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AN OBJECTIVE QUALITY EVALUATION METHOD FOR HAPTIC RENDERING SYSTEM: TAKING HARDNESS RENDERING AS AN EXAMPLE

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Abstract

Quality evaluation is very important for haptic rendering. In this paper, an objective evaluation method for a haptic rendering system based on haptic perception features is proposed. In the method, the haptic rendering process is compared to the real world perception process in a simple standardized procedure based on feature extraction and data analysis. A complete evaluation process for a simple haptic rendering task of pressing a virtual spring is presented as an example to explain the method in detail. Compared with the traditional objective method based on error statistics, the method is more concerned about the consistency of human subjective feelings rather than physical parameters, which makes the evaluation process more consistent with the haptic perception mechanism. The results of comparative analysis show that the method presented in this paper is simple, gives reliable results reflecting the consistency with subjective feeling and has a better discrimination ability for different kinds of devices and algorithms compared with the traditional evaluation methods.

Keywords: haptic rendering evaluation, human-machine interaction, haptic perception, measurement method, quality evaluation, bionic method.

1. Introduction

Virtual haptic rendering is a process that simulates the haptic interaction between human and environment through haptic devices. The aim of haptic rendering is to obtain good sense of reality consistent with the real world for users. The realistic quality evaluation of haptic rendering effect is an unavoidable topic in the design and improvement of haptic rendering algorithms and devices. A haptic rendering system usually consists of haptic rendering algorithms and haptic rendering hardware. There are two main types of evaluation methods in previous studies: methods based on physical parameters (objective methods) and methods based on psychophysical experiment (subjective methods). Many basic physical parameters such as mean square error of force and accuracy are proposed to assess the algorithms and devices based on physical properties. The results are objective, reliable and repeatable. However, realistic quality of a haptic rendering focuses on consistency of subjective feeling with the interaction process of the real world. Researches of human perception characteristics and mechanism have shown that consistency with physical stimuli (physical parameters) does not mean consistency with subjective perception in many cases. An evaluation method based on subjective perception experiments could measure the consistency with subjective feeling directly, but the results are easily influenced by experimental conditions and individual perceptual differences. Thus it is meaningful to find out a method with proper and reliable metrics to measure the objective quality of a haptic rendering system while taking human perceptual characteristics into consideration. So in this paper, an objective evaluation method for a haptic rendering system based on haptic perception features is proposed to simulate the user's subjective psychophysical evaluation process by extraction of perception features and consistency analysis.

In early exploration of evaluating performance of a haptic rendering system, researchers tried to choose parameters mostly used in robot mechanical devices, especially in teleoperation, to evaluate the performance of haptic devices [1–3]. Hayward and Astley [4] chose series of physical parameters defined as transparency to measure the performance of a device, including working space, inertia, stiffness, damping, refresh frequency *etc*. Colgate and Brown [5] from Northwestern University thought that stability was a fundamental problem and proposed the concept of Z-width as an important rationale in the evaluation. However, these evaluation methods based on physical property mainly focus on displacement or force tracking devices, and are hard to be applied to the evaluation of other different kinds of interactive forms such as vibration [6], electrical stimulation [7], pneumatic [8], and magneto rheological [9] with the rapid development of haptic interfaces. With the development of new kinds of haptic devices, it is difficult to define just several parameters that are complete and applicable to describe the performance of all kinds of haptic rendering systems.

At the same time, evaluation metrics based on psychophysical experiment technology were also proposed. Though some haptic rendering systems [10–12] could also be evaluated by physical parameters such as computation time, output error and so on, the subjective scores obtained from psychophysical experiments were more integrated and intuitive evaluation results. So the subjective evaluation methods have been widely used. Samur proposed a generic and systematic evaluation method of haptic interfaces based on test beds [13], different kinds of evaluation metrics were proposed based on haptic rendering tasks. For example, parameter *index of performance* (IP) was used for travel and manipulation, weber fraction was used for discrimination of force, texture and hardness, *information transfer* (IT) was used for identification of size and shape. Kirkpatrick and Douglas [14] proposed an application-driven approach to the evaluation of haptic interfaces and they considered that the evaluation method that exercises a single haptic mode tests the hardware and software of the interface on a task that has a clear relationship to actual applications. Kocsis and Cholewiak [15] in 2013 tried to use a similar discrimination ability of real and virtual surfaces with sinusoidal and triangular gratings using the fingertip and stylus as metrics to show the rendering effect based on psychological experiments. Evaluation methods based on psychological experiments take full account of human haptic perception characteristics but need a large number of manpower and resources. And the evaluation results are easily influenced by individual differences. In addition, experimental environment, controlled parameters and human perceptual habits could also greatly affect the results. So this method is usually unstable and difficult to repeat.

Some researchers used interaction information from the real world as a reference and calculated similarity between virtual and real interaction data as the evaluation metrics. Compared with psychophysical experiments, this kind of method is objective, more repeatable and stable. Okamura [16] evaluated their surgical cutting rendering system by comparing the haptic recordings from real cutting tasks with an empirical model. Similarly, Kuchenbecker *et al.* [17] assessed their event-based rendering approach using subjects tapping on real and virtual samples. Leskovsky [18] in 2006 used multidimensional scaling to identify different perceptual cues in discriminating real and virtual objects to evaluate the fidelity of haptic rendering. A standardized evaluation method for haptic rendering systems was also proposed by Ruffaldi, Emanuele and Morris in 2006 [19]. They compared data of virtual and real interaction processes by building and releasing several sets of position and force information, collected by physical scanning a set of real world objects, along with virtual models of those objects and provided a series of example analyses that demonstrate their approach's ability to quantitatively assess haptic rendering systems. In 2009, Swindells [20] compared the similarity of automatically estimated model parameters to manually tuned parameters from humans to infer the rendering effect. Hassen [21] also proposed HSSIM as a measure for haptic force-feedback signals inspired by the evaluation method of visual information quality, which provided new ideas for objective evaluation methods. In conclusion, most objective evaluation methods were based on mathematical error analysis between real and virtual interaction processes, which could not reflect the consistency with subjective feeling in many cases because of ignoring the human perception characteristics.

Therefore, it is useful and necessary to take advantages of these kinds of methods and make a balance: using stable and reliable parameters to describe the performance of a haptic rendering system while taking human perception characteristics into consideration in a simple evaluation process. So in this paper, an objective evaluation method for haptic rendering based on human haptic characteristics is proposed. Interaction data and parameters of the real world are the evaluation reference. Similarity of haptic rendering interaction data to the real world is the final metrics to measure the rendering effect. What is important is that, during the evaluation process, error analysis is carried out based on integration of effective perception features extracted according to some important human perception characteristics and that makes the similarity represent the consistency with human subjective feelings. The method is introduced in Section 2. In Section 3, a haptic rendering example of pressing a real and virtual spring is chosen to illustrate how the proposed objective evaluation method works. The results show that, compared with the traditional methods, it can avoid the impact of human subjectivity and other uncertain factors such as experiment conditions. Moreover, it is a simple and standardized process suitable for different kinds of devices and more stable and reliable.

2. Objective evaluation process

The popular subjective evaluation method is actually a consistency analysis process carried out by the users through comparison of the haptic rendering process with the real feeling process or daily experience, as shown in the left half of Fig. 1. Similarly, the objective evaluation method proposed in this paper based on human haptic characteristics is also essentially a process of comparing the haptic rendering process with the standard process (the interaction process in the real world). Haptic stimuli are perceived by different kinds of sensors instead of haptic receptors, comprehensive analysis is done by consistency analysis in the perception space by a computer instead of the human neural centre. Compared with the traditional objective evaluation method, the error analysis in this paper is performed based on integration of effective perception features extracted according to some important human perception characteristics, which make the similarity analysis results represent the consistency with human subjective feelings rather than with physical parameters. There are several aspects of work to be done in this method (shown in the right half of Fig. 1): data collection, extraction of haptic perception features and comprehensive error analysis in the perception space.

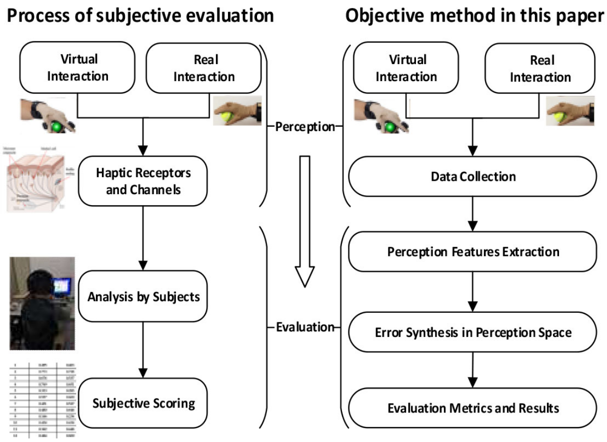

Fig. 1. An objective evaluation method proposed in this paper compared with the subjective evaluation method. For the subjective evaluation process, the perception and evaluation processes are completed by human haptic receptors and human analysis respectively. In the objective method in this paper, the perception process is performed by data collection via sensors (corresponding with haptic receptors and channels), the evaluation process is carried out by feature extraction and similarity analysis in the perception space (corresponding to subjective analysis in the central nervous system).

2.1. Data collection and representation

Different kinds of haptic rendering tasks may have different interactive modes. But there are several common physical parameters that can reflect the interaction process because the users usually perceive stimuli via hand and arm in most situations: force, displacement, velocity, acceleration, angle and so on. In the method proposed in this paper, data of these parameters should be collected by corresponding sensors selected according to different haptic rendering tasks. For example, for a grabbing task, force information is suggested being measured by flexible force sensors that can fit close to the palm and fingers and usually the maximum range of 3 N and resolution of 0.01 N are good enough for the grabbing task. Displacement, velocity and acceleration information can be measured and calculated by a micro-accelerometer such as MPU-6050. And finger bending angle information could be measured by a fibre-optic sensor such as 5DT Data Glove Ultra. In the data collection process, all the information should be collected synchronously and integrated into a multidimensional matrix changing with time. The probable general form of the haptic data matrix is shown in (1) and (2). One of the advantages of the haptic data matrix is that it can visualize interaction data in multidimensional space to help us learn more details about the interaction process, as shown in Fig. 2, which shows an example of a haptic data matrix reflecting the process of pressing a spring. What is more, it is convenient to analyse the interrelationships between these parameters and process these data using methods of matrix transform:

$$M = (M_t), \qquad t = 1, 2, \ldots, T, \tag{1}$$

$$M_t = \begin{pmatrix} F_t^i & S_t^i & A_t^i & a_t^i & \ldots \\ \dot{F}_t^i & \dot{S}_t^i & \ddot{S}_t^i & \dot{A}_t^i & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \end{pmatrix}, \tag{2}$$

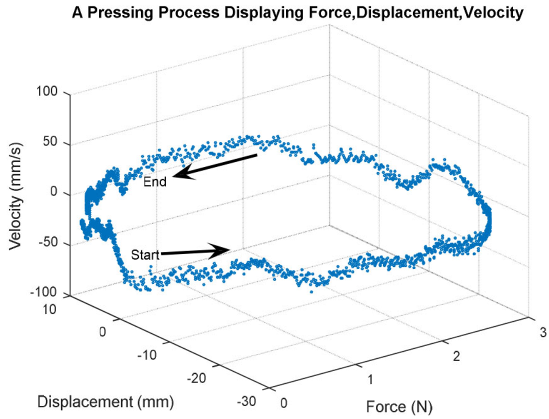

Fig. 2. An example of interaction process of pressing a spring. Force, displacement and velocity information of the pressing spring process are collected in the haptic data matrix. It is a multi-dimensional matrix changing with space and time, reflecting the changing of physical parameters and environment in the haptic rendering tasks.

where M_t is the observation state of haptic image M at time t, F_t^i, S_t^i, A_t^i, a_t^i are popular physical parameters measured in a haptic rendering interaction task, where F_t^i is the force information, S_t^i is the displacement information, A_t^i is the bending angle information of user fingers or arms, a_t^i is the acceleration information, i stands for different measuring parts of the users, $\dot{F}_t^i$, $\dot{S}_t^i$, $\ddot{S}_t^i$, $\dot{A}_t^i$ express potential physical relationships of these parameters. (1) and (2) are just an abstract universal mathematic description of a haptic data matrix. For different haptic rendering tasks, physical parameters should be chosen according to actual situations. Generation of a haptic data matrix is the preliminary task of the objective evaluation method based on human perception features. In this way, methods in other fields may be applied to the analysis of haptic data matrix and extraction of perception features.

2.2. Filtering and extraction of haptic perception features

Research results have shown that the human brain obtains the feeling regularity of stimuli through a multi-layer network model instead of processing the data directly when receiving external signals [22]. The perceptual system in this kind of hierarchy can greatly reduce the amount of data processed by brain and keep useful information. Considering the perception characteristics, haptic perception filters are used to extract useful perception features or metrics that dominate in haptic perception. This step is aiming to remove redundant information and express interaction information sparsely, just like the transformation process from physical stimulation to subjective perception accomplished by the human haptic perception channels and nervous system.

As a result of previous fundamental research on human haptic perception system, various types of characteristics such as Aristotle's illusion [23], Cutaneous Rabbit [24], Tau Effect [25] and Kappa Effect [26] have been well known to people. Memory properties [27] and the stochastic resonance phenomenon [28] in haptic perception were also discovered. At the same time, various mathematical models were built to describe these characteristics. For example, an appropriate white noise signal could be added into the filters to improve the realism of haptic rendering according to the human stochastic resonance phenomenon in haptic perception [29]. And for the Cutaneous Rabbit phenomenon, advantage of a Bayesian perceptual model [30] which is equivalent to a kind of filter can be taken in the objective evaluation method to simulate the real output feeling.

In the human haptic perception mechanism, each type of haptic perception characteristics is equipped with respectively perceiving, transmitting and comprehending channels. So each basic haptic perception characteristic and feature should be filtered and extracted based on corresponding perception models. Thus a perceptual model set for filtering and feature extraction can be expressed as:

$$\{fr(t);\ OF(t);\ JND;\ \ldots\}, \tag{3}$$

where $fr(t)$ represents a frequency characteristic filter; $OF(t)$ represents models or algorithms for feature extraction of force perception; JND is a filter designed according to different threshold characteristics of various stimuli. Just several perceptual models are listed here; proper models may vary for different haptic rendering tasks.

There are two kinds of models usually employed in this process. One kind is filters designed for data pre-processing to compress data or make data closer to human subjective feeling, such as the JND filter. And the other kind is models or algorithms for extracting features that are useful in the consistency analysis, such as models for extracting useful feature metrics for force perception.

In order to explain the process more comprehensively, some common filters or models are given here as examples.

The Stevens's power law [31]:

$$\phi(I) = kI^{\alpha}, \tag{4}$$

where I is the intensity or strength of a haptic stimulus (force, displacement, weight); $\phi(I)$ is the intensity or strength perceived by the human haptic perception system; α is an exponent that depends on the type of stimulation or sensory modality; and k is a proportionality constant that depends on the units used. This filter is usually used to transform data of objective stimuli into human subjective perceptual feeling, which is more accordant with the human perception system.

The perceptual-characteristic filter for hardness perception:

$$ERH = \frac{\max(f(t)')}{v_0}, \tag{5}$$

where ERH is a useful feature metric for haptic hardness perception proposed by Hans [32]; $\max(f(t)')$ is the maximum rate of changing force and v_0 is the initial penetration velocity in the interaction process. This filter can be used for perceptual feature extraction for haptic perception tasks related to the hardness perception.

2.3. Consistency analysis in perception space

In the objective evaluation method, the evaluation of haptic rendering is a process of analysing consistency between the evaluation reference (real process) and the target to be evaluated (haptic rendering task) in a perception space based on proper metrics. The haptic perception space is a vector space generated by dimensionality reduction based on perception features similar to the *Vector Space Model* (VSM) [33] widely used in text similarity analysis. Both the evaluation reference and target can be expressed in the space, where the recorded interaction information may reflect more of subjective feeling than the objective stimuli. For the original interaction information (Im_o) collected during any haptic rendering process in the perception space, it can be expressed as the perception information (Im_p):

$$\mathrm{Im}_p = (w_{1t},\ w_{2t},\ \ldots,\ w_{nt}) = \mathrm{Im}_o \cdot T, \tag{6}$$

where unit vectors $w_{1t}, w_{2t}, \ldots, w_{nt}$ are a group of base vectors in the perception space; w_{nt} is the value in n-th perception dimension; T is the transform matrix. The number of dimensions depends on whether the perception space can keep enough useful information and express the haptic rendering tasks effectively. For different haptic rendering tasks, the effective haptic perception features may vary, so the perception spaces may be different. A perfect perception space means that the same haptic rendering tasks of different devices and algorithms can be distinguished effectively when they are expressed in the perception space. In the perception space, both real and virtual perception processes have their own expression forms (in most situations they are coordinates), recorded as $\mathrm{Im}_R(t)$ and $\mathrm{Im}_V(t)$. Thus, the similarity S can be expressed as:

$$S = Dist(\mathrm{Im}_R(t), \mathrm{Im}_V(t)), \tag{7}$$

where the metrics are the feature distances corresponding to the perception space and features. In practical haptic rendering tasks, the distance metric may be more complex and in a high-dimensional space. A small value of distance means a better haptic rendering reality effect.

Generally speaking, the Euclidean distance is the simplest measure for the consistency analysis between real and virtual interaction processes, which can be expressed as:

$$S_{Eu} \equiv \left|\mathrm{Im}_R(t) - \mathrm{Im}_V(t)\right|. \tag{8}$$

However, for different features and haptic rendering tasks, distance measures should be selected and defined accordingly. For example, if a feature of *perceptual mean square error* (PMSE) was used for the consistency analysis, as Chaudhari [34] proposed, in the perception space the distance metric could be defined as:

$$S_{pmse} = \frac{1}{N}\sum_{i=0}^{N-1}\left(\mathrm{Im}_R(i) - \mathrm{Im}_V(i)\right)^2, \tag{9}$$

where N is the number of feature points. For instance, for Force-Feedback Signals, Hassen [35] also proposed a distance measure named HSSIM inspired by a quality assessment method for digital image, adopting the Minkowski pooling approach:

$$S_p = \left[l(\mathrm{Im}_R, \mathrm{Im}_V)\right]^{\alpha} \cdot \left[c(\mathrm{Im}_R, \mathrm{Im}_V)\right]^{\beta} \cdot \left[s(\mathrm{Im}_R, \mathrm{Im}_V)\right]^{\gamma}, \tag{10}$$

$$HSSIM = \frac{1}{N}\sum_{i=1}^{N} S_p^r, \tag{11}$$

where $l(\mathrm{Im}_R, \mathrm{Im}_V)$; $c(\mathrm{Im}_R, \mathrm{Im}_V)$; and $s(\mathrm{Im}_R, \mathrm{Im}_V)$ are the luminance feature, contrast feature, and structure feature defined and calculated analogically with the HSSIM measure for image quality assessment [36], $\alpha > 0$, $\beta > 0$ and $\gamma > 0$ are parameters used to determine the relative importance of the three comparison features; N is the number of samples in S_p, and r is the Minkowski power.

3. Example of objective quality evaluation for haptic rendering system

In this part, a simple haptic interaction process is given as an example to show the general process of objective haptic rendering evaluation method of this paper. Though not all kinds of processing steps and methods mentioned above are used in this example, it offers a template of the objective evaluation method of haptic rendering realism for more complex haptic interaction processes, which may have more complex parameters and data in a high-dimensional space.

Virtual mass spring is a basic but widely used model in virtual reality modelling such as virtual surgery, virtual remote operations and virtual reality games. So the processes of pressing a spring in the real and virtual environments are chosen as an example of rendering system to be evaluated in this paper. A haptic rendering system consists of two parts: the virtual rendering algorithms and the hardware devices. So in the objective evaluation method proposed in this paper, haptic rendering realism effects from both algorithms and hardware performance are considered, which is consistent with the common experience about human haptic perception: in most situations the users cannot distinguish whether the feeling change of haptic rendering is caused by a change of algorithms or hardware. So in this paper, haptic rendering changes both in algorithms and hardware are performed as contrasts. Three kinds of haptic devices (*Geomagic Touch, Geomagic Touch X and Omega.7*) and two kinds of rendering models of spring ($F = k\Delta x$ and $F = k\Delta x^2$) are used in this paper. *Touch* and *Touch X* are produced by the Geomagic company and *Omega.7* is produced by the Force Dimension company. Their details are shown in Table 1. For a spring model, the linear one $F = k\Delta x$ is natural and popular, whereas the nonlinear one $F = k\Delta x^2$ exhibits unnatural feeling and it was selected as a contrast because the unnatural rendering effect or a small degree of similarity to the real normal spring are also important metrics to evaluate the rendering effect. For a device with worse performance, it is possible that the user cannot distinguish which kind of spring (natural or unnatural) is being rendered while they can easily tell the difference on a better device. Thus, there are 6 kinds of haptic rendering conditions to be evaluated in total: {$F = k\Delta x$ *by Omega.7*, $F = k\Delta x$ *by Touch X*, $F = k\Delta x$ *by Touch*, $F = k\Delta x^2$ *by Omega.7*, $F = k\Delta x^2$ *by Touch X* and $F = k\Delta x^2$ *by Touch*}.

Table 1. Details of three kinds of haptic devices.

Devices	***Omega.7***	***Touch X***	***Touch***
Workspace (mm)	Φ 160 × 110	160 W 120 H 120 D	160 W 120 H 120 D
Max Force (N)	12	7.9	3.3
Stiffness (N/mm)	14.5	X 1.86 Y 2.35 Z 1.48	X 1.26 Y 2.31 Z 1.12
Resolution (mm)	0.01	0.023	0.055

3.1. Data collection

The data collection process of interaction in the virtual environment is shown in Fig. 3. When the user is pressing the virtual spring through a haptic device, the feedback forces will be put on the user's finger by the device in real time according to the spring models ($F = k\Delta x$ and $F = k\Delta x^2$). During the pressing process, the position and force information of the user's finger is recorded by the device and sensors with a frequency of 1000 Hz. For the process of pressing a spring in the real world, a baffle is placed between the fingertip and the spring to make the pressure on the spring more uniformly distributed, as shown in Fig. 4. The time information is recorded to make sure that every pressing process spends nearly the same length of time, which is

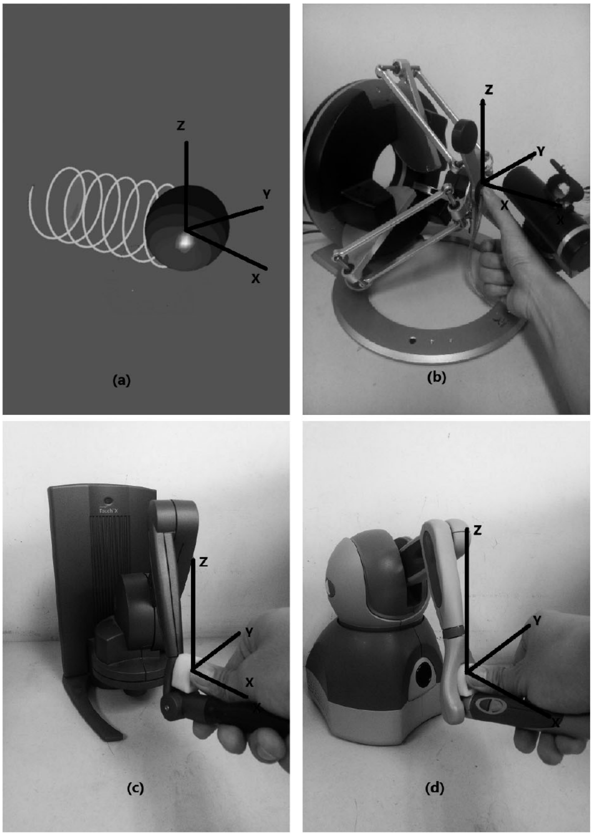

Fig. 3. Data collection of interaction on haptic devices. a) Virtual spring. The black ball is the proxy of haptic device in the virtual environment; when the subject presses the haptic device along X axis direction, the ball will press the spring along the X axis synchronously; b) *Omega.7*; c) *Touch X*; d) *Touch*. In the experiment of pressing a spring in this paper, only the DOF along the X axis is useful because the spring is restrained to produce deformation in only one direction.

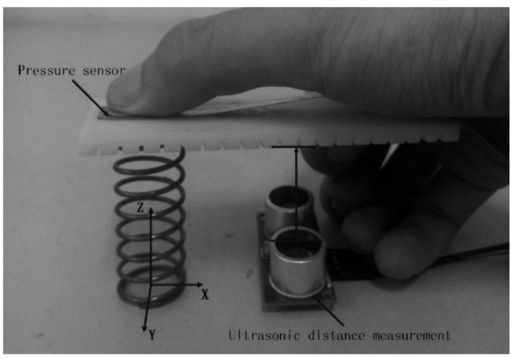

Fig. 4. Pressing a spring. The force on the subject's fingertip is measured by a flexi-force pressure sensor on the baffle when the spring is pressed. Meanwhile, an ultrasonic distance measuring module is placed at the bottom of the spring to record the distance from bottom of the spring to the baffle, which can be translated to the deformation displacement of the spring.

more convenient for the standardization and normalization process in the following steps. Fig. 5 gives the force acquisition results of pressing process in different conditions.

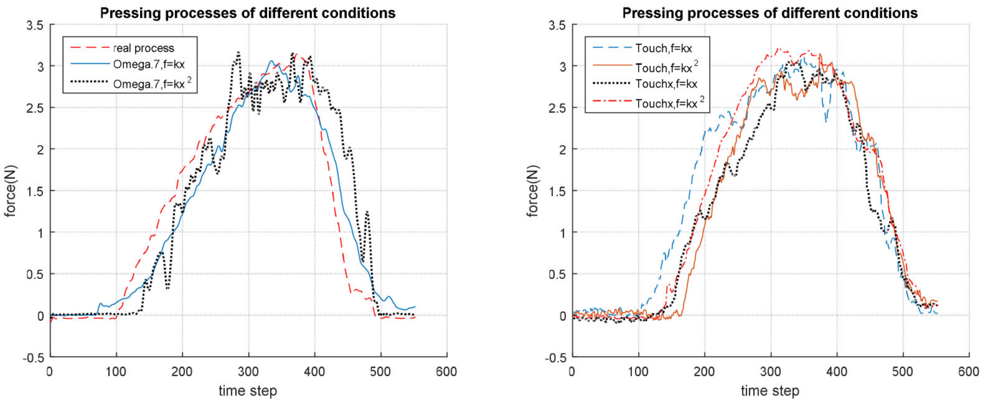

Fig. 5. The virtual and real interactive forces. Lines represent the interactive force curves of a complete real and virtual interaction process in the same experiment environment.

3.2. Pre-processing and filtering

Many fundamental researches have focused on the psychophysical experiments of human haptic perception characteristics, which can provide a lot of valuable references in the design of perception filters. Their results show that the changing rate of force and deformation play the key role in haptic discrimination of hardness [37]. So in the example of this paper, haptic perception characteristics of hardness perception about force and position are taken into account. Moreover, many researches [38, 39] have employed the *Just Noticeable Differences* (JND) as a dead-band in perceptual analysis to eliminate unperceivable force, position or velocity interaction data, which makes the data streamlined and more in line with data processing in the human perception system. So there are two main haptic perception characteristics used in the design of perception filters: JND perception filters for force and displacement and capable output frequency bandwidth of the human fingertip. The absolute force perception threshold of human fingertip is 0.06 N [40], which is much smaller than the force dead zone (0.6 N) of a pressure sensor. So the absolute force perception threshold can be ignored. The relative perception difference is 7% over the range of 2.5–10 N [41] and 15–27% for forces smaller than 2.5 N [39]. And the output force frequency bandwidth of human fingertip is 2–6 Hz [1, 42, 43]. For the JNDs, according to the Weber-Fechner's Law, the JND filter [44] is designed as:

$$\Delta I/I = K, \qquad 0 < K < 1, \tag{12}$$

where I means the magnitude of stimulus, which may be kinaesthetic distance, force, temperature or other physiological stimulus; ΔI represents the changing value of two adjacent continuous stimulations. A parameter K is called JND coefficient which usually is a constant percentage in a specific situation. When $\Delta I/I$ is smaller than K, it means that the user cannot feel the change of the stimulus magnitude. In this paper, ΔI means the change of fingertip force or displacement, I stands for the value of fingertip force. K is 20% (15–27%) when the force value is in the range of 0.06–2.5 N, and it is 7% when the force value is bigger than 2.5 N. Fig. 6 shows the results of perception filtering for all conditions.

Metrol. Meas. Syst., Vol. 26 (2019), No. 3, pp. 475–496
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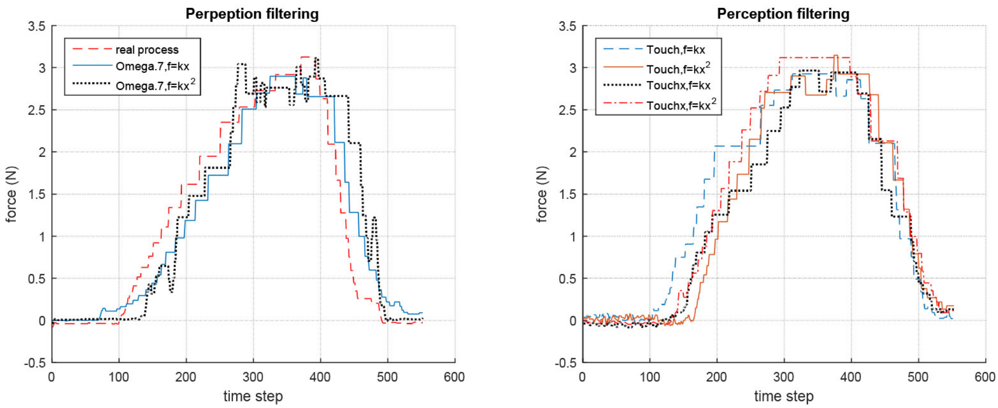


Fig. 6. Filtering results. The lines are interaction data after filtering. During this process, a continuous stimulus is transformed into discrete time series compared with original data in Fig. 5, which is closer to the real human perception feeling.

3.3. Effective perception features in hardness perception

In this step there are extracted effective features that can be used to build the perception space. Different features may reflect different characteristics of different devices and algorithms. A good perception space should have as much as possible effective features. For the perception of hardness, force, deformation and their changing rates there are usually useful features for human hardness perception [45]. Han [32] examined the relevance of 7 kinds of physical variables obtained in the experiment to the hardness perception of virtual surface. And the results show that the force changing rate, deformation changing rate and extended rate-hardness are effective metrics and features for perceived hardness. Besides, frequency is always an important parameter to measure the performance of a device. So in this paper, there are four useful features being extracted from the haptic data matrix of the pressing spring example:

Force and displacement changing features, Rate-hardness feature and *Frequency feature*.

Force and displacement changing features

A lot of researches show that the changing rate of force and displacement play the key role in haptic discrimination of hardness, which is consistent with the human common sense. So the rates of force and displacement changing with time are selected to see whether they are useful perception features in pressing spring perception. And they can be calculated as:

$$F_v = \frac{F_t(n+1) - F_t(n)}{\Delta T}, \tag{13}$$

$$S_v = \frac{S_t(n+1) - S_t(n)}{\Delta T}, \tag{14}$$

where n is the number of sampling points; ΔT is the number of sampling points between adjacent values after filtering. Fig. 7 shows F_v and S_v of virtual and real processes, in which the overall trends of change are similar, and the local features are different between the real and virtual processes in four conditions. Both the changes of devices and algorithms could lead to this result and that means that the series of pressing force and displacement changing with time are effective in reflecting the effect of haptic virtual reality rendering.

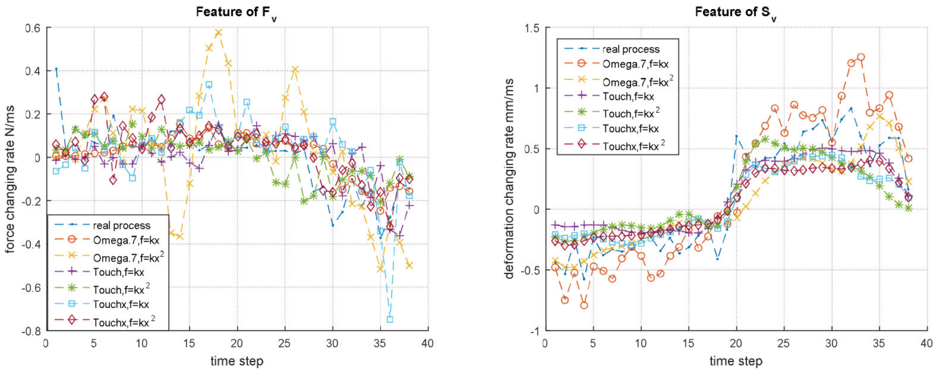


Fig. 7. a) F_v and b) S_v of the real process and of the virtual process for six haptic rendering conditions, respectively.

Rate-hardness feature

The concept of rate-hardness was proposed by Lawrence *et al*. [46] in 2000 as a feature to find out which features were dominating in perception of hardness rendering. In their research, they find out that even if the conditions of force and displacement are stable or exhibit high-level stiffness (out of perception range), the subjects can still distinguish different kinds of hardness rendering. In their following researches, they proposed the concept of rate-hardness, which is proved to be effective in perception of hardness by lots of experiments. In their work, rate-hardness is calculated only in the initial contact stage. In the example of pressing spring of this paper, the concept of rate-hardness is expanded to the whole compressing stage to examine more details of the interaction process:

$$H_R = \frac{(F_t(n+\Delta_i) - F_t(n))/(t_{n+\Delta_i} - t_n)}{(S_t(n+\Delta_i) - S_t(n))/(t_{n+\Delta_i} - t_n)}, \tag{15}$$

where F_t, S_t are the time series of force and displacement; Δ_i is the time window width, leading to different resolution and stability of rate-hardness. A small Δ_i means a high resolution but is easily affected by the measurement error, whereas a large Δ_i means a small computation volume but a low resolution where rate-hardness is nearly constant. So only a proper time window width can reflect the rendering effect of different devices and algorithms. Fig. 8 shows rate-hardness

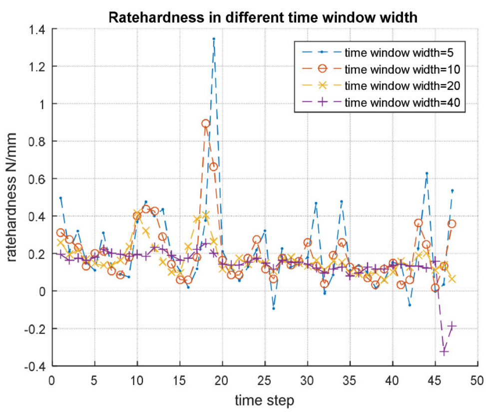


Fig. 8. Rate-hardness of the real process in different time window widths. Red, blue, dark and green lines are the rate-hardness series for the time window widths of 5, 10, 20, 40 sample points.

Metrol. Meas. Syst., Vol. 26 (2019), No. 3, pp. 475–496
DOI: 10.24425/mms.2019.129579

of the real process in time window widths of 5, 10, 20, and 40 sampling points. Analysis shows that rate-hardness has a good discrimination ability for different devices and algorithms in a time window of 10 sampling points in the pressing spring example, as shown in Fig. 9. Rate-hardness is a kind of local characteristics based on a small scale compared with F_v and S_v, and could reflect the stability and reality of haptic hardness rendering. Better devices and algorithms can achieve more stable rate-hardness.

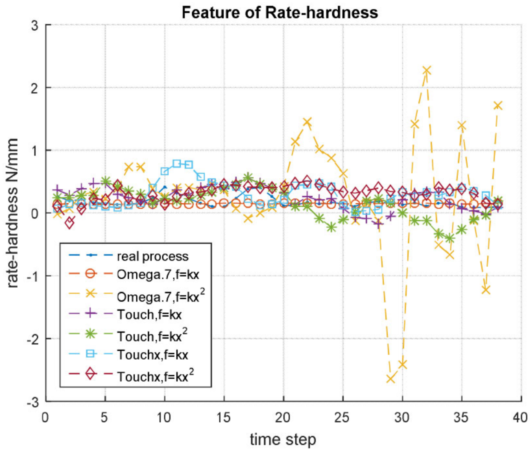


Fig. 9. Rate-hardness feature of the real process and for six conditions of virtual haptic rendering.

Frequency feature

The pressing spring process is a dynamic process. The effect of different devices and algorithms may be reflected in a frequency spectrogram. So in order to present global features of pressing spring in different conditions, force time series of pressing a spring multiple times in succession are used in the spectrum analysis. As shown in Fig. 10, the processes are composed of low-frequency signals, the different conditions are mainly reflected in the harmonic components.

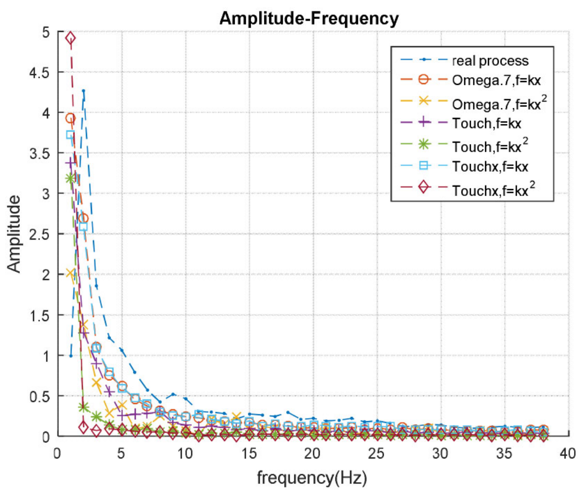


Fig. 10. Frequency feature of the real process and for six conditions of virtual haptic rendering.

So the spectrum series are selected as the frequency domain feature in the comparison of real and virtual processes. Better devices and algorithms may have a more concentrated harmonic component distribution and worse devices and algorithms may have a more dispersed harmonic component distribution, which reflects the stability and rendering effect in different conditions.

3.4. Similarity comparison in perception space

In Subsection 3.3, there are 4 different effective features (*Force and displacement changing features, Rate-hardness feature, Frequency feature*), which can be expressed as F_i, S_j, R_m, f_n. So each pressing process information can be expressed as:

$$\mathrm{Im}_k = (F_i,\ S_j,\ R_m,\ f_n), \tag{16}$$

where the *Principal Component Analysis* (PCA) method is used to build a simplest and effective perception space based on the features extracted above, which is a simple sparse representation method widely used in other fields. So in the process, each haptic perception condition is regarded as an independent input sample vector Im_k consisting of feature vectors F_i, S_j, R_m and f_n. k, i, j, m and n are dimensions of the input sample vector and four feature vectors respectively and $k$equals the sum of i, j, m and n. So the input sample set can be expressed as $\{\mathrm{Im}_1,\ \mathrm{Im}_2,\ \ldots,\ \mathrm{Im}_N\}$, N is the number of conditions, which is 7 in total in this paper. So the average vector of the sample set is:

$$\overline{\mathrm{Im}} = \frac{1}{N}\sum_{i=1}^{N} \mathrm{Im}_k \tag{17}$$

and the covariance matrix of the sample set is:

$$\sum = \frac{1}{N}\sum_{i=1}^{N} \left(\mathrm{Im}_k - \overline{\mathrm{Im}}\right)\left(\mathrm{Im}_k - \overline{\mathrm{Im}}\right)^T. \tag{18}$$

Then, if we calculate the feature vector u_i of the covariance matrix and the corresponding eigenvalues λ_i, the matrix T composed of these feature vectors is the orthogonal basis of the perception space. And the interaction information of different samples is kept in the feature vectors with larger eigenvalues. If we sort the eigenvalues of the covariance matrix in the descending order ($\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_d \geq \ldots \geq \lambda_i$) and select λ_i greater than λ_d as the principal components, the transition matrix T that is mapping perception features into the perception space can be expressed as:

$$T = (u_1,\ u_2,\ \ldots,\ u_d), \tag{19}$$

where u_i is the corresponding feature vector of the covariance matrix and d is the final dimensionality of the perception space.

In the evaluation process of this paper, the analysis results show that the first three principal components could keep more than 90% information of Im_k, which means that a three-dimensional perception space is good enough to express the pressing process:

$$\mathrm{Im}_i = (x_1,\ x_2,\ x_3) = (F_i,\ S_j,\ R_m,\ f_n)\cdot T. \tag{20}$$

Here, T is the transition matrix, through which the original feature matrix can be mapped into the perception space, $(x_1,\ x_2,\ x_3)$ are positions of features in the perception space, *e.g.*, the real

Metrol. Meas. Syst., Vol. 26 (2019), No. 3, pp. 475–496
DOI: 10.24425/mms.2019.129579

pressing process can be expressed in the perception space as (0.9280, 4.4891, −2.7224). And the distance between the real and virtual processes can be expressed as:

$$S_i = \|\mathrm{Im}_{vi} - \mathrm{Im}_r\|, \tag{21}$$

where S_i means the distance of i-th haptic rendering condition; Im_{vi} and Im_r are coordinate values of the virtual and real perception processes expressed in the perception space, respectively. Coordinate values in different conditions in the perception space are shown in Fig. 11 and Table 2.

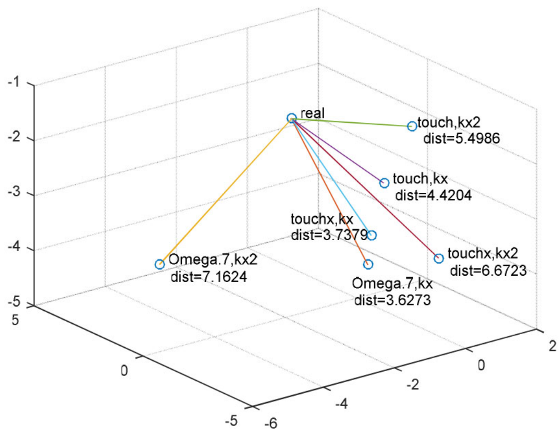


Fig. 11. Distances in the perception space. Information on six conditions of haptic rendering and the real process is expressed in a 3D perception space. Distances to the real process reflect the rendering effect of different devices and algorithms.

Table 2. Coordinate values of different conditions in the perception space (Sorted by distances to the real process).

Conditions	**Coordinate values** $(x_1,\ x_2,\ x_3)$	**Distance to real process in perception space**
Real process	(0.9280, 4.4891, −2.7224)	0
Omega.7 and $F = k\Delta x$	(1.3232, 1.6353, −4.9262)	3.6273
Touch X and $F = k\Delta x$	(1.0800, 1.0796, −4.2468)	3.7397
Touch and $F = k\Delta x$	(0.8210, 0.0836, −3.0685)	4.4204
Touch and $F = k\Delta x^2$	(0.9788, −0.9437, −1.8757)	5.4986
Touch X and $F = k\Delta x^2$	(1.0591, −2.0408, −4.0871)	6.6723
Omega.7 and $F = k\Delta x^2$	(−5.0980, 0.7248, −3.6258)	7.1624

As the real process is the evaluation reference, a smaller distance to the position of real process means a better haptic rendering effect. The results show that all the distances (3.6273, 3.7379 and 4.4204) of the linear spring model on three kinds of devices ($F = k\Delta x$) are obviously smaller than the distances (5.4986, 6.6723 and 7.1624) of the nonlinear model ($F = k\Delta x^2$). That means that the method in this paper can easily distinguish the effect of different algorithms. The natural spring model feels more similar to a real normal spring while the unnatural nonlinear spring models are not as expected. Specifically, *Omega.7* has the best rendering effect for $F = k\Delta x$ (distance = 3.6273) and the worst rendering effect for $F = k\Delta x^2$ (distance = 7.1624) compared with the real pressing process, *Touch* has the worst rendering effect for $F = k\Delta x$ (distance = 4.4204) and the best rendering effect for $F = k\Delta x^2$ (5.4986) compared with the real pressing process, while *Touch X* is in the middle of *Omega.7* and *Touch* for both $F = k\Delta x$ and $F = k\Delta x^2$. That means that the method in this paper can also easily distinguish the effect of different devices. *Omega.7* can achieve the most similar feeling when rendering linear spring models and the most dissimilar feeling when rendering nonlinear spring models compared with a real spring. Then, it can be concluded that *Omega.7* has the best rendering performance for hardness, *Touch* has the worst rendering performance while *Touch X* is in the middle. This conclusion can also be inferred by experiences according to the device details listed in Table 1.

4. Comparative analysis

To verify the rationality and effectiveness of the proposed objective evaluation method, a subjective perception experiment has also been carried out and a psychological scale was made in this paper as a guideline. Ten subjects participated in this experiment. Their ages ranged from 20 to 30, and all of them were right-handed and reported to have no known cutaneous or kinaesthetic problems. Informed consent was signed by all the subjects before the experiment. In the experiment, the devices (*Omega.7, Touch X* and *Touch*) and algorithms ($F = k\Delta x$ and $F = k\Delta x^2$) were combined pairwise, so there were 6 conditions in total to be perceived. Two conditions appeared each time and the subjects were asked to tell which of the rendering effect gave more similar feeling to that perceived with a real normal spring. Each condition should be compared with other conditions once in one try, so there were 15 comparisons in one try. A reverse order comparison try was carried out to eliminate the order error, so there were two tries for each subject. A psychometric curve of the psychological scale was fitted to the data to evaluate the rendering effect in 6 conditions, as shown in Fig. 12.

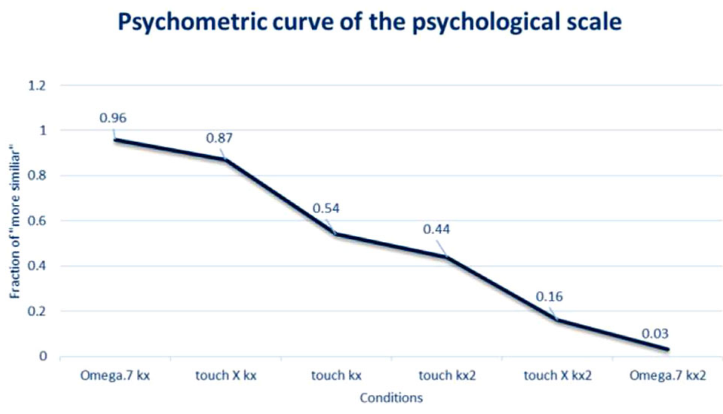


Fig. 12. A psychometric curve of the psychological scale. A higher fraction means a better rendering effect for spring and a lower fraction means a worse rendering effect.

The results presented in Fig. 12 indicate that the subjects thought that the condition of $F = k\Delta x$ by *Omega.7* had a better rendering effect in 96% comparisons and only for 3% of comparisons in the experiments the subjects thought that $F = k\Delta x^2$ by *Omega.7* had a better rendering effect. A higher fraction means a better rendering effect and a lower fraction means a worse rendering effect. So the psychometric curve indicates that for the linear spring model, *Omega.7* has the best

Metrol. Meas. Syst., Vol. 26 (2019), No. 3, pp. 475–496
DOI: 10.24425/mms.2019.129579

rendering effect and *Touch* has the worst rendering effect, while *Touch X* is in the middle. And for the nonlinear spring model, *Omega.7* has the most dissimilar rendering effect and *Touch* has the most similar rendering effect, while *Touch X* is in the middle compared with a real spring, which exhibits the same trend as the method in this paper.

In this paper, evaluation methods based on statistical analysis are also presented as a contrastive analysis. Evaluation methods based on statistical analysis [47, 48] usually select parameters such as *peak value, maximum absolute error, mean square error, distance measurement, correlation coefficient* to describe the similarity between the real and virtual pressure time series.

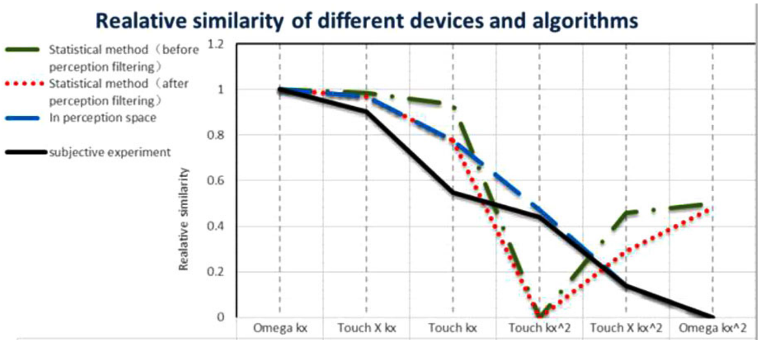
Table 3 lists the two most intuitive indexes (the *Euclidean distance* and *Pearson correlation coefficient*) to reflect the similarity to the real process. Both the *Euclidean distance* and *Pearson correlation coefficient* are calculated by statistical analysis and reflect the output errors of the rendering effect, just like the *output force error* and *output position error* used by Ruffaldi et al. in their method presented in 2006 [19]. Smaller distances and a higher Pearson correlation coefficient (ranging from 0 to 1) mean a better haptic rendering effect. As shown in Table 3, for the linear spring model, *Omega.7* has the best rendering effect and *Touch* has the worst rendering effect, while *Touch X* is in the middle, which is consistent with the results of the method in this paper. However, for the nonlinear spring model, the effect of *Omega.7* feels most like a normal real spring and *Touch* feels most unlike a normal real spring, which is totally opposite to the results obtained with the method in this paper. So more analysis is needed to explain this phenomenon.

Table 3. Statistical analysis results of the virtual process (compared with the real process).

Conditions	**Output Error (Euclidean Distance)**	**Pearson Correlation Coefficient**	**Output Error (after filtering)**	**Pearson Correlation Coefficient (after filtering)**
Real process	0	1	0	1
Omega.7 and $F = k\Delta x$	7.3078	0.9664	8.0194	0.9526
Touch X and $F = k\Delta x$	7.7797	0.9649	8.8416	0.9497
Touch and $F = k\Delta x$	8.6782	0.9595	9.1567	0.9317
Omega.7 and $F = k\Delta x^2$	10.6449	0.9144	9.8345	0.9041
Touch X and $F = k\Delta x^2$	11.6782	0.9099	12.1334	0.8864
Touch and $F = k\Delta x^2$	14.6449	0.8621	14.8596	0.8591

In order to express the similarity of these methods in the same scale, similarity results of different conditions and the perception experiment (*similarity of statistical method before perception filtering, similarity of statistical method after filtering, distances in the perception space, the subjects' scorings in the perception experiment*) are normalized into 0 to 1, which are equivalent to the relative similarity of different devices and algorithms. 0 means the worst rendering effect and 1 means the best rendering effect of the six conditions, as is seen in Fig. 13.

The results show that the algorithm of $F = k\Delta x$ and the device *Omega.7* have the best rendering effect in all four evaluation methods. But the relative significance degrees are different. As it is seen in Fig. 13, perception filtering (red line) reduces a great deal of noise and makes distances of different conditions larger, which means that the differences are easier to distinguish. The evaluation method proposed in this paper (blue line) gives a more obvious distinguishing result and is more sensitive than the method based on statistical analysis (green line) for the change of both virtual haptic rendering algorithms and haptic rendering devices because of extraction of perception features and their mapping into the perception space. Moreover, the method proposed in this paper exhibits a high similarity trend compared with the result of perception experiment



	Omega kx	Touch X kx	Touch kx	Touch kx^2	Touch X kx^2	Omega kx^2
Statistical method（before perception filtering）	1	0.9856	0.9338	0	0.4583	0.5014
Statistical method（after perception filtering）	1	0.969	0.7765	0	0.292	0.4813
In perception space	1	0.9687	0.7757	0.4706	0.1386	0
subjective experiment	1	0.9032	0.5484	0.4409	0.1398	0

Fig. 13. The relative similarity of different devices and algorithms. Green, red, blue and dark lines are the relative similarity of different evaluation methods in the same scale: similarity of statistical method before perception filtering, similarity of statistical method after filtering, distances in the perception space (the method of this paper) and the subjects' psychometric curve in the perception experiment.

(dark line), whereas statistical methods (before and after perception filtering) give absolutely opposite results for evaluation of the nonlinear spring model. These consistent evaluation results of objective and subjective evaluation methods also prove that the method proposed in this paper is more accordant with the human haptic perception mechanism, which is focused on the consistency of subjective feeling rather than stimuli.

5. Discussion

The objective evaluation method for haptic rendering based on human haptic perception features proposed in this paper has two main advantages compared with other previous methods. Firstly, the objective evaluation is a method that simulates the process of evaluation by the human perception system. This method can avoid the effect of experimental environment, the human subjective individual differences if proper parameters and features are selected and extracted. It analyses a similarity between the virtual and real haptic rendering processes in the haptic perception space based on data collection and analysis, which means that it has a good repeatability and stability. Secondly, the data processing is taking into account the human haptic perception characteristics. The data obtained from the haptic rendering interaction processes are not just simply analysed by the ANOVA, but are translated into the human perception space based on perception feature extraction, where the features and distances in the perception space can reflect the human subjective feelings more veritably.

Certainly, there are several key problems and limitations that should be discussed and solved in detail in the future research to make the objective evaluation method more complete and convincing. Firstly, how to collect multiform interaction data steadily. Some information is hard to obtain by proper sensors in some conditions. For example, in the case of grabbing a soft object, the deformation of the object is hard to measure accurately. This may limit the application of the method proposed in this paper to some less complex tasks. Another important problem is how to extract effective perception features that can reflect the actual feeling changing in some complex haptic rendering tasks. Effective features can also provide useful guidance for the improvement of haptic rendering devices and models. At the same time, more fundamental studies on human haptic perception characteristics are necessary to help making extraction of human haptic perception features more reasonable, especially characteristics and mathematical models of some haptic perception illusions. What is more, the inherent correlation and redundancy of the data acquired from the process of haptic rendering tasks should be taken into consideration to make the analysis simple and convenient.

6. Conclusion

In this paper an objective evaluation method for haptic rendering based on the human haptic perception features is proposed. The key steps of the objective method have been introduced. A simple haptic rendering example of pressing springs has been used to show the general process of objective haptic rendering evaluation method based on the human haptic perception characteristics. In the example, a haptic image matrix has been generated according to the regulation of data changing with time and space in the haptic rendering process. Haptic features reflecting essential characteristics of pressing virtual springs have been extracted. Similarity has been analysed in a perception space built based on haptic perception features to evaluate the effect of haptic rendering produced by different devices and algorithms. The final results show that the method proposed in this paper is closer to the human subjective perception and evaluation process. The objective evaluation method based on the human perception features proposed in this paper is simple and gives reliable results, reflecting the consistency with subjective feeling and has a better discrimination ability for different kinds of devices and algorithms compared with the used evaluation method. Certainly, only a simple haptic rendering task is presented in this paper to prove the validation of the proposed process. More complex examples are required to confirm the effectiveness of the method, which is the research aim of our current work (comprehensive perception of haptic rendering combining hardness and texture) that may be published in the future.

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References

[1] Brooks, T.L. (1990). Telerobotic response requirements. *Systems, Man and Cybernetics, 1990. Conference Proceedings, IEEE International Conference*, 113–120.

[2] McAffee, D.A., Fiorini, P. (1991). Hand controller design requirements and performance issues in telerobotics. *Advanced Robotics, 1991.'Robots in Unstructured Environments', 91 ICAR., Fifth International Conference*, 186–192.

[3] Hunter, I.W., Hollerbach, J.M., Ballantyne, J. (1991). A comparative analysis of actuator technologies for robotics. *Robotics Review*, 2, 299–342.

[4] Hayward, V., Astley, O.R. (1996). Performance measures for haptic interfaces. *Robotics research*, London, Springer, 195–2016.

[5] Colgate, J.E., Brown, J.M. (1994). Factors affecting the z-width of a haptic display. *Robotics and Automation, Proc.*, 3205–3210.

[6] Wu, J., Ding, Y., Ni, D., Song, G., Liu, W. (2013). Vibrotactile representation of three-dimensional shape and implementation on a vibrotactile pad. *Sensors and Materials*, 25(1), 79–97.

[7] Kim, U., Kang, J., Lee, C., Kwon, H.Y., Hwang, S., Moon, H., Choi, H.R. (2013). A transparent and stretchable graphene-based actuator for tactile display. *Nanotechnology*, 24(14), 145501.

[8] Shang, W., Su, H., Li, G., Fischer, G.S. (2013). Teleoperation system with hybrid pneumatic-piezoelectric actuation for MRI-guided needle insertion with haptic feedback. *Intelligent Robots and Systems (IROS), 2013 IEEE/RSJ International Conference*, 4092–4098.

[9] Chen, D., Song, A., Tian, L. (2015). A novel miniature multi-mode haptic pen for image interaction on mobile terminal. *Haptic, Audio and Visual Environments and Games (HAVE), 2015 IEEE International Symposium*, 1–6.

[10] Hayward, V., Armstrong, B. (2000). A new computational model of friction applied to haptic rendering. *Experimental Robotics VI*, Springer, London, 403–412.

[11] Kim, S.Y., Park, J., Kwon, D.S. (2005). The real-time haptic simulation of a biomedical volumetric object with shape-retaining chain linked model. *IEICE transactions on information and systems*, 88(5), 1012–1020.

[12] Lang, J., Andrews, S. (2011). Measurement-based modeling of contact forces and textures for haptic rendering. *IEEE Transactions on Visualization and Computer Graphics*, 17(3), 380–391.

[13] Samur, E., Wang, F., Spaelter, U., Bleuler, H. (2007). Generic and systematic evaluation of haptic interfaces based on testbeds. *2007 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS'07)*, 2113–2119.

[14] Kirkpatrick, A.E., Douglas, S.A. (2002). Application-based evaluation of haptic interfaces. *Haptic Interfaces for Virtual Environment and Teleoperator Systems, HAPTICS 2002*, 32–39.

[15] Kocsis, M.B., Cholewiak, S.A., Traylor, R.M., Adelstein, B.D., Hirleman, E.D., Tan, H.Z. (2013). Discrimination of real and virtual surfaces with sinusoidal and triangular gratings using the fingertip and stylus. *IEEE transactions on haptics*, 6(2), 181–192.

[16] Okamura, A.M., Webster, R.J., Nolin, J.T., Johnson, K.W., Jafry, H. (2003). The haptic scissors: Cutting in virtual environments. *IEEE International Conference on Robotics and Automation*, 1, 828–833.

[17] Kuchenbecker, K.J., Fiene, J., Niemeyer, G. (2005). Event-based haptics and acceleration matching: Portraying and assessing the realism of contact. *Eurohaptics Conference, 2005 and Symposium on Haptic Interfaces for Virtual Environment and Teleoperator Systems, World Haptics 2005. First Joint*, 381–387.

[18] Leškovský, P., Cooke, T., Ernst, M., Harders, M. (2006). Using multidimensional scaling to quantify the fidelity of haptic rendering of deformable objects. *Proc. of EUROHAPTICS*, 289–295.

[19] Ruffaldi, E., Morris, D., Edmunds, T., Barbagli, F., Pai, D.K. (2006). Standardized evaluation of haptic rendering systems. *Haptic Interfaces for Virtual Environment and Teleoperator Systems, 2006 14th Symposium*, 225–232.

[20] Swindells, C., MacLean, K.E., Booth, K.S. (2009). Designing for feel: Contrasts between human and automated parametric capture of knob physics. *IEEE transactions on haptics*, 2(4), 200–211.

[21] Hassen, R., Steinbach, E. (2018). HSSIM: An Objective Haptic Quality Assessment Measure for Force-Feedback Signals. *Tenth International Conference on Quality of Multimedia Experience (QoMEX)*, 1–6.

[22] Amari, S. (2003). *The handbook of brain theory and neural networks*. MIT Press.

[23] Rivers, W.H.R. (1894). A modification of aristotle's experiment. *Mind*, 3(12), 583–b.

[24] Geldard, F.A., Sherrick, C.E. (1972). The cutaneous "rabbit": a perceptual illusion. *Science*, 178(4057), 178–179.

[25] Helson, H., King, S.M. (1931). The tau effect: an example of psychological relativity. *Journal of Experimental Psychology*, 14(3), 202.

[26] Price-Williams, D.R. (1954). The kappa effect. *Nature*, 173(4399), 363.

[27] Harris, J.A., Miniussi, C., Harris, I.M., Diamond, M.E. (2002). Transient storage of a tactile memory trace in primary somatosensory cortex. *Journal of Neuroscience*, 22(19), 8720–8725.

[28] Dinse, H.R., Kalisch, T., Ragert, P., Pleger, B., Schwenkreis, P., Tegenthoff, M. (2005). Improving human haptic performance in normal and impaired human populations through unattended activation-based learning. *ACM Transactions on Applied Perception (TAP)*, 2(2), 71–88.

[29] Repperger, D.W., Phillips, C.A., Berlin, J.E., Neidhard-Doll, A.T., Haas, M.W. (2005). Human-machine haptic interface design using stochastic resonance methods. *IEEE Transactions on systems, man, and cybernetics-part A: Systems and Humans*, 35(4), 574–582.

[30] Goldreich, D. (2007). A Bayesian perceptual model replicates the cutaneous rabbit and other tactile spatiotemporal illusions. *PloS one*, 2(3), 333.

[31] Stevens, S.S. (1957). On the psychophysical law. *Psychological review*, 64(3), 153.

[32] Han, G., Choi, S. (2010). Extended rate-hardness: a measure for perceived hardness. *International Conference on Human Haptic Sensing and Touch Enabled Computer Applications*, 117–124.

[33] Salton, G., Wong, A., Yang, C.S. (1975). A vector space model for automatic indexing. *Communications of the ACM*, 18(11), 613–620.

[34] Chaudhari, R., Steinbach, E., Hirche, S. (2011). Towards an objective quality evaluation framework for haptic data reduction. *2011 IEEE World Haptics Conference*, 539–544.

[35] Hassen, R., Steinbach, E. (2018). HSSIM: An objective haptic quality assessment measure for force-feedback signals. *2018 Tenth International Conference on Quality of Multimedia Experience (QoMEX)*, 1–6.

[36] Wang, Z., Bovik, A.C., Sheikh, H.R., Simoncelli, E.P. (2004). Image quality assessment: from error visibility to structural similarity. *IEEE transactions on image processing*, 13(4), 600–612.

[37] Tiest, W.M.B., Kappers, A.M. (2009). Cues for haptic perception of compliance. *IEEE Transactions on Haptics*, 2(4), 189–199.

[38] Hinterseer, P., Hirche, S., Chaudhuri, S., Steinbach, E., Buss, M. (2008). Perception-based data reduction and transmission of haptic data in telepresence and teleaction systems. *IEEE Transactions on Signal Processing*, 56(2), 588–597.

[39] Nadjarbashi, O.F., Abdi, H., Nahavandi, S. (2015). Applying inverse just-noticeable-differences of velocity to position data for haptic data reduction. *Systems, Man, and Cybernetics (SMC), 2015 IEEE International Conference*, 440–445.

[40] Jones, L.A. (2000). Kinesthetic sensing. *Human and Machine Haptics*.

[41] Pang, X.D., Tan, H.Z., Durlach, N.I. (1991). Manual discrimination of force using active finger motion. *Perception & Psychophysics*, 49(6), 531–540.

[42] Jones, L.A., Hunter, I.W. (1990). Influence of the mechanical properties of a manipulandum on human operator dynamics. *Biological Cybernetics*, 62(4), 299–307.

[43] Partridge, L.D. (1966). Signal-handling characteristics of load-moving skeletal muscle. *American Journal of Physiology-Legacy Content*, 210(5), 1178–1191.

[44] Kammerl, J., Chaudhari, R., Steinbach, E. (2011). Combining contact models with perceptual data reduction for efficient haptic data communication in networked VEs. *IEEE Transactions on Instrumentation and Measurement*, 60(1), 57–68.

[45] Ambrosi, G., Bicchi, A., De Rossi, D., Scilingo, E.P. (1999). The role of contact area spread rate in haptic discrimination of softness. *Robotics and Automation, 1999 IEEE International Conference*, 1, 305–310. IEEE.

[46] Lawrence, D.A., Pao, L.Y., Dougherty, A.M., Salada, M.A., Pavlou, Y. (2000). Rate-hardness: A new performance metric for haptic interfaces. *IEEE Transactions on Robotics and Automation*, 16(4), 357–371.

[47] Mastmeyer, A., Hecht, T., Fortmeier, D., Handels, H. (2014). Ray-casting based evaluation framework for haptic force feedback during percutaneous transhepatic catheter drainage punctures. *International Journal of Computer Assisted Radiology and Surgery*, 9(3), 421–431.

[48] Mastmeyer, A., Fortmeier, D., Handels, H. (2017). Evaluation of direct haptic 4d volume rendering of partially segmented data for liver puncture simulation. *Scientific reports*, 7(1), 671.