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EDITORIAL

SPECIAL SECTION

Fractional Signals and Systems

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1. Introduction

The special section in the current volume of the Bulletin of the Polish Academy of Sciences, entitled “Fractional Signals and Systems”, includes selected papers from the FSS17 International Conference, which was held in Łódź, Poland on October 9–11, 2017. The founder of the conference is Manuel Duarte Ortigueira from the New University of Lisbon, Portugal. The FSS17 is yet another in a series of conferences, which had previously taken place in:

1. Caparica, Portugal, 2009
2. Coimbra, Portugal, 2011
3. Ghent, Belgium, 2013
4. Cluj-Napoca, Romania, 2015.

The FSS17 conference addressed a broad spectrum of the Fractional Calculus (FC) applications in technical sciences. Main topics included the fractional-order continuous-, and discrete-time linear or non-linear fractional-order control, dynamic system identification via fractional models, fractional order filtering, as well as image processing using fractional methods. The conference’s main organizers included the Institute of Applied Computer Science (Instytut Informatyki Stosowanej Politechniki Łódzkiej), the Lodz University of Technology (Politechnika Łódzka) and the Polish Information Processing Society – Łódź Branch (Polskie Towarzystwo Informatyczne – Oddział Łódzki).

2. Fractional calculus

The FC is a generalization of the conventional calculus that emerged was raised in 1695 from an idea by Leibniz, to be found first in an exchange of letters between him and Bernoulli [1]. Although not as accessible as the standard calculus, the FC leads to similar concepts and tools, but it enjoys wider generality and applicability. Because it allows integral/derivative operations of arbitrary order (real or complex one), it consists of an upgrade similar to the generalization from the integer to real or complex numbers. In the almost 200 years that have passed since Liouville had published his works [2], the fractional derivative was considered a curious, interesting yet abstract mathematical concept. The main developments of the FC were accomplished by mathematicians without finding any real world applications. During this period several definitions of derivative and integral operators were formulated, not necessarily compatible in the sense of giving of always yielding the same results, which created difficulties when trying to extend specific tools based on the traditional integer order to the more general arbitrary order context. Since early 1990s, scientists and engineers, with the perspective of practical applications in mind, have been working with those different forms and obtaining novel interesting results [3]. However, this progress does not exclude the need for converging on formalisms [4]. We must remark that these developments were reached in an analogical domain. The first discrete formulations appeared as a result of numerical approximations to continuous variables [5, 6]. Recently, several advances in fractional discrete-time techniques were proposed [7–11]. The above-mentioned referred fractional operators are left or right, which can be considered causal or anti-causal when the independent variable is time. However, this is not mandatory, as Riesz showed when he proposed what is usually considered as “Riesz potential”, i.e. a two-sided operator [12]. The same is applicable to happens with the Riesz-Feller potential. Versions of the two-sided derivatives based on fractional incremental ratia were proposed in [13, 14]. These are equivalent to the Riesz and Riesz-Feller potentials.

When speaking in signals and systems, we are considering a very important set of tools responsible for many of the realizations of our modern life, such as in the areas of communication, bio-medicine and biology, electrical and mechanical engineering, economy and finance, and many others. We are considering analysis, modelling, and, very importantly, synthesis of systems. There are many integer order tools that need to be extended to the fractional framework while maintaining, but keeping a backward compatibility. In the set of operators we include the impulse, step and frequency responses. Apparently, not all proposed formulations for fractional derivatives and integrals are suitable for doing this correctly.

2.1. Fractional calculus – short historical review. In 1695, Gottfried Leibniz raised the question about generalizing the orders of derivatives and integrals to non-integer values. In the four centuries that followed many important mathematicians contributed to the theoretical development of FC (see Fig. 1 for a time line) for the time line of FC. In most recent decades practical implementations emerged and FC is now recognized as an important tool for describing phenomena that the integer-order calculus overlooks. The evolution of FC under the light of indices such as the number of books published is an assertive measure of scientific and technological progress. The data collected up to the year 2016 are represented in Fig. 2.

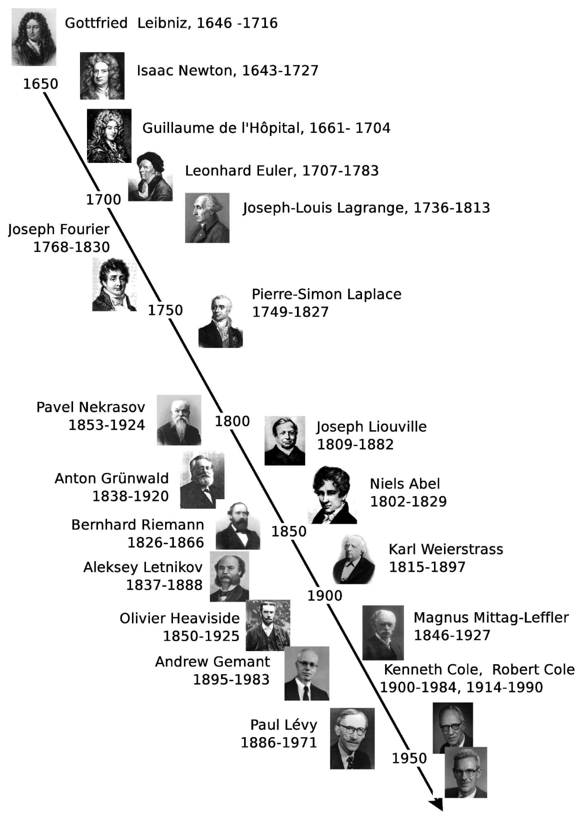

Fig. 1. FC timeline (1650–2016)

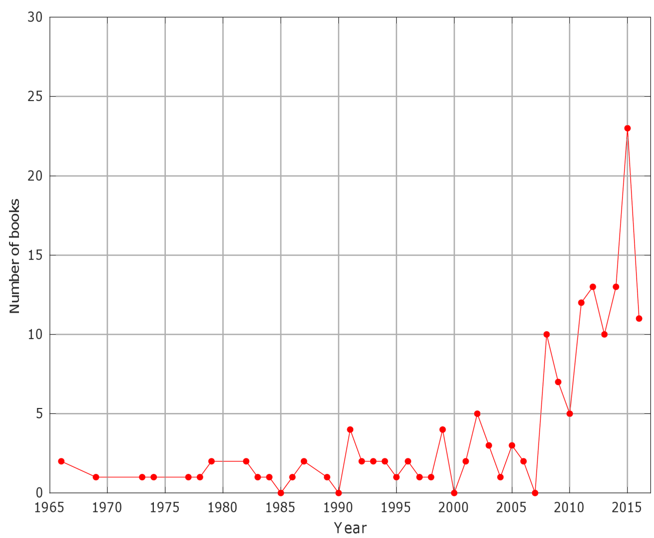

Fig. 2. Number of books per authors in 1965–2016

3. Fractional signals

The presence of fractional behavior in nature and in many man-made systems is unquestionable. Many signals caught when observing such systems have spectra that exhibit increasing/decreasing slopes in Bode diagrams that are not multiples of 20 dB per decade. This can be found, for example, in ECG, speech, electronic noise in junctions, network traffic, and others [15]. Designations such as $1/f$ noise, long range dependence, fractional Gaussian noise and fractional Brownian motion (fBm) appear many times in scientific literature. Fractional Gaussian noise is the output of a causal Liouville derivative [2,4] excited by Gaussian white noise. The integral of this noise is the fBm [16, 17].

A digital image can be treated as a two-dimensional discrete-space signal. In the processing of such signals we can also apply FC, for image edge detection and filtering. Here we apply the fractional-order differences [11]. In the edge detection the masks are more sophisticated then when using the classical approach [18, 19].

4. Fractional systems

There is a belief in the literature that fractional calculus is adequate only in a description of dynamical systems with so-called "memory". Here we can mention electricity (with electrical circuits with supercapacitors and inductive phenomena based on the skin effect), mechanics (with relaxation-oscillation problems and viscoelasticity), thermal engineering (with heat transfer phenomena), fluid dynamics (with anomalous diffusion), biology, etc. Similarity between different types of dynamical systems is mentioned in Table 1.

Table 1
Physical phenomena equivalence

System type	Flow variable	Effort variable	Compliance	Inductance	Resistance
Mechanical	velocity	force	spring factor	mass	damper factor
Electrical	current	voltage	capacitance	inductance	resistance
Thermal	heat flow rate	temperature change	thermal capacitance	thermal inductance	heat conduction
Fluid	volume flow rate	pressure	tank	mass	valve

The Table above permits to state that the known systems with a "memory" have their equivalence in other types of systems. Moreover, all dynamical systems that are considered classical can also be described by the fractional-order linear or non-linear differential equations. Such equations take into account hidden physical phenomena such as inductive and capacitive couplings between the electrical circuit elements and friction in mechanical ones. At this point, let us foretell that in the future fractional calculus is highly likely to supersede the so-called classical one with only integer derivatives and multiple integrals. The latter will be treated as an approximation or a special case of fractional calculus.

4.1. Continuous-time fractional systems. All real dynamical plants in closed-loop control systems are continuous-time. Classical approach to all control systems shall encompass:

1. Identification of the plant (mathematical modeling) by the fractional-order linear or non-linear differential equations, and fractional-order state space models.
2. Analysis of the closed-loop system (stability, observability, reachability, controllability, system response, frequency characteristics).
3. Control supported by the fractional calculus algorithms, PID control, robust control, CRONE control.

4.2. Discrete-time fractional systems. Long-term global trends in control theory and general practise point to the huge dominance of discrete-time systems. Discrete-time fractional systems exist independently of their continuous counterpart and are generalizations of the classical approach represented by difference equations [9]. However, they acquire relevance in approximating continuous-time systems for digital implementations. In fact, their applications in control strategies realization and continuous signal measurement and acquisition become easier. Yet, similarly to the continuous-time closed-loop control system mentioned above, problems arise in a discrete-time case. To quote but a few here:

1. Approximation of the fractional-order derivative by means of the fractional-order difference with a finite sampling period.
2. Limited accuracy of microprocessor calculations, liquidated made partly obsolete by sophisticated calculation algorithms.
3. The so-called "finite calculation tail", caused by the finite sampling period of the discrete-time system.

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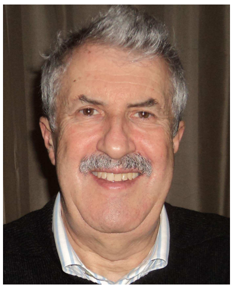

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