

## APPLICATION OF FRACTAL ANALYSIS METHODS FOR LIFT HEIGHT OPTIMIZATION IN MAGNETIC FORCE MICROSCOPY MEASUREMENTS

The paper presents results of a research on simulation of magnetic tip-surface interaction as a function of the lift height in the magnetic force microscopy. As expected, magnetic signal monotonically decays with increasing lift height, but the question arises, whether or not optimal lift height eventually exists. To estimate such a lift height simple procedure is proposed in the paper based on the minimization of the fractal dimension of the averaged profile of the MFM signal. In this case, the fractal dimension serves as a measure of distortion of a pure tip-surface magnetic coupling by various side effects, e.g. thermal noise and contribution of topographic features. Obtained simulation results apparently agree with experimental data.

*Keywords:* Lift height optimization, magnetic force microscopy, fractal analysis

### 1. Introduction

Scanning Probe Microscopy (SPM) is a powerful surface characterization tool on the atomic level for a wide range of natural and engineering materials. It is sensitive to a large variety of interactions occurring between the scanning tip and the probed surface maintained at a distance from 10 pm to 1 mm. Apart from 3-dimensional topographical maps, SPM images can also contain spatially resolved information on physical properties of the surface, for example: friction coefficient, adhesion forces, reduced Young's modulus, spontaneously magnetized domains etc. Obtained data in the form of 2-dimensional matrices are further processed numerically to investigate geometrical, statistical and functional properties of the surface. Since the early 1990's, extended numerical analysis based on correlation and fractal methods has gained increasing attention [1], although so far it is still mainly used for description of topographical features of the surface of various materials: from engineering to medical ones [2,3]. It might be interesting to extend this technique into studies on magnetic domains and stray magnetic fields, spatial modifications of the friction coefficient, adhesion forces etc.

Fractal analyses of images of magnetic interactions were firstly attempted in the studies of the Barkhausen noise observed in silicon steel sheets [4], and in the multiscale analysis of the domain walls in the Pt/Co/Pt trilayer systems obtained using polar Kerr microscopy [5]. Our previous works in this field involved fractal characterization of the magnetic domains in maraging steel samples [6], largely extended into fractal studies of the decay of the magnetic stray field [7]. Magnetic stray field turned out

to decay very slowly with increasing tip-surface separation (lift height), and could be extracted from the background noise even up to 1 mm from the surface. At very low lift heights, however, topographic features of the surface were found to emerge from magnetic interactions. This naturally raised the question about optimal settings for the lift height, and eventual procedure of its estimation. In the following paragraphs a simple method is proposed that relies on calculations of the fractal dimension of the maps of magnetic interactions recorded at various lift heights.

### 2. Methods

Fractal analysis can be carried out using, for example, structure function, autocorrelation function, roughness data, and the cube-count method, which were compared in details in previous work [8]. Here, the three-step method is employed that involves: (1) calculation of the autocorrelation function (ACF)  $R(t)$ , (2) calculation of the structure function (SF)  $S(t)$ , (3) estimation of the fractal parameters  $F$  (fractal dimension), and  $K$  (quasi-topothesy).

Assume a 2-dimensional MFM data matrix containing phase shifts  $\varphi$  between the tip oscillation and its driving piezo signal. The profile ACF is an averaged sum of terms each of which is the product of the entries with their delayed counterparts:

$$R(\tau) = \frac{1}{N} \sum_{x=0}^{N-1} [\varphi(x) - \varphi_z][\varphi(x + \tau) - \varphi_z] \quad (1)$$

where:  $\tau$  – denotes the discrete spatial lag between matrix en-

\* UNIVERSITY OF WARMIA AND MAZURY IN OLSZTYN, FACULTY OF TECHNICAL SCIENCES, 11 OCZAPOWSKIEGO STR., 10-719 OLSZTYN, POLAND

\*\* UNIVERSITY OF WARMIA AND MAZURY IN OLSZTYN, FACULTY OF MATHEMATICS AND COMPUTER SCIENCE, 54 SŁONECZNA STR., 10-710 OLSZTYN, POLAND

# Corresponding author: cezary.senderowski@uwm.edu.pl

tries,  $\varphi_z$  – is the averaged phase shift, while  $N$  – is the number of profile samples. Now, the profile structure function  $S(t)$  can be computed according to [9]:

$$S(\tau) = 2\sigma^2(1 - R(\tau)) \quad (2)$$

where:  $\sigma$  – is the root-mean-square of the MFM signal. Within the limit of the self-similar behavior, any profile of the structure function is assumed to obey the scaling law:

$$S(\tau) = \Lambda^{2(D-1)}\tau^{2(2-D)} \quad (3)$$

where  $D$  – is the profile fractal dimension, while  $\Lambda$  – is the topothesy. Fractal parameters can then be estimated from a log-log plot of the above structure function vs. separation length. For more or less isotropic surfaces, their areal fractal dimension  $D_A$  is simply profile fractal dimension  $D$  plus one. Unfortunately, the topothesy often turns out to be smaller than the instrumental resolution, and even goes below the atomic scale lengths, hence it is replaced with the  $K$  parameter referred to as the pseudo-topothesy defined as [10]:

$$K = \frac{\pi G^{2(D-1)}}{2\Gamma(5-2D)\sin[(2-D)\pi]} \quad (4)$$

where:  $G$  – is a scale-dependent constant, while  $\Gamma$  – is the Euler's function. In such a case, the structure function is given as:

$$S(\tau) = K\tau^{2(2-D)} \quad (5)$$

Similar to Eq. (3), fractal parameters can be estimated from a log-log plot of the structure function vs. separation length.

### 3. Model of a noisy MFM signal

Suppose we have a sample containing a series of parallel stripe domains as in, for example, amorphous magnetic materials (Fig. 1). Upon scanning with magnetized tip, sinusoid-like phase shift signal is produced, the period of which corresponds to the domain widths.

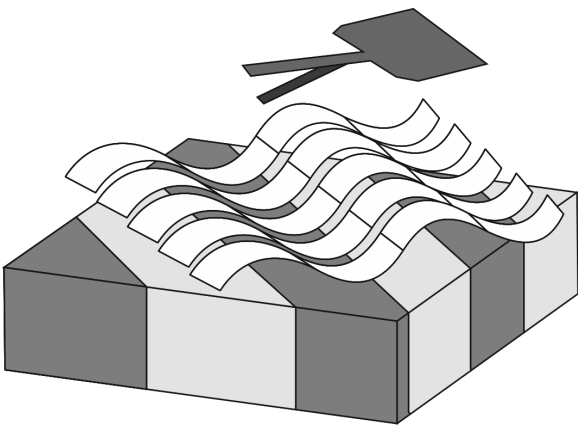


Fig. 1. Schematic picture of stripe domains probed by the magnetized tip. Sinusoid profiles above the surface correspond to phase deviation of the main signal from the driving piezo due to magnetic tip-surface interactions in the presence of stripe domains.

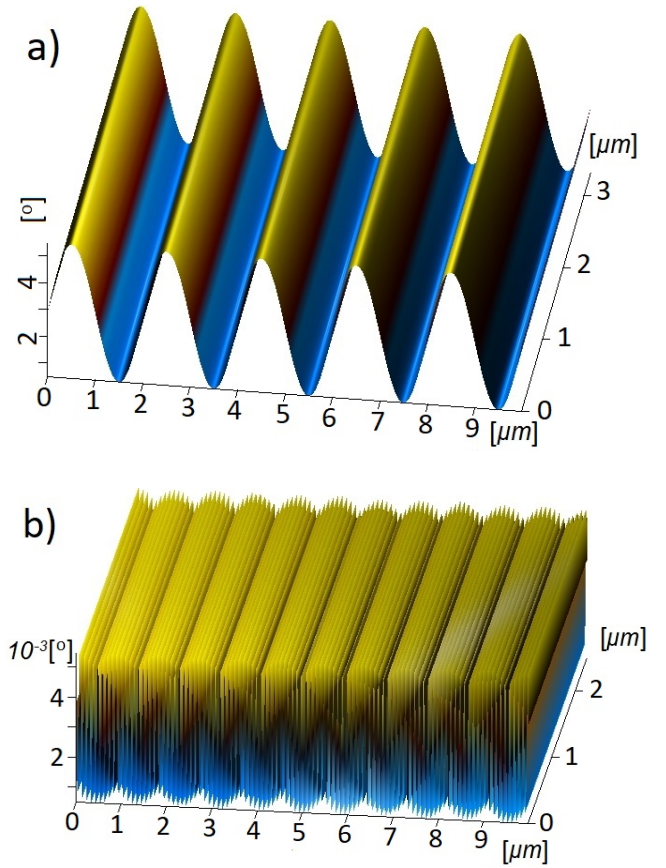


Fig. 2. Simulated maps of phase shifts of pure magnetic signal (a), and a signal with strong background noise (b)

The signal amplitude obviously depends on the tip-sample distance due to varying overlap between the tip coating and magnetic stray field. At small lift heights, however, the tip can be interfered by non-magnetic residual forces (for example: electrostatic, capillary etc.), the effect of which is enhanced by the inevitable horizontal drift of the piezoelectric scanner. Magnetic interactions are usually imaged using the two-pass method that involves completion of the topography profile in the first scan, and then mapping magnetic forces tracking the tip along the recorded surface profile (i.e. maintaining constant tip-surface distance). Such a procedure is believed to separate short-range non-magnetic Van der Waals forces from long-range magnetic forces as if the surface were flat. Unfortunately, unavoidable thermal drift shifts the scanner with respect to the stored profile, which results in increasing contribution of the surface topography to the recorded phase shift. On the other hand, the tip also suffers from thermal noise so that at larger lift heights main magnetic signal vanishes being replaced by its white-noise counterpart.

Obtaining an exact equation that describes observed phase shifts of the MFM signal as a function of the lift height including the effects of thermal noise, drift of the piezo driver, quality factor and others, is a very difficult task, and therefore we propose a simple model. Assuming that the material is made of parallel stripe domains, in-plane components of the stray magnetic field are governed by sinus-like functions, whereas the normal compo-

ment vanishes exponentially. This periodic signal corresponds to a fundamental domain period, which is additionally modulated by a high-frequency component corresponding to the thermal noise (initially set at 2 per cent of the amplitude of the main signal). According to Kong et al. [11], gradient of the stray magnetic field influences the phase shift between driven and actual tip oscillations. Maps and profiles of these two components are shown in Fig. 2. It was noted that the topographic contribution to the MFM signal, which is seen at small lift heights, is not included in the presented model.

#### 4. Results and discussion

In our previous paper [7], results of fractal analysis of the magnetic domains recorded at various lift heights (i.e. the tip-sample distances) were reported. Fig. 3 schematically depicts most important results presented therein.

MFM images were recorded using MESP probe (Bruker) with the tip curvature radius about 35 nm, and the coercivity 400 Oe. The scan length was 20 m2 with 256 equidistant samples along each scan axis. The wear resistant martensitic steel (XAR400) was cut off and taken as a sample.

As seen in Fig. 3a, the plot of fractal dimension vs. lift height is a curve bent downward, approaching its minimum value  $D_{\min}$  at the lift height  $h_0$ . What is even more surprising, the profile fractal dimension falls at  $h_0 = 350$  nm below one ( $D_{\min} = 0,966$ ) despite the fact that it should remain somewhere between  $1 < D < 2$  according to previous statements [10]. Remaining curves behave in a quite different manner since none of them exhibits any extreme. Fig. 3b shows that the plot of the pseudo-topothesy exponentially decreases with increasing lift height, whereas Fig. 3c actually exhibits the opposite trend of the correlation length.

Having experimental data, Fig. 4 presents results of the fractal analysis of simulated MFM signal plotted as a function of the lift height. In order to find out whether or not the lift height itself affects the estimation process, a comparison between noisy and noise-free signals is made. Fig. 4a shows that the fractal dimension computed from a noisy signal exponentially increases with increasing lift height from  $D_{\min} = D(h_0)$  up to around 1.53, which establishes asymptotic limit of the fractal dimension for a pure pseudo-noise. This agrees very well with experimental data shown schematically in the right-hand side in Fig. 3a, that is in the range  $h > h_0$ . Without the noise, however, the signal remains constant at  $D_{\min}$  approximately equal to 0.93, which means that changes in  $D$  observed previously are indeed caused by increasing contribution of the pseudo-noise to the overall MFM signal. Similar behavior can be observed looking at the plot of the correlation length. Open dots in Fig. 4c corresponding to a noisy signal follow a curve that exponentially increases with increasing lift height in agreement with data in Fig. 3c. On the other hand, open triangles corresponding to a noise-free signal form an almost flat line, exhibiting significant influence of the pseudo-noise on final estimations.

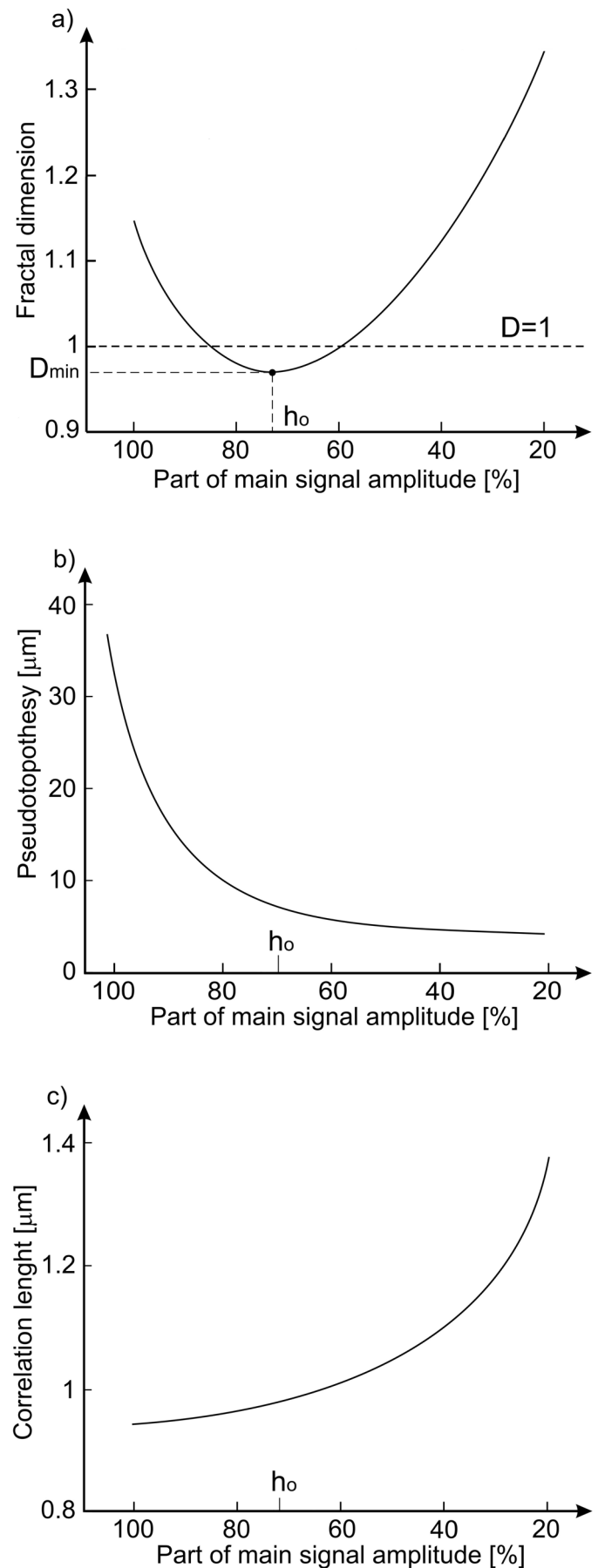


Fig. 3. Schematic plots of changes in fractal parameters, respectively: profile fractal dimension, pseudo-topothesy, and correlation length, versus lift heights (source data published in [7])

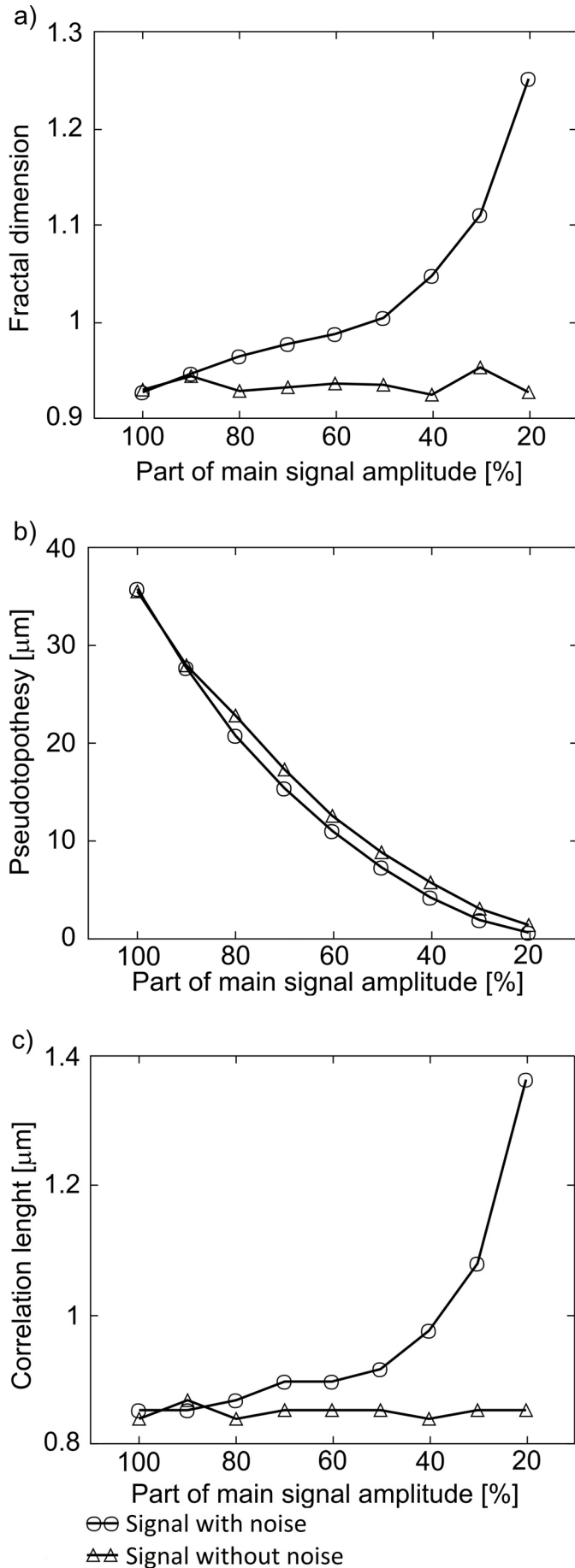


Fig. 4. Changes in fractal parameters of the simulated MFM signal as a function of the lift height of the scanning probe

Unfortunately, calculations of the pseudo-topothesy seen in Fig. 3b confirm these findings only in part. As before, results for the noisy signal are in accordance with experimental data in Fig. 3b, that is they asymptotically decrease upon increasing lift height. However, pseudo-topothesy of a noisy signal is actually identical to that of a noise-free signal. According to Eq. (5), the topothesy is a scaling factor that defines the intercept in a log-log plot of the structure function vs. separation length. Then, it should characterize the variability in data series in a way that neither depends on the signal magnification nor describes any spatial organization. Hence, simulated MFM signals composed of several sinusoids of different periods cannot be separated using the topothesy itself.

According to the statements given in the preceding paragraph, despite the fact that the model does not cover changes in the MFM signal occurring at small lift heights ( $h < h_0$ ), the top oscillations in this range are likely to deviate due to non-magnetic short-range forces such as Van der Waals ones. However, these interactions gradually become weaker with the lift height increasing up to  $h_0$ , where it evens with a bunch of long-range forces (magnetic, capillary etc.) Further increase in the lift height results in raising contribution of the thermal noise.

Results discussed above allow us to propose a simple and efficient procedure for optimizing lift height settings in the MFM measurements, which relies on recording MFM maps at several different lift heights  $h_1 < h_2 < \dots < h_k$ , followed by an estimation of their fractal dimension  $D(h)$  using the autocorrelation method. Note, however, that various relations between obtained pairs  $(h_k, D_k)$  are possible, hence approximation of  $h_0$  requires further numerical processing. Simplest algorithm assumes parabolic dependence of the fractal dimension  $D$  on the lift height  $h$ :

$$D(h) = a + b \cdot h + c \cdot h^2 \quad (6)$$

where:  $a$ ,  $b$ , and  $c$  are unknown coefficients of the least-squares parabola. To find these coefficients, only three MFM measurements at three different lift heights ( $h_1 < h_2 < h_3$ ) are required, which ends up in three fractal dimensions  $D_1$ ,  $D_2$ , and  $D_3$ , respectively. Coefficients of the best fitting curve  $D(h)$  can be obtained by solving a set of linear equations in a matrix form:

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} 1 & h_1 & h_1^2 \\ 1 & h_2 & h_2^2 \\ 1 & h_3 & h_3^2 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} \quad (7)$$

Providing that given lift heights are different ( $h_1 \neq h_2 \neq h_3$ ), this system has unique solution. Optimum lift height  $h_0$  corresponds to the x-coordinate of the vertex, hence it is given by the equation:

$$h_0 = -\frac{b}{2c} = -\frac{\det \begin{bmatrix} 1 & D_1 & h_1^2 \\ 1 & D_2 & h_2^2 \\ 1 & D_3 & h_3^2 \end{bmatrix}}{2 \cdot \det \begin{bmatrix} 1 & h_1 & D_1 \\ 1 & h_2 & D_2 \\ 1 & h_3 & D_3 \end{bmatrix}} \quad (8)$$

In such a case, optimum lift height can be expressed explicitly as follows:

$$h_0 = \frac{D_1(h_3^2 - h_2^2) + D_2(h_1^2 - h_3^2) + D_3(h_2^2 - h_1^2)}{2 \cdot [D_1(h_3 - h_2) + D_2(h_1 - h_3) + D_3(h_2 - h_1)]} \quad (9)$$

Obtained result allows us to scan the MFM signal at a height, which is a compromise between inevitable drift of the scanner and thermal noise.

## 5. Conclusions

Summarizing, the main point of this paper concerns the problem of estimation of optimal lift height settings in a two-pass MFM scanning mode. Presented simulation results agree well with experimental data published elsewhere [7] that confirms the usability of proposed rough model of the MFM signal.

Performed simulations proved that probing noise-free magnetic signal at various lift heights neither affects the fractal dimension nor correlation length, but exerts strong influence on the pseudo-topothesy. Observed decay in the latter upon increasing lift height casts some light into vanishing long-range magnetic interaction between the tip of a scanning probe and magnetic stray field. On the other hand, fractal analysis of noisy signals exhibited quasi-parabolic dependence of the fractal dimension, and monotonic although opposite trends in the pseudo-topothesy and correlation length as a function of the lift height.

Observed minimum fractal dimension, which temporarily fell below 1, that is below lower limit for a profile, drawn our special attention. Most likely explanation for that laid in unaware peculiarity of the method used for estimation of the fractal dimension, though, numerical accuracy appeared to have no influence on the procedure of estimation of optimal lift height proposed in this paper.

Further investigation will be driven towards expansion of the presented model of the MFM signal into the effect of short-range forces and explain observed changes in fractal properties covering the whole range of possible lift heights.

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