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An improved GM(1.1) model with background value optimization and Fourier-series residual error correction and its application in cost forecasting of coal mine

Introduction

Forecasting is both qualitative and quantitative to anticipate future events. More specifically, qualitative forecasting is a subjective judgment of the future, while quantitative forecasting is a process of making predictions for the future by extrapolating historical data with a mathematical model. When past numerical data are not available, qualitative forecasting techniques such as the Delphi method and market research are applied to short and/or

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long-range decisions (Lee et al. 2016). When a large amount of historical data is available and it is reasonable to make some strict assumptions regarding statistical distribution of the data, quantitative forecasting methods such as time series analysis (Jandoc et al. 2015), regression analysis (Ghysels and Ozkan 2015), neural network models (Patra et al. 2016) and input-output modelling (Zhang and Anadon 2014) are appropriate.

However, it is challenging to collect sufficient historical data in many circumstances. Then, on the context of situations with few data, extreme values, emerging changes, and numerous uncertainties during the forecasting process, prediction by the above approaches is almost impossible. Grey forecasting theory, as a powerful method to manage uncertain systems with few observations or data, was initially developed by Deng (1982) to analyze and understand the system via generating, excavating, and extracting partially available information, and thus is the most adaptive approach in such situations (Lu 2018). Moreover, a core exists in grey forecasting theory, namely grey forecast modelling. At present, the basic GM(1.1) has been applied in numerous fields, including the energy field (Deb et al. 2017; Aydoğdu and Yildiz 2017; Xie et al. 2015; Tsai 2016), business and finance field (Wang 2013; Sun et al. 2016; Xia and Wong 2014; Zhao et al. 2012), and engineering field (Chen and Yu 2014; Lin et al. 2016; Yu et al. 2016; Özdemir and Özdagoglu 2017).

It is important to note that GM(1.1) is primarily adaptable to fit and predict data sequences with the pattern of exponential growth. If the data sequence is highly variable, the fitting values of GM(1.1) will not be accurate, not to mention the predictive results. Consequently, much research has been done to strengthen the forecasting capability of the GM(1.1) model. For example, Liu (1997) developed and studied the characteristic of the buffer operator. This led to a new approach in finding a way out of the trap in GM(1.1). To transform original sequence into some kinds of data, some universal weakening operators such as weighted average weakening buffer operator, weighted geometric average weakening buffer operator and the geometric average weakening buffer operator are applied (Dang et al. 2007).

Moreover, other researchers mainly focused on improving approaches in calculating the variables of GM(1.1) as well as establishing reformed grey forecasting model combined with other techniques. Specifically, Hao-jun and Yuan (2009) improved GM(1.1)'s background value and initial value through the method of cubic spline interpolation. Bahrami, Hooshmand, and Parastegari (2014) utilized the algorithm of particle swarm optimization to generate the coefficient of GM and applied a model on the basis of the combination between wavelet transform and GM short-term electric load forecasting problem. Chang et al. (2015) proposed a box plot to comprehend data characteristics and apply the novel approach to optimize background values in GM to modify the prediction performance. Liu et al. (2016) proposed a combination prediction model on the basis of grey models and BP (Back Propagation) neural network, and modified the residual sequence via Markov Chain.

Despite all the work done to amend the predictive capability of grey models, the precision of existing grey forecasting models may not always be satisfying in different scenarios. This paper demonstrates the validation of the prediction model combined grey forecasting model with Fourier-series residual error correction. On the one side, the optimal GM(1.1) is obtained from the optimization of the background value based on the golden segmentation searching method and taking the metabolism for the regression sequence into account. On the other hand, the Fourier-series correction approach improves predictive performance from the input data series and does not have an impact on the local features of grey models (Tan and Chang 1996). Thus, this approach is adopted aimed at modifying the residual error of the optimal GM(1.1).

In order to verify the effectiveness of the proposed model, both the fluctuation data of the numerical example in the reference of Hou et al. (2013) and the practical application are used. All these simulation results indicated that the proposed model could offer a more precise forecast than several different kinds of grey forecasting models. Through simulation results, this study offers an effective model in dealing with the high fluctuation sequence. Its application on forecasting the unit costs in coal mining is meaningful for decision-making and enhancing the competitiveness of enterprises.

The rest of this paper is summarized as follows. In the next section, the dynamic grey model (DGM(1.1)) based on background value optimization and Fourier-series residual error correction is proposed. Based on this model, in Section 3, the presented new model is employed in the prediction of unit costs in coal mining, and model simulation and model validation are conducted in this section. The results demonstrate that the presented new model has better performance when compared with other grey models. Finally, Section 4 summarizes this paper.

1. Research methodology

Grey forecast modelling and Fourier-series residual error correction are used as the primary research methodology. Figure 1 presents an overview for developing the improved GM(1.1) forecasting model with background value optimization and Fourier-series residual error correction; the specific modelling steps are summarized as follows.

2.1. Dynamic GM(1.1) with background value optimization

Model effectiveness and accuracy are essential for any prediction model. The regular forecasting model is applicable to the developing tendency of short-term prediction; and a dynamic forecasting model that can mitigate abrupt disturbance will be better. The dynamic GM(1.1) model with background value optimization is an improved time-series prediction model on the basis of the traditional GM(1.1) model, and it is derived as below.

Suppose there is a time sequence described by the following formulation:

$$c^{(0)} = \left\{c^{(0)}(1), c^{(0)}(2), \ldots, c^{(0)}(n)\right\} \tag{1}$$

- $c^{(0)}(t) \geq 0$, $t = 1, 2, \ldots, n$; $c^{(0)}(t)$ is a non-negativity sequence and n is the amount of the data.

Utilizing the sequence, the following basic operations to construct the dynamic GM(1.1) with background value optimization need to be conducted.

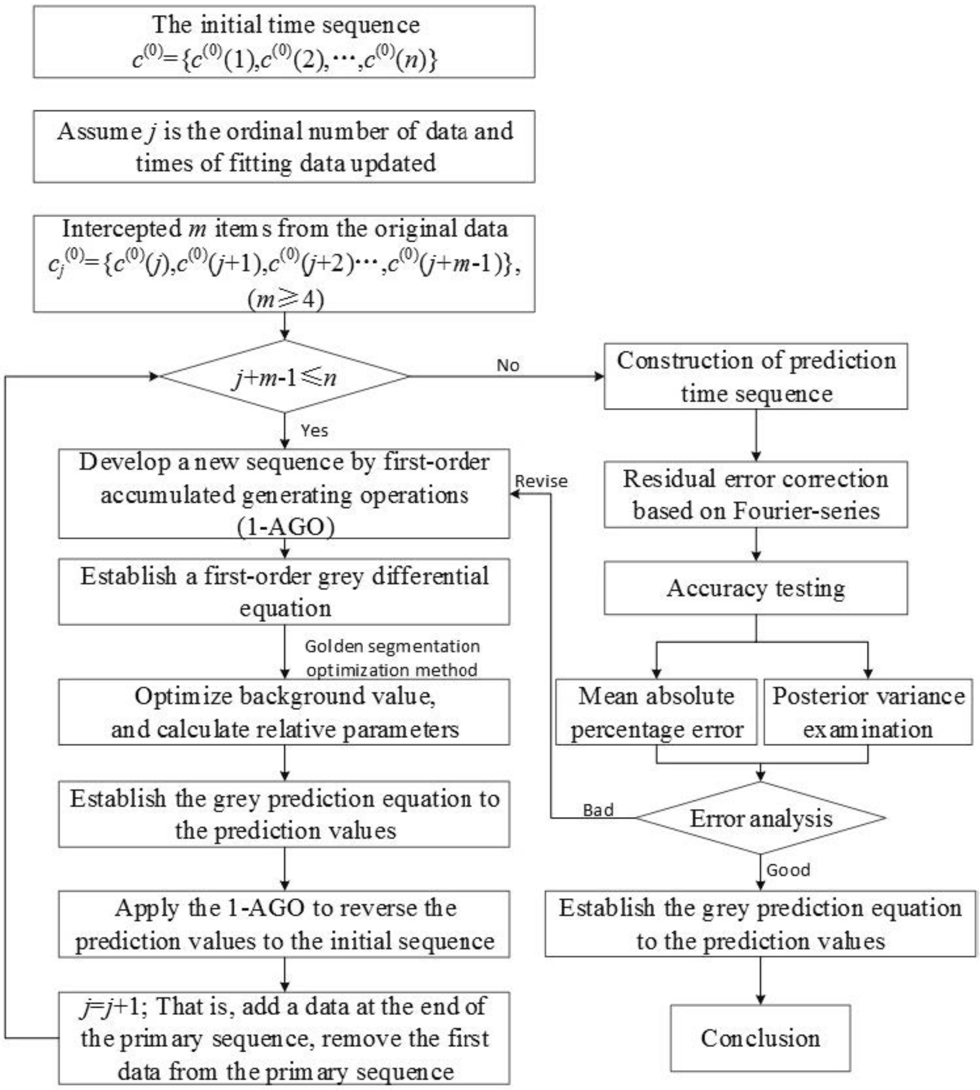

Fig. 1. Scheme of dynamic GM(1.1) with background value optimization and Fourier-series residual error correction

Rys. 1. Schemat dynamiczny szarego modelu pierwszego rzędu GM(1.1) z optymalizacją wartości tła i korektą błędów resztkowych szeregów Fouriera

◆ Step 1: Selection of the primary data sequence

Assume j is the ordinal number of data in (1) and times of fitting data updated. First, intercept m items from (1), the primary data sequence for prediction is described as follows:

$$c_j^{(0)} = \left\{c^{(0)}(j), c^{(0)}(j+1), c^{(0)}(j+2), \ldots, c^{(0)}(j+m-1)\right\} \tag{2}$$

$m \geq 4$ and $j + m - 1 \leq n$; the value of j is defined starting from 1.

◆ Step 2: Development of the 1-AGO (first-order accumulated generating operations) sequence

To figure out the regularity, a new pattern of the sequence is created via first-order accumulated generating operation (1-AGO):

$$c^{(1)}(t) = \sum_{i=j}^{t} c^{(0)}(i) \qquad t = j, j+1, \ldots, j+m-1 \tag{3}$$

Therefore,

$$\begin{aligned} &c^{(1)}(j) = c^{(0)}(j) \\ &c^{(1)}(j+1) = c^{(0)}(j) + c^{(0)}(j+1) \\ &\ldots \\ &c^{(1)}(j+m-1) = c^{(0)}(j) + c^{(0)}(j+1) + \ldots + c^{(0)}(j+m-1) \end{aligned} \tag{4}$$

Then, the 1-AGO sequence is formed:

$$c_j^{(1)} = \left\{c^{(1)}(j), c^{(1)}(j+1), c^{(1)}(j+2), \ldots, c^{(1)}(j+m-1)\right\} \tag{5}$$

◆ Step 3: Establish a first-order grey differential equation

The first-order grey differential equation can be denoted as follows:

$$c^{(0)}(t) + a_j x_j^{(1)}(t) = b_j, t = j, j+1, \ldots, j+m-1 \tag{6}$$

$x_j^{(1)} = \{x^{(1)}(j), x^{(1)}(j+1), x^{(1)}(j+2), \ldots, x^{(1)}(j+m-1)\}$ is the adjoining mean created sequence of $c_j^{(1)}$, and $x^{(1)}(t)$ is the background value. Usually, in the traditional grey model, $x^{(1)}(t)$ can be calculated by:

$$x^{(1)}(t)=\left[c^{(1)}(t)+c^{(1)}(t-1)\right]\Big/2,\ t=j+1,\ j+2,\ldots,\ j+m-1 \tag{7}$$

The albino formula is therefore, as follows:

$$\frac{\mathrm{d}c^{(1)}(t)}{\mathrm{d}t}+a_j c^{(1)}(t)=b_j \tag{8}$$

↳ a_j is the developing coefficient; b_j is the controlled variable; a_j, b_j are parameters of the differential formula.

Notably, an underlying assumption exists in (7), in which the parameter $x^{(1)}(t)$ has no change during the interval $\Delta t = 1$. Whereas Δt is a relatively short-time concept, in many dynamic situations, changes occur during the interval Δt at times leading to the fluctuation of the data series.

Then, during the interval of $[t–1, t]$, both sides of (8) can be expressed by the integration method as below:

$$\int_{t-1}^{t}\frac{\mathrm{d}c^{(1)}(t)}{\mathrm{d}t}\mathrm{d}t+a_j\int_{t-1}^{t}c^{(1)}(t)\mathrm{d}t=b_j\int_{t-1}^{t}\mathrm{d}t \tag{9}$$

Then simplify (9),

$$c^{(0)}(t)+a_j\int_{t-1}^{t}c^{(1)}(t)\mathrm{d}t=b_j \tag{10}$$

According to (6) and (10), we could know that the real background value should be as follows:

$$x^{(1)}(t)=\int_{t-1}^{t}c^{(1)}(t)dt \tag{11}$$

Equation (11) could also be expressed by the linear combination of $c^{(1)}(t)$, $c^{(1)}(t–1)$ and variable p. Namely,

$$\exists p\in[0,1],\ x^{(1)}(t)=pc^{(1)}(t)+(1-p)c^{(1)}(t-1),\ t=j+1,\quad j+2,\ldots,\ j+m-1 \tag{12}$$

Therefore, using (12) to determine the background value rather than (7) can avoid unnecessary error.

- Step 4: Calculate the parameters

Through the adoption of the least-square method, a_j and b_j can be generated as:

$$\left[a_j,\ \ b_j\right]^{\mathrm{T}} = \left(B_j{}^{\mathrm{T}} B_j\right)^{-1} B_j{}^{\mathrm{T}} Y_j \tag{13}$$

Among them, $B_j = \begin{bmatrix} -x^{(1)}(j+1) & 1 \\ -x^{(1)}(j+2) & 1 \\ \cdots & \cdots \\ -x^{(1)}(j+m-1) & 1 \end{bmatrix}$, $Y_j = \begin{bmatrix} c^{(0)}(j+1) \\ c^{(0)}(j+2) \\ \cdots \\ c^{(0)}(j+m-1) \end{bmatrix}$, Then

$$a_j = \frac{\sum_{t=j+1}^{j+m-1} x^{(1)}(t) \sum_{t=j+1}^{j+m-1} c^{(0)}(t) - (m-1) \sum_{t=j+1}^{j+m-1} x^{(1)}(t) c^{(0)}(t)}{(m-1) \sum_{t=j+1}^{j+m-1} \left[x^{(1)}(t)\right]^2 - \sum_{t=j+1}^{j+m-1} \left[x^{(1)}(t)\right]^2}, \quad t = j+1, j+2, \ldots, j+m-1 \tag{14}$$

$$b_j = \frac{\sum_{t=j+1}^{j+m-1} \left[x^{(1)}(t)\right]^2 \sum_{t=j+1}^{j+m-1} c^{(0)}(t) - \sum_{t=j+1}^{j+m-1} x^{(1)}(t) \sum_{t=j+1}^{j+m-1} x^{(1)}(t) c^{(0)}(t)}{(m-1) \sum_{t=j+1}^{j+m-1} \left[x^{(1)}(t)\right]^2 - \sum_{t=j+1}^{j+m-1} \left[x^{(1)}(t)\right]^2}, \tag{15}$$

$$t = j+1, j+2, \ldots, j+m-1$$

- Step 5: Construct the forecasting model for the new pattern of sequence

$$c'^{(1)}(t+1) = (c^{(0)}(1) - \frac{b_j}{a_j}) \mathrm{e}^{-a_j t} + \frac{b_j}{a_j} \tag{16}$$

- Step 6: Construct the forecasting model for the primary data sequence

Since the forecasting model for the new pattern of sequence is constructed based on the sequence of 1-AGO but not the primary data, the predictive value of sequence $c'^{(0)}$ can be obtained via inversing 1-AGO, expressed as:

$$\begin{aligned} c'^{(0)}(t) &= c'^{(1)}(t) - c'^{(1)}(t-1) \\ &= (c^{(0)}(1) - \frac{b_j}{a_j})e^{-a_j(t-1)} + \frac{b_j}{a_j} - (c^{(0)}(1) - \frac{b_j}{a_j})e^{-a_j(t-2)} - \frac{b_j}{a_j} \\ &= (c^{(0)}(1) - \frac{b_j}{a_j})(1 - e^{a_j})e^{-a_j(t-1)} \end{aligned} \tag{17}$$

↳ $c'^{(0)}(j) = c^{(0)}(j)$ is the initial value of prediction model. Then, if $t = j + 1, j + 2, \ldots, j + m - 1$, $c'^{(0)}(t)$ is the fitting results of $c^{(0)}(t)$. While $t > j + m - 1$, $c'^{(0)}(t)$ is the predicted value of $c^{(0)}(t)$.

- Step 7: Optimize the background value

Actually, $x^{(1)}(t)$, which in (12) is an unknown value because it depends on the value of p. Thus, the accurate prediction values cannot be determined.

Since p serves as a background value generating variable, there is an assumption that we can obtain the optimum p (p^*) until the average relative error of prediction results reaches the minimum, and thus the optimum background value is correspondingly calculated, namely:

$$Q(p^*) = \min_{0 \le p \le 1} Q(p) \tag{18}$$

$$Q(p) = \frac{1}{m} \sum_{t=j}^{j+m-1} \left| \frac{c'^{(0)}(t) - c^{(0)}(t)}{c^{(0)}(t)} \right| \tag{19}$$

Then, the golden segmentation searching method is adopted to find p^* since the method requires only objective function evaluations and does not use the derivative of the function.

Figure 2 shows the flowchart to calculate p^* by the golden segmentation searching method. More specifically, it is assumed that the original uncertain space of p [0, 1] includes the optimal value and ε is the desirable accuracy to calculate the optimal value firstly. Secondly, choose two points from the interval [0, 1], evaluate and compare objective function at these testing points, then a part of uncertain space will be eliminated. The parameter of k means times of the uncertain space updated. Repeat the process till the uncertain space in each dimension below has the desired accuracy. Finally, the optimal value of p can be calculated according to the last renewal of uncertainty space, which can be expressed as $g_{opt} = (U_k + L_k)/2$.

- Step 8: Metabolize the data sequence for prediction

Make j equals to $j + 1$, and then repeat Step 1–7 until $j + m - 1 \le n$. That is, add data at the end of the primary sequence, remove the first data from the primary sequence, and then begin the next predictive procedure.

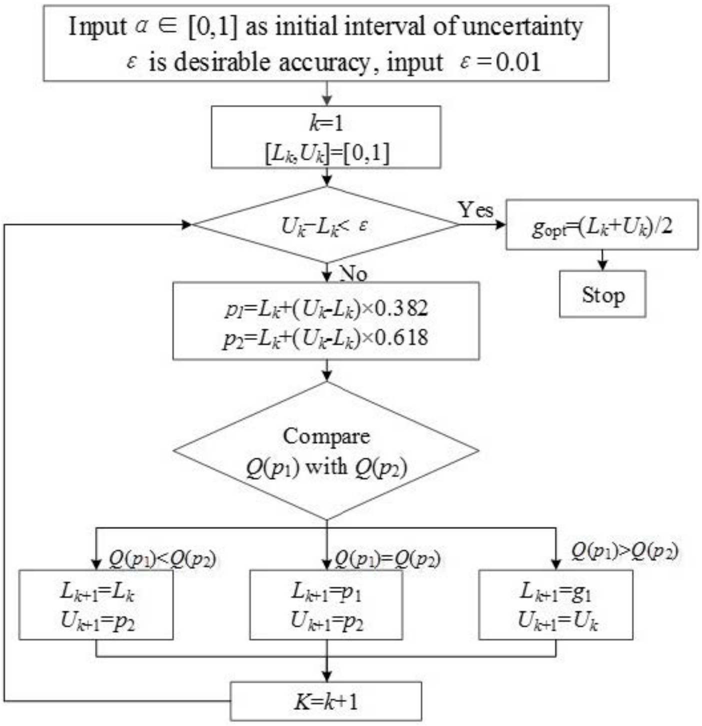

Fig. 2. Flowchart for golden segmentation searching method

Rys. 2. Schemat blokowy metody poszukiwania złotego podziału

The above processes can be implemented steadily to obtain new fitting data for each period. Also, the basic principle is to maintain the same items of data continuously, no less than four, during each predictive procedure without bypassing any data.

1.2. Fourier-series residual error correction

Fourier-series is a periodic function, which can extract the periodic information implied in the data sequence and play a role of noise reduction. In fact, any periodic function or an extendable non-periodic function satisfying the condition can be expanded into the Fourier-series and expressed as an infinite series of sine and cosine functions to become a special trigonometric function. The trigonometric functions, in turn, can be converted into exponential form according to Euler's formula. Therefore, Fourier-series can also be called an exponential-series, which could combine with the grey prediction model to extract different indexes from the data sequence and improve the grey prediction model. Steps of residual error correction based on the Fourier-series are as follows:

- Step 1: Definition of a residual sequence

Assuming that an initial residual error sequence of dynamic GM(1.1) with background value optimization is $e^{(0)}$, which can be expressed as below:

$$e^{(0)} = \left\{ e^{(0)}(2), e^{(0)}(3), \ldots, e^{(0)}(n) \right\}^{\mathrm{T}} \tag{20}$$

$$e^{(0)}(t) = c^{(0)}(t) - c'^{(0)}(t), \;\; t = 2, 3, \ldots, n \tag{21}$$

- Step 2: Development of the Fourier-series expansion

Based on initial residual sequence, the Fourier-series can be approximately expanded as:

$$E(t) = \frac{1}{2} a_0 + \sum_{i=1}^{k_a} \left[a_i \cos\left(\frac{i \cdot 2\pi}{T} t\right) + b_i \sin\left(\frac{i \cdot 2\pi}{T} t\right) \right], \;\; t = 2, 3, \ldots, n \tag{22}$$

$T = n - 1$; $k_a = [(n - 1)/2] - 1$, “[]”represents no more than the nearest integer.

- Step 3: Calculate the parameters

According to (22), the coefficient matrix D can be described as follows:

$$D = \left[a_0, a_1, b_1, \ldots, a_{k_a}, b_{k_a} \right]^{\mathrm{T}} \tag{23}$$

Through the adoption of the least-square method, D can be generated as

$$D = \left(W^{\mathrm{T}} W^{-1} \right) W^{\mathrm{T}} e^{(0)} \tag{24}$$

Where:

$$W = \begin{bmatrix} \frac{1}{2} & \cos\left(\frac{2\pi \cdot 2}{T}\right) & \sin\left(\frac{2\pi \cdot 2}{T}\right) & \cos\left(\frac{2 \cdot 2\pi \cdot 2}{T}\right) & \sin\left(\frac{2 \cdot 2\pi \cdot 2}{T}\right) & \ldots & \cos\left(\frac{k_a \cdot 2\pi \cdot 2}{T}\right) & \sin\left(\frac{k_a \cdot 2\pi \cdot 2}{T}\right) \\ \frac{1}{2} & \cos\left(\frac{2\pi \cdot 3}{T}\right) & \sin\left(\frac{2\pi \cdot 3}{T}\right) & \cos\left(\frac{2 \cdot 2\pi \cdot 3}{T}\right) & \sin\left(\frac{2 \cdot 2\pi \cdot 3}{T}\right) & \ldots & \cos\left(\frac{k_a \cdot 2\pi \cdot 3}{T}\right) & \sin\left(\frac{k_a \cdot 2\pi \cdot 3}{T}\right) \\ \ldots & \ldots & \ldots & \ldots & \ldots & & \ldots & \ldots \\ \frac{1}{2} & \cos\left(\frac{2\pi \cdot n}{T}\right) & \sin\left(\frac{2\pi \cdot n}{T}\right) & \cos\left(\frac{2 \cdot 2\pi \cdot n}{T}\right) & \sin\left(\frac{2 \cdot 2\pi \cdot n}{T}\right) & \ldots & \cos\left(\frac{k_a \cdot 2\pi \cdot n}{T}\right) & \sin\left(\frac{k_a \cdot 2\pi \cdot n}{T}\right) \end{bmatrix} \tag{25}$$

- Step 4: Residual error correction for initial predicted sequence using DGM(1.1) with background value optimization:

$$\begin{cases} c''^{(0)}(1) = c'^{(0)}(1) \\ c''^{(0)}(t) = c'^{(0)}(t) + E(t) \qquad t = 2, 3, \ldots \end{cases} \tag{26}$$

Then, when $t > n$, $c''^{(0)}(t)$ is the modified prediction results of $c'^{(0)}(t)$.

1.3. Accuracy testing

The error measures are applied to estimate the predictive precision and provide an analysis of effectiveness for the forecasting model. Two of the most widely used criteria in accessing prediction performance are summarized as below.

- Mean absolute percentage error (*MAPE*)

Since *MAPE* demonstrates that the percentage errors generated from the prediction model, the value of *MAPE* need to be as low as possible. The mathematical equation of *MAPE* and criteria can be expressed as seen in (27) and Table 1, respectively.

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\frac{\left|c^{(0)}(t) - c''^{(0)}(t)\right|}{c^{(0)}(t)} \tag{27}$$

↳ $c^{(0)}(t)$ and $c''^{(0)}(t)$ mean original value and prediction value, respectively.

Table 1. *MAPE* criteria for model assessment

Tabela 1. Kryteria *MAPE* do oceny modelu

MAPE (%)	Forecasting capability
<10	High precision prediction capability
10–20	Good prediction capability
20–50	Reasonable prediction capability
>50	Poor prediction capability

- Posterior variance examination

The posterior variance ratio C is the ratio of the error variance to the original data variance, which could be described by the following formulation:

$$C = \frac{S_e}{S_x} \tag{28}$$

Among them, S_e is the residual variance and S_x is the original data variance.

$$S_e{}^2 = \frac{1}{n}\sum_{t=1}^{n}\left[d^{(0)}(t) - \bar{d}\right]^2 \tag{29}$$

$$S_x{}^2 = \frac{1}{n}\sum_{t=1}^{n}\left[c^{(0)}(t) - \bar{c}\right]^2 \tag{30}$$

$d^{(0)}(t)$ is equal to the difference between the prediction value and original value; $\bar{d}$ is the average of $d^{(0)}(t)$; and $\bar{c}$ is equal to the average of $c^{(0)}(t)$.

Lower C values indicate a smaller S_e and a larger S_x. The smaller S_e is, the smaller the dispersion of the prediction error is. At the same time, larger S_x shows that the dispersion of raw data is large and the regularity of raw data is poor. Therefore, under the premise of large S_x, S_e should be as small as possible. That is to say, the lower the C value resulting in the better results produced by the prediction model.

In addition, the little probability of error (P) is described as follows:

$$P = P\left\{\left|d^{(0)}(t) - \bar{d}\right| < 0.6745 S_x\right\} \qquad t = 1, 2, \ldots, n \tag{31}$$

The larger P value is, the stronger generalization ability of the prediction model for original data is. The accuracy standard of posteriori error ratio C and little error probability P are listed in Table 2.

Table 2. Accuracy criteria of C and P

Tabela 2. Kryteria dokładności C i P

C	P	Forecasting ability
$C < 0.35$	$P > 0.95$	High accurate predictability
$C < 0.45$	$P > 0.80$	Good predictability
$C < 0.50$	$P > 0.70$	Reasonable predictability
$C \geq 0.65$	$P \leq 0.70$	Weak and inaccurate predictability

2. Case study

An application of the proposed forecasting model with background value optimization and Fourier-series residual error correction is demonstrated in this part; it involves the prediction of unit mining costs in a coal mine.

2.1. Variables and data

In the process of cost management in coal mines, not only the cost of coal mines should be analyzed, but also the scientific and reasonable prediction of the cost changes should be made. However, it is difficult to forecast coal costs by analyzing its numerous influencing factors including not only the external market and technology but internal factors related to geological and mining conditions in the mine.

Grey's prediction based on grey theory is an effective method to forecast coal costs, which directly figures out regular pattern from history datum of unit costs in coal mines. Hou et al. (2013) improved the basic GM(1.1) to forecast coal costs and tested its validation via comparing the prediction results of improved GM(1.1) with the basic GM(1.1). Then, in order to express the validity of the forecasting model proposed in this paper more clearly and compare prediction results of different forecasting models conveniently, the dataset used in the case study is consistent with the data sources in the reference of Hou et al. (2013) as shown in Table 3.

Table 3. Data of unit cost of coal mining in a mine (2003–2018)

Tabela 3. Dane kosztów jednostkowych wydobycia węgla w kopalni (2003–2018)

Year	2003	2004	2005	2006	2007	2008	2009	2010
Unit Cost (¥/t)	111.48	123.66	147.10	171.35	166.01	140.58	128.16	127.23
Year	2011	2012	2013	2014	2015	2016	2017	2018
Unit Cost (¥/t)	128.02	147.01	154.77	158.02	167.22	151.36	146.74	145.89

Taking the inflation caused by price rise into account, the national wage adjustment and other external factors, also with production price level in 2003 as the base, the original data is adjusted in accordance with the price indexes (Yang and Guo 2012). Hence, the data available for prediction can be obtained as follows:

2.2. Application of the traditional GM(1.1) model to forecast coal costs

Define $c^{(0)}$ as the data source. There are 15 years of data about unit costs for a coal mining operation from 2004 to 2018. After dealing with the sequence of $c^{(0)}$ via 1-AGO, Figure 3 presents that $c^{(1)}$ is smoother than $c^{(0)}$.

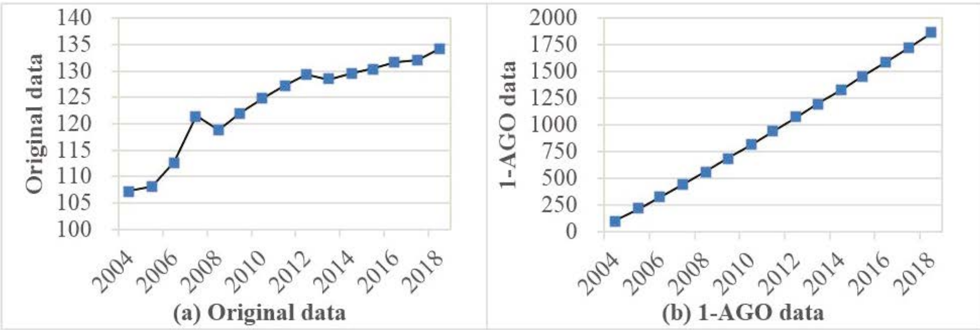

Fig. 3. Comparison of original sequence and 1-AGO sequence
(a) original data, (b) 1-AGO data

Rys. 3. Porównanie oryginalnej sekwencji i sekwencji 1-AGO
(a) dane pierwotne, (b) dane 1-AGO

Construct grey equation (similar to (6)), and the reflection equation which is similar to (8) is formed. Calculate coefficients $[a\ b]^{\mathrm{T}}$ based on least square approach and obtain results: $\begin{cases} a = -0.0134 \\ b = 112.2833 \end{cases}$. The albino equation should be:

$$\frac{\mathrm{d}c^{(1)}(t)}{\mathrm{d}t} - 0.0134c^{(1)}(t) = 112.2833 \tag{32}$$

Further, discrete time response function can be achieved as:

$$\begin{cases} c'^{(0)}(1) = c^{(0)}(1) \\ c'^{(0)}(t) = (c^{(0)}(1) + 8379.3507)(1 - \mathrm{e}^{-0.0134})\mathrm{e}^{0.0134(t-1)}, \quad t = 2, 3, \ldots, n \end{cases} \tag{33}$$

Next, validate the effectiveness by indicators such as *MAPE* and posterior variance. The results are listed in Table 4. The above processes can be resolved by Matlab software.

Table 4. Results of the traditional GM(1.1) prediction model

Tabela 4. Wyniki tradycyjnego modelu prognozowania GM(1.1)

No.	Year	Original data	Predicted values	Absolute error	Relative error
1	2004	107.26	107.26	0.0000	0.00%
2	2005	108.13	114.49	6.3564	5.88%
3	2006	112.80	116.03	3.2312	2.86%
4	2007	121.49	117.60	3.8932	3.20%
5	2008	118.77	119.18	0.4136	0.35%
6	2009	122.15	120.79	1.3583	1.11%
7	2010	124.86	122.42	2.4384	1.95%
8	2011	127.19	124.07	3.1166	2.45%
9	2012	129.37	125.75	3.6224	2.80%
10	2013	128.46	127.44	1.0157	0.79%
11	2014	129.58	129.16	0.4161	0.32%
12	2015	130.43	130.91	0.4767	0.37%
13	2016	131.68	132.67	0.9931	0.75%
14	2017	132.07	134.46	2.3933	1.81%
15	2018	134.22	136.28	2.0576	1.53%
MAPE 1.75%		*C* 0.37		*P* 0.933	

2.3. Application of the proposed GM(1.1) model to forecast coal costs

Firstly, take the first four items from $c^{(0)}$ and denote it as $c_1^{(0)}$, that is 4 years data of unit cost of coal mining in a mine from 2004 to 2007. Namely,

$$c_1^{(0)} = \left\{c_1^{(0)}(1), c_1^{(0)}(2), c_1^{(0)}(3), c_1^{(0)}(4)\right\} = \{107.26, 108.13, 112.80, 121.49\}$$

According to the traditional GM(1.1) modeling process, the first-order grey differential equation and the albino equation can be obtained after developing the 1-AGO sequence according to the theory of background value optimization. Then, the value of p is calculated by Matlab software based on the golden segmentation optimization method. The result shows that when p is 0.5747, the average simulation relative error reaches the minimum. Therefore, the optimization background value can be formed as follows:

$$x_1^{(1)} = 0.5747c^{(1)}(t) + (1 - 0.5747)c^{(1)}(t-1), \quad t = 2, 3, 4 \tag{34}$$

The coefficients $[a_1\ b_1]^{\mathrm{T}}$ are determined on the basis of least square approach, and thus obtain the following results: $\begin{cases} a_1 = -0.0589 \\ b_1 = 98.0134 \end{cases}$. The albino equation should be:

$$\frac{\mathrm{d}c_1^{(1)}(t)}{\mathrm{d}t} - 0.0589c_1^{(1)}(t) = 98.0134 \tag{35}$$

Furthermore, discrete time prediction function is expressed below:

$$\begin{cases} c'^{(0)}_1(1) = c_1^{(0)}(1) \\ c'^{(0)}_1(t) = (c_1^{(0)}(1) + 1664.0645)(1 - \mathrm{e}^{-0.0589})\mathrm{e}^{0.0589(t-1)}, \quad t = 2, 3, 4 \end{cases} \tag{36}$$

Table 5. Intermediate results during prediction process

Tabela 5. Wyniki pośrednie uzyskane podczas procesu prognozowania

No.	Year	Original data	Intermediate values	Error
1	2004	107.26	107.26	0.0000
2	2005	108.13	107.95	0.1837
3	2006	112.8	114.52	–1.7190
4	2007	121.49	121.49	–0.0018
5	2008	118.77	119.07	–0.3003
6	2009	122.15	121.28	0.8652
7	2010	124.86	124.86	–0.0005
8	2011	127.19	127.19	–0.0005
9	2012	129.37	129.37	0.0005
10	2013	128.46	128.65	–0.1935
11	2014	129.58	129.29	0.2873
12	2015	130.43	130.43	–0.0001
13	2016	131.68	131.68	0.0001
14	2017	132.07	132.07	0.0001
15	2018	134.22	134.22	0.0001

Secondly, attach the latest information $c^{(0)}(5)$ on the end of data sequence $c_1^{(0)}$ and remove the first data from $x_1^{(0)}$. Thus, a new sequence is formed as follows:

$$c_2^{(0)} = \left\{c_2^{(0)}(2),\ c_2^{(0)}(3),\ c_2^{(0)}(4),\ c_2^{(0)}(5)\right\} = \{108.13,\ 112.80,\ 121.49,\ 118.77\}$$

This operation could be repeated until the last data to be forecasted is included. And the prediction result for each period could be implemented steadily based on the above calculation process. The intermediate results of the prediction are presented in Table 5.

From Table 5, the residual series from 2005 to 2018 can be obtained as follows:

$$\begin{aligned} e^{(0)} = [&0.1837, -1.7190, -0.0018, -0.3003, 0.8652, -0.0005, -0.0005,\\ &0.0005, -0.1935, 0.2873, -0.0001, 0.0001, 0.0001, 0.0001] \end{aligned}$$

According to the theory of Fourier-series residual error correction, the modified matrix W and D can be obtained according to (24)–(25) and expressed as follows:

$$W = \begin{bmatrix} \frac{1}{2} & \cos\left(\frac{2\pi\cdot 2}{14}\right) & \sin\left(\frac{2\pi\cdot 2}{14}\right) & \cos\left(\frac{2\cdot 2\pi\cdot 2}{14}\right) & \sin\left(\frac{2\cdot 2\pi\cdot 2}{14}\right) & \dots & \cos\left(\frac{6\cdot 2\pi\cdot 2}{14}\right) & \sin\left(\frac{6\cdot 2\pi\cdot 2}{14}\right) \\ \frac{1}{2} & \cos\left(\frac{2\pi\cdot 3}{14}\right) & \sin\left(\frac{2\pi\cdot 3}{14}\right) & \cos\left(\frac{2\cdot 2\pi\cdot 3}{14}\right) & \sin\left(\frac{2\cdot 2\pi\cdot 3}{14}\right) & \dots & \cos\left(\frac{6\cdot 2\pi\cdot 3}{14}\right) & \sin\left(\frac{6\cdot 2\pi\cdot 3}{14}\right) \\ \dots & \dots & \dots & \dots & \dots & & \dots & \dots \\ \frac{1}{2} & \cos\left(\frac{2\pi\cdot 15}{14}\right) & \sin\left(\frac{2\pi\cdot 15}{14}\right) & \cos\left(\frac{2\cdot 2\pi\cdot 15}{14}\right) & \sin\left(\frac{2\cdot 2\pi\cdot 15}{14}\right) & \dots & \cos\left(\frac{6\cdot 2\pi\cdot 15}{14}\right) & \sin\left(\frac{6\cdot 2\pi\cdot 15}{14}\right) \end{bmatrix}$$

$$\begin{aligned} D = [&-0.1255, -0.1074, -0.2121, 0.2901, -0.1654, 0.0205, 0.3160, -0.1575, 0.0949,\\ &-0.0974, 0.0005, -0.0700, -0.4139]^{\mathrm{T}} \end{aligned}$$

Then, $E(t)$ is calculated according to (22), which is described as follows:

$$\begin{aligned} E = [&-0.0009, -1.5344, -0.1864, -0.1157, 0.6806, 0.1841, -0.1851, 0.1851, -0.3781,\\ &0.4719, -0.1847, 0.1847, -0.1845, 0.1847]^{\mathrm{T}} \end{aligned}$$

Put the results into (26), the simulation values of the proposed model are seen in Table 6. Validate the effectiveness by indicators such as *MAPE*, posterior variance, etc. The simulation outcomes are also listed in Table 6.

Table 6. Results of the proposed model

Tabela 6. Wyniki proponowanego modelu

No.	Year	Original data	Predicted values	Absolute error	Relative error
1	2004	107.26	107.26	0.0000	0.00%
2	2005	108.13	107.95	0.1846	0.17%
3	2006	112.80	112.98	0.1846	0.16%
4	2007	121.49	121.31	0.1847	0.15%
5	2008	118.77	118.95	0.1847	0.16%
6	2009	122.15	121.97	0.1846	0.15%
7	2010	124.86	125.04	0.1847	0.15%
8	2011	127.19	127.01	0.1846	0.15%
9	2012	129.37	129.55	0.1847	0.14%
10	2013	128.46	128.28	0.1846	0.14%
11	2014	129.58	129.76	0.1847	0.14%
12	2015	130.43	130.25	0.1847	0.14%
13	2016	131.68	131.86	0.1847	0.14%
14	2017	132.07	131.89	0.1846	0.14%
15	2018	134.22	134.40	0.1846	0.14%
MAPE 0.14%		*C* 0.02		*P* 1	

2.4. Comparison with alternative models

To validate the prediction efficiency of the model proposed in the research, another two grey models are introduced as reference models in comparison, including the traditional GM(1.1) model (denoted as GM(1.1)) and the improved GM(1.1) model proposed by Hou et al. (2013) (denoted as improved GM(1.1)). More details on these two models are presented in section 3.2 and the reference of Hou et al. (2013), respectively.

However, it should be noted that in the reference of Hou et al. (2013), there were only 8 years' prediction values of unit cost of coal mining in a mine from 2004 to 2011. Then, to ensure consistency of data comparison, the selected prediction values of different grey prediction models, including GM(1.1), improved GM(1.1) and the model presented in this paper (denoted as new grey model), are 8 years' prediction values of unit cost of coal mining in a mine from 2004 to 2011 as summarized in Table 7. Table 8 presents prediction accuracy of different grey models and Figure 4 shows these models' relative error distribution, demonstrating that the new proposed model is validated. Considering the model's simulation performance, the graph presenting original data sequence as well as three graphs presenting fitting situations of different grey models to original data are shown in Figure 5. As seen in Figure 5, the new grey model offers significant improvement in the prediction of coal mining costs in comparison to the other grey models.

Table 7. Prediction values of different grey prediction models for coal costs

Tabela 7. Wartości prognozowane dla różnych szarych modeli prognozy kosztów jednostkowych pozyskania węgla

Years	GM(1.1)		Improved GM(1.1)		New grey model	
	Prediction values	PE	Prediction values	PE	Prediction values	PE
2004	107.26	0.00%	107.26	0.00%	107.26	0.00%
2005	114.49	5.88%	107.57	0.52%	107.95	0.17%
2006	116.03	2.86%	112.65	0.13%	112.98	0.16%
2007	117.60	3.20%	120.85	0.53%	121.31	0.15%
2008	119.18	0.35%	119.92	0.97%	118.95	0.16%
2009	120.79	1.11%	121.15	0.81%	121.97	0.15%
2010	122.42	1.95%	124.54	0.26%	125.04	0.15%
2011	124.07	2.45%	127.54	0.27%	127.01	0.15%

Table 8. Prediction accuracy of different grey models

Tabela 8. Dokładność prognozowania różnych szarych modeli

Indices	GM(1.1)	Improved GM(1.1)	New grey model
MAPE	1.75%	0.44%	0.14%
C	0.37	0.09	0.02
P	0.933	1	1

From Tables 7–8 and Figures 4–5, three observations can be made:

- Overall, these grey models, achieved an accurate forecasting level combined with the criteria of *MAPE* and the accuracy standard of *C* and *P*. Among these models, the traditional GM(1.1) model shows relatively good prediction capability while the other grey models have a higher level of prediction accuracy.

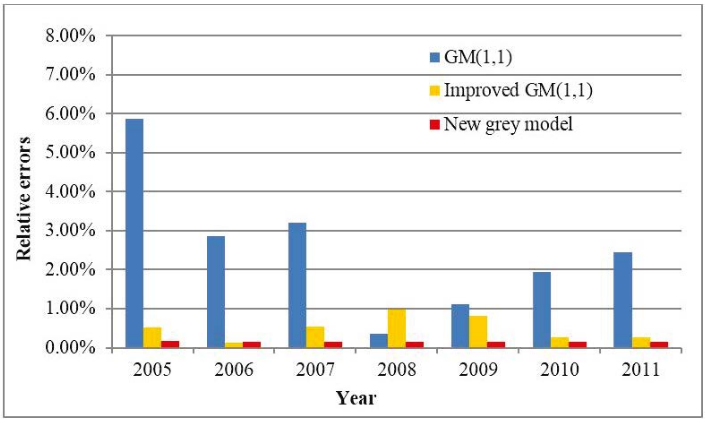

Fig. 4. Relative error distribution chart of three different grey models

Rys. 4. Wykres względnego rozkładu błędów dla trzech różnych szarych modeli

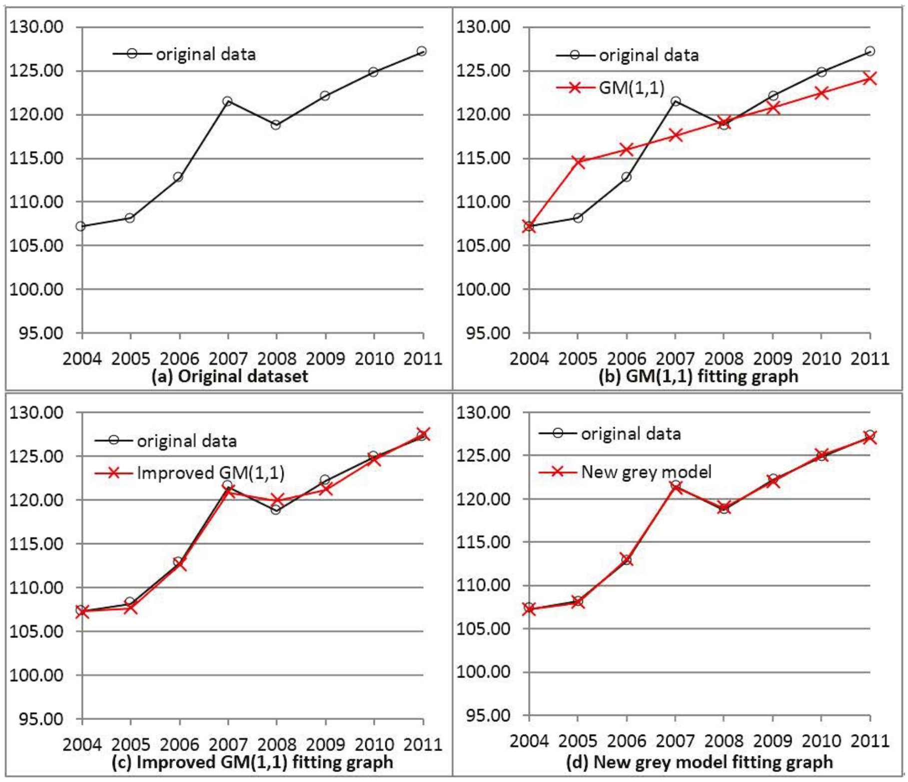

Fig. 5. Fitting curves of different grey models

Rys. 5. Krzywe dopasowania różnych szarych modeli

- As shown in Table 8, *MAPE* and *C* of the new grey model are 0.14% and 0.02, respectively, lower than the other two grey models. Thus, the new prediction algorithm presented in this paper is a very effective tool in forecasting.
- Figure 4 indicates that these grey models are robust predictors of original data because most of their relative errors are less than 5%. In most instances, however, relative errors of the new grey model are the lowest. From Figure 5, it can see that the new grey model performs better than the traditional GM(1.1) and the improved GM(1.1). Therefore, the proposed dynamic GM(1.1) model is a better choice to produce forecasting values for analyzing unit cost in coal mining operation.

Conclusions

This paper has shown three critical improvements in the presented model. Firstly, the fitting precision of GM(1.1) is enhanced by optimizing background value, where the background value is easy to obtain by searching the generation parameter p through golden segmentation searching method. Secondly, in grey forecasting model, the metabolism of data is introduced, which reflects the trend of sequence in a timely manner. Finally, the application of the Fourier-series method to extract periodic information and optimize the changing rate for correcting the residual error improves the adaptability and flexibility of the new prediction algorithm.

In the case study, the predictive results show that the forecasting precision of the grey model presented in this paper is better than the traditional GM(1.1) and the improved model. The new grey model gives a lower *MAPE* & *C* value of 0.14% and 0.02, respectively, compared to 1.75% and 0.37 respectively for the traditional GM(1.1) as well as 0.44% and 0.09 respectively for the improved GM(1.1).

Despite the value of our method, it has several limitations. On the one hand, putting the new grey model proposed in this paper to a larger data application and wider range of application is needed. On the other hand, a quantitative comparison of new grey model with other types of forecasting models should also be considered. Both of these are areas for our future research.

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REFERENCES

Aydoğdu, G. and Yildiz, O. 2017. Forecasting the annual electricity consumption of Turkey using a hybrid model [In:] *2017 25th Signal Processing and Communications Applications Conference (SIU)*, IEEE, pp. 1–4.

Bahrami et al. 2014 – Bahrami, S., Hooshmand, R.A. and Parastegari, M. 2014. Short term electric load forecasting by wavelet transform and grey model improved by PSO (particle swarm optimization) algorithm. *Energy* 72, pp. 434–442.

Chang et al. 2015 – Chang, C.J., Li, D.C., Huang, Y.H. and Chen, C.C. 2015. A novel gray forecasting model based on the box plot for small manufacturing data sets. *Applied Mathematics and Computation* 265, pp. 400–408.

Chen, P.Y. and Yu, H.M. 2014. Foundation settlement prediction based on a novel NGM model. *Mathematical Problems in Engineering* 2014, pp. 1–8.

Dang et al. 2007 – Dang, Y.G., Liu, S.F. and Mi, C.M. 2007. Study on characteristics of the strengthening buffer operators. *Control and Decision* 22, pp. 730.

Deb et al. 2017 – Deb, C., Zhang, F., Yang, J., Lee, S.E. and Shah, K.W. 2017. A review on time series forecasting techniques for building energy consumption. *Renewable and Sustainable Energy Reviews* 74, pp. 902–924.

Deng, J.L. 1982. Control problems of grey systems. *Sys. & Contr. Lett.* 1, pp. 288–294.

Chysels, E. and Ozkan, N. 2015. Real-time forecasting of the US federal government budget: A simple mixed frequency data regression approach. *International Journal of Forecasting* 31, pp. 1009–1020.

Li et al. 2009 – Li, D.W., Xu, H.J., Liu, D.L. and Xue, Y. 2009. Improved Grey Markov Model and Its Application in Prediction of Flight Accident Rate. *China Safety Science Journal (CSSJ)* 19(9), pp. 53–57.

Hou et al. 2013 – Hou, Y.B., Wang, J.M., Zhang, X., Shi, S.S. and Li, Z.D. 2013. The application of improved grey model on coal cost forecasting. *China Mining Magazine* 22(5), pp. 49–52.

Jandoc et al. 2015 – Jandoc, R., Burden, A.M., Mamdani, M., Lévesque, L.E. and Cadarette, S.M. 2015. Interrupted time series analysis in drug utilization research is increasing: systematic review and recommendations. *Journal of Clinical Epidemiology* 68(8), pp. 950–956.

Lee et al. 2016 – Lee, S., Cho, C., Hong, E.K. and Yoon, B. 2016. Forecasting mobile broadband traffic: Application of scenario analysis and Delphi method. *Expert Systems with Applications* 44, pp. 126–137.

Lin et al. 2016 – Lin, C.C., Deng, D.J., Kang, J.R., Chang, S.C. and Chueh, C.H. 2016. Forecasting rare faults of critical components in LED epitaxy plants using a hybrid grey forecasting and harmony search approach. *IEEE Transactions on Industrial Informatics* 12(6), pp. 2228–2235

Liu, S. 1997. The trap in the prediction of a shock disturbed system and the buffer operator. *Journal-Huazhong University of Science and Technology Chinese Edition* 25, pp. 25–27.

Liu et al. 2016 – Liu, X., Moreno, B. and García, A.S. 2016. A grey neural network and input-output combined forecasting model. Primary energy consumption forecasts in Spanish economic sectors. *Energy* 115, pp. 1042–1054.

Lu, S.L. 2019. Integrating heuristic time series with modified grey forecasting for renewable energy in Taiwan. *Renewable Energy* 133, pp. 1436–1444.

Özdemir, A. and Özdagoglu, G. 2017. Predicting product demand from small-sized data: grey models. *Grey Systems: Theory and Application* 7(1), pp. 80–96.

Patra et al. 2016 – Patra, A.K., Gautam, S., Majumdar, S. and Kumar, P. 2016. Prediction of particulate matter concentration profile in an opencast copper mine in India using an artificial neural network model. *Air Quality, Atmosphere & Health* 9(6), pp. 697–711.

Sun et al. 2016 – Sun, X., Sun, W., Wang, J., Zhang, Y. and Gao, Y. 2016. Using a Grey–Markov model optimized by Cuckoo search algorithm to forecast the annual foreign tourist arrivals to China. *Tourism Management* 52, pp. 369–379.

Tan, C. and Chang, S. 1996. Residual correction method of Fourier series to GM (1, 1) model [In:] *Proceedings of the first national conference on grey theory and applications, Kauhsiung, Taiwan*, pp. 93–101.

Tsai, S.B. 2016. Using grey models for forecasting China's growth trends in renewable energy consumption. *Clean Technologies and Environmental Policy* 18(2), pp. 563–571.

Wang, Z.X. 2013. An optimized Nash nonlinear grey Bernoulli model for forecasting the main economic indices of high technology enterprises in China. *Computers & Industrial Engineering* 64(3), pp. 780–787.

Xia, M. and Wong, W.K. 2014. A seasonal discrete grey forecasting model for fashion retailing. *Knowledge-Based Systems* 57, pp. 119–126.

Xie et al. 2015 – Xie, N.M., Yuan, C.Q. and Yang, Y.J. 2015. Forecasting China's energy demand and self-sufficiency rate by grey forecasting model and Markov model. *International Journal of Electrical Power & Energy Systems* 66, pp. 1–8.

Yang, M.H. and Guo, D.Y. 2012. The application of grey forecasting model in the cost forecast of coal mine. *Value Engineering* 31, pp. 128–129.

Yu et al. 2016 – Yu, T., Xiang, L. and Wu, D. 2016. Grey system and BP neural network model for industrial economic forecasting. *Recent Patents on Computer Science* 9(1), pp. 40–45.

Zhang, C. and Anadon, L.D. 2014. A multi-regional input–output analysis of domestic virtual water trade and provincial water footprint in China. *Ecological Economics* 100, pp. 159–172.

Zhao et al. 2012 – Zhao, Z., Wang, J., Zhao, J. and Su, Z. 2012. Using a grey model optimized by differential evolution algorithm to forecast the per capita annual net income of rural households in China. *Omega* 40(5), pp. 525–532.

AN IMPROVED GM(1.1) MODEL WITH BACKGROUND VALUE OPTIMIZATION AND FOURIER-SERIES RESIDUAL ERROR CORRECTION AND ITS APPLICATION IN COST FORECASTING OF COAL MINE

Keywords

cost forecasting, dynamic grey model, background value optimization, Fourier-series residual error correction

Abstract

This paper researches the application of grey system theory in cost forecasting of the coal mine. The grey model (GM(1.1)) is widely used in forecasting in business and industrial systems with advantages of minimal data, a short time and little fluctuation. Also, the model fits exponentially with increasing data more precisely than other prediction techniques. However, the traditional GM(1.1) model suffers from the poor anti-interference ability. Aimed at the flaws of the conventional GM(1.1) model, this paper proposes a novel dynamic forecasting model with the theory of background value optimization and Fourier-series residual error correction based on the traditional GM(1.1) model. The new model applies the golden segmentation optimization method to optimize the background value and Fourier-series theory to extract periodic information in the grey forecasting model for correcting the residual error. In the proposed dynamic model, the newest data is gradually added while the oldest is removed from the original data sequence. To test the new model's forecasting performance, it was applied to the prediction of unit costs in coal mining, and the results show that the prediction accuracy is improved compared with other grey forecasting models. The new model gives a MAPE & C value of 0.14% and 0.02, respectively, compared to 1.75% and 0.37 respectively for the traditional GM(1.1) model. Thus, the new GM(1.1) model proposed in this paper, with advantages of practical application and high accuracy, provides a new method for cost forecasting in coal mining, and then help decision makers to make more scientific decisions for the mining operation.

ULEPSZONY MODEL GM(1,1) Z OPTYMALIZACJĄ WARTOŚCI TŁA I KOREKCJĄ BŁĘDÓW RESZTKOWYCH SZEREGÓW FOURIERA ORAZ JEGO ZASTOSOWANIE W PROGNOZOWANIU KOSZTÓW KOPALNI WĘGLA KAMIENNEGO

Słowa kluczowe

prognozowanie kosztów, dynamiczny model szary, optymalizacja wartości tła,
korekcja błędów resztkowych z szeregów Fouriera

Streszczenie

W pracy zbadano zastosowanie teorii szarego systemu w prognozowaniu kosztów kopalni węgla. Szary model (GM(1,1)) jest szeroko wykorzystywany w prognozowaniu w systemach biznesowych i przemysłowych z niewielką ilością danych, krótkim czasem i nieznacznymi wahaniami. Ponadto model dopasowuje wykładniczo dane bardziej dokładnie niż inne techniki prognozowania. Jednak tradycyjny model GM(1,1) ma słabą zdolność przeciwdziałania zakłóceniom. Mając na uwadze wady konwencjonalnego modelu GM(1,1), w artykule zaproponowano – w oparciu o tradycyjny model GM(1,1) – nowy model dynamicznego prognozowania z teorią optymalizacji wartości tła i korektą błędów resztkowych szeregów Fouriera. Nowy model stosuje metodę optymalizacji złotej segmentacji do optymalizacji wartości tła oraz teorię szeregów Fouriera w celu wyodrębnienia okresowych informacji w szarym modelu prognozowania, aby skorygować błąd resztkowy. W proponowanym modelu dynamicznym najnowsze dane są stopniowo dodawane, podczas gdy najstarsze – usuwane z oryginalnej sekwencji danych. Aby przetestować dokładność prognozowania nowego modelu, zastosowano go do prognozowania kosztów jednostkowych pozyskania węgla, a wyniki pokazują, że dokładność prognozowania jest lepsza w porównaniu z innymi szarymi modelami prognozowania. Nowy model daje wartości *MAPE & C* wynoszące odpowiednio 0,33% i 0,07, w porównaniu z odpowiednio 1,1% i 0,3 dla tradycyjnego modelu GM(1,1). Zatem zaproponowany w artykule, ulepszony model GM(1,1) z zaletami praktycznego zastosowania i wysoką dokładnością, jest nową metodą prognozowania kosztów w górnictwie węgla, która ułatwia decydentom podejmowanie decyzji ugruntowanych naukowo dotyczących operacji pozyskania węgla.