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Effect of diffusion and internal heat source on a two-temperature thermoelastic medium with three-phase-lag model

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Abstract The purpose of this paper is to depict the effect of diffusion and internal heat source on a two-temperature magneto-thermoelastic medium. The effect of magnetic field on two-temperature thermoelastic medium within the three-phase-lag model and Green-Naghdi theory without energy dissipation i discussed. The analytical method used to obtain the formula of the physical quantities is the normal mode analysis. Numerical results for the field quantities given in the physical domain are illustrated on the graphs. Comparisons are made with results of the two models with and without diffusion as well as the internal heat source and in the absence and presence of a magnetic field.

Keywords: Diffusion; Green-Naghdi theory; Internal heat source; Three-phase-lag model; Two-temperature

Nomenclature

a	–	measure of thermodiffusion effect
b	–	measure of diffusive effect
C	–	mass concentration
C_E	–	specific heat at constant strain
d	–	thermo-diffusion constant
e_{ij}	–	components of strain

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$e = e_{kk}$	–	dilatation
f	–	complex constant
i	–	imaginary unit, $\mathrm{i} = \sqrt{-1}$
J	–	current density vector
K^*	–	additional material constant
K	–	coefficient of thermal conductivity
m	–	wave number in the x-direction
Q	–	moving internal heat source
Q_0	–	magnitude of an internal heat source
P	–	chemical potential per unit mass
T	–	temperature above the reference temperature T_0
t	–	time
$\dot{u}$	–	particle velocity of the medium
V_0	–	velocity of a moving internal heat source
u, w	–	displacement component
x, y, z	–	Cartesian coordinates

Greek symbols

α_c	–	linear diffusion expansion coefficient
α_t	–	linear thermal expansion coefficient
δ	–	constant called a two-temperature parameter, $\delta > 0$
δ_{ij}	–	Kronecker delta
Φ	–	conductive temperature
ε_0	–	electric permeability
θ	–	thermal temperature
$\lambda,\ \mu$	–	Lame' constant
μ_0	–	magnetic permeability
ρ	–	mass density
σ_{ij}	–	components of stress
τ_ν	–	phase-lag of thermal displacement gradient
$\tau_T,\ \tau_q$	–	phase-lag of temperature gradient and the phase-lag of heat flux respectively

$\beta_1 = (3\lambda + 2\mu)\alpha_t, \quad \beta_2 = (3\lambda + 2\mu)\alpha_c, \quad \hat{T} = T - T_0, \quad \tau_\nu^* = K + \tau_\nu K^*$

1 Introduction

Diffusion is defined as a random walk of the ensemble of particles, from regions of high concentration to a region of lower concentration. It occurs as a result of the second law of thermodynamics which states that the entropy or disorder of any system must always increase with time. Diffusion is important in many life processes. There is now a great deal of interest in the study of this phenomenon, due to its many applications in geophysics and industry. In integrated circuit fabrication, diffusion is used to introduce dopants in controlled amounts into the semiconductor substrate. In particular, diffusion is used to form the base and emitter in bipolar transistors, integrated resistors, the source/drain regions in metal-oxide semiconductor (MOS) transistors and dope polysilicon gates in MOS transistors. In most of these applications, the concentration is calculated using what is known as the Fick's law [1]. This is a simple law that does not take into consideration the mutual interaction between the introduced substance and the medium into which it is introduced or the effect of the temperature on this interaction. The phenomenon of diffusion is used to improve the conditions of oil extractions (seeking ways of more efficiently recovering oil from oil deposits). These days, oil companies are interested in the process of thermoelastic diffusion for more efficient extraction of oil from oil deposits. The thermodiffusion process also helps the investigation in the field associated with the advent of semiconductor devices and the advancement of microelectronics.

Thermodiffusion in the solids is one of the transport processes that has great practical importance. Most of the research associated with the presence of concentration and temperature gradients has been made with metals and alloys. The first critical review was published in the work of Oriani [2]. With the advancement of a nuclear energy, the interest in thermodiffusion has returned to metallic oxides that often heats up in the inhomogeneous temperature field, Fryxel and Aitken [3], in connection with technological conditions. Thermodiffusion in the elastic solid is due to the coupling of the fields of temperature, mass diffusion and that of strain. The heat and mass are exchanged with the environment during the process of thermodiffusion in an elastic solid. The concept of thermodiffusion is used to describe the process of thermomechanical treatment of metals (carbonizing, nitriding steel, etc.), these processes are thermally activated, and their diffusing substances being, e.g, nitrogen, carbon, etc. They are accompanied by deformations of the solid. The coupled thermoelastic model was used to develop the theory of thermoelastic diffusion by Nowacki [4–7]. Sherief *et al.* discussed the theory of generalized thermoelastic diffusion with one relaxation time [8]. This implies a finite speed of propagation of waves. Othman *et al.* studied the effect of diffusion on the two-dimensional problem of generalized thermoelasticity with Green and Naghdi theory [9]. Karmakar and Kanoria discussed elasto-thermo-diffusive response in a spherically isotropic hollow sphere [10]. Sherief and Hussein studied a two-dimensional problem for a thick plate with axi-symmetric distribution in the theory of generalized thermoelastic diffusion [11].

It is well-known that the usual theory of heat conduction based on Fourier's law predicts an infinite heat propagation speed. It is also known that heat transmission at low temperature propagates by means of waves. These aspects have caused intense activity in the field of heat propagation. Extensive reviews on the second sound theories (hyperbolic heat conduction) given in Hetnarski and Ignaczak [12,13]. A two-phase-lag with both the heat flux vector and the temperature gradient was introduced by Tzou [14]. According to this model, classical Fourier's law $q = - K\,\nabla \mathrm{T}$ is replaced by $q(P, t{+}\tau_{\mathrm{q}}) = - K\nabla \mathrm{T}(P, t{+}\tau_T)$, where the temperature gradient $\nabla \mathrm{T}$ at a point P of the material at time $\mathrm{t}{+}\tau_{\mathrm{T}}$ corresponds to the heat flux vector q at the same point at time $\mathrm{t}{+}\tau_{\mathrm{q}}$. Here K is the thermal conductivity of the material. The delay time τ_{T} interpreted as that caused by the microstructural interactions and is called the phase-lag of the temperature gradient. The other delay time τ_{q} interpreted as the relaxation time due to the fast transient effects of thermal inertia and is called the phase-lag of the heat flux. Recently, Choudhuri has proposed a theory with the three-phase lag (3PHL) which is able to contain all the previous theories at the same time [15]. In this case Fourier's law $q = - K\,\nabla \mathrm{T}$ is replaced by $q(P, t{+}\tau_{\mathrm{q}}) = - \; [K\nabla \mathrm{T}(P, t{+}\tau_T){+}K^*\nabla\nu(P, t{+}\tau_\nu)]$, where $\nabla\nu$ ($\dot{\nu} = T$) is the thermal displacement gradient, K^* is the additional material constant and τ_ν is the phase-lag for the thermal displacement gradient. The purpose of the work of Choudhuri was to establish a mathematical model that includes 3PHL in the heat flux vector, the temperature gradient and in the thermal displacement gradient [15]. For this model, we can consider several kinds of Taylor approximations to recover the previously cited theories. In particular, the thermoelasticity without energy dissipation and thermoelasticity with energy dissipation were introduced by Green and Naghdi [16–18]. A three-phase-lag model is very useful in the problems of nuclear boiling, exothermic catalytic reactions, phonon-electron interactions, phonon-scattering, etc. Quintanilla and Racke [19], Kar and Kanoria [20], and Said and Othman [21] have solved different problems applying the 3PHL model. Othman and Eraki studied the generalized magneto-thermoelastic half-space with diffusion under initial stress using three-phase-lag model [22]. The development of the effect of rotation and magnetic field is available in many studies, such as [23–34].

The present paper is concerned with the investigations related to the effect of diffusion and magnetic field on a two temperature thermoelastic medium with the 3PHL model and thermoelasticity without energy dissipation (G-N II) theory. The variations of the considered variables with the horizontal distance are illustrated graphically. Comparisons made between the two models in the absence and presence of the diffusion as well as magnetic field. Also a comparison is made with the results of the two models with and without an internal heat source.

2 Formulation of the problem and basic equations

The problem is considered as a generalized thermodiffusion problem for a medium with an internal heat source being permeated into the uniform magnetic field with a constant intensity $H = (0, H_0, 0)$ which is acting parallel to the $y-$axis at uniform temperature T_0 in the undisturbed state. A fixed Cartesian coordinate system (x, y, z) with origin on the surface $z = 0$, which is stress free, and z-axis directed vertically into the medium. The region $z > 0$ is occupied by the elastic solid with generalized thermodiffusion. We are interested in a plane strain in the xz-plane with displacement vector $u = (u, 0, w)$. The governing equations in the absence of the body force are as in Sherief *et al.* [8], Youssef [35], Choudhuri [15]:

1. The equation of motion

$$\rho\, u_{i,tt} = (\lambda + \mu)u_{j,ij} + \mu\, u_{i,jj} - (\beta_1 \hat{T}_{,i} + \beta_2\, C_{,i}) + F_i\,. \tag{1}$$

2. The generalized heat conduction equation in the 3PHL model

$$K^* \nabla^2\Phi + \tau_\nu^* \nabla^2\Phi_{,t} + K\tau_T \nabla^2\Phi_{,tt} = \left[1 + \tau_q\frac{\partial}{\partial t} + \frac{1}{2}\tau_q^2\frac{\partial^2}{\partial t^2}\right]\Big(\rho\, C_E T_{,tt} + \beta_1\, T_0\, e_{,tt} + a\, T_0\, C_{,tt} - Q\Big)\,. \tag{2}$$

The relation between the conductive temperature and the thermodynamic temperature is

$$\Phi - T = \delta\,\Phi_{,ii}\,. \tag{3}$$

3. The generalized diffusion equation

$$d\beta_2 e_{kk,ii} + da\hat{T}_{,ii} + \left[1 + \tau_q\frac{\partial}{\partial t} + \frac{1}{2}\tau_q^2\frac{\partial^2}{\partial t^2}\right] C - dbC_{ii} = 0\,. \tag{4}$$

The constitutive law of the theory of generalized thermoelasticity is

$$\sigma_{ij} = \lambda\, e_{kk}\, \delta_{ij} + 2\,\mu\, e_{ij} - (\beta_1 \hat{T} + \beta_2\, C)\delta_{ij}\ ,$$

$$e_{kk} = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}, \quad e_{ij} = \frac{1}{2}(u_{i,\,j} + u_{j,\,i}), \quad i, j = x,\, z. \tag{5}$$

$$P = -\,\beta_2\, e_{kk} -\ a\,\hat{T} +\ b\,C\ . \tag{6}$$

The variations of the magnetic and electric fields are perfectly conducting slowly moving medium and are given by Maxwell's equation in [36]

$$J = \operatorname{curl} h -\ \varepsilon_0 E_{,t},\ \operatorname{curl} E = -\ \mu_0 h_{,t},\ E = -\ \mu_0\, (\mathrm{u}_{,t}\, \mathrm{x}\, H\,),\ \nabla \cdot h = 0\ , \tag{7}$$

F is the Lorentz force given by $F_i{=}\mu_0(J \times H)_i$. The components of the Lorentz force will be

$$F_1 = -\mu_0\, H_0 \frac{\partial h}{\partial x} - \mu_0^2\, H_0^2\, \varepsilon_0 \frac{\partial^2 u}{\partial t^2}, \quad F_2 = 0, \quad F_3 = -\mu_0\, H_0 \frac{\partial h}{\partial z} - \mu_0^2\, H_0^2\, \varepsilon_0 \frac{\partial^2 w}{\partial t^2},$$

where the small effect of the temperature gradient on J is also ignored. Due to application of the initial magnetic field H, there is an induced magnetic field $h = (0, h, 0)$ and an induced electric field E, as well as the simplified equations of electrodynamics of a slowly moving medium for a homogeneous, thermal and electrically conducting, elastic solid. In the above equation a comma followed by a suffix denotes partial derivative with respect to the corresponding coordinates.

The field Eqs. (1) and (5) become

$$\begin{aligned}\rho\, \frac{\partial^2 u}{\partial t^2} &= A \frac{\partial^2 u}{\partial x^2} + B \frac{\partial^2 w}{\partial x \partial\, z} + \mu\, \frac{\partial^2 u}{\partial\, z^2} \\ &\quad -\beta_1 \frac{\partial \hat{T}}{\partial\, x} - \beta_2\, \frac{\partial C}{\partial\, x}\ - \mu_0 H_0 \frac{\partial\, h}{\partial\, x} - \varepsilon_0 \mu_0^2 H_0^2 \frac{\partial^2 u}{\partial\, t^2}\ ,\end{aligned} \tag{8}$$

$$\begin{aligned}\rho\, \frac{\partial^2 w}{\partial t^2} &= \mu\, \frac{\partial^2 w}{\partial\, x^2} + B \frac{\partial^2 u}{\partial\, x\, \partial\, z} + A\, \frac{\partial^2 w}{\partial\, z^2} \\ &\quad -\beta_1 \frac{\partial \hat{T}}{\partial\, z} - \beta_2 \frac{\partial C}{\partial\, z} - \mu_0 H_0 \frac{\partial\, h}{\partial\, z} - \varepsilon_0 \mu_0^2 H_0^2 \frac{\partial^2 w}{\partial\, t^2}\ ,\end{aligned} \tag{9}$$

$$\sigma_{zz} = \lambda \frac{\partial u}{\partial x} + A \frac{\partial w}{\partial z} - \beta_1 \hat{T} - \beta_2 C\ , \tag{10}$$

$$\sigma_{xz} = \mu\, (\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x})\ , \tag{11}$$

where $A = \lambda + 2\mu$ and $B = \lambda + \mu$.

Introducing the following non-dimensional quantities:

$$(x',\, z',\, u', w') = (c_1 \eta\, x,\ c_1 \eta\, z,\, c_1 \eta\, u,\, c_1 \eta\, w)\ ,$$

$$(\, t',\, \tau_q',\, \tau_\nu',\, \tau_T'\,) = (\, c_1^2 \eta\, t,\, c_1^2 \eta\, \tau_q,\, c_1^2 \eta\, \tau_\nu,\, c_1^2 \eta\, \tau_T\,)\ ,$$

$$h' = \frac{h}{H_0}\ ,\quad \sigma_{ij}' = \frac{\sigma_{ij}}{\mu}\ ,\quad \theta = \frac{\beta_1 \hat{T}}{(\lambda + 2\mu)}\ ,\quad C' = \frac{C}{\rho},$$
$$P' = \frac{P}{\beta_2}\ ,\quad Q' = \frac{\beta_1}{\rho\, C_E\, c_1^4\, \eta^2 (\lambda + 2\mu)}\, Q\ ,\quad \Phi' = \frac{\beta_1\, (\Phi - T_0)}{(\lambda + 2\mu)}, \tag{12}$$

where $\eta = \frac{\rho\, C_E}{K^*}$, and $c_1^2 = \frac{(\lambda + 2\mu)}{\rho}$. Using the above non-dimension variables, followed by employing $h = -\, H_0\, e$, and introducing the potential functions defined by expressions

$$u = \frac{\partial\, \psi_1}{\partial\, x} - \frac{\partial\, \psi_2}{\partial\, z}\ ,\qquad w = \frac{\partial\, \psi_1}{\partial\, z} + \frac{\partial\, \psi_2}{\partial\, x}\ , \tag{13}$$

leads to obtaining

$$\alpha \frac{\partial^2 \psi_1}{\partial\, t^2} = B_{11} \nabla^2 \psi_1 - \theta - a_1\, C\ , \tag{14}$$

$$\mu_1 \nabla^2 \psi_2 - \alpha \frac{\partial^2 \psi_2}{\partial\, t^2} = 0\ , \tag{15}$$

$$C_K\, \Phi_{,ii} + C_\nu\, \dot{\Phi}_{,ii} + C_T\, \ddot{\Phi}_{,ii} =$$
$$\left[1\, + \tau_q \frac{\partial}{\partial\, t} + \frac{1}{2} \tau_q^2 \frac{\partial^2}{\partial\, t^2} \right] \left(\ddot{\theta} + \varepsilon\, \nabla^2 \ddot{\psi}_1 - a_2\, \ddot{C} - Q \right)\ , \tag{16}$$

$$\Phi - T = \beta_0\, \Phi_{,ii}\ , \tag{17}$$

$$\nabla^4\, \psi_1 + a_3 \nabla^2 \theta + \left[1\, + \tau_q \frac{\partial}{\partial t} + \frac{1}{2} \tau_q^2 \frac{\partial^2}{\partial t^2} \right]\, a_4 C - b_1 \nabla^2 C = 0\ , \tag{18}$$

where: $B_{11} = B_1 + \mu_1 + h_0 H_0$, $(\, A_1,\, B_1, \mu_1,\ h_0\,)\ = \frac{(\, A,\ B, \mu, \mu_0\, H_0^2)}{\rho\, c_1^2}$,

$$a_1 = \frac{\beta_2}{c_1^2},\quad C_K = \frac{K^*}{\rho\, C_E\, c_1^2},\quad C_\nu = \frac{\eta\, K}{\rho\, C_E} + C_K\, \tau_\nu\ ,$$

$$C_T = \frac{\eta\, K \tau_T}{\rho\, C_E}, \quad \beta_0 = \delta\, c_1^2 \eta^2, \quad \varepsilon = \frac{\beta_1^2\, T_0}{\rho\, C_E\,(\lambda + 2\mu)}\,, \quad b_1 = \frac{b\rho}{\beta_2}\,,$$

$$a_2 = \frac{a\;\beta_1\, T_0}{C_E\,(\lambda + 2\mu)}, \quad a_3 = \frac{a\;(\lambda + 2\mu)}{\beta_1 \beta_2}, \quad a_4 = \frac{\rho}{d\,\beta_2\, c_1^2\, \eta^2}, \quad \alpha = 1 + \frac{\varepsilon_0 \mu_0^2 H_0^2}{\rho}\,.$$

In Eq. (16) the dots refer to time differentiation.

3 Normal mode analysis

Solution of the considered physical variable can be decomposed in terms of normal modes as in the following form:

$$\Big[u, w, \psi_1, \psi_2, \theta, \Phi, \sigma_{ij}, C\Big](x, z, t) =$$
$$\Big[u^*, w^*,\ \psi_1^*,\ \psi_2^*, \theta^*,\ \Phi^*,\ \sigma_{ij}^*\ , C^*\Big](z)\ \exp(f\,t + \mathrm{i}\, m\, x),$$

$$Q = Q^* \exp(f\,t + \mathrm{i}\, m\, z), \qquad Q^* = Q_0 V_0, \tag{19}$$

where $u^*(z)$, $w^*(z)$, $\psi_1^*(z)$, $\psi_2^*(z)$, $\theta^*(z)$, $\Phi^*(x)$, $\sigma_{ij}^*(z)$, and $C^*(z)$ are the amplitudes of the field quantities.

Introducing Eqs. (19) in Eqs. (14)–(18), we obtain

$$\Big[B_{11}\mathrm{D}^2 - N_1\Big]\,\psi_1^* - a_1 C^* = \theta^*, \tag{20}$$

$$\Big[\mu_1 \mathrm{D}^2 - N_2\Big]\psi_2^* = 0\,, \tag{21}$$

$$\Big[\mathrm{D}^4 - 2m^2\mathrm{D}^2 + m^4\Big]\,\psi_1^* + a_3\Big[\mathrm{D}^2 - m^2\Big]\,\theta^* - [b_1\,\mathrm{D}^2 - N_4]\,C^* = 0\,, \tag{22}$$

$$\Big[N_5\,\mathrm{D}^2 - N_5\, m^2\Big]\,\psi_1^* - N_6\, C^* = \Big[N_7\,\mathrm{D}^2 - N_8\Big]\,\Phi^* + N_3\, Q_0\, V_0 - N_3\, f^2 \theta^*, \tag{23}$$

$$\theta^* = \Big(N_0 - \beta_0\,\mathrm{D}^2\Big)\Phi^*, \tag{24}$$

where: $N_0 = 1 + \beta_0\, m^2$,
$N_1 = B_{11}\, m^2 + \alpha\, f^2$,
$N_2 = \mu_1\, m^2 + \alpha\, f^2$,
$N_3 = 1 + \tau_q\, f + \frac{1}{2}\tau_q^2\, f^2$,
$N_4 = N_3\, a_4 + b_1\, m^2$, $N_5 = \varepsilon\, f^2 N_3$,
$N_6 = a_2\, f^2 N_3$,

$N_7 = C_K + C_\nu\, f + C_T\, f^2$,
$N_8 = m^2 \quad N_7$, $\mathrm{D} = \frac{\mathrm{d}}{\mathrm{d}z}$.
Eliminating $C^*(x)$ and $\Phi^*(x)$ between Eqs. (20), (22), (23), and (24), we obtain the sixth-order ordinary differential equation satisfied with $\psi_1^*(x)$

$$\left[\mathrm{D}^6 - L_1\,\mathrm{D}^4 + L_2\,\mathrm{D}^2 - L_3\right]\psi_1^*(z) = \frac{-\,N_0 N_3 Q_0 V_0 a_9}{L_0}\,, \tag{25}$$

where $L_0 = -a_8\, a_{10} + a_5\, a_{13}$,
$L_1 = (-\,a_8\, a_{11} - a_9\, a_{10} + a_{13}\, a_6 + a_{14}\, a_5)/L_0$,
$L_2 = (-\,a_8\, a_{12} - a_9\, a_{11} + a_{13}\, a_7 + a_{14}\, a_6)\,/L_0$,
$L_3 = (-\,a_9\, a_{12} + a_{14}\, a_7)/L_0$,
$a_5 = 1 + a_3 B_{11}$, $a_6 = a_3 B_{11} m^2 + N_1 a_3 + 2m^2$,
$a_7 = a_3 N_1 m^2 + m^4$, $a_8 = a_1 a_3 + b_1$,
$a_9 = a_1 a_3 m^2 + N_4$, $a_{10} = B_{11} N_9 + N_5 \beta_0$,
$a_{11} = N_1 N_9 + B_{11} N_{10} + N_0 N_5 + m^2 N_5 \beta_0$, $a_{12} = N_1 N_{10} + N_0 N_5 m^2$,
$a_{13} = a_1 N_9 + N_6 \beta_0$, $a_{14} = a_1 N_{10} + N_6 N_0$,
$N_9 = N_7 + N_3\, f^2\, \beta_0$, $N_{10} = N_8 + N_0 N_3\, f^2$.
Equation (25) can be factored as

$$\left(\mathrm{D}^2 - k_1^2\right)\left(\mathrm{D}^2 - k_2^2\right)\left(\mathrm{D}^2 - k_3^2\right)\,\psi_1^*(z) = \frac{-N_0 N_3 Q_0 V_0 a_9}{L_0}, \tag{26}$$

where k_n^2 $(n = 1,\,2,\,3)$ are the roots of the following characteristic equation

$$k^6 - L_1\, k^4 + L_2 k^2 - L_3 = 0\,. \tag{27}$$

Solution of Eq. (25), bound as $z \to \infty$, given by

$$\psi_1^*(z) = \sum_{n=1}^{3} R_n \exp(-\,k_n z) + \frac{N_0 N_3 Q_0 V_0 a_9}{L_0 L_3}\,. \tag{28}$$

In a similar manner, we get that

$$C^*(z) = \sum_{n=1}^{3} H_{1n} R_n \exp(-\,k_n z) - \frac{N_0 N_3 Q_0 V_0 a_7}{L_0 L_3}\,, \tag{29}$$

$$\Phi^*(z) = \sum_{n=1}^{3} H_{2n} R_n \exp(-\,k_n z) - \frac{N_3 Q_0 V_0 (N_1 N_4 - a_1 m^4)}{L_0 L_3}\,, \tag{30}$$

where $H_{1n} = \dfrac{a_5 k_n^4 - a_6 k_n^2 + a_7}{a_8 k_n^2 - a_9}$, and $H_{2n} = \dfrac{B_{11} k_n^2 - N_1 - a_1 H_{1n}}{N_0 - \beta_0 k_n^2}$.

Introducing Eq. (30) in Eq. (24), we get

$$\theta^*(z) = \sum_{n=1}^{3} H_{3n} R_n \exp(-k_n z) - \frac{N_0 N_3 Q_0 V_0 (N_1 N_4 - a_1 m^4)}{L_0 L_3}\ , \tag{31}$$

where, $H_{3n} = (N_0 - \beta_0\, k_n^2)\, H_{2n}$.
Solution of Eq. (21), bound as $z \to \infty$, given by

$$\psi_2^*(z) = \xi_2 \exp(-k_4 z), \qquad k_4 = \sqrt{N_2/\mu_1}\ . \tag{32}$$

Introducing Eq. (28) and (32) in Eq. (13), yields

$$u^*(z) = \sum_{n=1}^{3} \mathrm{i}m\, R_n \exp(-k_n z) + \frac{\mathrm{i}m\, N_0 N_3 Q_0 V_0 a_9}{L_0 L_3} + \xi_2 k_4 \exp(-k_4 z)\ , \tag{33}$$

$$w^*(z) = \sum_{n=1}^{3} -R_n\, k_n \exp(-k_n z) + \mathrm{i}m \xi_2 \exp(-k_4 z)\ . \tag{34}$$

Introducing Eqs. (12), (19), (29), (31), (33), and (34) in Eqs. (10), (11), and (6), we get

$$\sigma_{zz}^* = \sum_{n=1}^{3} H_{4n} R_n \exp(-k_n z) + \xi_3 \xi_2 \exp(-k_4 z) + G_1, \tag{35}$$

$$\sigma_{xz}^* = \sum_{n=1}^{3} H_{5n} R_n \exp(-k_n z) - \xi_4 \xi_2 \exp(-k_4 z), \tag{36}$$

$$P^* = \sum_{n=1}^{3} H_{6n} R_n \exp(-k_n z) + \mathrm{i}m\, \xi_2 k_4 \exp(-k_4 z) + G_2, \tag{37}$$

where $R_n (\mathrm{n} = 1, 2, 3)$ are some coefficients, $\xi_3 = \mathrm{i}m\, k_4 (\lambda - A)/\mu$,
$\xi_4 = (k_4^2 + m^2)\ , \quad H_{4n} = \frac{1}{\mu}\, [-m^2 \lambda + A\, k_n^2 - (\lambda + 2\mu)\, H_{3n} - \beta_2 \rho H_{1n}]$,
$H_{5n} = -2\mathrm{i}m k_n\ , \quad H_{6n} = m^2 - k_n^2 - a_3\, H_{3n} + b_1 H_{1n}\ ,$
$G_1 = \dfrac{[-a_1 a_3 m^4 \lambda - N_4 m^2 \lambda + (\lambda + 2\mu)(N_1 N_4 - a_1 m^4) + \beta_2 \rho\, a_7] N_0 N_3 Q_0 V_0}{\mu\, L_0 L_3}$,
$G_2 = N_0 N_3 Q_0 V_0 \left[N_4 m^2 + a_3\, N_1 N_4 - b_1 a_3 N_1 m^2 - b_1 m^4 \right]$.

4 Boundary conditions

In the physical problem, we should suppress the positive exponentials that are unbounded at infinity. The four constants ξ_2 and R_n $(n = 1, 2, 3)$ can be obtained by using the following boundary conditions on the surface at $z = 0$:

$$\sigma_{zz} = -g\,(x{,}t) = -g_0\, e^{f\,t+\mathrm{i}\,m\,x}\ , \quad \sigma_{xz} = 0, \Phi = 0, \frac{\partial\, C}{\partial\, z} = 0\ , \tag{38}$$

where $g\,(x{,}t)$ is a function of x and t, and g_0 is the magnitude of the mechanical force. Using the expressions of the variables considered into the above boundary conditions, Eqs. (38), we can obtain the following equations satisfied with the parameters:

$$-\sum_{n=1}^{3} k_n H_{1n} R_n = 0\ , \qquad \sum_{n=1}^{3} H_{2n} R_n = 0\ ,$$

$$\sum_{n=1}^{3} H_{4n} R_n + \xi_3 \xi_2 = -\, g_0 - G_1\ , \quad \sum_{n=1}^{3} H_{5n} R_n - \ \xi_4\, \xi_2 = 0\ . \tag{39}$$

Invoking Eqs. (39), we obtain a system of four equations. After applying the inverse of matrix method (or Cramer's rule using Matlab programming), we have the values of the four constants ξ_2 and R_n, $(n = 1, 2, 3)$. Hence, we obtain the expressions of the displacement components, the conductive temperature, the thermal temperature, the chemical potential, the mass concentration and stress components.

$$\begin{pmatrix} R_1 \\ R_2 \\ R_3 \\ \xi_2 \end{pmatrix} = \begin{pmatrix} k_1\, H_{11} & k_2 H_{12} & k_3 H_{13} & 0 \\ H_{21} & H_{22} & H_{23} & 0 \\ H_{41} & H_{42} & H_{43} & \xi_3 \\ H_{51} & H_{52} & H_{53} & -\xi_4 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ \xi_5 \\ -g_0 - \ G_1 \\ 0 \end{pmatrix}, \tag{40}$$

where $\xi_5 \ = \ \dfrac{N_3 Q_0 V_0 (N_1 N_4 - a_1 m^4)}{L_0 L_3}$.

5 Particular cases

1. The corresponding equations for a two-temperature generalized thermoelastic medium with diffusion, with internal heat source and without the magnetic field have been mentioned in cases above by taking $H_0 = 0$.

2. The corresponding equations for a two-temperature generalized thermoelastic medium with diffusion, with magnetic field and without internal heat source can be obtained from the mentioned cases above by taking $Q_0 = 0$.
3. Equations of the 3PHL model when, K, τ_T, τ_q, τ_ν are greater then zero and the solutions are always (exponentially) stable if $\frac{2K\tau_T}{\tau_q} > \tau_\nu^* > K^*\tau_q$, as in Quintanilla and Racke [19].
4. Equations of the GN-II theory can be obtained, when $K = \tau_T = \tau_q = \tau_\nu = 0$.
5. The corresponding equations for a two-temperature generalized thermoelastic medium with magnetic field, with internal heat source and without diffusion, can be obtained by taking $C = a = b = \beta_2 = 0$, thus we have

$$\left[B_{11}\mathrm{D}^2 - N_1\right]\psi_1^* = \left[N_0 - \beta_0\,\mathrm{D}^2\right]\Phi^* \,, \tag{41}$$

$$\left[\mu_1\mathrm{D}^2 - N_2\right]\psi_2^* = 0 \,, \tag{42}$$

$$\left[N_5\,\mathrm{D}^2 - N_5\,m^2\right]\psi_1^* = \left[N_9\,\mathrm{D}^2 - N_{10}\right]\Phi^* + N_3\,Q_0\,V_0 \,, \tag{43}$$

$$\theta^* = \left(N_0 - \beta_0\,\mathrm{D}^2\right)\Phi^* \,. \tag{44}$$

Eliminating $\Phi^*(x)$ between Eqs. (41) and (43), we obtain the fourth-order ordinary differential equation satisfied with $\psi_1^*(x)$

$$\left[\mathrm{D}^4 - L_5\,\mathrm{D}^2 + L_6\right]\psi_1^* z) = \frac{-N_0 N_3 Q_0 V_0}{L_4} \,, \tag{45}$$

where $L_4 = B_{11}\,N_9 + N_5\,\beta_0$,
$L_5 = \dfrac{N_1\,N_9 + N_{10}\,B_{11} + N_5\,m^2\beta_0 + N_0\,N_5}{L_4}$,
$L_6 = \dfrac{N_1\,N_{10} + N_0\,N_5\,m^2}{L_4}$.
Equation (45) can factored as

$$\left(\mathrm{D}^2 - s_1^2\right)\left(\mathrm{D}^2 - s_2^2\right)\psi_1^*(z) = \frac{-N_0 N_3 Q_0 V_0}{L_4} \,, \tag{46}$$

where s_n^2 $(n = 1,\, 2)$ are the roots of the following characteristic equation:

$$s^4 - L_5\,s^2 + L_6 = 0 \,. \tag{47}$$

Solution of Eq. (45), bound as $z \to \infty$, is given by

$$\psi_1^*(z) = \sum_{n=1}^{2} I_n \exp(-s_n z) - \frac{N_0 N_3 Q_0 V_0}{L_4 L_6} \,. \tag{48}$$

In a similar manner, we get that

$$\Phi^*(z) = \sum_{n=1}^{2} H_{7n} I_n \exp(-s_n z) + \frac{N_1 N_3 Q_0 V_0}{L_4 L_6} \,. \tag{49}$$

Introducing Eq. (49) into Eq. (44), we get

$$\theta^*(z) = \sum_{n=1}^{2} H_{8n} I_n \exp(-s_n z) + \frac{N_0 N_1 N_3 Q_0 V_0}{L_4 L_6} \,, \tag{50}$$

where I_n (n = 1, 2) are some coefficients, $H_{7n} = \dfrac{B_{11} s_n^2 - N_1}{N_0 - \beta_0 s_n^2}$, and $H_{8n} = (N_0 - \beta_0 s_n^2) H_{7n}$.

Solution of Eq. (42) is the same as in Eq. (32) and

$$u^*(z) = \sum_{n=1}^{2} \mathrm{i}m\, I_n \exp(-s_n z) + \xi_2 k_4 \exp(-k_4 z) - \frac{\mathrm{i}m\, N_0 N_3 Q_0 V_0}{L_4 L_6} \,, \tag{51}$$

$$w^*(z) = \sum_{n=1}^{2} -I_n s_n \exp(-s_n z) + \mathrm{i}m\, \xi_2 \exp(-k_4 z) \,, \tag{52}$$

$$\sigma_{zz}^* = \sum_{n=1}^{2} H_{9n} I_n \exp(-s_n z) + \xi_3 \xi_2 \exp(-k_4 z) + G_3 \,, \tag{53}$$

$$\sigma_{xz}^* = \sum_{n=1}^{2} H_{10n} I_n \exp(-s_n z) - \xi_4 \xi_2 \exp(-k_4 z) \,, \tag{54}$$

where $H_{9n} = \frac{1}{\mu}\Big[-m^2\lambda + A s_n^2 - (\lambda + 2\mu) H_{8n}\Big]$, $\quad H_{10n} = -2\mathrm{i}m s_n$,

$$G_3 = \frac{[m^2 \lambda - (\lambda + 2\mu) N_1] N_0 N_3 Q_0 V_0}{\mu L_4 L_6} \,.$$

In this case the boundary conditions are

$$\sigma_{zz} = -g(z,t) = -g_0 e^{ft + \mathrm{i}mx}, \quad \sigma_{xz} = 0, \Phi = 0 \,. \tag{55}$$

Using the expressions of the variables considered into the above boundary conditions Eqs. (55), we can obtain the following equations satisfied with the parameters:

$$\sum_{n=1}^{2} H_{7n} I_n + G_4 = 0, \sum_{n=1}^{2} H_{9n} I_n + \xi_3 \xi_2 = -g_0 - G_3, \sum_{n=1}^{2} H_{10n} I_n - \xi_4 \xi_2 = 0, \tag{56}$$

where $G_4 = \dfrac{N_1 N_3 Q_0 V_0}{L_4 L_6}$.

Invoking Eqs. (56), we obtain a system of three equations. After applying the inverse of matrix method, we have the values of three constants $I_n,\ \xi_2\ (n = 1, 2)$. Hence, we obtain the expressions of the displacement components, conductive temperature, thermal temperature and stress components.

$$\begin{pmatrix} I_1 \\ I_2 \\ \xi_2 \end{pmatrix} = \begin{pmatrix} H_{71} & H_{72} & 0 \\ H_{91} & H_{92} & \xi_3 \\ H_{101} & H_{102} & -\xi_4 \end{pmatrix}^{-1} \begin{pmatrix} -G_4 \\ -g_0 - G_3 \\ 0 \end{pmatrix}. \tag{57}$$

6 Numerical calculation and discussion

In order to illustrate the theoretical results obtained in the preceding section, and to compare these in the context of the 3PHL model and the GN-II theory, we now present some numerical results for the physical constants as [37].
$\lambda = 7.76 \times 10^9\,\mathrm{N\,m^{-2}},\ \mu = 3.86 \times 10^{10}\,\mathrm{N\,m^{-2}},\ \rho = 8954\ \mathrm{kg\,m^{-3}},$
$f = 0.25,\ C_E = 383.1\ \mathrm{J\,kg^{-1}.K^{-1}},\ Q_0 = 3\,\mathrm{K},\ T_0 = 293\,\mathrm{K},\ g_0 = 3\,\mathrm{N\,m^{-2}},$
$V_0 = 0.2\,\mathrm{m\,s^{-1}},\ \tau_T = 7 \times 10^{-5}\,\mathrm{s},\ \tau_q = 9 \times 10^{-5}\,\mathrm{s},\ \tau_\nu = 6 \times 10^{-5}\,\mathrm{s},$
$\alpha_t = 3.78 \times 10^{-4}\,\mathrm{K^{-1}},\ \alpha_c = 1.98 \times 10^{-5}\ \mathrm{K^{-1}},\ K^* = 386\ \mathrm{w\,m^{-1}K^{-1}},$
$\mu_0 = 1.9\,\mathrm{N\,A^{-2}},\ \varepsilon_0 = 0.5\ \mathrm{F\,m^{-1}},\ K = 150\ \mathrm{w\,m^{-1}K^{-1}},\ \omega = \omega_0 + i\zeta,$
$\zeta = 0.7,\ \omega_0 = -0.3,\ \delta = 1.3 \times 10^{-15},\ a = 1.2 \times 10^4\,\mathrm{m^2\,s^{-2}K^{-1}},$
$b = 0.9 \times 10^6\,\mathrm{m^5\ kg^{-1}\ s^{-2}},\ d = 0.85 \times 10^{-8}\,\mathrm{kg\,s\ m^{-3}}.$

The computations carried out for a value of the time $t = 0.1\,\mathrm{s}$. The variations of the thermal temperature θ, the conductive temperature Φ, the displacement component w, the chemical potential P, the mass concentration C and the stress components $\sigma_{zz},\ \sigma_{xz}$ with distance z for the value of x, namely $x = -1.3$, substituted in performing the computation. The results are shown in Figs. 1–14. The graphs show four curves predicted by two different theories of thermoelasticity. In these figures, the solid lines represent the solution in the 3PHL model, and the dashed lines represent the solution derived using the G-N II theory. Here all the variables are taken in nondimensional forms.

6.1 The effect of diffusion

Figures 1–5 show comparisons between the displacement component, w, the thermal temperature, θ, the conductive temperature, Φ, and the stress components σ_{zz} and σ_{xz} with and without diffusion.

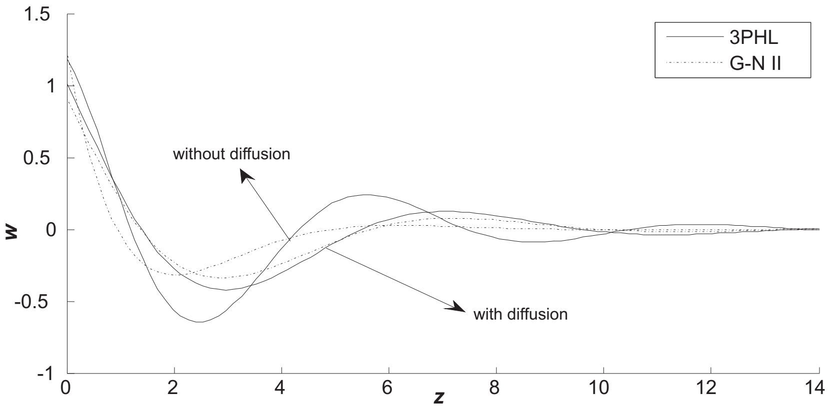

Figure 1: Vertical displacement distribution with and without diffusion.

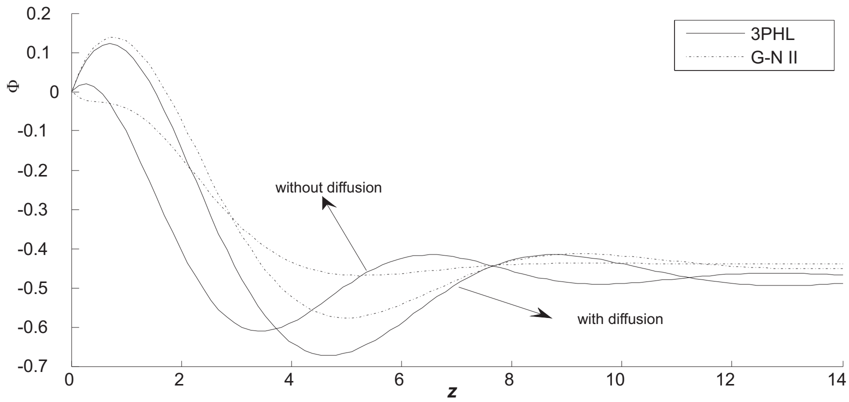

Figure 2: Conductive temperature distribution with and without diffusion.

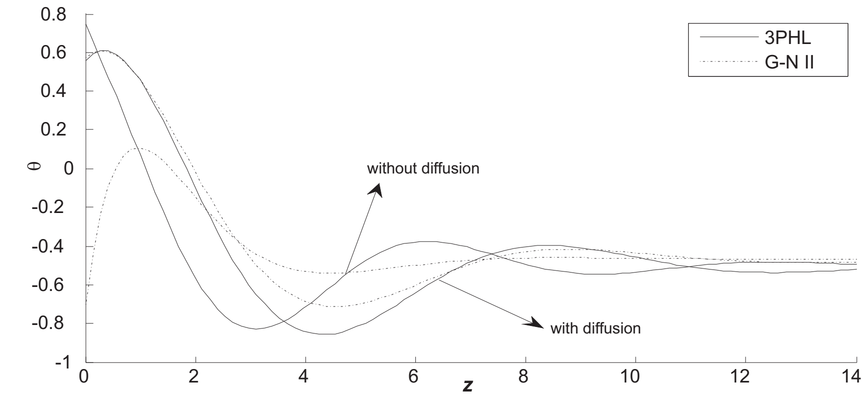

Figure 3: Thermal temperature distribution with and without diffusion.

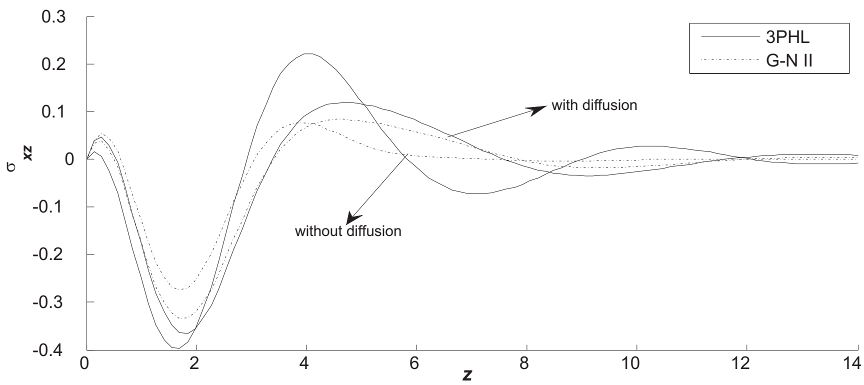

Figure 4: Distribution of stress component σ_{xz} with and without diffusion.

Figure 1 displays distribution of the vertical displacement w. In the context of the two models with diffusion, w starts with decreasing, then increases, and again decreases. However, in the context of the two theories without diffusion, w decreases to a minimum value, then increases, and moves in wave propagation. Figure 2 shows that distribution of the conductive temperature Φ, begins from a zero value and satisfies the boundary condition at $z = 0$. In the context of two theories with diffusion, Φ increases to a maximum value, then decreases, and moves in wave propagation. However, in the context of the two theories without diffusion, Φ increases, then decreases to a minimum value, and moves in wave propagation. Figure 3 explains the distribution of the thermal temperature θ. In the context of two theories with diffusion, θ increases, then decreases to a minimum value,

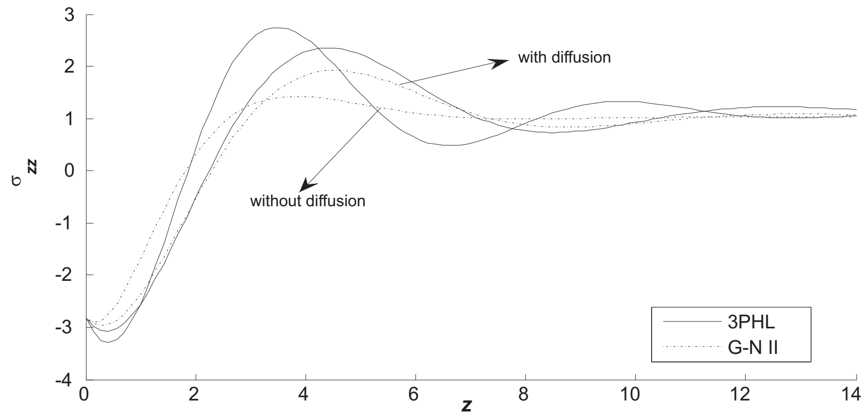

Figure 5: Distribution of stress component σ_{zz} with and without diffusion.

and moves in wave propagation direction. However, in the context of the 3PHL model without diffusion, θ decreases, then increases, and moves in wave propagation direction. In the context of the G-N II model without diffusion, θ increases, then decreases, and moves in wave propagation. Figure 4 depicts the distribution of the stress component σ_{xz} and demonstrates that it reaches a zero value and satisfies the boundary condition at $z = 0$. In the context of the two theories with diffusion, σ_{xz} increases, then decreases, and moves in wave propagation. However, in the context of two theories without diffusion, σ_{xz} increases, and then decreases, and moves in wave propagation. Figure 5 explains that distribution of the stress component σ_{zz} begins from a negative value and satisfies the boundary condition at $z = 0$. In the context of the two theories with diffusion, σ_{zz} increases, then decreases, and moves in wave propagation. However, in the context of the two theories without diffusion, σ_{zz} decreases, then increases, and moves in wave propagation. All physical quantities begin to coincide when the vertical distance increases reach the reference temperature of the solid.

6.2 The effect of the internal heat source

Figures 6–10 show comparisons between the thermal temperature, θ, the conductive temperature, Φ, chemical potential, P, mass concentration, C, and the stress component σ_{zz} with $(Q_0 = 3)$ and without $(Q_0 = 0)$ the internal heat source.

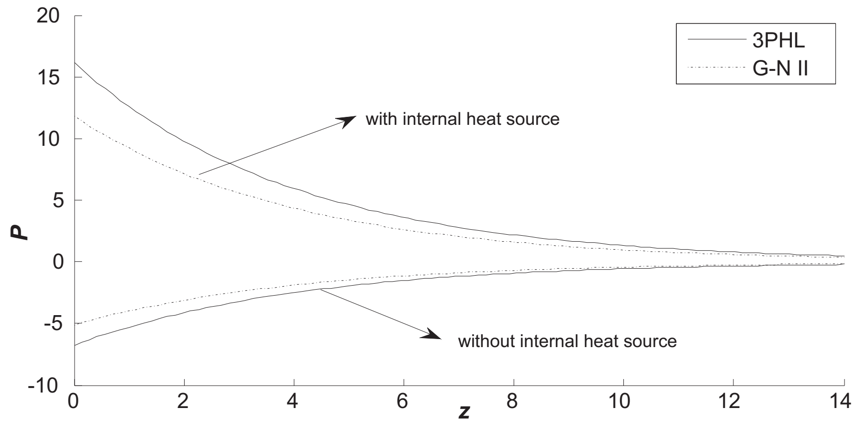

Figure 6: Chemical potential P with and without internal heat source.

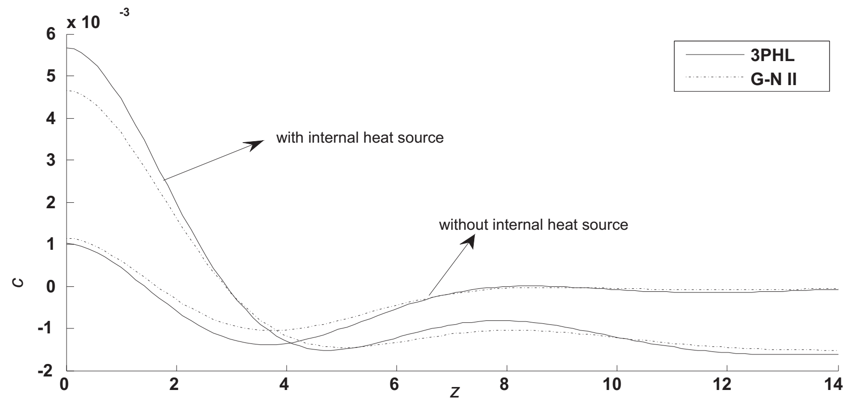

Figure 7: Mass concentration C with and without internal heat source.

Figure 6 displays that the values of chemical potential P increase with heat source. Figure 7 depicts that the values of mass concentration C increase with heat source then decrease. Figures 8 and 9 show that, the values of thermal temperature θ and the conductive temperature Φ, decrease with heat source. Figure 10 explains that the values of the stress component σ_{zz} decrease with the heat source increasing.

6.3 The effect of magnetic field

Figures 11–14 show comparisons between the displacement component, w, the conductive temperature, Φ, chemical potential, P, and mass concen-

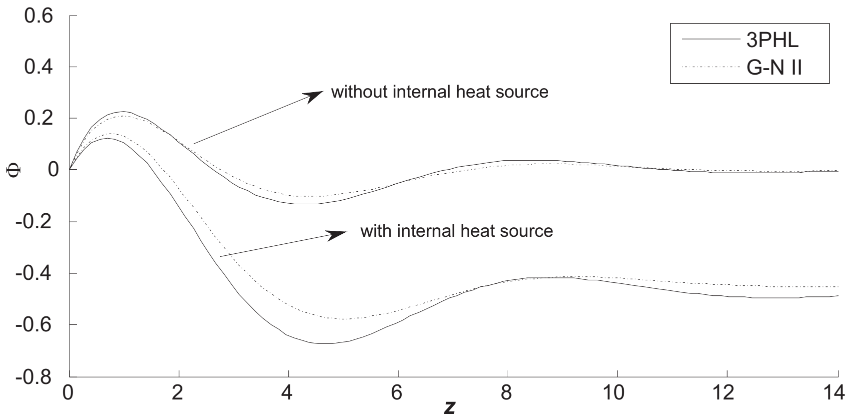

Figure 8: Conductive temperature distribution Φ with and without internal heat source.

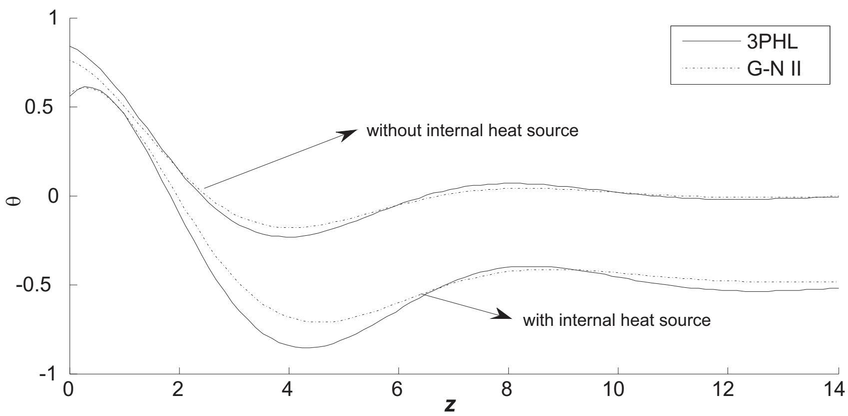

Figure 9: Thermal temperature distribution θ with and without internal heat source.

tration C in the absence ($H_0 = 0$) and presence ($H_0 = 100$) of a magnetic field.

Figure 11 describes the distribution of the chemical potential P. In the context of the two theories, P decreases in the range $0 \leq z \leq 14$. The values of P increase in the presence of a magnetic field. Figure 12 exhibits the distribution of mass concentration C. In the context of the two theories, C decreases and then increases for $H_0 = 0$. The values of C increase in the presence of a magnetic field in the first, then decrease, again increase and last decrease. Figure 13 shows the distribution of the vertical displacement w. In the context of the two models, w starts with decreasing,

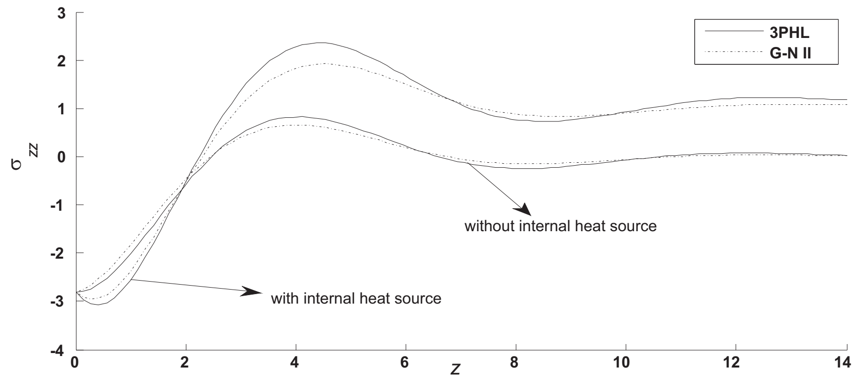

Figure 10: Distribution of stress component σ_{zz} with and without internal heat source.

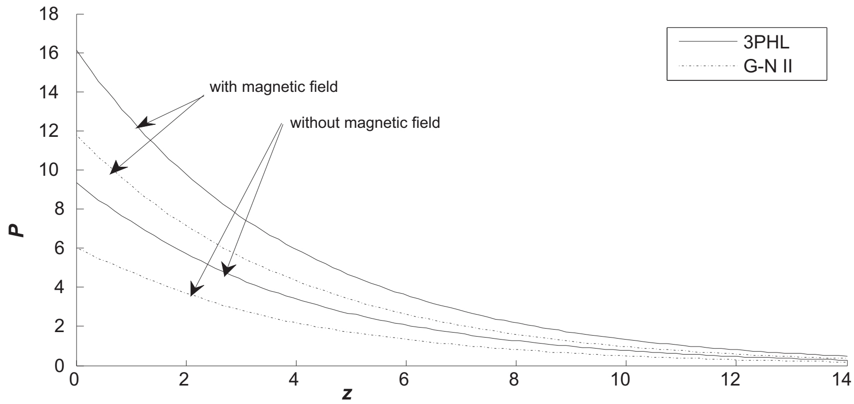

Figure 11: Chemical potential P with and without magnetic field.

and then increases for $H_0 = 0$. Figure 14 explains that the distribution of the conductive temperature Φ begins from a zero value and satisfies the boundary condition at $z = 0$. In the context of the two theories, Φ increases to a maximum value, then decreases, and moves in wave propagation for $H_0 = 0$. The values of the w and Φ decrease in the presence of a magnetic field in the first, then increase and again decrease.

7 The effect of horizontal distance

Figures 15 and 16 are giving 3D surface curves for the physical quantities, i.e., the stress components, σ_{xz}, and conductive temperature, Φ, to study

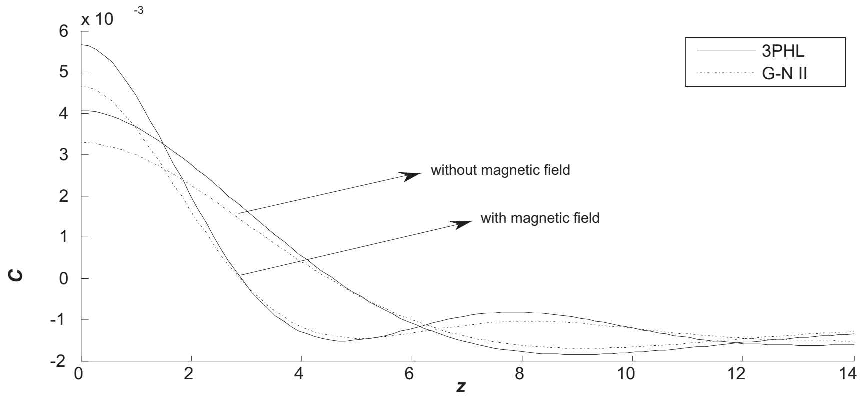

Figure 12: Mass concentration C with and without magnetic field.

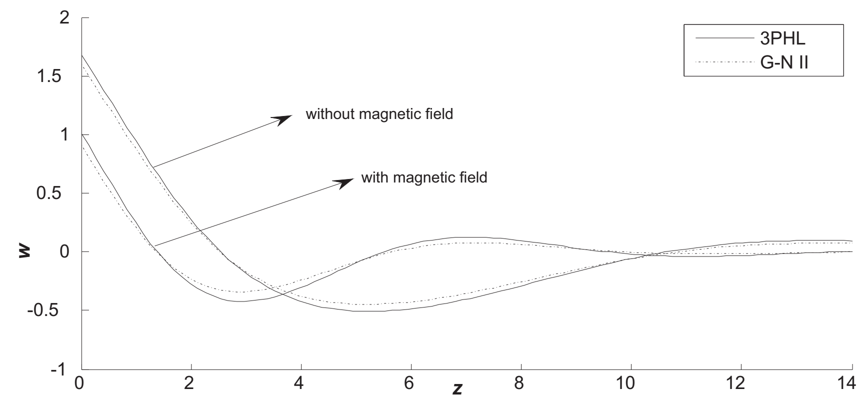

Figure 13: Vertical displacement distribution w with and without magnetic field.

the effect of a magnetic field and the diffusion on wave propagation in a generalized thermoelastic problem for a medium with an internal heat that is moving with a constant speed in the context of the 3PHL model. These figures are very important to study the dependence of these physical quantities on the horizontal component of distance. The curves obtained are highly depending on the vertical distance from origin, all the physical quantities satisfy boundary condition and are moving in wave propagation.

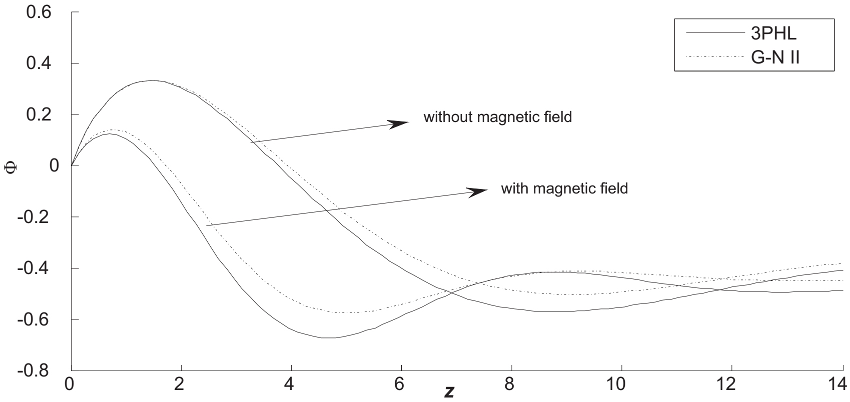

Figure 14: Conductive temperature distribution Φ with and without magnetic field.

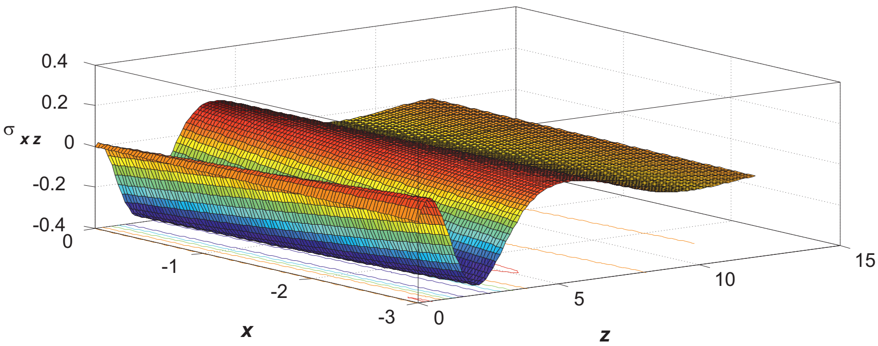

Figure 15: Distribution of stress component σ_{xz} against both components of distance based on the 3PHL model.

8 Conclusions

The following conclusions based on the above analysis can be drawn:

1. It is clear that the diffusion, internal heat source and a magnetic field have important roles in the distribution of the displacement component w, thermal temperature θ, conductive temperature Φ, and stress components σ_{zz}, σ_{xz}.
2. The analytical solutions based upon the normal mode analysis of the thermoelastic problem in solids have been developed and used.
3. There are significant differences in the field quantities between the G-N II theory and the 3PHL model due to the phase-lag of temper-

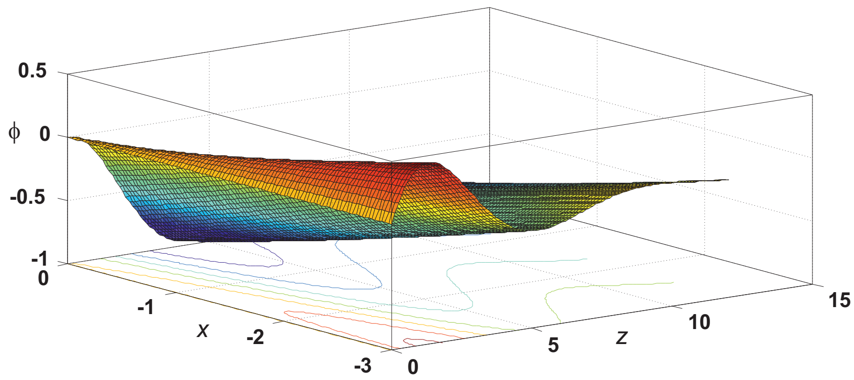

Figure 16: Conductive temperature distribution Φ against both components.

ature gradient and the phase-lag of heat flux.

4. The physical quantities are very depending on the vertical distance and horizontal distance.

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