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Applying a fractional coil model for power system ferroresonance analysis

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Abstract. This paper addresses the problem of modeling the nonlinear coil used for ferroresonant circuit analysis. The effect of ferroresonance is described and a general modeling approach is presented. The hysteresis modeling problem is also shortly discussed, on the example of a ferromagnetic coil. A brief overview of available literature and contributors to this area are provided. A series RLC circuit supplied from an AC source is discussed. The application of the fractional derivative in the modeling of an iron core coil is presented and suggestions of model implementations are given. The computations performed are illustrated by means of waveform data obtained from computer simulations and compared with those obtained from measurements performed in a specially prepared laboratory setup.

Key words: ferroresonant circuit, fractional derivative, ferromagnetic core coil, hysteresis, modeling, measurement verification, nonlinear dynamics, power system.

1. Introduction

Reliable computations and simulations of a power system under failure-free and failure conditions depend directly on the credibility of the model against the real system. The variety of potential events in the power system requires flexibility from the models the variability of its structure and parameters is concerned. One of the methods for determining the abovementioned structures and parameters is their estimation based on a recorded steady state and dynamic waveforms of electrical quantities, especially during routine measurements, for instance, when launching or performing maintenance tests [1].

Determination of the mathematical model parameters is done by means of an approximation of the recorded measurement waveforms through equations defining the model.

In the estimation process, an error is computed (e.g. mean-square error as in the study presented herein) between the measured (approximated) and computed (approximating) waveforms. Minimization of the objective function formulated in such a way is a nonlinear problem which can be solved by numerical methods [2, 3].

One of the problems that require reliable models of a real object and careful analyses along with multivariate simulations is the phenomenon of ferroresonance [4].

The term "ferroresonance" describes a wide variety of interactions taking place between capacitors and iron-core inductors and resulting in high overvoltages or overcurrents. This term also refers to an oscillating phenomenon occurring in an electric circuit containing at least a voltage source, a nonlinear inductance, a capacitance and a damping element of some sort. In a circuit consisting of these elements, any change in the parameters of any electrical component can lead to sudden current and voltage changes at the terminals of other components [5, 6].

Ferroresonance is a phenomenon which particularly affects power networks. Modern power systems contain multiple capacitances and nonlinear inductances. Operating conditions vary widely and this results in an almost endless amount of possible scenarios where ferroresonance may emerge [7].

Ferroresonance generally occurs during system imbalance, usually during switching events where capacitances are connected in series or in parallel with a transformer's magnetizing inductance. This is possible for any transformer core configuration. The capacitive effect may be due to underground cable or transmission line capacitances, capacitor banks, coupling capacitances between double circuit lines or even voltage grading capacitors in HV circuit breakers [6, 7].

Ferroresonance can cause electrical equipment damage (failures in transformers, cables, and arresters) and severe power quality problems. Ferroresonant overvoltage, overcurrent and the distortion level in waveforms all depend on many aspects such as individual circuit sensitivity to parameter values and initial conditions [8]. Values such as residual remanence in the transformer core, initial capacitor charge or source phase shift can affect initiation of the phenomenon and will influence the final response. In addition, the nonlinear inductance characteristic, which is due to the hysteresis curve of the transformer, greatly affects ferroresonance overvoltage.

Due to the potential cost of damage caused by this phenomenon, issues related to it need attention and support in the form of simulation analysis and improvement of tools in this area [5, 6].

Over the years, the phenomenon of ferroresonance has been widely researched. The main areas of interest have focused on improving methods of analysis and prediction as well as iron core coil models. Still, a vast amount of work

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is needed in order to improve the understanding of ferroresonance.

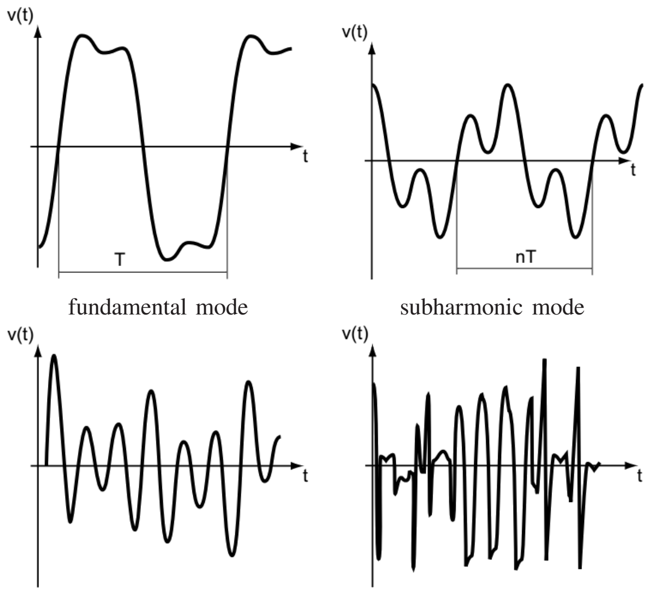
The objectives of this paper are as follows:

- investigation of steady state and dynamic circuit behavior under ferroresonance conditions and exploration of possibilities arising from the application of new mathematical and numerical tools in the description of the nonlinear core coil,
- application of waveforms obtained from measurements for validation of the iron core coil model applied,
- development of a fractional derivative model used in modeling of the hysteresis effect and its consequences in ferroresonance analysis,
- simulations of a ferroresonance circuit (including the fractional coil model) and verification of obtained results against measurement data.

2. State of the art in ferroresonance analyses

2.1. Ferroresonant modes. To prevent the consequences of ferroresonance, it is necessary to understand the phenomenon and to be able to predict and identify it. The correct classification of ferroresonance is needed to choose the proper mathematical approach. Basing on a literature survey, one can distinguish four ferroresonant modes [4, 9, 10]:

- fundamental mode, where voltages and currents are periodic (with a period T, equal to the source voltage waveform period) and can contain a varying number of harmonics,
- subharmonic mode, where signals are periodic with a period *nT*, which is a multiple of the source voltage period; this state is also known as subharmonic n or harmonic $1/n$, and its harmonic content is normally of odd order,
- quasi-periodic mode (also called pseudo-periodic), where signals are not periodic. The harmonic spectrum is discontinuous,
- chaotic mode, where the signals display irregular and unpredictable behavior.



fundamental mode | subharmonic mode

quasi-periodic mode | chaotic mode

Fig. 1. Graphical representation of ferroresonant modes [4]

Figure 1 includes a graphical representation of the above-mentioned ferroresonant modes.

2.2. Analysis tools. Ferroresonance as a phenomenon is so complex that it cannot be analyzed or predicted by means of computation methods based on linear approximation. Because of nonlinearities and hysteresis, a ferroresonant circuit is usually analyzed using time-domain methods. Typically, a numerical integration method is used [6, 11–13].

Ferroresonant behavior can be identified by means of many techniques, ranging from traditional fast Fourier transform (obtaining the frequency spectrum) to modern nonlinear theory techniques.

Some sophisticated analyses have been performed with the use of Poincaré maps and bifurcation theory [6, 14] for the classification and prediction of ferroresonance. As for simulation tools, one can mention e.g. EMTP-based analysis [5, 6] and the harmonic balance approach [15].

Continuously performed studies and a wide range of methods and tools already developed seem to only confirm that the ferroresonance phenomenon is diverse and interesting.

The structure and parameters of the model, as a simplified mathematical reflection of the real and complex phenomenon, should allow for updates based on measurements that are relatively simple to perform. This work focuses on this issue.

3. Ferroresonance circuit approach

The ideal starting point for ferroresonance analysis is the discussion of a simple circuit (see Fig. 2a) where ferroresonance can appear. The basic circuit elements required for the ferroresonance effect to occur are: a voltage source, nonlinear inductance, some capacitance and a damping element. Moreover, the RLC circuit structure presented herein may later be adapted to many problems appearing when modeling a power system (Fig. 2b, where an RLC circuit constitutes a fragment of a simplified model of a power system) [7].

A ferroresonant circuit, composed of a linear resistor, capacitor and nonlinear coil, has been set up in a laboratory (this is discussed in detail in Sec. 6).

The most characteristic features of the ferroresonant circuit are those related to a given frequency [4]:

- resonance is possible over a wide range of capacitance values (C),
- frequency of the voltage and current waveforms may be different than that of the sinusoidal voltage source,
- it is possible to have several stable steady state solutions for a particular set of conditions, depending on the initial conditions of charge and flux (in capacitive and inductive elements, respectively),
- jump phenomenon can occur where the circuit moves from one stable state to another as a result of changes in parameter values.

a)

b)

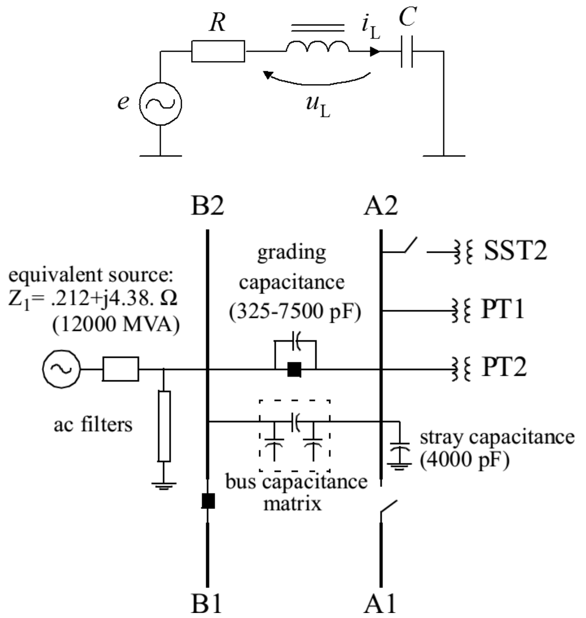


Fig. 2. Concept circuit with ferromagnetic coil (a) and an exemplary power system model circuit with ferroresonance situation (b) [7]

Circuit analysis consists in the observations of changes in the response of the circuit caused by an increase in the source voltage. Figure 3 presents the source voltage and coil voltage responses in a steady state and during ferroresonance (transient state). The results were obtained using a sampling step $\Delta t = 0.1$ µs. Measurements for the analysis are discussed further in Sec. 6.

a)

b)

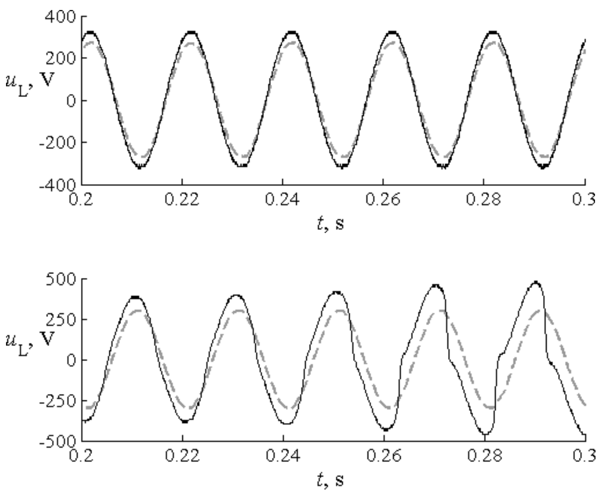


Fig. 3. Measured waveforms of the coil voltage (solid lines) and source voltage (dotted lines) in a steady state (a) and when ferroresonance occurs (b)

4. Modeling the nonlinear coil

The ability to simulate ferroresonance depends primarily on the accuracy of the iron core coil model used in computer simulation.

Modeling the magnetic core of the coil for ferroresonance analysis requires:

- selection of a frequency range for the transients,
- implementation of an adequate representation of losses,
- minimization of the number of components in order to minimize probability of insufficient or inaccurate modeling, and to reduce simulation time,
- idealization of some components if the simulated system is too complex, since such simplification will facilitate computer simulation [16].

This paper deals with the modeling of the magnetic core of a coil that may involve slow-front transients. Therefore, the transformer's iron-core and any other ferromagnetic nonlinear inductances are modeled in a standard manner. In EMTP (electromagnetic transient program) typical models for such analyses suffer from low accuracy. Advanced models require detailed measurements for the estimation of their parameters, while manufacturer data sheets and basic measurements (e.g. during routine maintenance checks) usually constitute the only source of data. An accurate representation of the losses (which are nonlinear and frequency dependent) is also a required feature of an appropriately accurate model.

The problem of modeling a coil with a ferromagnetic core is related to an accurate resemblance of behavior which results from a number of complex phenomena such as hysteresis and eddy currents losses [17].

Hysteresis is a special type of dynamic nonlinearity. It can be found as a research topic in both science and engineering. This is mainly due to the fact that in most applications hysteresis effects have a harmful or at least negative impact on device performance, therefore they should be appropriately included. Mathematicians, physicists and engineers have all proposed numerous models to describe hysteresis phenomena.

Some well-known hysteresis models (four of which originate from the magnetic hysteresis modeling problem) are [18–20]:

- the Jiles-Atherton model [21],
- the Preisach model [21, 22],
- the Chua model [12],
- the Coleman-Hodgdon model [23],
- the Bouc-Wen model [24] (this model was originally used in vibration mechanics).

Different models are often restricted to particular applications. They also must be investigated keeping in mind their potential use (availability) as well as their complexity and difficulty of implementation [21]. Some apparently accurate models may prove inadequate when used in circuit simulation because of discontinuities which can cause convergence problems and therefore, alter the simulation results. For many known models used in circuit simulators, it is difficult to identify the correct correspondence between the parameters of a real device and model parameters [25].

The author also aims to investigate simpler models for ferromagnetic coils at this point. Two models are studied in this paper and presented in the section that follows.

5. Ferromagnetic coil fractional model

A frequently applied conventional model of the ferromagnetic coil [26, 27] is shown in Fig. 4.

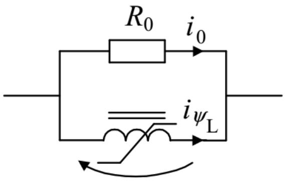


Fig. 4. Classical (conventional) model of the ferromagnetic coil

The model does not contain additional capacitances as it is applied for circuits with supply voltages with a dominant 50–60 Hz frequency [7, 28].

It has been shown in some initial studies [29–31] that up to a certain saturation of the hysteresis the model can accurately resemble the behavior of a nonlinear coil. In a study performed in parallel to the one presented herein [32], this analysis has been complemented by trials aiming at resembling hysteresis but for different supply voltage levels and for greater saturation. The results for the model (for smaller and greater saturation of the hysteresis) have been presented in Fig. 5.

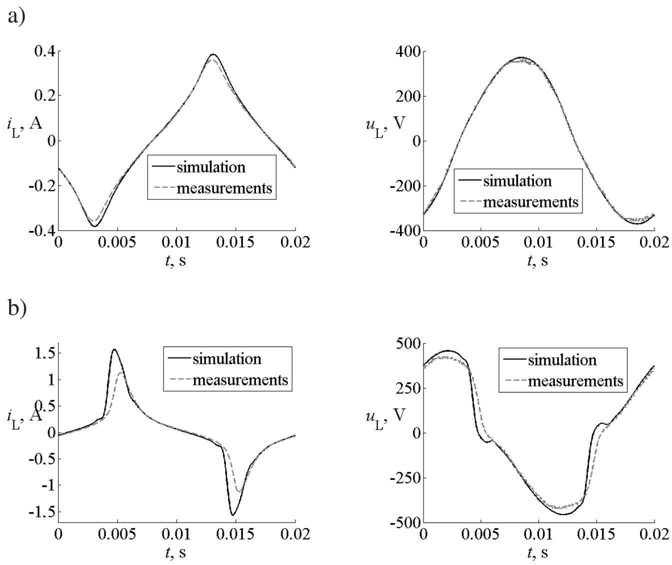


Fig. 5. Waveforms of coil current and voltage for the conventional coil model: small iron core saturation (a) and large iron core saturation (b)

Better accuracy can be obtained by applying a simple model based on the fractional derivative, which was first introduced in the previously mentioned parallel study [32]. This has been presented in Fig. 6. Fractional derivatives have many applications including circuit theory [28, 33, 34, 35] but also e.g. control theory, due to fractional PID controllers [36], heat conduction analyses [37] and continuum mechanics [38].

Figure 7 presents the symbol applied for the nonlinear, fractional coil.

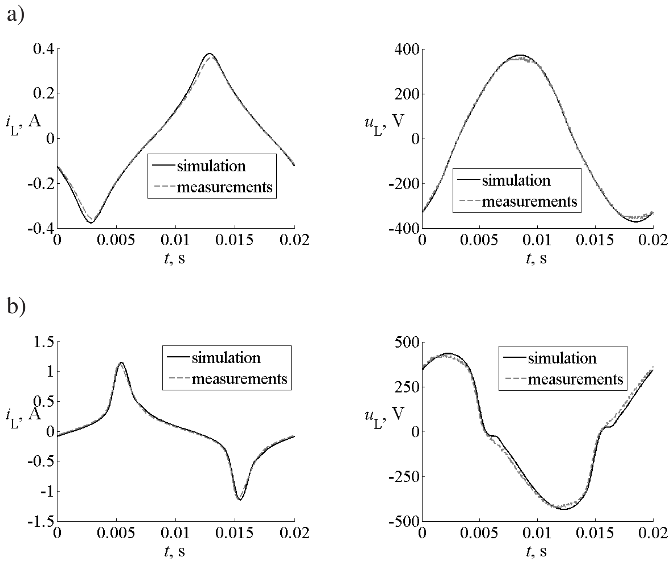


Fig. 6. Waveforms of coil current and voltage for the fractional coil model: small iron core saturation (a) and large iron core saturation (b)

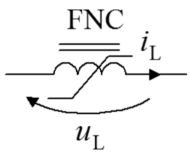


Fig. 7. The symbol applied for FNC (fractional, nonlinear coil)

The definition of the fractional derivative that has been applied in the analysis is that of Caputo [39] for the order:

$$ {}_{t_0}^{C}D_t^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \int_{\alpha}^{t} \frac{f^{(1)}(\tau)}{(t-\tau)^{\alpha}} \mathrm{d}\tau \tag{1} $$

with $\Gamma(\cdot)$ being the gamma function:

$$ \Gamma(x) = \int_{0}^{\infty} e^{-t} t^{x-1} \mathrm{d}t. \tag{2} $$

The fractional differential equation describing the element is as follows:

$$ {}_{t_0}^{C}D_t^{\alpha} \Psi_L = u_L. \tag{3} $$

The dependency between Ψ and i_L is given by a nonlinear function $\Psi(i_L)$ defined through (i_L, Ψ) value pairs.

The variable Ψ (when $\alpha \neq 1$) is an artificial variable with the unit of $\mathrm{Wb} \cdot \mathrm{s}^{\alpha-1}$.

6. Test circuit and measurements performed

The measurements have been performed in a specially designed laboratory setup (Fig. 8a). This allows to observe the response in a circuit with a saturable inductor. The elements of the circuit have been deliberately adjusted to assure that one can observe the ferroresonance phenomenon for high enough supply voltage. The voltage and current waveforms on all circuit components were recorded for selected levels of the supply voltage. During the measurements, the coil operated in saturation conditions of the magnetic core.

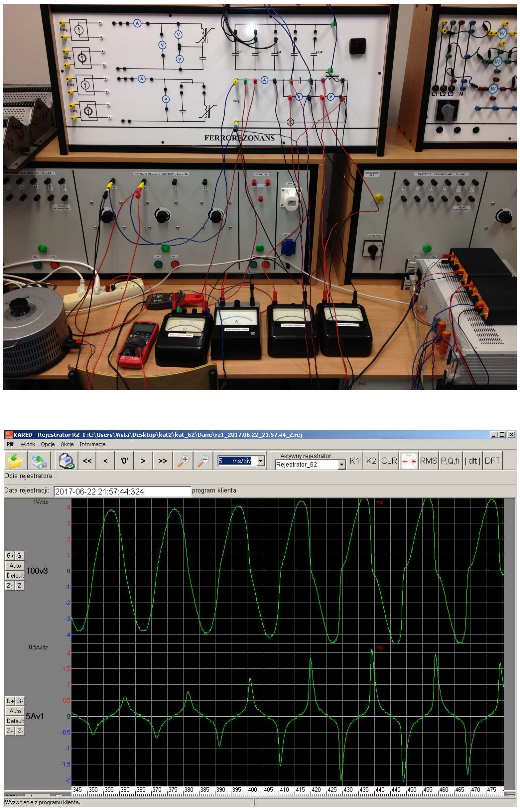
a)
b)


Fig. 8. Ferroresonance test circuit measurements: laboratory set-up (a), screenshot of exemplary measurement records (b)

Digital interference and event recorder RZ1 (designed by Kared [40]) were used for the collection of measurements (Fig. 8b).

Figure 9 presents waveforms of the current through the circuit and the coil voltage collected for high enough supply voltage, which resulted in ferroresonance.

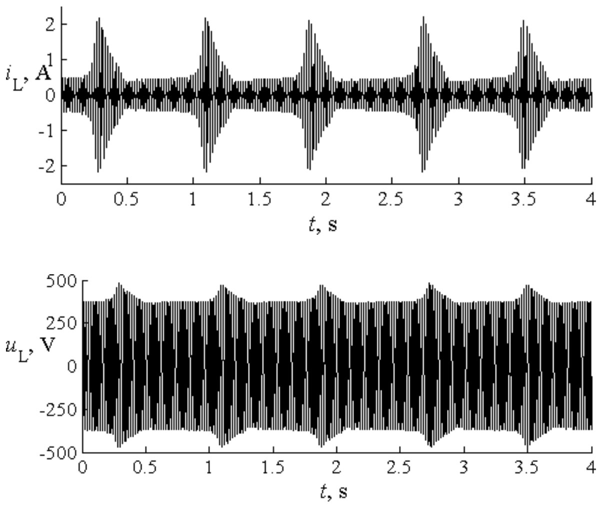
a)
b)


Fig. 9. Exemplary waveforms of current and voltage of the nonlinear coil

From both waveforms one can classify the phenomenon as subharmonic ferroresonance (in reference to the modes mentioned in Sec. 2). The period in which the phenomenon occurs is much greater than the base frequency of the source voltage ($n \approx 38$).

7. Computation results

The computations required for the analysis have been performed at Matlab [41]. They consist of two stages:

- the estimation part, where the parameters of the circuit elements are obtained,
- the transient circuit analysis part, where a time dependent problem is solved.

The estimation part has taken into account a set of steady state measurements (each performed for a different level of the supply voltage). Capacitance C has been estimated by applying linear least squares regression.

The parameter α and the (i_L, Ψ) value pairs for the nonlinear coil have been obtained by means of a trust region dogleg algorithm [42] (which is implemented at Matlab). The hysteresis obtained for the measurement set with the greatest supply voltage is depicted in Fig. 10 along with the results that can be obtained for the model introduced in Sec. 5.

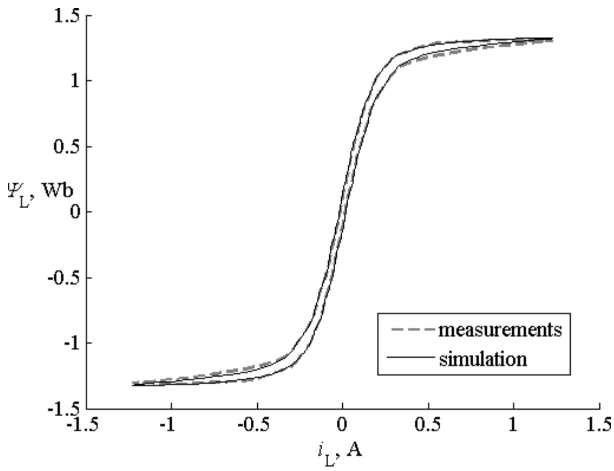


Fig. 10. Hysteresis modeling results for the fractional model

Resistance R has been handled differently than the parameters for the coil and capacitor, because of it taking a certain range of values depending on the level of the supply voltage and (hence) the current through the circuit. For the obtained measurement samples the $u - i$ dependency is depicted in Fig. 11. The variability has been no doubt caused by temperature dependency of the R parameter.

The varying resistance values have been recorded and kept for the second part of the computations. The other parameters of the circuit (concerning the capacitor and nonlinear coil) have been assumed as constant.

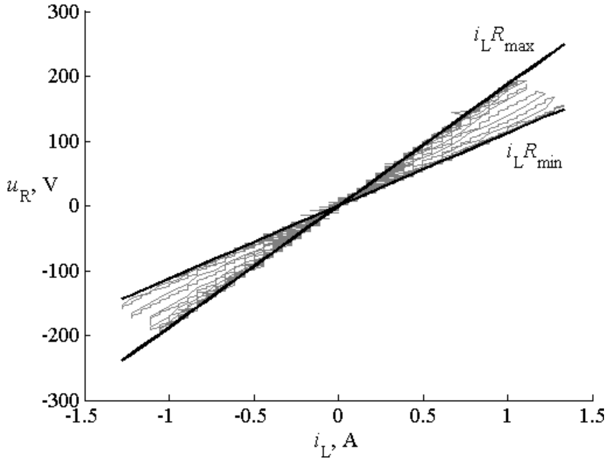


Fig. 11. Current-voltage dependency of resistive element along with representations of $i_L \cdot R_{\min}$ and $i_L \cdot R_{\max}$

The transient circuit simulation aimed to recreate the phenomena occurring in the real circuit. The analyzed circuit is depicted in Fig. 12. The time dependent problem was made up from the ordinary differential equation for the capacitor, the fractional differential equation for the coil (3) and the nonlinear equation:

$$\Psi = \Psi(i_L) \tag{4}$$

with the equation emerging from Kirchhoff's voltage law.

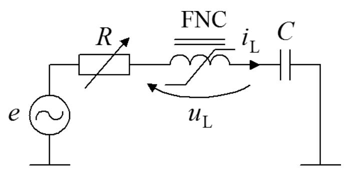


Fig. 12. Considered ferroresonance circuit

In the time stepping solver, zero initial conditions have been assumed. In order to handle the differential equation for the capacitor, an implicit method using BDF (backward differentiation formulae) has been applied. To deal with the fractional time derivative of the nonlinear coil model, the method known as SubIval [43–45] has been applied. It is implemented in the DLL available in [46], which can be applied at Matlab.

The above-mentioned methods simplified the requirements of the solver in such a way that a system of equations needed to be solved for each time step has been developed.

At the beginning of the simulation process 5 periods have been computed to obtain a steady state for a constant R value. Then the R variability was recreated according to the values recorded from the measurements.

The time functions obtained for the current and coil voltage are presented in Fig. 13 along with their comparison against the measurement results.

The values obtained through the transient solver are close to the measurement data. An assessment of these results' accuracy has been performed according to the following formula:

$$\varepsilon = \sqrt{\sum_{i=1}^{n} (f_{i(\mathrm{m})} - f_{t(\mathrm{s})})^2} \cdot \frac{100\%}{nF_{\max}}, \tag{5}$$

where $f_{i(\mathrm{m})}$, $f_{i(\mathrm{s})}$ – voltage or current value for the i-th time instance, $F_{\max}$ – maximum value of the respective waveform. The index m denotes measured values, while s denoted those obtained in the time dependent simulations.

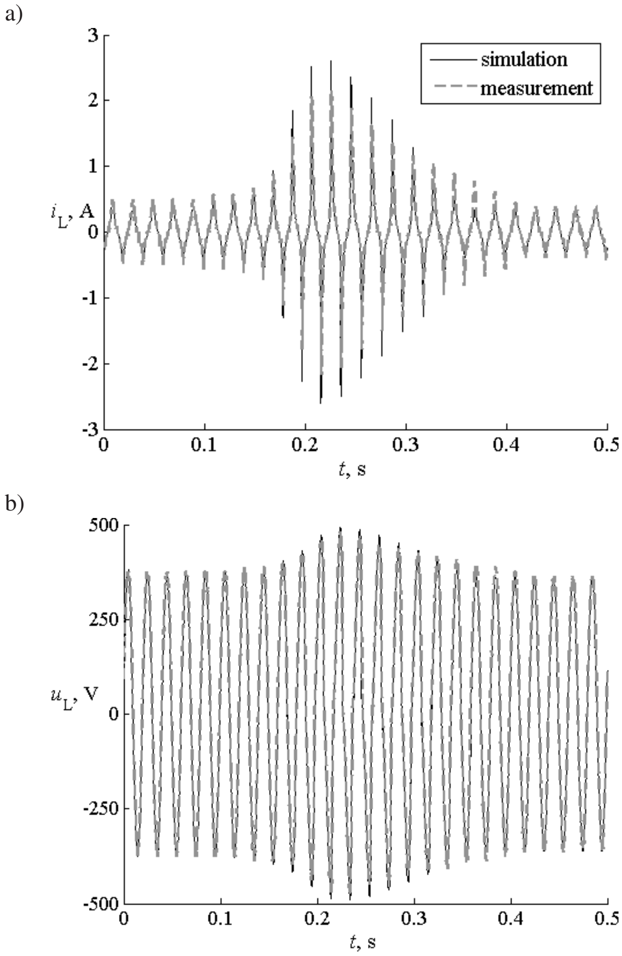


Fig. 13. Comparison between coil current and voltage time functions (obtained from measurements and simulations)

The values for this error definition – for the coil voltage u_L and the current i_L – are given in Table 1. Additionally, the errors are also given for when the integer-order model of the nonlinear coil is applied.

Table 1
Error values illustrating the mismatch between measured and simulated waveforms of the nonlinear coil current and voltage

Model type	Fractional-derivative	Integer-order
Coil voltage u_L waveform error, %	0.30	1.27
Coil current i_L waveform error, %	0.44	1.51

8. Concluding remarks

A study concerning a ferroresonance circuit has been concluded. A simple series circuit has been studied, consisting of voltage supply, resistance, a capacitor and an iron core coil. This simple circuit represents the first step in a larger study, though it can already resemble a fragment of a real configuration in a power system (as discussed in Sec. 3).

The study has relied much on measurements that have been performed in a specially prepared laboratory setup (with the iron core working in saturation conditions).

The model of the iron core coil has been taken into account as a significant part of the study (the modeling of nonlinear coils has been discussed in Secs. 4 and 5), where a model using the Caputo fractional derivative has been introduced.

The parameters of the circuit consisted of the recorded history of changes of resistance R, capacitance C (assumed as constant) and the parameters for the fractional, nonlinear coil model:

- (i_L, Ψ) value pairs,
- α, being the order of the fractional derivative.

The circuit parameters have been obtained through estimation procedures (in Matlab).

After obtaining the parameters, a time-dependent solver applying the backward differentiation formulae (for first order differential equations) and a method known as SubIval [43–45] (a numerical method for fractional differential equations) have both been used for simulations.

The results obtained are promising as the values obtained have been very close to those obtained through measurements. This can be concluded through the observation of waveforms (Fig. 13) and through the error defined by (5).

Future research will be directed at:

- potential improvements to the model of the nonlinear coil using the Caputo fractional derivative (the research will aim to avoid greater complexities in modeling in order not to generate problems for numerical solvers),
- addition of some features to the simulations that have so far been simplified, e.g. the resistance change has been simulated by recreating recorded values, while it would be best if the above-mentioned temperature dependence were found in the solver,
- cases where other ferroresonance modes can be observed. Because the fractional model has proven to give such satisfactory results, these studies can also be supported by theoretical analyses for resonances in fractional circuits [47].

REFERENCES

[1] Ł. Majka and S. Paszek, "Mathematical model parameter estimation of a generating unit operating in the Polish National Power System", *Bull. Pol. Ac.: Tech.* 64(2), 409–416 (2016).
[2] M. Lewandowski, Ł. Majka, and A. Świetlicka, "Effective estimation of angular speed of synchronous generator based on stator voltage measurement", *International Journal of Electrical Power & Energy Systems* 100, 391–399 (2018).
[3] S. Paszek and A. Nocoń, "Parameter polyoptimization of PSS2A power system stabilizers operating in a multi-machine power system including the uncertainty of model parameters", *Applied Mathematics and Computation* 267, 750–757 (2015).
[4] Ph. Ferracci, Ferroresonance, Schneider-electric technical book, 190, Schneider's Group Technical collection, 1998.
[5] IEEE working group on modeling and analysis of systems transients, "Modeling and analysis guidelines for slow transients—part III: the study of ferroresonance", *IEEE Trans. Power Delivery* 15, Issue 1, 255–265 (2000).
[6] J.A. Corea-Araujo, F. Gonzalez-Molina, J.A. Martinez-Velasco, J.A. Barrado-Rodrigo, and L. Guasch-Pesquer, "Tools for characterization and assessment of ferroresonance using 3-D bifurcation diagrams", *IEEE Trans. Power Delivery* 29(6), 2543–2551 (2014).
[7] D.A.N. Jacobson, "Examples of Ferroresonance in a High Voltage Power System", *IEEE, Power Engineering Society General Meeting*, 2003, DOI: 10.1109/PES.2003.1270499.
[8] V. Valverde, G. Buigues, A.J. Mazón, I. Zamora, and I. Albizu, "Ferroresonant Configurations in Power Systems", Renewable Energies and Power Quality (ICREPQ'12), 1 (10), International Conference on, Santiago de Compostela, Spain, 2012, DOI: 10.24084/repqj10.351 474 RE&PQJ.
[9] Zhao-ming Lei, Zuo-jun Liu, He-xu Sun, and Hong-jun Chang, "Research on the Control and Application of Chaos in an Electrical System", *Advances in Machine Learning and Cybernetics*, LNAI 3930, Springer-Verlag, Berlin, 142–148 (2006).
[10] P. Sowa and K. Łuszcz, "Chaos behavior simulation of the nonlinear dynamic ferroresonance phenomena", *Electrical review* R.90(8), 116–121 (2014).
[11] S. Seker, T.C. Akinci, and S. Taskin, "Spectral and statistical analysis for ferroresonance phenomenon in electric power systems", *Electrical Engineering* 94(2), 117–124 (2012).
[12] K. Milicevic, E.K. Nyarko, and I. Biondic, "Chua's model of nonlinear coil in a ferroresonant circuit obtained using Dommel's method and grey box modelling approach", *Nonlinear Dyn.* 86, 51–63 (2016).
[13] J. Kolańska-Płuska and B. Grochowicz, "Modelling of a nonlinear coil with loss in iron using the Runge-Kutta methods", *Archives of Electrical Engineering* 65(3), 527–539 (2016).
[14] F.B. Amar and R. Dhifaoui, "Study of the Periodic Ferroresonance in the Electrical Power Networks by Bifurcation Diagrams", *International Journal of Electrical Power and Energy Systems* 33(1), 61–85 (2011).
[15] M. Moradi and A. Gholami, "Harmonic balance based stability domain analysis of period-1 ferroresonance", *Electric Power Components and Systems* 39(12), (2011).
[16] J.A. Martinez-Velasco (Ed.), *Transient Analysis of Power Systems: Solution Techniques, Tools and Applications*, Wiley-IEEE Press, John Wiley & Sons, 2015.
[17] A. Visintin, *Differential Models of Hysteresis*, Applied Mathematical Science, Springer-Verlag, New York, 1994.
[18] I. Biondić, R. Topalović, and K. Miličević, "Comparison of Basic Ferromagnetic Coil Hysteresis Models", pp. 67–77, Papers of 33rd International Scientific Conference: Science in Practice, Hochschule für Angewandte Wissenschaften Würzburg-Schweinfurt, 2015.
[19] S. Cundeva, "A transformer model based on the Jiles-Atherton theory of ferromagnetic hysteresis", *Serbian Journal of Electrical Engineering* 5(1), 21–30 (2008).
[20] K. Chwastek and J. Szczygłowski, "Estimation methods for the Jiles-Atherton model parameters – a review", *Electrical review* R.84(12), 145–148 (2008).
[21] A. Benabou, S. Clénet, and F. Piriou, "Comparison of Preisach and Jiles-Atherton models to take into account hysteresis phenomenon for finite element analysis", *Journal of Magnetism and Magnetic Materials* 261(1–2), 139–160 (2003).

[22] D. Carnevale, S. Nicosia, and L. Zaccarian, "Generalized constructive model of hysteresis", *IEEE Transactions on magnetics* 42(12), 3809–3817 (2006).

[23] J. Voros, "Modeling and identification of hysteresis using special forms of the Coleman-Hodgdon model", *Journal of Electrical engineering* 60(2), 100–105 (2009).

[24] J.P. Noel, A.F. Esfahani, G. Kerschen, and J. Schoukens, "A nonlinear state-space approach to hysteresis identification", *Mechanical Systems and Signal Processing* 84, 171–184 (2017).

[25] J.P.A. Bastos, N. Sadowski, J.V. Leite, N.J. Jhoe Batistela, K. Hoffmann, G. Meunier, and O. Chadebec, "A differential permeability 3-D formulation for anisotropic vector hysteresis analysis", *IEEE Transactions on Magnetics* 50(2), 341–344 (2014).

[26] K. Milicevic, I. Lukacevic, and I. Flegar, "Modeling of nonlinear coil in a ferroresonant circuit", *Electr. Eng. (Archiv fur Elektrotechnik)* 91, 51–59 (2009).

[27] K. Milicevic, D. Vinko, and Z. Emin, "Identifying ferroresonance initiation for a range of initial conditions and parameters", *Nonlinear Dyn.* 66, 755–762 (2011).

[28] I. Schafer and K. Kruger, "Modelling of coils using fractional derivatives", *Journal of Magnetism and Magnetic Materials* 307, 91–98 (2006).

[29] Ł. Majka, "Measurement based inductor modeling for the purpose of ferroresonance analyses", pp. V-3, Proc. International Conference AMTEE, Trebic, Czech Republic, 2015.

[30] Ł. Majka, "Measurement verification of the nonlinear coil models", pp. 89–90, Proc. 39th International Conference IC-SPETO, Ustroń, 2016.

[31] Ł. Majka, "Measurements and simulation for a ferroresonance circuit", pp. 47–48, Proc. 40th International Conference IC-SPETO, Ustroń, 2017.

[32] Ł. Majka, "Fractional derivative approach in modeling of a nonlinear coil for ferroresonance analyses", in *Non-integer order calculus and its applications*, pp. 135-147, eds. P. Ostalczyk, D. Sankowski, J. Nowakowski, 9th International Conference on Non-integer Order Calculus and its Applications, Łódź, Poland, Springer International Publishing, Cham, 2019.

[33] I. Schafer and K. Kruger, "Modelling of lossy coils using fractional derivatives", *J. Phys. D: Appl. Phys.* 41, 2008, DOI: 10.1088/0022-3727/41/4/045001.

[34] T. Kaczorek, "Minimum energy control of fractional positive continuous-time linear systems using Caputo-Fabrizio definition", *Bull. Pol. Ac.: Tech.* 65(1), 45–51 (2017).

[35] W. Mitkowski and P. Skruch, "Fractional-order models of the supercapacitors in the form of RC ladder networks", *Bull. Pol. Ac.: Tech.* 61(3), 580–587 (2013).

[36] D. Spałek, "Synchronous Generator Model with Fractional Order Voltage Regulator PIbDa", *Acta Energetica* 23(2), 78–84 (2015), DOI: 10.12736/issn.2300-3022.2015208.

[37] S. Kukla and U. Siedlecka, "An analytical solution to the problem of time-fractional heat conduction in a composite sphere", *Bull. Pol. Ac.: Tech.* 65(2), 179–186 (2017).

[38] W. Sumelka, "Fractional calculus for continuum mechanics - anisotropic non-locality", *Bull. Pol. Ac.: Tech.* 64(2), 361–372 (2016).

[39] M. Caputo, "Linear models of dissipation whose Q is almost frequency independent-II", *Geophysical Journal of the Royal Astronomical Society* 13(5), 529–539 (1967).

[40] http://www.kared.com.pl/Products/11/48/Cyfrowy-rejestrator-zaklocen-RZ-1.html

[41] https://www.mathworks.com/help/matlab/

[42] M.J.D. Powell, "A Fortran Subroutine for Solving Systems of Nonlinear Algebraic Equations", *Numerical Methods for Nonlinear Algebraic Equations*, Ch.7, 1970.

[43] M. Sowa, "A subinterval-based method for circuits with fractional order elements", *Bull. Pol. Ac.: Tech.* 62(3), 449–454 (2014).

[44] M. Sowa, "Application of SubIval in solving initial value problems with fractional derivatives", *Applied Mathematics and Computation*, 2017, DOI: 10.1016/j.amc.2017.01.047.

[45] M. Sowa, "Application of SubIval, a method for fractional-order derivative computations in IVPs", in *Theory and applications of non-integer order systems*, pp. 489–499, eds. A. Babiarz, A. Czornik, J. Klamka and M. Niezabitowski, 8th Conference on Non-integer Order Calculus and Its Applications, Zakopane, Poland, Springer International Publishing, Berlin, 2017.

[46] http://msowascience.com/

[47] D. Sierociuk, G. Sarwas, and M. Twardy, "Resonance phenomena in circuits with ultracapacitors", *IEEE Environment and Electrical Engineering (EEEIC)*, pp. 197–202, 12th International Conference on, Wroclaw, Poland, 2013, DOI: 10.1109/EEEIC.2013.6549616.