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SINGLE-FRAME ATTITUDE DETERMINATION METHODS FOR NANOSATELLITES

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Abstract

Single-frame methods of determining the attitude of a nanosatellite are compared in this study. The methods selected for comparison are: *Single Value Decomposition* (SVD), *q* method, *Quaternion ESTimator* (QUEST), *Fast Optimal Attitude Matrix* (FOAM) – all solving optimally the Wahba's problem, and the algebraic method using only two vector measurements. For proper comparison, two sensors are chosen for the vector observations on-board: magnetometer and Sun sensors. Covariance results obtained as a result of using those methods have a critical importance for a non-traditional attitude estimation approach; therefore, the variance calculations are also presented. The examined methods are compared with respect to their *root mean square* (RMS) error and variance results. Also, some recommendations are given.

Keywords: attitude determination, single-frame methods, algebraic method, covariance analysis, vector observation.

1. Introduction

The attitude determination and control subsystem of a nanosatellite is important for maintaining a required direction of the spacecraft and its instruments. There are several methods for determining the satellite's attitude using attitude sensors. Sun sensors and magnetometers are very common sensors for nanosatellites because of their cost-effectiveness and commercial availability on the market with various mass and size versions. After determination of its attitude, using the actuators, a satellite should be oriented towards a specified direction. For doing that, two or more vectors should be used as reference directions in a single-frame method which this paper is mainly focused on. Commonly used reference vectors are the Earth's magnetic field and unit vectors in the direction of the Sun, a known star or the centre of the Earth. Given a reference vector, the orientations of these vectors can be obtained from the measurement results of the attitude sensor.

The algebraic attitude determination methods [1−4] are based only on the vector observations. These methods are based on computing any two analytical vectors in the reference frame and measuring them in the body coordinate system [2]. The paper [3] deals with estimating and enhancing the accuracy of an algebraic method of attitude determination. This method was examined with the use of three different vector pairs: 1) Earth's magnetic field and Sun vectors; 2) Earth's magnetic field and nadir vectors; 3) Sun and nadir vectors. In order to determine the attitude accuracy, some analytical relations were found for the attitude angles (pitch, roll and yaw). These relations include terms of the measured and theoretical vectors, used in the attitude determination. The effects of various factors on the attitude determination were examined and those which most significantly affect the accuracy were determined. In order to increase the attitude determination accuracy, a redundant data processing algorithm,

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based on the Maximum Likelihood method, was used to carry out the statistical operation on the measurement results of three algorithms mentioned above and appropriate formulas were derived. As a result, the attitude was determined with a high accuracy in a wide range (even when the reference vectors were almost parallel). This method involves different vector pairs, therefore it may require redundant hardware and a substantial computational load.

The vectors obtained from selected sensor data and the developed models can be used in solving the Wahba's problem [2, 5]. Coordinate systems used as the reference frame and the body frame can be transformed to each other with necessary input parameters. The system uses single-frame methods: SVD, q, QUEST and FOAM to minimize the Wahba's loss function and to determine the attitude of the satellite. They are different from the algebraic method, because they use an unlimited number of direction vectors and can process all of them in one attitude determination algorithm. In [6], the algebraic and SVD methods are compared to find an optimum attitude determination method. Also, the effects of magnetometer biases are examined in the study.

Kalman filters can give more improved results than the single-frame methods. In [7], a sigma-point Kalman filter is derived using the modified Rodrigues parameters and the real data of attitude sensors of CBERS-2 (China Brazil Earth Resources Satellite). The unscented Kalman filter algorithm is used for attitude estimation and a gyro-based model is considered for attitude propagation. The estimated attitude is very similar to the one obtained by the Euler angles' propagation. Single-frame methods can also be used in filtering techniques as measurement of inputs in order to estimate the satellite's attitude with a high accuracy. Also, the covariance analysis can be used directly in a non-traditional method which is an integrated algorithm using linear measurements. In [8, 9], a non-traditional attitude estimation scheme has been presented and it is shown that the non-traditional methods give the attitude results for a satellite that are superior to the traditional Kalman filters, even in the eclipse period.

In [10], the performance of several methods is examined regarding their computational load and accuracy of used algorithms. Attitude determination and estimation methods are divided into two categories: those that use and those that do not use spacecraft attitude motion models inside their algorithms. The attitude determination methods which are considered in this paper as single-frame methods do not use knowledge about the attitude motion because they find the attitude at a single moment from the sensor-model data. In that paper, the attitude determination algorithms are characterized as ones with a low computation load in addition to a low accuracy of spacecraft attitude angles, in comparison with such attitude estimation methods as the extended Kalman filter, which is obvious. There were examined only methods based on observation of two vectors. Also, classification of the methods (attitude determination and estimation methods) is different; thus, there only the single-frame methods are compared to find the most robust and the fastest method in their classification.

The goal of this study is to examine the errors and variances of errors for most of the vector-observation-based satellite attitude determination methods which are single-frame methods. Also, based on this error and variance analysis, these attitude determination methods are compared.

2. Measurement models and attitude determination methods

To find the attitude of a spacecraft, minimum two vectors should be known. In order to find these vectors, many different sensors can be applied. In this study, the sensors of magnetic field and Sun direction vectors are used because these sensors are very common for on-board use in small satellites. In this paper, the orbital parameters are calculated using the orbit propagation from the *Two Line Elements* (TLE) data for the TIMED satellite. Using mathematical models, the Sun vector (S_R) and the Earth's magnetic field vector (H_R) are calculated in the orbital frame

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(see Fig. 1). A Sun sensor and a magnetometer measure those vectors in the body frame. In order to transform data between these frames, a transformation matrix must be known. From the dynamic and kinematic equations, the Euler angles (θ is the pitch angle, ϕ is the roll angle and ψ is the yaw angle) are calculated to form this matrix. The transformation- attitude matrix (A) can be created using the Euler angles [11]. Numerical or analytical methods can be used to solve the kinematic and dynamic equations for the spacecraft attitude propagation [12].

2.1. Measurement models

International Geomagnetic Reference Field (IGRF) 12 is a basic magnetic field model defining 4-input variable (r, θ, ϕ, t) in nT, using numerical Gauss coefficients (g, h) – global variables in the IGRF algorithm [13]. In (1), a is a magnetic reference spherical radius $a = 6371.2$ km, θ is a colatitude (deg) and ϕ is a longitude (deg). The transformed magnetic field model in the body coordinates with added a defined noise matrix forms the measurement model. The mathematical ($\boldsymbol{B_o}$) and the measurement models ($\boldsymbol{B_b}$) of the magnetic field can be written as in the (1) and (2), respectively. B indicates the magnetic field, whereas S indicates the Sun direction vector.

$$B_o(r,\theta,\phi,t) = -\nabla\{a\sum_{n=1}^{N}\sum_{m=0}^{n}(\frac{a}{r})^{n+1}[g_n^m(t)\cos m\phi + h_n^m(t)\sin m\phi]\times P_n^m(\cos\theta)\}. \tag{1}$$

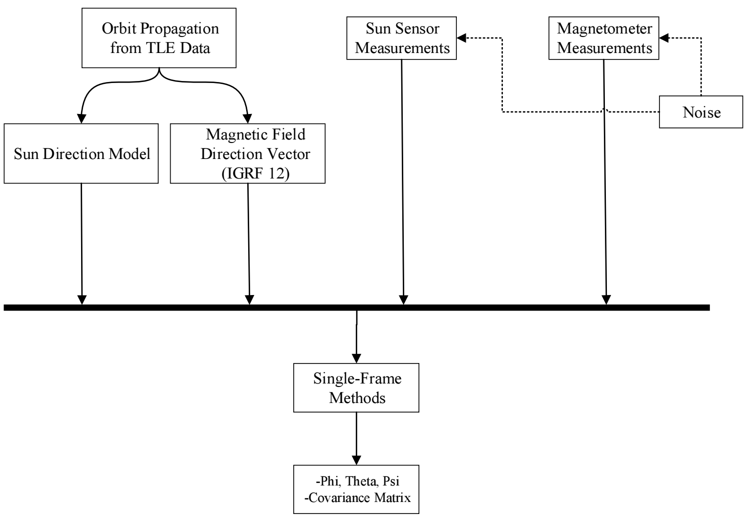

Fig. 1. A block diagram of the Sun and Earth's magnetic field vectors-based single-frame attitude determination methods.

$$\boldsymbol{B_b} = A\boldsymbol{B_o} + noise_B. \tag{2}$$

The Sun direction in the *Earth Centred Inertial* (ECI) frame can be modelled. The ecliptic longitude of the Sun is $\lambda_{ecliptic}$ and a linear model of the ecliptic longitude of the Sun is ε [14]. A unit Sun direction vector ($\boldsymbol{S}_{ECI}$) in the ECI frame can be obtained as in (3). Measurements can be modelled in the (4) with transforming data into the body coordinates and adding a defined noise matrix. The subscript notation used in the equations defines their coordinate systems as the orbital, body or ECI frames.

$$\boldsymbol{S}_{ECI} = \begin{bmatrix} \cos\lambda_{ecliptic} \\ \sin\lambda_{ecliptic}\cos\varepsilon \\ \sin\lambda_{ecliptic}\sin\varepsilon \end{bmatrix}, \tag{3}$$

$$\boldsymbol{S}_b = A\boldsymbol{S}_o + \eta_2 . \tag{4}$$

2.2. Algebraic method

In the algebraic method, transformation-attitude matrix (A) is determined by the observations of two vectors. $\hat{u}$ and $\hat{v}$ are any vectors which define an orthogonal coordinate system. In this study $\hat{u}$ and $\hat{v}$ are chosen as the Sun direction (S) and magnetic field (B) vectors.

The equations can be defined as follows [15]:

$$\hat{q} = \hat{u}, \tag{5}$$

$$\hat{r} = \frac{\hat{u}\times\hat{v}}{|\,\hat{u}\times\hat{v}\,|}, \tag{6}$$

$$\hat{s} = \hat{q}\,x\,\hat{r}. \tag{7}$$

A reference matrix M_R can be calculated using two reference vectors in the orbital coordinates, $\hat{u}_R$ and $\hat{v}_R$.

$$M_R = \left[\hat{q}_R \,\vdots\, \hat{r}_R \,\vdots\, \hat{s}_R\right]. \tag{8}$$

A body matrix M_B can be calculated using two measured vectors in the spacecraft body coordinates, $\hat{u}_B$ and $\hat{v}_B$.

$$M_B = \left[\hat{q}_B \,\vdots\, \hat{r}_B \,\vdots\, \hat{s}_B\right]. \tag{9}$$

An attitude matrix is calculated as:

$$AM_R = M_B,\; A = M_B M_R^{-1}. \tag{10}$$

To calculate the attitude covariance matrix [1] which is in the Euler form, first the Cartesian attitude covariance matrix ($P_{\theta\theta}$) must be known.

$$P_{\theta\theta} = \sigma_1^2 I + \frac{1}{\left|\vec{u}_B\times\vec{v}_B\right|^2}\left[\left(\sigma_2^2-\sigma_1^2\right)\vec{u}_B\vec{u}_B{}^T + \sigma_1^2\left(\vec{u}_B.\vec{v}_B\right)\left(\vec{u}_B\vec{v}_B{}^T + \vec{v}_B\vec{u}_B{}^T\right)\right], \tag{11}$$

where σ_1^2 is a variance of the magnetometer; σ_2^2 is a variance of the Sun sensor and I is a unit matrix with a dimension of 3 × 3.

The attitude covariance matrix is a set of Euler angles:

$$P_{\phi\phi} = BP_{\theta\theta}B^T, \tag{12}$$

where B^{-1}:

$$B^{-1} = \frac{1}{2}\sum_{k=1}^{3}\left(\frac{\partial A_k}{\partial\phi_j}\times A_k\right)_i, \tag{13}$$

ϕ_j are the Euler angles (ϕ, θ, ψ), respectively [1] .

2.3. SVD method

In 1965, Wahba defined a problem which aims to minimize the loss ($L(A)$) between chosen reference and measured unit vectors [5]. In the (14), b_i (a set of unit vectors in the body frame)

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and r_i (a set of unit vectors in the reference frame) with their a_i (a non-negative weight) are the loss function variables obtained for instant time intervals.

$$L(A) = \frac{1}{2}\sum_i a_i \,|\, \mathrm{b}_i - \mathrm{Ar}_i \,|^2, \tag{14}$$

$$B^* = \sum a_i b_i r_i^T, \tag{15}$$

$$L(A) = \lambda_0 - tr(AB^{*T}). \tag{16}$$

To simplify the loss function, B^* matrix can be defined. The (16) shows that the trace of the product of transformation matrix A and transposition of the defined matrix B^* in (15) should be maximized using statistical methods. In this study, the *Singular Value Decomposition* (SVD) Method is chosen to minimize the loss function problem as the optimal statistical method [16, 17].

$$B^* = U\Sigma VT = Udiag\left|\Sigma_{11}\ \Sigma_{22}\ \Sigma_{33}\right| VT, \tag{17}$$

$$A_{opt} = Udiag[1 \quad 1 \quad \det(U)\det(V)]V^T. \tag{18}$$

The matrices U and V are orthogonal left and right matrices, respectively, and the primary singular values ($\Sigma_{11}, \Sigma_{22}, \Sigma_{33}$) obey the inequalities $\Sigma_{11} \ge \Sigma_{22} \ge \Sigma_{33} \ge 0$. To find the rotation angles of the satellite, a transformation matrix should be found from the (18) first with the determinant of one.

A rotation angle error covariance matrix (P_{SVD}) is necessary for determining the instant time intervals which give higher error results than desired.

$$P_{SVD} = Udiag[(\mathrm{s}_2 + \mathrm{s}_3)^{-1} \quad (\mathrm{s}_3 + \mathrm{s}_1)^{-1} \quad (\mathrm{s}_1 + \mathrm{s}_2)^{-1}]U^T, \tag{19}$$

where the secondary singular values are $s_1 = \Sigma_{11}$, $s_2 = \Sigma_{22}$, $s_3 = \det(\mathrm{U})\det(\mathrm{V})\Sigma_{33}$. The satellite has only two sensors (*e.g.* Sun and magnetic field sensors), thus the SVD-method fails when the satellite is in eclipse and when two observations are parallel with the same trend of the absolute error results.

2.4. q method

In (14), a Wahba's loss function has been defined. The attitude matrix can be parameterized with quaternions. Davenport suggested a useful solution with a unit quaternion denoted q [16, 18]:

$$q = \begin{bmatrix} q \\ q_4 \end{bmatrix}, \tag{20}$$

$$A(q) = (\mathrm{q}_4{}^2 - |q_v|^2)\,\mathrm{I} + 2\,\mathrm{q}_v \mathrm{q}_v{}^T - 2q_4\,[q_v \times], \tag{21}$$

$$tr(\mathrm{AB}^{*T}) = q^T Kq. \tag{22}$$

Because of the quaternion definition by the Euler theorem, there will be a rotation of the axis and the angle. The quadratic function from (21) includes scalar and vector quaternion elements. If K is defined as a traceless matrix, the eigenvector corresponding to the maximum eigenvalue is the optimum quaternion vector q_{opt} in (26).

$$K \equiv \begin{bmatrix} S - Itr(B) & z \\ z^T & tr(B^*) \end{bmatrix}, \tag{23}$$

$$S \equiv B^* + B^{*T}, \tag{24}$$

$$z \equiv \begin{Bmatrix} B^*_{23} - B^*_{32} \\ B_{31} - B^*_{13} \\ B^*_{12} - B^*_{21} \end{Bmatrix} = \sum_i a_i b_i \times r_i, \tag{25}$$

$$Kq_{opt} \equiv \lambda_{\text{maksimum}} q_{opt}. \tag{26}$$

There is only one problem: if the eigenvectors are equal, then the correct solution cannot be obtained. Markley stated in [19] that it is not a problem of the q method, since in this situation the available data are not suitable to determine attitude (2010). The expected results in this situation are, like the results with a Sun sensor in the eclipse, going to infinity. The q method is used in various projects and studies [16, 20] .

The rotation angle error covariance matrix can be found from the (19). A covariance goes to infinity if the eigenvectors are equal. Also, if the attitude cannot be observed then the covariance would be infinite, too.

2.5. QUEST method

The QUEST method as one of the single-frame methods aims to minimize the Wahba's loss function in (14). Iterative techniques can be used to solve the characteristic equation in the *q* method. Additionally, some assumptions can be made to obtain the solutions faster. QUEST is one of methods that uses numerical iterative techniques. In this paper, QUEST is using the Newton Raphson method as an iterative approach with a Gibbs vector. However, with the Gibbs vector a singularity problem is associated that studies like [21] are working on to remove. The *q* and QUEST methods use only quaternions to obtain the attitude, but the SVD method can solve the Wahba's problem with Euler angles directly besides using quaternions. An advantage to the method can be brought by comparing both results obtained in the same conditions.

$$\alpha \equiv \lambda_{max}^2 - (trB^*)^2 + tr(adjS), \tag{27}$$

$$\beta \equiv \lambda_{max} - trB^*, \tag{28}$$

$$\gamma \equiv det\left[\left(\lambda_{max} + trB^*\right) I - S\right] = \alpha\left(\lambda_{max} + trB^*\right) - det\, S, \tag{29}$$

$$\mathbf{x} \equiv \left(\alpha I + \beta S + S^2\right)\mathbf{z}, \tag{30}$$

$$q_{opt} = \frac{1}{\sqrt{\gamma^2 + |\mathbf{x}|^2}} \begin{bmatrix} \mathbf{x} \\ \gamma \end{bmatrix}. \tag{31}$$

To find $\lambda_{maximum}$, from the $det(K - \lambda_{max} I) = 0$ characteristic equation, a defined λ_0 can be used as the initial value for simplicity [16]. The parameters are the same as in the q method.

Also, from the reference [19], the covariance matrix can be obtained as follows:

$$P_{QUEST} = \left[\sum_i a_i \left(I - b_i b_i^T\right)\right]^{-1}. \tag{32}$$

The covariance matrix (P_{QUEST}) is a result that can be used as the initial value for filtering approaches like EKF, UKF or variance values for the whole mission period. Besides, instantaneous time intervals when the algorithm should be switched to another one can be found out by the covariance analysis.

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2.6. FOAM method

The loss function in (14) can be also minimized using the FOAM method [22]. First of all, the Frobenius norm should be defined in (33) using the G symbol. From this definition, the optimal attitude matrix can be determined (35):

$$\|G\|_F^2 = \sum_{i,j} G_{i,j}^2 = tr\left(GG^T\right), \tag{33}$$

$$\kappa = \tfrac{1}{2}\left(\lambda_{max}^2 - \|B\|_F^2\right), \tag{34}$$

$$A_{opt} = \left[\kappa\lambda_{max} - det\left(B\right)\right]^{-1}\left[\left(\kappa + \|B\|_F^2\right)B + \lambda_{max} adj\left(B^T\right) - BB^T B\right], \tag{35}$$

$$\lambda_{max} = tr\left(A_{opt}B^T\right). \tag{36}$$

Using the FOAM method, the optimal attitude matrix can be found, and from that a quaternion or Euler angle representation can be used.

The covariance Matrix (P_{FOAM}):

$$P_{FOAM} = \left[\kappa\lambda_{max} - det\left(B\right)\right]^{-1}\left(\kappa I + BB^T\right). \tag{37}$$

The matrix B defined in (17) is directly used in (37) to find the error covariance.

3. Simulation results

The simulations were performed in order to estimate the attitude of the satellite and compare the methods to find the optimum one. The simulations were based on the orbital parameters of TIMED satellite. The algorithm was run for almost one orbital period (6000 seconds) with 1 second sampling time of the sensors. Direction cosines of standard deviations for the magnetometer and Sun sensors were taken as 0.008 and 0.002, respectively. The attitude angle errors found by using single-frame methods are presented in Figs. 3–7. In Fig. 2, the angles between the vector observations coming from the sensors and the pitch angle propagation can be seen in the respective frames. The angles between the vectors are close neither to 0 degree nor 180 degrees; therefore, they are not parallel to each other and will not affect the attitude of the satellite by being not observable vectors. On the other hand, the pitch angle is closing up to 90 degrees (at about 1000th sec and 3800th sec) which causes oscillations because of the trigonometric calculations in the methods, especially in the algebraic method.

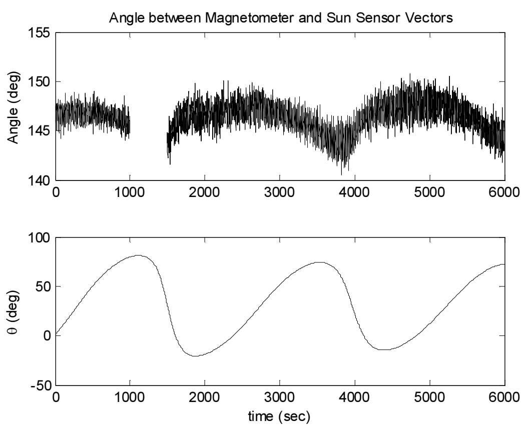

Fig. 2. From top to bottom: An angle between measurement vectors and a pitch angle.

In Figs. 3−7, the absolute attitude errors obtained by using the algebraic, SVD, *q*, QUEST and FOAM methods are presented. All three axes as frames in the figures can be seen as the roll, pitch and yaw angles, respectively. Inside the dotted lines the eclipse period is defined as a 1000−1500 second time interval. In Fig. 3, the attitude of the satellite found by the algebraic method can be seen – with a variance propagation given in deg2 units – in the bottom frame. It should be kept in mind that single-frame methods are not capable to find accurate results for the eclipse period because of no data are available from the Sun sensor.

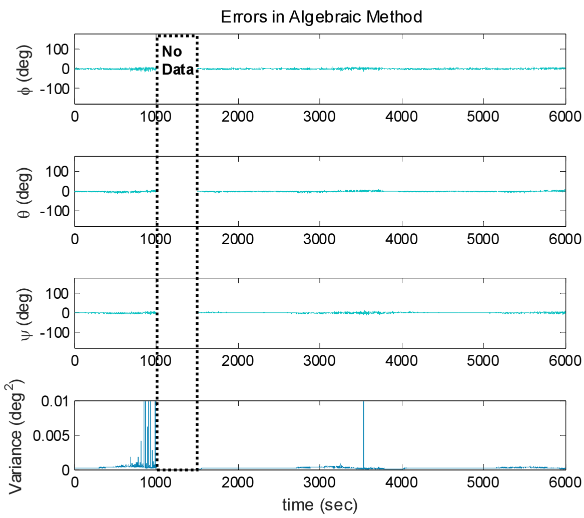

Fig. 3. The attitude error and variance results obtained by the algebraic method.

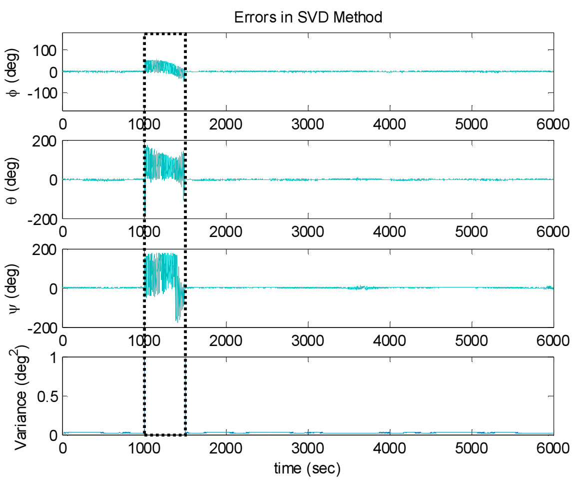

Fig. 4. The attitude error and variance results obtained by the SVD method.

In Fig. 4, the SVD method with its attitude and variance results is presented in four frames. The variance is having the same trend with the error of attitude angles as it can be seen from the graphs. In Fig. 5, attitude angles determined by the q method can also be seen. The variance can be calculated using the same equation as with the SVD method, (19). After those methods with higher robustness, variance changes in time of the QUEST algorithm are presented

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in Fig. 6. Here, QUEST is not able to follow the trend of the error as the error covariance values. Lastly, the absolute attitude errors and variances found by the FOAM method are shown in Fig. 7. The results of FOAM have some gaps in their propagation even if the algorithm uses measurements at a single moment, and there are some jumps in the determined attitude.

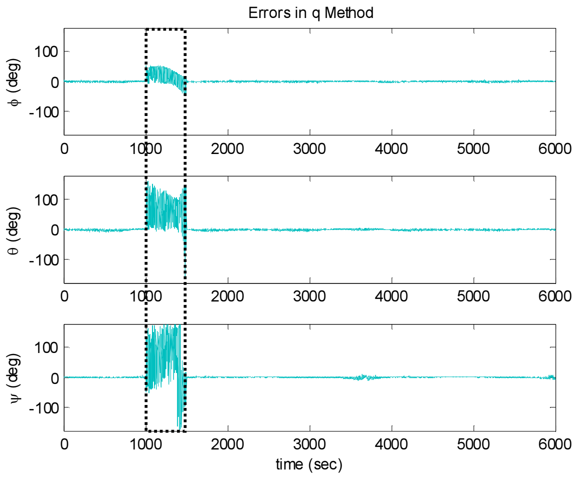

Fig. 5. The attitude error and variance results obtained by the *q* method.

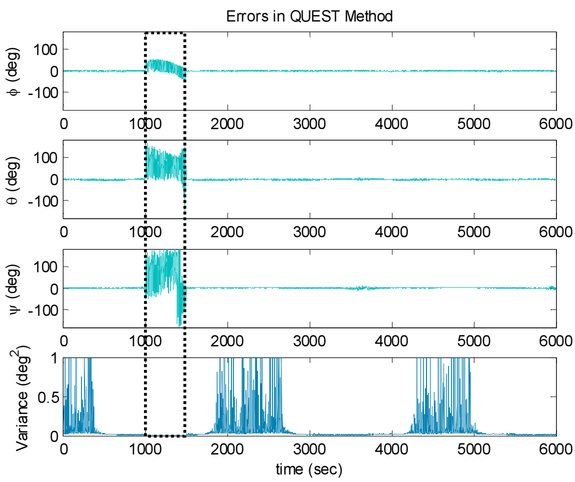

Fig. 6. The attitude error and variance results obtained by the QUEST method.

Both magnetometer and Sun sensors are assumed to be calibrated before running the algorithm. The *Root Mean Square* (RMS) error results for different time intervals are presented in Table 1 for each method. Therefore, it is possible to recognize the most reliable method for single-frame attitude determination.

In Table 1, the 2nd, 5th, 8th, 11th, and 14th rows (Error 1) represent the interval of 0–1000 sec. The eclipse period can be seen as Error 2 within the 1000–1500 sec interval. The last rows of all methods, denoted Error 3, concern the period between 1500 sec and 6000 sec which is outside of the eclipse. Those three ranges have been selected in order to separate the periods before, during and after the eclipse period which is a crucial time interval for nanosatellites having Sun sensors. Here, as seen from the table, the SVD and q methods are the most reliable ones regarding robustness. If the computational burden is concerned, then the QUEST or algebraic method can possibly be selected as the base method to determine the attitude of a satellite.

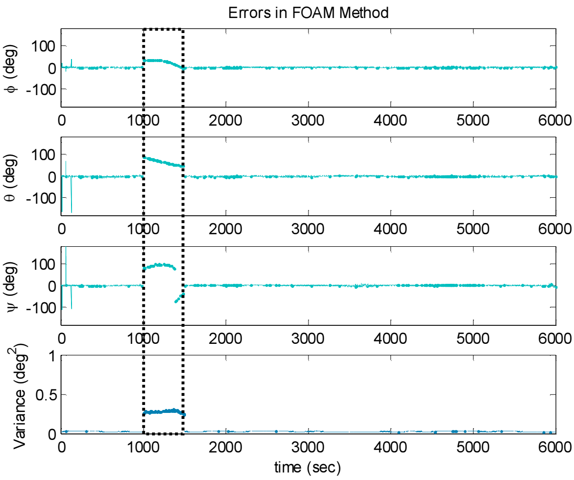

Fig. 7. The attitude error and variance results obtained by the FOAM method.

Table 1. The RMS results for attitude angles obtained with different single-frame methods.

RMS Error (deg)		Roll	Pitch	Yaw
Algebraic Method	Error 1	5,53	1,81	5,42
	Error 2	nd	nd	nd
	Error 3	4,07	1,70	1,14
SVD Method	Error 1	3,38	1,79	0,96
	Error 2	100,09	28,33	200,10
	Error 3	2,69	1,64	1,07
Q Method	Error 1	3,38	1,79	0,97
	Error 2	100,19	30,09	194,19
	Error 3	2,69	1,65	1,07
QUEST Method	Error 1	3,39	1,87	0,98
	Error 2	93,19	30,12	194,21
	Error 3	2,72	1,65	1,11
FOAM Method	Error 1	4,26	2,21	1,00
	Error 2	nd	nd	nd
	Error 3	2,93	0,84	1,98

4. Conclusions

The *Single Value Decomposition* (SVD), *q*, *Quaternion Estimator* (QUEST), *Fast Optimal Attitude Matrix* (FOAM) and algebraic methods can determine the attitude with using two vector observations. The orbit propagation of a satellite is achieved by using chosen satellite's orbital parameters for 1 period of the mission. In this study, magnetometer and Sun sensors are selected as attitude sensors because of their common usage on nanosatellites. According to the simulation results, the optimal attitude results can be obtained by using the SVD or q methods. However, the suggested methods may fail in the process of finding the desired solutions in some situations. Neither methods can estimate the attitude angles if at least one of the sensors cannot send any measurement data. Moreover, if the Sun direction and magnetic field vectors are parallel to each other, both algorithms would also fail. In the paper there is demonstrated that the SVD gives more accurate and robust results and the QUEST is the fastest of the other methods.

Filtering methods such as the extended Kalman filter or unscented Kalman filter can be used after those coarse attitude determination methods, to improve the results with an integration. Also, the variance information becomes an important issue to make the filter naturally adapt to the measurement results by direct using the attitude error covariance values.

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