in diagonal and off-diagonal pairing, settling time is considered. Closed loop performance at $t =$ (sec) is shown in Table 1.

This table demonstrates that off-diagonal pairing shows better performance than diagonal pairing.

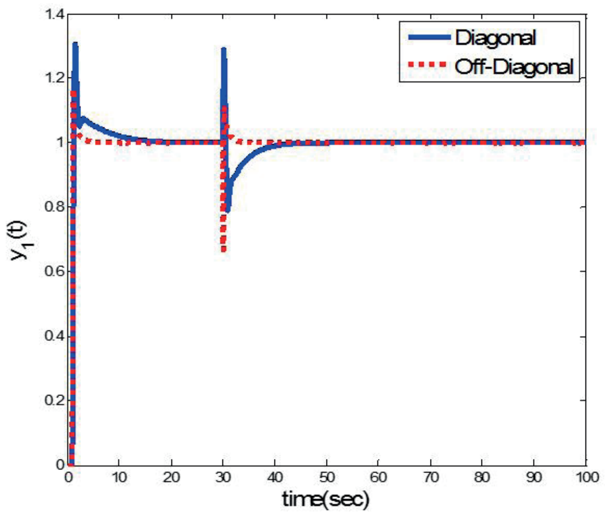

Fig. 1. Closed loop responses for y_1 in diagonal and off-diagonal pairings

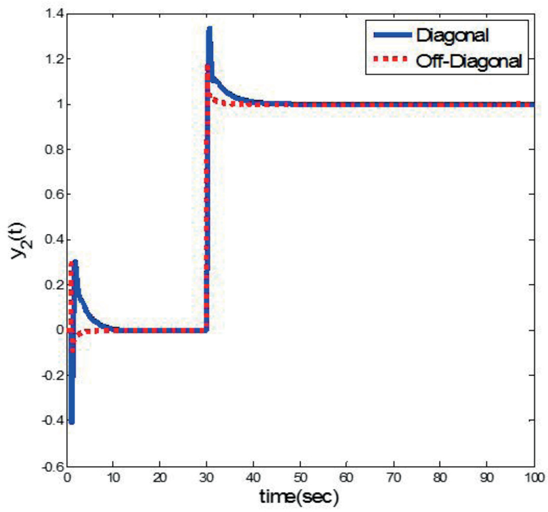

Fig. 2. Closed loop responses for y_2 in diagonal and off-diagonal pairings

Table 1
Settling times in closed loop system at t = 0 sec

pairing	outputs	settling time (sec)
diagonal	y_1	12
	y_2	9.5
off-diagonal	y_1	3
	y_2	6

Closed loop performance at t = 30 (sec) is shown in Table 2.

Table 2
Settling times in closed loop system at t = 30 sec

pairing	outputs	settling time (sec)
diagonal	y_1	10
	y_2	9
off-diagonal	y_1	3.5
	y_2	3

As the result shows, off-diagonal pairings has less settling time and reaches steady state sooner than diagonal pairing.

4. Conclusion

This paper has introduced and evaluated a novel approach for determining appropriate input-output pairs in unstable systems. First, certain conditions for stability and integrity are considered, and through an example, it is demonstrated that using certain conditions may not be useful for determining final pairs. On the other hand, in the proposed scheme optimal value of controllers are designed by genetic algorithm such that the relative error matrix is minimized. Since the relative error matrix is a measure of interaction in multivariable system, its maximum singular value, as the measuring magnitude of matrix, can be utilized for interaction measurement. Finally, it is demonstrated that there is a case in which the method presented in [6] cannot find suitable pairs, while the method proposed in this paper leads to pairs with less interaction. The accuracy of the proposed approach is demonstrated through closed loop simulation.

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