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# Thermal models of pulse electrochemical machining

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**Abstract.** Pulse electrochemical machining (PECM) provides an economical and effective method for machining high strength, heat-resistant materials into complex shapes such as turbine blades, die, molds and micro cavities. Pulse Electrochemical Machining involves the application of a voltage pulse at high current density in the anodic dissolution process. Small interelectrode gap, low electrolyte flow rate, gap state recovery during the pulse off-times lead to improved machining accuracy and surface finish when compared with ECM using continuous current. This paper presents a mathematical model for PECM and employs this model in a computer simulation of the PECM process for determination of the thermal limitation and energy consumption in PECM. The experimental results and discussion of the characteristics PECM are presented.

**Keywords:** micromachining, electrochemical dissolution, temperature distribution.

## 1. Introduction

The field of micro engineering, which consists of manufacture, assembly and use of very small components connected to microelectromechanical systems (MEMS), is rapidly gaining importance in all areas of industrial and consumer products [1]. Many existing precision engineering technologies in modified form are being tried to manufacture micro components [2–4]. The process technologies such as lithography, various etching techniques and the deposition of thin films are being used mainly to produce two-dimensional micro components [2]. However, only recently have been made attempts to investigate processes for producing 3-D micro components. Besides traditional machining techniques such as micro turning and milling, the attention is focused on non-traditional machining techniques such as micro electrical discharge machining and micro electrochemical micromachining (MECM) and laser beam machining because of their unique characteristics [3]. The ability of MECM in rapidly generating a stress-free and crack-free smooth surface on any electrically conductive material (irrespective of hardness) makes it an excellent choice as a micro production process [4–10]. Electrochemical Machining (ECM) is based on controlled anodic dissolution process of the workpiece (anode) with the tool as the cathode in an electrolyte cell. In the ECM process, a low voltage (2–30 V) is normally applied between electrodes with a small gap (usually 0.2–0.8 mm) producing a high current density of the order of the (10–100 A/cm$^2$), and a metal removal rate ranging from 0.1 mm$^3$/min to $10^4$ mm$^3$/min. Electrolyte (typically NaCl, $NaNO_3$ or acids aqueous solutions) is supplied to flow through the gap with a velocity of 5–50 m/s to maintain the electrochemical dissolution with a high rate to flush away the reactions products (usually gases and hydroxides) and heat generated caused by the passage of current and electrochemical reactions. It has been applied in diverse industries such as aerospace, automotive and electronics, to manufacture airfoils and turbine blades, die and mold, artillery projectiles, surgical implants and prostheses, etc. Moreover with recent advances in machining accuracy and precision, based on the development of advanced electrochemical metal-removal processes, demonstrate that the ECM can be effectively used for micro-machining components in the electronics and precision industries [3–10].

From theory and practice of ECM it follows that for improvement of shape accuracy and simplification of tool design, the gap size during machining should be as small as possible. All these requirements for ECM performance however are not achievable with continuous working voltage. The minimum practical interelectrode gap size, which may be employed, is constrained by the onset of unwanted electrical discharges as result of boiling and gas/vapor generation.

These constraints of continuous ECM can be reduced and the requirements in point of view of machining accuracy can be achieved by application of pulse working voltage in the electrochemical shaping and smoothing [9, 11–15]. In the pulse electrochemical machining (PECM), a pulse generator is used to supply the working voltage pulses across the two electrodes, typically in the form of pulse strings consisting of single pulses or grouped pulses (Fig. 1).

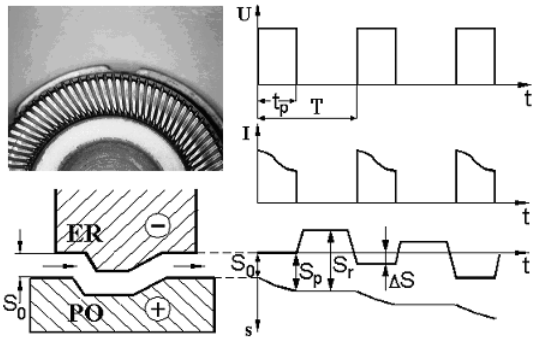


Fig. 1. Principal scheme of the PECM and example of machined part (a detail of a shaving head with slots [6])

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The anodic electrochemical dissolution occurs during the short pulse on-times $t_p$, each raging from 1 $\mu$s to 5 ms. The dissolution products (sludge, gas bubbles and heat) can be flushed away from the inter-electrode gap by the flowing electrolyte during the pulse off-times $t_0 = T - t_p$ between two pulses or two groups of pulses. To intensify the electrolyte flushing, the tool is retracted from the workpiece to enlarge the gap $S_r$ during the pulse off-times. The gap checking and tool repositioning can also be conducted during these pulse pauses to establish a given gap size $S_0$ before the arrival of the next pulse, leading to a significant reduction in the indeterminacy of the gap and, hence, improving the shaping accuracy. With PECM, it is possible to reproduce complex shapes, such as dies, turbine blades, and precision electronic components, with accuracy within 0.01–0.05 [mm].

However, in order to make PECM suitable for micro machining 3-D shapes, it is necessary to develop a modified ECM system which will provide a small gap below 10 $\mu$m and ultra short pulses of order a few or hundred nanoseconds for a highly localized and controlled electrochemical dissolution [7–10]. Instead of designing and fabricating a tool for a given component size and shape, a pulse electrochemical micro machining (PECMM) approach uses a universal tool electrode (e.g. cylindrical, flat-faced wire tool electrode) which is moved in a manner similar to that of a miniature-milling cutter, along the three axes using a computer controlled nano resolution drive system (Fig. 2)

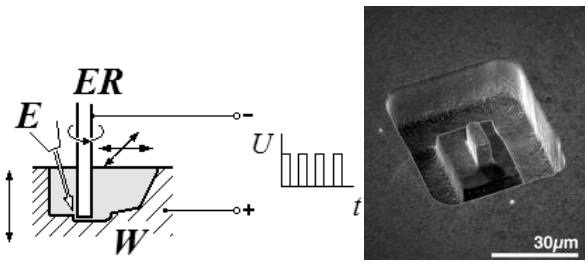


Fig. 2. Principal scheme of the Pulse Electrochemical Micro Machining (PECMM) and example of machined part (a micro-cavity in the copper sample [7]): ER – tool electrode (cathode), E – electrolyte, W – workpiece (anode)

The feasibility of such a system has been described in [7–10], a comprehensive study consisting of a mathematical model for the micro machining process and influence of machining conditions in the gap on the process performance has not been yet reported. The thermal mathematical models and computer simulation of PECM processes used for precision machining and micromachining are presented in the paper.

## 2. Mathematical modelling

The main task in PECM shaping, regardless of variant, is to calculate distribution of the material removed on the anode-workpiece surface after a single or pocket pulse. This is determined by the current density distribution in the gap during pulse on time, in particular in a medium of varying electrical conductivity, with complex processes occurring on the surface of the electrodes and with changes in shape of the machined surface during the course of machining.

Additional tasks are connected with describing the effects of the PECM conditions on the selection of pulse parameters, such as pulse on time $t_p$, pulse off-time $t_0$, and parameters for movements of the tool-electrode. The PECM limitations due to electrolyte boiling have also been determined particularly critical pulse time $t_p^*$, causing electrolyte boiling in the gap of the given width $S$. That corresponds to the determination of the critical $S^*$ width causing boiling for the given $t_p$ pulse time.

In PECMM with application of ultra-small gap size below 10 $\mu$m, and ultra-short pulses of nano/microsecond level, the effect of the gas bubble on the process in gap is not significant and can be neglected. The main changes in electrolyte properties are result of the heating.

Depending on the assumptions concerning of heat transfer and flow velocity distribution, the models divided in at least two groups:

- Adiabatic model without heat transfer through the electrode surfaces. It is equivalent to assuming in a given cross-section of the gap: constant temperature and electrical conductivity. Its values are equal to the averaged values over a cross-section.
- Thermal model with heat transfer through the electrode surfaces.

The following conditions are assumed for the both, above models:

- Ohm's law is applicable in the whole gap, so as to relate the current density $i$ with the medium's electrical conductivity $\kappa$, pulse voltage $U$ and gap size $S$.
- Electrochemical and diffusion processes are accounted for by the characteristic of pulse anodic dissolution $K_V$ ($i$, $t_p$) and by introducing the total overpotential $E$ ($i$, $t_p$) into the boundary condition.
- Flow in the gap is neglected.

The metal removal rate, $V_n$ during pulse on-time can be obtained from the Faraday's law:

$$V_n = K_v i \tag{1}$$

where: $K_v$ is the coefficient electrochemical machinability, which is defined as the volume of material dissolved per unit electrical charge.

To simplify the calculations let us introduce a curvilinear coordinate system $(\xi, \zeta)$, connected with the tool-electrode in which a coordinate $\xi$ lies on the given electrode and is measured from the inlet of the electrolyte and let axis $\zeta$ overlap its normal $\bar{n}_c$ (Fig. 3). Taking into account that the gap size is much less than the length, width and the radius of surfaces, we can use one-dimensional approximation for the distribution of electrical potential along the distance $S$, for a given point of the anode from

the tool-electrode surface (Fig. 3). In this approximation, the current density, $i$, can be expressed by the Ohm's law:

$$i = \kappa_0 \frac{U - E}{\int_0^S \frac{d\zeta}{1 + \alpha_T \theta}} \tag{2}$$

where: $U$ and $E$ are amplitude of voltage pulse and overpotential, respectively, $\theta = T - T_0$ — the temperature increment, $T_0$ — initial temperature of the electrolyte, $\alpha_T$ — conductivity coefficient of the electrolyte at $T_0$, $\kappa_0$ is the electrolytic conductivity at $T_0$.

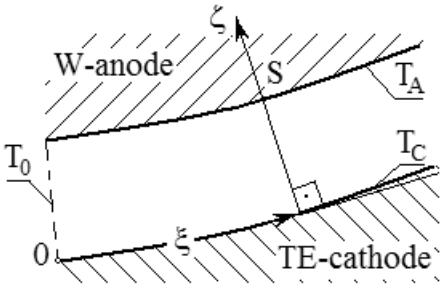


Fig. 3. Scheme of the curvilinear coordinate system

Generally, the temperature distribution in the gap with respect to Joule's heat generation, convection and heat transfer through the electrode surfaces is described as following:

$$\frac{\partial T}{\partial t} + W(\xi, \zeta, t)\frac{\partial T}{\partial \xi} = a\frac{\partial^2 T}{\partial \zeta^2} + \frac{i^2}{\kappa_0 \rho_e C_p [1 + \alpha_T (T - T_0)]} \tag{3}$$

where: $W$ is the flow electrolyte velocity, $a$ is the thermal diffusivity, and $\rho_e$, $C_p$ are density and heat capacity of electrolyte, respectively.

The boundary conditions are as follows: at the inlet: $T(\xi = 0) = To$, on the electrodes: $T(\xi, 0) = T_A$, $T(\xi, S) = T_C$, where: $T_A$ and $T_C$ are temperatures of the anode and cathode, respectively.

Mathematical approximation of the pulse electrochemical processes depends on the Strouhal number:

$$St = \frac{L}{wt_p} \tag{4}$$

where $L$ is characteristic length of the machining surface along flow path, and $w$ is average flow velocity in the gap.

For PECMM conditions of electrolyte flow $w < 1$ m/s, pulse on-time $t_p < 10^{-6}$ s, and gap length $L < 1$ mm, the Strouhal Number will be larger than 1000. The proceeding processes are visible nonstationary. In their mathematical description the local derivative plays the greatest role and the transfer of mass and energy by convection may altogether be neglected.

Mathematical modelling of PECM process also depends on the relation between the pulse off-time $t_0$, flow velocity w and characteristic length L i.e. Strouhal number for flushing period $St_0 = L/wt_0$.

If the time interval between the pulses $t_0 = t_{pp} - t_p$ is long enough to ensure a complete renewal of the electrolyte in the gap i.e. when $St_0 < 1$ or $t_0 > L/w$, it is reasonable to assume that the analysis of the process for a single pulse is valid over a series of pulses. However, if small pulse interval is used, in mathematical modelling an additional description is required to track the changes in the initial conditions from pulse to pulse in order to apply the models derived under single pulse conditions.

**2.1. Thermal model of PECM without heat transfer through the electrode surfaces.** For a single pulse on-time, the following system of equations can thus be established for describing the PECM process:

$$\begin{aligned} \frac{dS}{dt} &= \kappa_0 K_V (1 + \alpha_T \theta)\frac{U - E}{S} \\ \frac{d(\theta S)}{dt} &= \kappa_0 (1 + \alpha_T \theta)\frac{U - E}{S}\frac{U}{\rho_e C_p} \end{aligned} \tag{5}$$

with initial conditions: $S = S_0$, and $\theta = 0$ for $t = 0$.

By combining above equations, the following equation is obtained:

$$S\frac{d\theta}{dt} = \kappa_0 (1 + \alpha_T \theta)\frac{U - E}{S}\frac{U}{\rho_e C_p} \times \left(1 - \frac{\rho_e C_p K_V}{U}\theta\right). \tag{6}$$

For typical PECM conditions where $U$ = 10–20 V, $\rho_e C_p < 5 \times 10^6$ J/m$^3$K, $\theta < 80$°K, and $K_V < 4 \times 10^{-11}$ m$^3$/A s, the term $(\rho_e C_p K_V/U)\theta < 1.6 \times 10^{-3} < 1$ and, hence, can be neglected.

In this model only heating of the electrolyte is regarded with neglecting of hydrodynamic effects, therefore, it is reasonable to use in this case of modelling, the term: *thermal model of PECM.*

In PECM with short pulse on-time, the effects of changes in the gap size, $S$, on the current density and Joule heat generation are not significant and can be neglected, therefore, the integration of Eqs. (6), can be simplified by treating $S$ as a constant. Upon the integration, the temperature increment equation is

$$\theta = \frac{1}{\alpha_T}\left\{\exp\left[\frac{\alpha_T \kappa_e}{\rho_e C_p S_0^2}\int_0^t U(U - E)\,dt\right] - 1\right\} \tag{7}$$

where working voltage pulse depends on time i.e. $U = U(t)$.

For example, Fig. 4 presents the results for the triangle voltage pulse. The following parameters are constant throughout the computer simulation process: $\rho$ = 1070 kg/m$^3$, $C_p$ = 3800 J/kgK, $\alpha_T$ = 0.02 1/K, $E$ = 3.0 V. The typical current $i$ and the electrolyte temperature $\theta$ characteristics are presented in Fig. 4.

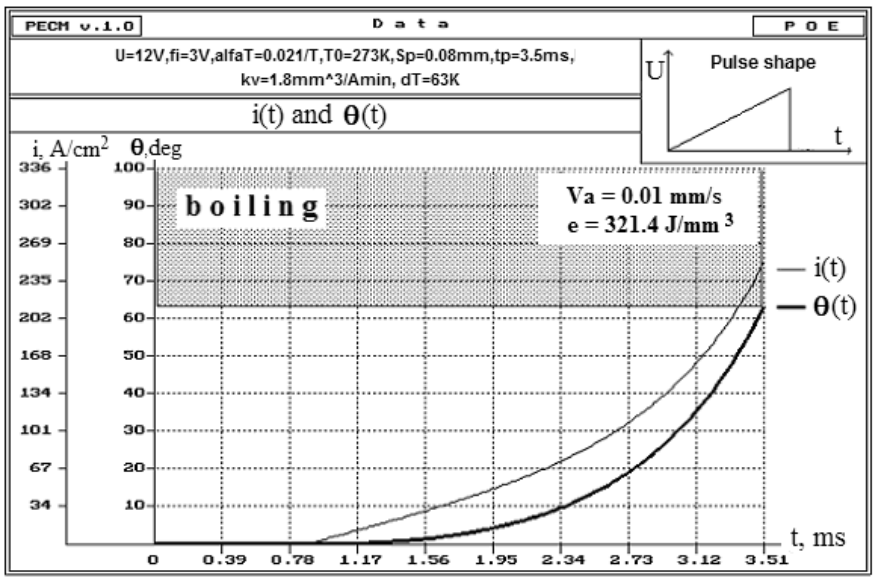


Fig. 4. The current and the electrolyte temperature changes during pulse on-time for triangular shaped pulse (::::::: boiling area)

If electrolyte boiling is considered as the limit for maintaining a stable machining process, the expression of the gap size, denoted as $S^*$ can be obtained from simulation with condition as following: $t = t_p^*$, $\theta = \theta_b$, where: $\theta_b = T_{boil} - T_0$, and $T_{boil}$ is the boiling temperature of the electrolyte under the gap conditions. The change of the critical gap size with pulse on-time for given voltage $U = 12$ V at different shape pulse, illustrates Fig. 5 [15].

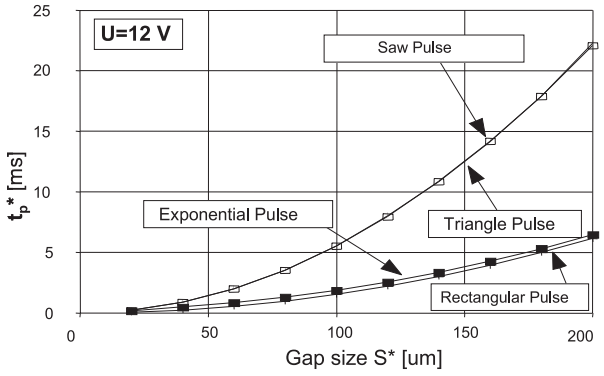


Fig. 5. Critical pulse on time vs. gap size at different shape of voltage pulse [15]

Let us consider the case of PECM with using rectangular pulse of voltage $U = \text{const}$. For rectangular pulse of voltage, the Eq. (7) becomes

$$\theta = \frac{1}{\alpha_T}\left\{\exp\left[\frac{\alpha_T \kappa_e}{\rho_e C_p S_0^2} U\,(U-E)\,t\right] - 1\right\}. \tag{8}$$

For this case, from Eq. (5), a differential equation $S$ and $\theta$ can be derived as:

$$\frac{dS}{S} = \frac{\rho_e C_p K_V}{U} d\theta. \tag{9}$$

The solution of this equation upon integration can be expressed as follows:

$$S = S_0 \exp\left(\frac{\rho_e C_p K_V}{U}\theta\right). \tag{10}$$

For typical PECM parameters mentioned earlier, the exponent in Eq. (10) is in order of $10^{-3}$, therefore, the Eq. (10) can be approximated as

$$S = S_0\left(1 + \frac{\rho_e C_p K_V}{U}\theta\right). \tag{11}$$

The material removal thickness $\Delta S = S - S_0$ during a single pulse is equal to

$$\Delta S = \frac{\rho_e C_p K_V}{U}\theta S_0. \tag{12}$$

If the temperature increment $\theta$ is known at the end of the pulse, denoted as $\theta_p$, the gap changes at the time $t = t_p$ can be calculated using Eq. (12). After substituting Eq. (8) into Eq. (12), removed thickness at time $t_p$ is equal to:

$$\Delta S = \frac{\rho_e C_p K_V}{\alpha_T U} \times S_0\left\{\exp\left[\frac{\alpha_T \kappa_e}{\rho_e C_p S_0^2} U\,(U-E)\,t_p\right] - 1\right\}. \tag{13}$$

The average material removal rate, $V_a = \Delta S/t_p$, during the pulse on-time is expressed as

$$V_a = \frac{\rho_e C_p K_V}{\alpha_T U t_p} \times S_0\left\{\exp\left[\frac{\alpha_T \kappa_e}{\rho_e C_p S_0^2} U\,(U-E)\,t_p\right] - 1\right\}. \tag{14}$$

With this equation, the tool-electrode averaged feed rate $V_f$, in quasi-steady state PECM can be determined by regarding of pulse duty cycle $\sigma = t_p/T$:

$$V_f = \sigma V_a. \tag{15}$$

If electrolyte boiling is considered as the limitation for maintaining a stable machining process, the expression of the minimum gap size for given $t_p$, denoted as $S^*$, can be obtained from Eq. (8) as

$$S^* = \sqrt{\frac{\alpha_T \kappa_0 U\,(U-E)}{\rho_e C_p \ln\left(1+\alpha_T \theta_b\right)}\, t_p}. \tag{16}$$

From the point of view of thermal limitation the critical pulse on-time is expressed as

$$t_p^* = \frac{\rho_e C_p \ln\left(1+\alpha_T \theta_b\right)}{\alpha_T \kappa_0 U\,(U-E)} S^2. \tag{17}$$

The change of the critical gap size at varies the amplitude voltage, $U$, for given pulse on-time, what illustrates Fig. 6.

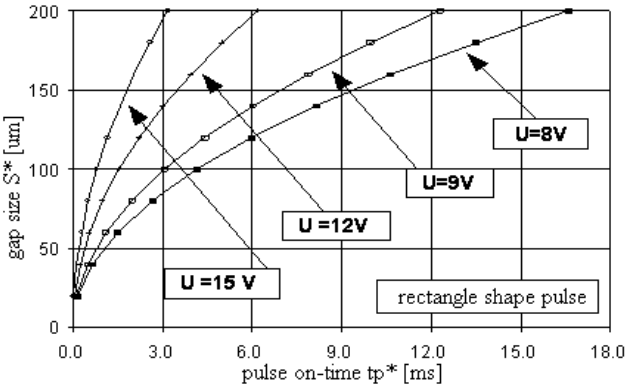


Fig. 6. The effect of on-time $t_p^*$ on the gap $S^*$ under different voltages $U$

**2.2. Thermal model of PECM with heat transfer through electrodes.** Neglecting the convection term also implies that, in this particular case, $\theta$ is dependent only on time and a coordinate $y$ in Fig. 7 (coordinate $y$ is corresponding to coordinate $\zeta$ in Fig. 3), and $S$ at a fixed coordinate $\xi$, depends only on time.

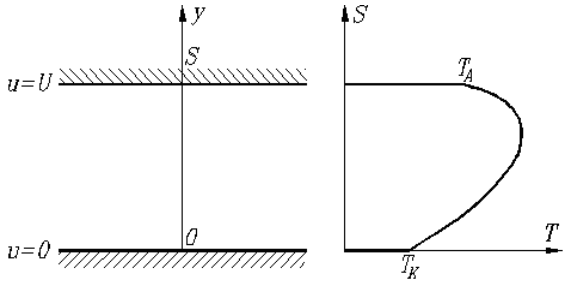


Fig. 7. Scheme for PECMM without a flow convection of heat

Let us consider a single pulse and a case of PECM with full renovation of the electrolyte in the gap, i.e. at Strouhal number for flushing period $St_0 = L/wt_0 > 1$.

According to Eq. (2 and 3), a mathematical model can be obtained describing the changes in the temperature and current density which occur in the gap as follows (18):

$$\frac{\partial \theta}{\partial t} = a\frac{\partial^2 \theta}{\partial y^2} + \frac{i^2}{\rho C_p \kappa_0 \left(1+\alpha_T \theta\right)}, \tag{18}$$

$$i = \frac{\kappa_0\,(U-E)}{\int\limits_0^S \frac{dy}{1+\alpha_T \theta}},$$

$$\theta\,[y=0] = \theta_K$$

$$\theta\,[y=S] = \theta_A.$$

The heat transfer is described by partial, nonlinear differential equation; therefore, a solution can be obtained by using numerical methods. For example, the change of the temperature in time and across the gap is shown in Fig. 8.

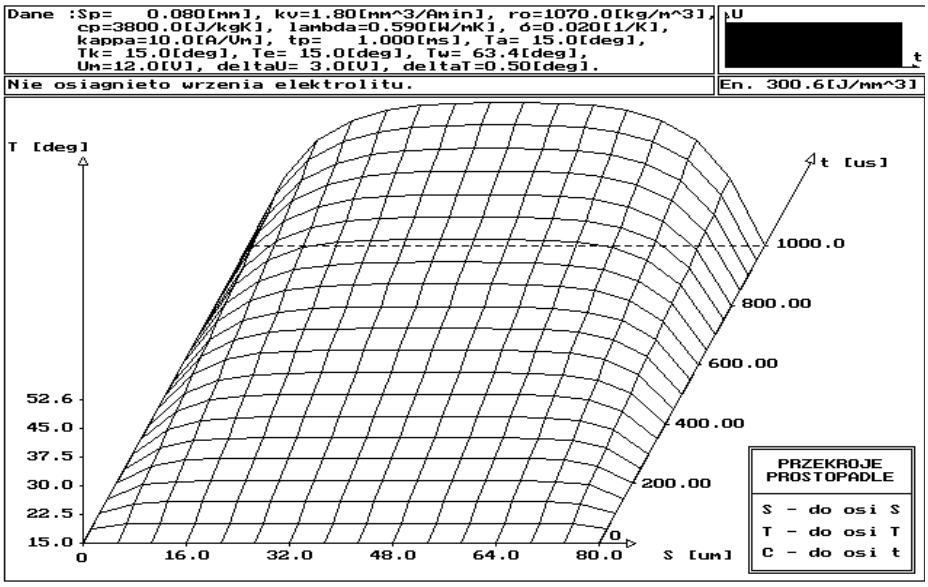


Fig. 8. Electrolyte temperature distribution across the gap size in time-on for rectangular shaped pulse

An estimated value of critical pulse on-time from the give adiabatic model is lower, in comparison with the estimated value, than in the model in which heat transfer through electrodes regarded. Therefore, application of the adiabatic thermal model to calculation critical conditions of PECM leads to "more safe" estimation.

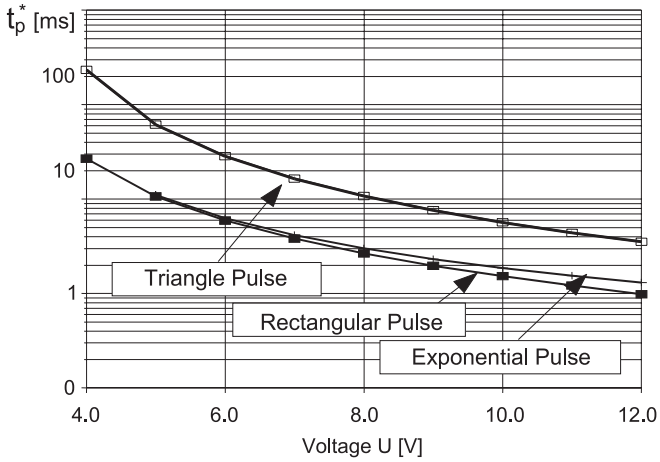


Fig. 9. Critical pulse on time vs. voltage amplitude

The Fig. 9 indicates that increasing of amplitude of pulse voltage leads to very significant decrease critical pulse on time. For example, at rectangular pulse increase of voltage from 6 V to 12 V, reduced pulse on time from 6 ms to 1 ms.

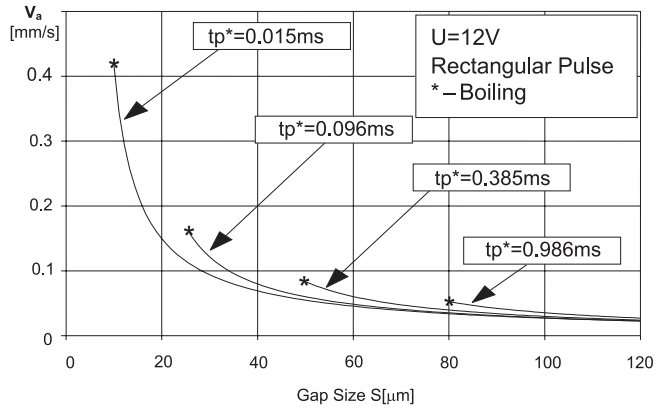


Fig. 10. The effect of the initial gap size on the average removal rate, with different pulse on times

In order to improve shaping accuracy and simplify tool design, the down-slope of curves describing average material removal rate $V_a = f(S)$ should be as steep as possible so that the material removal rate will decrease sharply for any increase in the gap and a high degree of anodic dissolution localization can be attained. It was proven that the slope of this function encountered in PECM is indeed steeper than that in the ECM with continuous current [11–15]. Typical curve $V_a = f(S)$ obtained from computer simulation based on thermal adiabatic model is shown in Fig. 10. Calculations were carried out for rectangular pulse of voltage and at assumption that avoid increment of temperature is equal to $\theta = 70$ K at $S = 0.08$ mm (i.e. final temperature for $S = 0.08$ mm is 5 degree below of the boiling point of electrolyte).

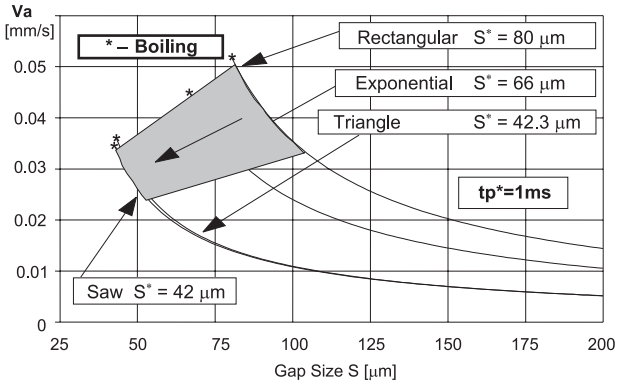


Fig. 11. The effect of the initial gap size on the average MRR, with different shape of voltage

Curves of function $V_a = f(S)$ for the different shape of voltage pulses at constant the pulse on time $t_p = 1$ ms, illustrates Fig. 11.

The specific energy consumption $e$ is one of very important factor in ECM process. The specific energy consumption, is defined as

$$e = \frac{\int_0^{t_p} U(t)\, i(t)\, dt}{\Delta S}. \tag{19}$$

It can be concluded from diagram $e^*(S^*)$ presented in Fig. 12 that under critical conditions ($t_p^*$ and $S^*$) the changes in specific energy consumption is not significant for a given pulse shape (excluding the exponential-shaped pulse due to the pulse time constant, $\tau$, effect) and it amounts to the highest values for the rectangular shape pulse.

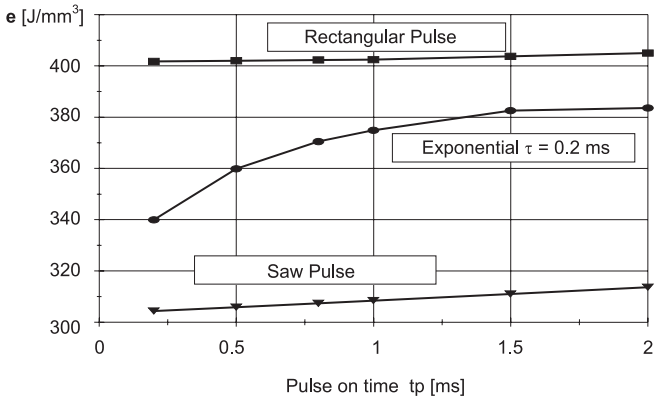


Fig. 12. The effect of pulse on time on specific energy consumption at different pulse shape

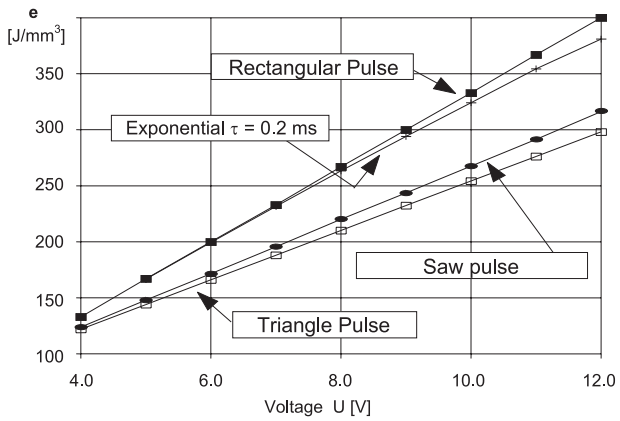


Fig. 13. The effect of voltage $U$ on the energy consumption ratio $e^*$

The energy consumption ratio is decreased according to voltage amplitude decrease and that is depicted in Fig. 13. One should be also aware that the decrease of voltage amplitude $U_0$ means the increase of the pulse time $t_p^*$.

The obtained results from the theoretical model have been verified by experimental results. The experiments were performed on a PECM machine, which is fitted with step motor driven by programmable tool feed driver. The ring is supplied from DC supply unit via a thyristor connector with forced communication, which generates controlled current pulses. The technical data for the power supply are pulse on-time $t_p = 0.1$–$10$ ms, pulse off-time $t_0 = 5$ ms to 50 s, voltage $U = 3$–$30$ V, maximum pulse current $I = 1000$ A. Solution of 15% $NH_4NO_3$ in water were used as the electrolyte. The workpiece is made of steel 45 (0.45%C) and treated to a hardness of 55 HRC.

Figure 14 shows the comparison between experimental and calculated results of the average removal rates for on-time $t_p = 2.0$ ms.

A comparison between results of the energy consumption shows that theoretical ones and the regressed experimental results have a fairly good agreement with the theoretical results (Fig. 15).

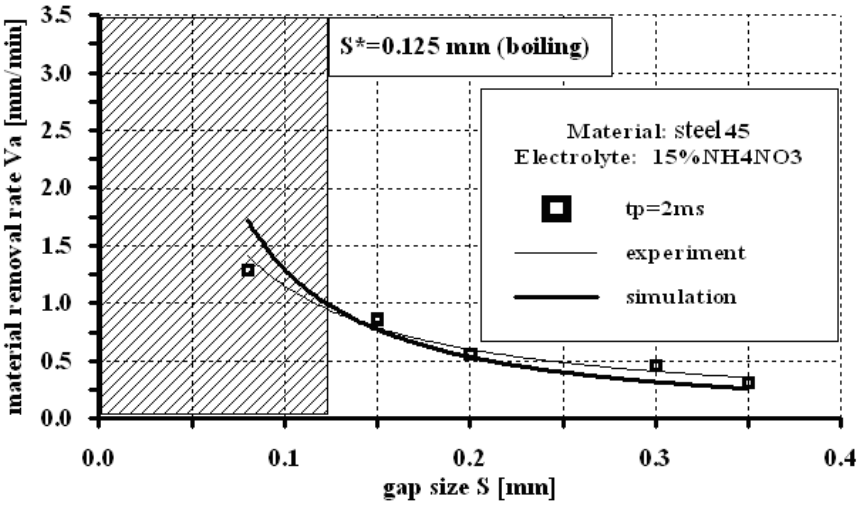


Fig. 14. The effect of gap with on the material removal rate (simulation and experiment)

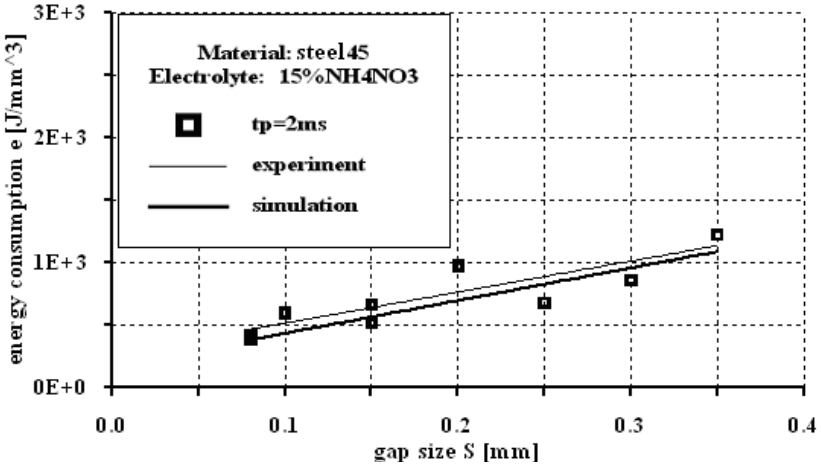


Fig. 15. The effect of gap with on the energy consumption (simulation and experiment)

## Conclusions

Setting and maintaining a small yet stable gap size in PECMM is very important to achieve a better dimensional accuracy control. The allowable minimum gap size, however, is limited by interelectrode gap conditions such as electrolyte boiling. A model developed to estimate the minimum gap size reveals that a shorter pulse on-time would allow a smaller minimum gap size without electrolyte boiling. Experimental investigations confirmed the theoretical results, which permits to recommend mathematical models and software for simulation of PECM for practical applications.

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