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Arc models for simulating processes in circuits with a SF_6 circuit breaker

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Abstract: This paper demonstrates that if a linear dependence of arc dissipated power on power supplied is introduced at an initial stage of analysis, then, with some simplifying assumptions, the classical Mayr model is obtained. Similarly, if this dependence is taken into account in a model with residual conductance, the modified Mayr model is obtained. The study takes into consideration the local phenomenon of sudden voltage drop accompanying linear current decrease occurring in the circuit breaker. To account for this phenomenon, the Dirac delta function and its approximation by a Gaussian function, representing power or enthalpy disturbances, are introduced to the power balance equation. It is demonstrated that both variants yield the same effect, leading to identical differential equations. Macromodels of the circuit-breaker arc are created and connected with the power source circuit with linearly decreasing current. The results obtained were found to be consistent with experimental data available in the literature. The models presented are based on a fairly uncomplicated 1^{st} order differential equation and offer a straightforward physical interpretation of the phenomena in question.

Key words: switching arc, high-voltage circuit breaker, Mayr model

1. Introduction

Electric arc plasma undergoes various internal and external disturbances. In the majority of electric devices some steps are undertaken to stabilize the arc discharge and to ensure uninterrupted operation of the technological process. Such steps include selecting appropriate characteristics of the power supply and screening the discharge area. In the case of switchgear, it is usually impossible to control parameters of the supply systems and actions are taken to intensify arc disturbances in order to accelerate arc quenching. As evident, the conditions of arc burning in these two cases are essentially different. In practice, however, the mathematical models created for the sake of accounting for a stable arc are also used for modelling the processes occurring in a non-stable arc in circuit breakers. To make the approximation more accurate, some modifications are introduced to simple models, or – as the second option – such models are combined together. In the former case it is only at the final stage of modelling that the nonlinear character of electric characteristics is taken into account [1, 2]. In the latter case, two or even three different models are connected in series [1, 3] or in parallel [4, 5], with weight functions being used. Each complication of this kind, however, leads to inevitable difficulties in interpreting results and in obtaining arc parameters on the basis of experiments. What is required, is a compromise between accuracy and simplicity.

The objective of this paper is to present a few simple mathematical models of the electric arc obtained by modifying the initial assumptions underlying Mayr's model. These modified models take into account both the linear increase in thermal dissipated power accompanying increase in power supplied and the sudden local voltage drop in the arc during the linear current decrease.

2. Selected physical properties of a circuit breaker arc

Results of investigation on current interruption that are considered to be reliable in the literature [5–7] are those obtained in KEMA Laboratories in the Netherlands. They were conducted on a SF_6 circuit breaker with the parameters 245 kV/50 kA/50 Hz. Voltages and currents were registered by means of a measuring system with parameters 10 MHz/12 bits. Current and voltage curves are shown in Fig. 1, where it can be noticed that linear current decrease from the value of about 1 800 A is initially accompanied by a relatively mild decrease in voltage down to the value of about 1 700 V, until the current reaches the critical value $i_{cr} = 10^3$ A. At this point, voltage drops almost in one step from 1 600 V to about 1 200 V, and subsequently increases steeply up to 2 400 V, despite further decrease in current. Then, in the range of current decrease from about 100 A to zero, voltage dramatically drops to zero, too.

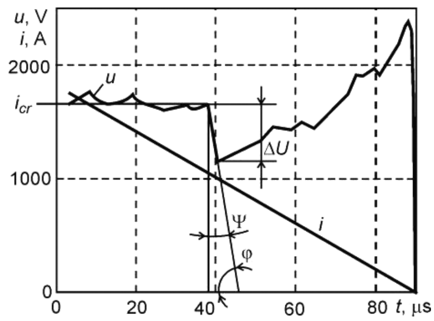


Fig. 1. Voltage u and current i curves during the circuit breaker operation
(source: own work, based on [5])

In the standard Mayr model it is assumed that dissipated power is constant. As experimental research indicates, however [5], dissipated power increases quasi-linearly together with increase in supplied electric power (Fig. 2). As can be seen on the curve, when the input power is about $ui = 1.2{-}1.6\ 10^6$ W, the real plot slightly diverges from its approximation. It corresponds to an almost stepwise, sudden arc voltage drop (Fig. 1). Repeated tests confirmed that the value of the critical current is in the narrow range $i_{cr} \approx 1{-}1.11$ kA. This phenomenon can be explained by a sudden change of the arc length [7].

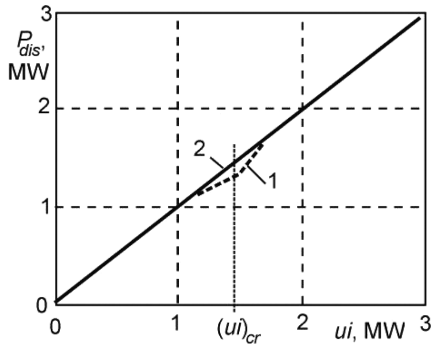


Fig. 2. Dependence of dissipated power P_{dis} on the supplied electric power *ui*
(1 – experiment; 2 – approximation) (source: own work, based on [5])

The increase in voltage on the arc when the current intensity approaches zero results from the shape of the static curve characteristics. In the classical Mayr model the power $UI = P_M$ = const, so when the current decreases, the voltage increases.

Interaction between the arc and the electric network during circuit interruption is often accounted for in terms of black box models, which are usually derived from the simple Mayr and Cassie models [1, 5–7]. Because of the simplifying assumptions adopted in these simple models, they can be utilised for describing the phenomena occurring in the low current range and in the high current range, respectively. Since the ultimate effect of arc quenching occurs at low current, the analysis typically focus on this current range. Voltages in the commuted circuits are relatively high, and because of that voltage drops near the electrodes are negligible.

Depending on the objectives and needs of the analysis of processes, various simplifying assumptions are made in electrical apparatus in relation to the observed physical phenomena. More accurate multidimensional mathematical models take into account the combination of simultaneously and very quickly occurring electrical, thermal, magnetohydrodynamic processes, etc. [8, 9]. However, they lead to very complex models with complicated parameters, described by nonlinear partial differential equations. According to the Pareto principle, in many practical cases more simplified one-dimensional and even linear models are sufficient [1, 10–12]. As a result, they often provide sufficient accuracy and speed of process simulation in electrical circuits in which commutations take place using modeled electrical devices.

3. Properties of standard and modified Mayr's models of the electric arc

Mathematical models can describe three different time intervals in arc occurrence: the current flow stage, the current decay stage and the no-current stage (after a switch-off). At the first stage, local thermodynamic equilibrium occurs in the arc plasma. At the second stage the balance is upset, but it is assumed that it can be restored. At the third stage there is no thermodynamic balance. The mathematical models of arc based on these assumptions are usually developed for the first and the second time intervals [13].

In the case of Mayr's model, the following simplifying assumptions are typically made [1]:

– $g(Q)$ = var. – the arc conductance depends on its enthalpy,

– P_{dis} = const. – the dissipated arc power is constant.

Heat is transferred from the area where the plasma temperature is the highest, i.e. at the arc axis, to the area near the plasma surface (about 6 000 K) by means of conduction. As analysis of experimental data [14] indicates, further heat transfer takes place mostly by thermal radiation, although conduction and convection channel from the lateral arc area to the environment also take place. Increase or decrease in the amount of electric energy supplied affects the arc enthalpy Q. Such arc burning conditions occur at low currents.

In the arc model presented below, the conical part of the arc near the cathode is disregarded (and so is the conical part of the arc near the anode), so plasma is not pumped through the arc column and this channel of heat dissipation by convection is thus neglected. In reality, the temperature distribution along the arc is non-homogeneous, with the lowest temperatures at the electrodes, playing the cooling role. Hence, heat transfer by conduction from the arc to the electrodes is neglected, too.

One of the fundamental assumptions of Mayr's model is the condition tying the conductance g with the arc enthalpy Q:

$$\frac{g}{g_0} = \exp\left(\frac{Q}{Q_0}\right), \tag{1}$$

where: g_0 – reference conductance, Q_0 – reference enthalpy. The base size Q_0 is the substitute of the exponential function. The characteristic of the exponential function is Q_0 = const. On the basis of the arc power balance, the following can be written:

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = p_{el} - P_{dis}\,, \tag{2}$$

where: $p_{el} = ui$ – electric power supplied, P_{dis} – thermal power dissipated. After substituting (1) into (2) and performing transformations, Mayr's model is obtained:

$$\frac{\mathrm{d}\ln g}{\mathrm{d}t} = \frac{1}{g}\,\frac{\mathrm{d}g}{\mathrm{d}t} = \frac{1}{\tau}\left(\frac{ui}{P_M} - 1\right), \tag{3}$$

where: $\tau = Q_0/P_M$ – time constant of Mayr model, $P_{dis} = P_M$ – constant power of Mayr's model. The model is linear and can be also represented as:

$$\tau\frac{\mathrm{d}g}{\mathrm{d}t} + g = \frac{i^2}{P_M}\,. \tag{4}$$

Assuming as it is done in [5, 7] that the arc dissipated power P_{dis} varies linearly:

$$P_{dis} = P_0 + p_1 ui \tag{5}$$

and assuming (1), Eq. (2) can be transformed into:

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = \frac{Q_0}{g}\,\frac{\mathrm{d}g}{\mathrm{d}t} = p_{el} - P_{dis} = ui - (P_0 + p_1 ui)\,, \tag{6}$$

from which it follows that Mayr's model takes the form:

$$\tau \frac{\mathrm{d}g}{\mathrm{d}t} + g = \frac{i^2}{P_{Mp}}, \tag{7}$$

or

$$\tau \frac{\mathrm{d}\ln g}{\mathrm{d}t} = \frac{ui}{P_{Mp}} - 1, \tag{8}$$

where:

$$\tau = Q_0/P_0, \qquad P_{Mp} = P_0/(1 - p_1) \quad \text{and} \quad 0 \le p_1 < 1.$$

P_0 is the constant component of linear dissipation power approximation; p_1 is the coefficient of approximation of linear dissipated power. As evident, introducing the linear approximation of the power function $P_{dis}(ui)$ also leads to Mayr's model, without the assumption, however, that the dissipated power is constant.

Like in [5, 7] we assume that the arc dissipated power varies linearly with the power supplied, and when the latter decreases, the arc residual conductance g_M appears:

$$P_{dis} = P_0 \left(1 - \frac{g_M}{g}\right) + p_1 ui. \tag{9}$$

Introduction of g_M conductance results from the modification of Mayr's model. It consists in replacing the hyperbolic static characteristics of the arc ($UI = P_M =$ const) with a new static characteristic [15]:

$$U = \frac{P_M I}{I^2 + I_M^2} = \frac{P_M I}{I^2 + P_M g_M}, \tag{10}$$

passing through the origin of the system of coordinates $(U,\ I)$, where $g_M P_M = I_M^2$.

After substituting (9) into Eq. (2) and carrying out transformations, the differential equation is obtained:

$$\frac{\mathrm{d}\ln g}{\mathrm{d}t} = \frac{1}{g}\frac{\mathrm{d}g}{\mathrm{d}t} = \frac{1}{\tau}\left(\frac{ui}{P_{Mg}} + \frac{g_M}{g} - 1\right), \tag{11}$$

or alternatively in the form analogous to (4):

$$\tau \frac{\mathrm{d}g}{\mathrm{d}t} + g = \frac{i^2}{P_{Mg}} + g_M, \tag{12}$$

where: $\tau = Q_0/P_0$ is the time constant of the modified Mayr model; $P_{Mg} = P_0/(1 - p_1)$ is the constant power of the modified Mayr model.

From Eq. (9) it follows that in the steady state for the current $i = I = 0$ the dissipation power $P_{dis} = 0$, because according to the Eq. (12) the condition $g = g_M$ is then fulfilled, thus $\mathrm{d}g/\mathrm{d}t = 0$. The fact that

$$g|_{I=0} = \frac{I_M^2}{P_M} = g_M,$$

also results from the shape of static characteristics.

4. The modified mathematical Mayr model of the circuit-breaker arc with a disturbance in the linear function of dissipated power

Already at the beginning of the formula derivation, the disturbance in the dissipated power due to the passing of the current through its critical value i_{cr} is introduced:

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = Q_0 \frac{\mathrm{d}\ln g}{\mathrm{d}t} = p_{el} - P_{dis} = ui - (P_0 + p_1 ui) + \Delta P \cdot \delta\left(i - i_{cr}\right). \tag{13}$$

If current decrease is linear, it is convenient to use the notation:

$$\left.\frac{1}{u}\frac{\mathrm{d}u}{\mathrm{d}i}\frac{\mathrm{d}i}{\mathrm{d}t}\right|_{i_{cr}} = -\frac{\alpha}{U_{cr}}\left.\frac{\mathrm{d}u}{\mathrm{d}i}\right|_{i_{cr}}, \tag{14}$$

where: $U_{cr} = u(i_{cr})$, $\mathrm{d}i/\mathrm{d}t = -\alpha$. After integrating Eq. (13) in the neighbourhood of the sudden voltage change corresponding to the change of current in the interval from i_{cr}^- to i_{cr}^+, where $i_{cr}^- < i_{cr} < i_{cr}^+$ and $i_{cr}^+ - i_{cr}^- \geq 0$, one obtains:

$$\frac{\Delta P}{P_0} = \frac{\alpha\tau}{U_{cr}}\Delta U, \tag{15}$$

where ΔU is the voltage drop corresponding to the increase in power ΔP, which can be obtained directly from experimental data.

After substituting (14) and (15) into Eq. (13), the following modified Mayr model is obtained:

$$\tau\frac{\mathrm{d}\ln g}{\mathrm{d}t} = \frac{ui}{P_{Mp}} - 1 + \frac{\alpha\tau}{U_{cr}}\Delta U \cdot \delta\left(i - i_{cr}\right), \tag{16}$$

where $P_{Mp} = P_0/(1 - P_1)$ or in a different form:

$$\tau\frac{\mathrm{d}g}{\mathrm{d}t} + \left[1 - \alpha\tau\frac{\Delta U}{U_{cr}}\delta\left(i - i_{cr}\right)\right]g = \frac{i^2}{P_M}. \tag{17}$$

These equations can be transformed into Bernoulli's equation with respect to voltage u variation:

$$\alpha\tau\frac{\mathrm{d}u}{\mathrm{d}i} + \left[1 - \alpha\tau\left(\frac{1}{i} + \frac{\Delta U}{U_{cr}}\delta(i - i_{cr})\right)\right]u = \frac{i}{P_M}u^2. \tag{18}$$

The derivative $\left.(\mathrm{d}u/\mathrm{d}i)\right|_{cr}$ in the neighbourhood of the current i_{cr}, where a sudden voltage drops occurs, behaves similarly to the Dirac delta function. To simplify the analysis, the Dirac delta function is approximated by means of a Gaussian function:

$$\delta(i - i_{cr}) \approx \sqrt{\frac{\beta}{\pi}}\exp\left(-\beta(i - i_{cr})^2\right). \tag{19}$$

When this modification is taken into account, (18) becomes:

$$\alpha\tau\frac{\mathrm{d}u}{\mathrm{d}i} + \left[1 - \alpha\tau\left(\frac{1}{i} + \frac{\Delta U}{U_{cr}}\sqrt{\frac{\beta}{\pi}}e^{-\beta(i - i_{cr})^2}\right)\right]u = \frac{i}{P_M}u^2. \tag{20}$$

Since for the assumed approximation β is large, in a small neighbourhood of i_{cr}, the voltage jump is sudden and the conductance g has a sharp peak. If the free terms in the environment of the point $i = i_{cr}$ are neglected, a formula for the voltage derivative in this neighbourhood is obtained:

$$\left.\frac{\mathrm{d}u}{\mathrm{d}i}\right|_{i_{cr}} = \Delta U \sqrt{\frac{\beta}{\pi}} \exp\left(-\beta (i - i_{cr})^2\right), \tag{21}$$

hence

$$\beta = \frac{\pi}{(\Delta U \cdot \alpha \cdot \tan\psi)^2}\,. \tag{22}$$

Substituting the above-given formula into (20) and carrying out transformations leads to [16]:

$$\tau \frac{\mathrm{d}g}{\mathrm{d}t} + \left[1 - \frac{\tau \cot\psi}{U_{cr}} e^{-\beta(i-i_{cr})^2}\right] g = \frac{i^2}{P_M}, \tag{23}$$

which, after being divided by g, can be brought to an equivalent form:

$$\tau \frac{\mathrm{d}\ln g}{\mathrm{d}t} = \frac{ui}{P_M} - 1 + \frac{\tau \cot\psi}{U_{cr}} e^{-\beta(i-i_{cr})^2}. \tag{24}$$

5. The modified mathematical Mayr model of the circuit-breaker arc with a disturbance of the plasma enthalpy function

One of the possible explanations for the causes of the sudden arc voltage drop during circuit interruption can be disturbance of plasma enthalpy. Such an effect will be taken into account at the initial stage of constructing the model. Then, on the basis of (1):

$$\frac{Q}{Q_0} = \ln \frac{g}{g_0} + \frac{\Delta Q}{Q_0} \mathbf{1}(i - i_{cr}), \tag{25}$$

where $\mathbf{1}$ is the Heaviside step function. After differentiating the equation:

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = Q_0 \frac{\mathrm{d}\ln g}{\mathrm{d}t} + \Delta Q \frac{\mathrm{d}\mathbf{1}(i - i_{cr})}{\mathrm{d}t}\frac{\mathrm{d}i}{\mathrm{d}t} = Q_0 \frac{\mathrm{d}\ln g}{\mathrm{d}t} + \Delta Q \delta\,(i - i_{cr})\,\frac{\mathrm{d}i}{\mathrm{d}t}, \tag{26}$$

where δ is the Dirac delta function. This formula has been substituted into Eq. (6), which has been integrated with respect to the current in the neighbourhood of the critical current i_{cr}, as a result one obtains:

$$\frac{\Delta U}{U_{cr}} = \frac{\Delta Q}{Q_0}\,. \tag{27}$$

The following formula obtained from the power balance holds for any current variation:

$$Q_0 \frac{\mathrm{d}\ln g}{\mathrm{d}t} = p_{el} - P_{dis} - \Delta Q \delta\,(i - i_{cr})\,\frac{\mathrm{d}i}{\mathrm{d}t} = ui - (P_0 + p_1 ui) - \Delta Q \delta\,(i - i_{cr})\,\frac{\mathrm{d}i}{\mathrm{d}t}\,. \tag{28}$$

If linear current decrease is assumed, and when (15) is taken into account, the following is obtained after transformations:

$$\frac{\mathrm{d}\ln g}{\mathrm{d}t} = \frac{1}{\tau}\left(\frac{ui}{P_{Mp}} - 1\right) + \alpha \frac{\Delta U}{U_{cr}} \delta\,(i - i_{cr})\,, \tag{29}$$

where $P_{Mp} = P_0/(1 - p_1)$ is the constant power of the modified Mayr model.

After approximation (19) is applied to Eq. (29), the resulting form of the formula is identical to (20). Further considerations lead to the ultimate form of the equation, which is (24).

As can be seen, introducing a jump disturbance to the dissipated power or to plasma enthalpy both lead to the same differential equation, i.e. to the same effect being a step decrease in arc voltage.

6. The modified mathematical Mayr model of the circuit-breaker arc with residual conductance and with the plasma enthalpy function disturbance

Let dissipated power depend on the forcing power in accordance with (9). Additionally, the dependence between enthalpy and plasma conductance satisfies the condition (25). After differentiating this formula, (26) is obtained. When the resulting formula is substituted into (2) and integrated with respect to current in the neighbourhood of the critical current i_{cr}, (27) is obtained. From the power balance equation it follows that:

$$Q_0 \frac{\mathrm{d}\ln g}{\mathrm{d}t} = p_{el} - P_{dis} - \Delta Q \delta(i - i_{cr}) \frac{\mathrm{d}i}{\mathrm{d}t} = ui - P_0 \left(1 - \frac{g_M}{g}\right) - p_1 ui - \Delta Q \delta(i - i_{cr}) \frac{\mathrm{d}i}{\mathrm{d}t}. \tag{30}$$

When (27) is applied and further transformations are performed:

$$\frac{\mathrm{d}\ln g}{\mathrm{d}t} = \frac{1}{\tau}\left(\frac{ui}{P_M} + \frac{g_M}{g} - 1\right) - \frac{\Delta U}{U_{cr}} \delta\left(i - i_{cr}\right) \frac{\mathrm{d}i}{\mathrm{d}t}. \tag{31}$$

This equation can be represented in an equivalent form as:

$$\tau \frac{\mathrm{d}g}{\mathrm{d}t} + \left(1 + \tau \frac{\Delta U}{U_{cr}} \delta(i - i_{cr}) \frac{\mathrm{d}i}{\mathrm{d}t}\right) g = \frac{i^2}{P_M} + g_M . \tag{32}$$

Eq. (31) can be transformed to an equation with unknown voltage:

$$-\frac{\mathrm{d}\ln i}{\mathrm{d}t} \tau \frac{\mathrm{d}u}{\mathrm{d}i} + \left[1 + \frac{\mathrm{d}\ln i}{\mathrm{d}t} \tau \left(\frac{1}{i} + \frac{\Delta U}{U_{cr}} \delta(i - i_{cr})\right)\right] u = \left(\frac{i}{P_M} + \frac{g_M}{i}\right) u^2. \tag{33}$$

If current decreases linearly, then after applying the dependence $\mathrm{d}i/\mathrm{d}t = -\alpha$ and carrying out transformations the following is obtained:

$$\frac{\mathrm{d}\ln g}{\mathrm{d}t} = \frac{1}{\tau}\left(\frac{ui}{P_M} + \frac{g_M}{g} - 1\right) + \alpha \frac{\Delta U}{U_{cr}} \delta\left(i - i_{cr}\right), \tag{34}$$

or

$$\tau \frac{\mathrm{d}g}{\mathrm{d}t} + \left(1 - \alpha\tau \frac{\Delta U}{U_{cr}} \delta(i - i_{cr})\right) g = \frac{i^2}{P_M} + g_M . \tag{35}$$

This can also be represented in a form analogous to (18) as:

$$\alpha\tau \frac{\mathrm{d}u}{\mathrm{d}i} + \left[1 - \alpha\tau \left(\frac{1}{i} + \frac{\Delta U}{U_{cr}} \delta(i - i_{cr})\right)\right] u = \left(\frac{i}{P_M} + \frac{g_M}{i}\right) u^2. \tag{36}$$

When the approximation (19) is applied, the model is obtained [16]:

$$\tau\frac{\mathrm{d}\ln g}{\mathrm{d}t}=\frac{ui}{P_M}+\frac{g_M}{g}-1+\frac{\tau\cot\psi}{U_{cr}}e^{-\beta(i-i_{cr})^2},\tag{37}$$

or, after multiplying (37) by g, the model can be obtained in an equivalent form:

$$\tau\frac{\mathrm{d}g}{\mathrm{d}t}=\frac{i^2}{P_M}+g_M-\left(1-\frac{\tau\cot\psi}{U_{cr}}e^{-\beta(i-i_{cr})^2}\right)g.\tag{38}$$

When the formula $g_M = I_M^2/P_M$ is substituted into Eq. (38), it can also be written in an equivalent form:

$$\tau\frac{\mathrm{d}g}{\mathrm{d}t}=\frac{i^2+I_M^2}{P_M}-\left(1-\frac{\tau\cot\psi}{U_{cr}}e^{-\beta(i-i_{cr})^2}\right)g.\tag{39}$$

It can be observed that, as was expected, after assuming that $g_M = 0$ S, Eq. (38) is reduced to Eq. (23), and after assuming that $I_M = 0$ A, Eq. (39) is reduced to (24).

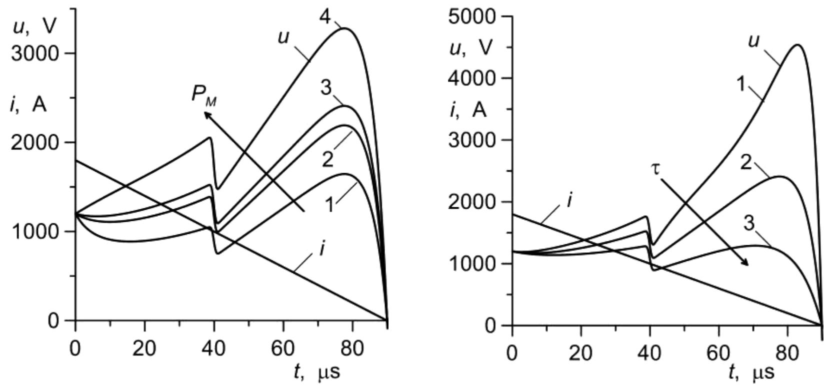
If the value of the voltage drops $\Delta U \to 0$ V, then according to Formula (22) $\beta \to \infty$, which leads to the transformation of the modified Mayr model (23) into the classic Mayr model (7). Then, Formula (24) is transformed into Formula (8). The voltage loss ΔU also leads to the transformation of the model with the residual conductance g_M of the form (38) into the model of the form (12) or the model of the form (37) into the model of the form (11).

7. Simulations of connecting processes in dc circuits with modified Mayr's models of the electric arc

Power sources are characterised by very small internal impedance, and because of that they can be classified as real voltage sources. Hence, the values and shape of the current waveform is largely affected by the characteristics of load impedance. For high supply voltages and low load impedances, the phase currents reach very high values. Under such circumstances, operation of the circuit breaker results in an intensive quenching of a large power arc. A factor conducive to breaking the circuit is passing the zero value by the current. Due to a high mass of the electrodes and inertia of the drive systems, the separation of the contacts does not occur immediately. The time of arc quenching depends on the parameters of the circuit and the circuit breaker type. It is desired that the current should decay completely without the arc being reignited.

The arc represented by the mathematical Mayr model should satisfy the conditions of a low-current arc. The simulations described in the study concern a circuit breaker of rated current 50 kA. This determines the construction of the breaker, which has to interrupt even higher short-circuit currents. Because of that it is equipped with electrodes of high mass and a large-size quenching chamber. The object of the scrutiny is the final stages of arc burning, when current decreases down to zero. In this case, the current range below 2 kA can be considered low.

In the simulations the forced current was applied in the same way as in [6, 7, 17], i.e. with a linear decrease from 1 800 A at the rate $20\cdot 10^6$ A/s to 0 A. An arc macromodel constructed on the basis of Eq. (23) was connected into the circuit. The initial value of the conductance G_0 and other arc parameters were selected in such a way that the plots shown in Fig. 3 were obtained. The initial value of the conductance G_0 affects the initial values of voltages in the family of plots.



(a) (b)

Fig. 3. Current and voltage curves in a circuit with the arc model (23) during circuit interruption ($\cot\psi$ = 386.1 V/μs, U_{cr} = 1 250 V, i_{cr} = 1 000 A, $\beta = 4.683 \cdot 10^{-3}$ A^{-2}, G_0 = 1.5 S): (a) the case of various powers in the arc model (τ = 9 μs, 1 – P_M = 1.5 MW, 2 – P_M = 2 MW, 3 – P_M = 2.5 MW, 4 – P_M = 3 MW); (b) the case of various time constants in the arc model (P_M = 2.2 MW, 1 – τ = 5 μs, 2 – 9 μs, 3 – 15 μs)

In order to test the possibility of improving the simulation results, an arc macromodel was constructed on the basis of (39). The additional parameter g_M mainly affects the processes occurring at low current. The results of calculations are presented in Fig. 4, where it can be observed that increasing the value of the parameter $I_M = \sqrt{P_M g_M}$ leads to decreasing the maximal value of voltage, which in turn makes the arc voltage decay less steep. In this way, the simulation results of the interruption process become more discrepant with the experimental data. This can be predicted since the construction and operation of the circuit breaker are designed in such a way that the residual conductance g_M is minimised to guarantee a quick quenching of the arc.

When simulating arc quenching processes with the use of the model (39), the dependence (22) can be used in an altered form for greater convenience:

$$\beta = \pi \left(\frac{\cot\psi}{\Delta U \cdot \alpha} \right)^2 . \tag{40}$$

At the present stage of the investigations the simulation results shown in Fig. 5 are the most congruent with experimental data. They capture the sudden voltage drop occurring next to the critical current as well as increase in the voltage up to the maximal value before the ultimate current decay.

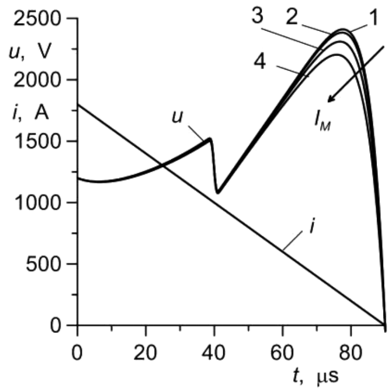


Fig. 4. Current i and voltage u curves in a circuit with the arc model (39) during circuit interruption (P_M = 2.2 MW, τ = 9 µs, $\cot\psi$ = 386.1 V/µs, U_{cr} = 1 250 V, i_{cr} = 1 000 A, $\beta = 4.683 \cdot 10^{-3}$ A^{-2}, G_0 = 1.5 S) with various residual currents in the model (1 – I_M = 0 A, 2 – I_M = 50 A, 3 – I_M = 100 A, 4 – I_M = 150 A)

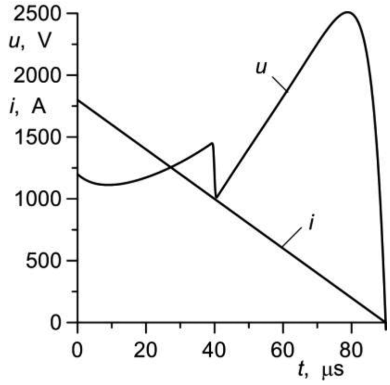


Fig. 5. Current and voltage curves in a circuit with the arc model (23) during circuit interruption (P_M = 2 MW, τ = 8 µs, $\cot\psi$ = 800 V/µs, U_{cr} = 1 250 V, i_{cr} = 1 000 A, ΔU = 500 V, G_0 = 1.5 S, I_M = 10 A)

8. Conclusions

1. Despite the fact that the model of the SF_6 circuit-breaker arc described in the literature [5–7] offers a good potential for approximating experimental data, it does not satisfy the fundamental power balance equation. Besides, it is quite complex and difficult to interpret in physical terms.
2. With rational initial assumptions following from the equation of arc power balance, it is possible to obtain simpler and more interpretively straightforward mathematical models of the circuit-breaker arc.

3. If the linear dependence between the electric supplied power and the thermal dissipated power is taken into account at the stage of formulating the power balance equation, it is possible to obtain a linear version of Mayr's model with constant parameters.
4. It is possible to represent the local sudden voltage drop occurring during the linear current decrease by introducing the local disturbance in dissipated power or arc enthalpy.
5. Simulations of the processes carried out by means of new macromodels of the circuit-breaker arc proved to be reliable tools for representing voltage during the circuit interruption by means of a SF_6 circuit breaker.
6. Despite very general and different input assumptions for the analysis of the impact of disturbances (dissipated power, plasma enthalpy) on the processes in the column of the arc, identical mathematical models were obtained. This indicates some capabilities of new models of mapping the influence of some physical phenomena on the electrical characteristics of circuit breakers.
7. Because the input assumptions of the presented models are consistent with the physical laws (meet the power balance equation), a much wider range of applications of the modified Mayr models can be predicted compared to the models in the cited papers

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