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Research paper

On the reduction of geodetic and gravimetric measurements on technogenic and geodynamic polygons

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Abstract: When conducting geodetic and gravimetric measurements, there is a problem of projecting them to the reference surface. Since the gravitational field is inhomogeneous under the real conditions, the problem arises of determining the corrections to the measured values of gravitational acceleration in order to use the obtained data for the subsequent solutions of projection problems. Currently, the solution to this problem is performed using a Bouguer reduction, which requires information about the internal structure of the upper layer of the earth's surface, topography, etc. The purpose of this study is to develop a methodological approach that would allow to determine the reduction (projection) corrections for gravitational acceleration on technogenic and geodynamic polygons without using data about the distribution of surface layer density and topography. The research process is based on the use of mathematical analysis methods and a wide range of experimental geodetic and gravimetric measurements. In the course of the performed researches, an algorithm was obtained and a practical implementation of the determination of the corrections in the measured values of gravitational acceleration on the basis of geodetic and gravimetric measurements was carried out at the certain geodynamic polygon in order to bring all corrections to one level surface.

Keywords: correction, gravitational acceleration, level surface, gravity reduction.

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1. Introduction

When conducting geodetic and gravimetric works, the corresponding measurements are reduced to the reference surface using the anomalies of the gravitational (free fall) acceleration in the Bouguer reduction (Ander et al., 1999).

According to the known values of gravitational acceleration, free from the influence of environment and structure of geological formation located between the observation point and the reference surface, project the measured values on this surface. That is, the results of geodetic and gravimetric measurements lead to one level surface.

Development of technologies and methods of processing gravimetric measurements, a tool park of gravimetric and geodetic instruments (Fairhead et al., 2003; Featherstone and Dentith, 1997) bring to the agenda a number of issues related to improving the accuracy of geodetic and gravimetric measurements, which in turn causes the need to research and develop new approaches to project geodetic and gravimetric measurements on the selected level surface, and hence to determine appropriate corrections.

It should be noted that at present the reduction of gravimetric data is carried out by means of a Bouguer reduction, which includes the corrections: "free air" correction (Faye correction); the density of the intermediate layer between the selected level surface and the observation point; for the influence of topographic masses which are surrounding the point of observation (corrections in topography).

The above corrections can bring their own mistakes in the resulting value of the gravitational acceleration anomaly. These errors can greatly distort the accuracy of the measurements. In particular, this concerns the correction for the intermediate layer density and the influence of topographic masses.

Thus, the development of a methodological approach to reducing (projecting) the gravitational acceleration to the influence of external and internal factors on geodynamic and technogenic polygons is an important and urgent task.

2. Analysis of recent research works and publications

The classical formula for calculating the gravitational acceleration anomalies in a Bouguer reduction is the following (Dvulit, 2002):

$$\Delta g_B = g_B - \gamma_0 + 0.3086H - 0.0419\sigma H + \delta g_m\,, \tag{1}$$

where:

g_B – observed gravitational acceleration, mGal;
H – normal height of the observation point, m;
γ_0 – normal value of gravitational acceleration on the surface of a level ellipsoid, mGal;
σ – intermediate layer density, g/cm^3;
δg_m – correction for the influence of topographic masses.

In gravimetry the following correction

$$\delta g_B = (0.3086 - 0.0419\sigma)\,H \tag{2}$$

is called a Bouguer reduction.

In expanded form, Faye's correction looks like:

$$\delta g_F = \left(0.3086 - 0.0044\sin^2 B\right)H - 7.2\times10^{-8}H^2, \tag{3}$$

where B – geodetic latitude of the observation point.

In practice, geodetic and gravimetric studies are accepted the following

$$\delta g_F = 0.3086H. \tag{4}$$

Due to technical improvements North American standard (Hinze, W.J. et al., 2005) provides an elaborate formula for Faye correction:

$$\delta g_F = \left(0.3087691 - 0.004398\sin^2 B\right)H - 7.2125\times 10^{-8}H^2. \tag{5}$$

A big discussion is being conducted on the introduction of a correction for the intermediate layer (Talwani, 1998). With the introduction of this correction ($0.0419\,\sigma H$) it is considered that this layer is a plane–parallel plate with constant density σ. This approach, on the one hand, does not take into account the heterogeneity of the density of the intermediate layer, and on the other hand, the sphericity of the Earth (Karl, 1971).

The Standard (Hinze et al., 2005) provides the replacement of the formula of a plane–parallel horizontal plate by the equation of a spherical segment with a radius of 166.7 km.

One of the main factors affecting the effectiveness of the determination of this correction is the lack of available information on the density of individual parts of the geological section in the total thickness of the intermediate layer. All this requires additional research that involves the creation of separate intermediate layer density models (Arafin, 2004; Chapin, 1996) and drilling of special gravity logging wells that require significant financial costs.

Since the approximation of the intermediate layer is a difficult procedure, it is proposed to refrain from introducing this correction and introduce it in conjunction with correction for the influence of topographic masses (Bychkov, 2010).

Various methods and, in particular, numerical solution methods are used in determining the correction for the influence of topographic masses, which differ from each other by different relief approximations by a set of elementary bodies, by which the gravimetric effect is expressed. The most rational method for determining the correction for the influence of topography at present is a technique based on the use of digital terrain models. The errors in the determination of the correction for the influence of topography are a function of two factors: the accuracy of the construction of digital terrain models and the accuracy of the planar-altitude anchorage of gravimetric points (Ander et al., 1999).

The correction for the density of the intermediate layer together with the correction for influence of topography are presenting a gravitational field of the rocks bounded below by the level surface and from above by the forms of relief. The errors in determining these corrections can be commensurate with the accuracy of the gravimetric observations themselves, which significantly reduces the efficiency of the use of gravimetric data in geodesy and geophysics.

In exploration and surveying works at local technogenic and geodynamic polygons, it would be advisable to refuse reducing (to project) gravimetric and geodetic measurements to the surface of a normal field and reduce the results to one level surface of a given research object (Marti, 2002; Ekman, 1989).

3. Problem statement

In view of the above, the purpose of this publication is to research a methodological approach for determining the integral correction, taking into account the influence of all external factors and the density of layers of geological formations, in measured values of gravitational acceleration when reducing (projecting) them to one level surface of a geodynamic or technogenic polygon.

4. Main research material

In the process of interpreting gravimetric data, one of the components is to consider the heterogeneity of the gravimetric field.

Therefore, the solution of the problem of determining the series of gravitational acceleration based on geodetic and gravimetric measurements without using knowledge of the density of geological formations between the reference surface and observation points, as well as topographical forms, is presented.

The difference in the potentials of gravitational acceleration at a fixed point A is represented in the form of a Taylor series. Limited to the second order of the development, we acquire the following (Brovar, 1983):

$$W_A - W_O = g_A H_A + \frac{1}{2}\frac{\mathrm{d}g}{\mathrm{d}n} H_A^2 + \cdots, \tag{6}$$

where:

g_A – gravitational acceleration at point A;

$\frac{\mathrm{d}g}{\mathrm{d}n}$ – vertical gradient of gravitational acceleration at point A;

H_A – normal height of point A.

From formula (6) we obtain:

$$\frac{\mathrm{d}g}{\mathrm{d}n} = [(W_A - W_O) - g_A H_A] \times \frac{2}{H_A^2}. \tag{7}$$

The difference in the potentials of gravitational acceleration at point A can be represented as (Dvulit, P.D., 2008):

$$W_A - W_O = \int_0^A g\,\mathrm{d}h, \tag{8}$$

where:

$\mathrm{d}h$ – measured height difference between the start point O and the end point A;

g – value of gravitational acceleration from point O to point A.

If the height difference between the points O and A is obtained by the method of geometric or trigonometric leveling, then the right side of formula (7) can be represented as:

$$\int_0^A g\,\mathrm{d}h = \sum_{i=1}^{n} \frac{g_i + g_{i+1}}{2} h_i, \tag{9}$$

where:

g_i – measured value of gravitational acceleration at point i;

h_i – height difference at the i-th leveling station.

Considering (8) and (9), formula (7) will be:

$$\frac{\mathrm{d}g}{\mathrm{d}n} = \left(\sum_{i=1}^{n} \frac{g_i + g_{i+1}}{2} h_i - g_A H_A\right) \times \frac{2}{H_A^2}\,. \tag{10}$$

The measured value of gravitational acceleration g_i is

$$g_i = g_O + \delta g_i\,, \tag{11}$$

where:

g_O – gravitational acceleration at point O;

δg_i – increment of gravitational acceleration at point i.

Thus, the first term of the right-hand side of formula (10) can be reduced to the form:

$$\begin{aligned}\sum_{i=1}^{n} \frac{g_i + g_{i+1}}{2} - g_A H_A = g_O\,(h_1 + h_2 + \cdots + h_n) + \frac{\delta g_1 + \delta g_2}{2} h_1 + \frac{\delta g_2 + \delta g_3}{2} h_2 + \\ + \cdots + \frac{\delta g_{n-1} + \delta g_A}{2} h_n - g_O H_A - \delta g_A H_A\,.\end{aligned} \tag{12}$$

When studying the figure of the Earth, mainly normal or geodetic heights are used.

However, when studying individual local technogenic and geodynamic polygons, it is advisable to reduce (to project) performed gravimetric or geodetic measurements to one, close to the real state, level surface when solving various engineering tasks. In this case, a dynamic height system starting at point O can be applied to technogenic and geodynamic polygons.

If the height of the starting point O is $H_O = 0.000$ m, then $h_1 + h_2 + \cdots + h_n = H_i$, and using $\dfrac{\delta g_i + \delta g_{i+1}}{2} = \delta g_{ic}$, the following will be acquired:

$$\sum_{i=1}^{n} \frac{\delta g_i + \delta g_{i+1}}{2} \cdot h_i - g_A H_A = \delta g_{1c} h_1 + \delta g_{2c} h_2 + \cdots + \delta g_{nc} h_n - \delta g_i H_i\,, \tag{13}$$

where H_i – dynamic height at point i.

It should be noted that in formula (13) there is no value of the gravitational acceleration at the starting point, and only the values of change in acceleration δg_{ic} appear.

Considering (13), formula (10) for determining the vertical gradient of gravitational acceleration will be:

$$\frac{\mathrm{d}g}{\mathrm{d}n} = \left(\sum_{i=1}^{n} \delta g_{ic} h_i - \delta g_i H_i\right) \times \frac{2}{H_i^2}\,. \tag{14}$$

It should be noted that in the obtained heights of points by the results of direct measurements, the corrections for the parallelism of level surfaces should be made. Thus, the formula for determining the dynamic height at any point i has the form (Perovych et al., 2018):

$$H_i = \int_O^i \mathrm{d}h_i - \frac{1}{2g_O}\delta g_i \int_O^i \mathrm{d}h_i . \tag{15}$$

Returning to expression (14), we note that $\frac{\mathrm{d}g}{\mathrm{d}n}$ is a change in gravitational acceleration with a height H_i at a fixed point i. Gravitational acceleration at point i will be:

$$g_i = g_O + \delta g_i = g_O + \left(\frac{\mathrm{d}g}{\mathrm{d}n}\right)_i H_i , \tag{16}$$

where $\left(\frac{\mathrm{d}g}{\mathrm{d}n}\right)_i H_i$ is the correction.

Hence, considering formula (14), the final formula for determining the correction in the results of gravimetric measurements for reduction to the initial level surface is obtained:

$$V_i = -\left(\frac{\mathrm{d}g}{\mathrm{d}n}\right)_i H_i = -\left(\sum_{i=1}^{n} \delta g_{iC} h_i - \delta g_i H_i\right)\frac{2}{H_i} , \tag{17}$$

or

$$V_i = 2\delta g_i - \frac{2}{H_i}\sum_{i=1}^{n} \delta g_{iC} h_i . \tag{18}$$

Formula (18) is a working formula to determine the correction in the measured value of gravitational acceleration during its reduction to the selected level surface of a geodynamic or technogenic polygon.

The practical application of the proposed methodological approach is noteworthy. To this end, the experimental studies performed on the Carpathian geodynamic polygon are used. Three profile lines with the length of 17.3 km, 2.6 km and 5.9 km were created at this polygon. Geodetic and gravimetric observations were made along these profile lines (Ilkiv, 1972).

Geodetic observations consisted in determining the height differences and heights of all points of the profile by the method of high-precision geometric leveling. Herewith, at the same time as determining the height differences at the points of staying of leveling rods, the gravitational acceleration was measured.

Three profile lines were laid in the study area: the first profile line with a length of 17.3 km included 723 leveling stations, the second profile line (2.6 km length) – 75 stations, and the third profile line (5.9 km length) – 67 stations.

As a result of the completed leveling works, the height differences between the starting point of the leveling and the end point were determined, which amounted to 1185.15 m for the first line, 293.53 m for the second, 514.60 m for the third.

As a result of processing, there were some discrepancies in the leveling lines: in the first – 43 mm; in the second – 19 mm; in the third – 1 mm, which corresponds to the permissible values for the second class of leveling.

It should be noted that a single reference point O with $H_O = 0.000$ m, which was considered stable, was selected as a starting point (benchmark) for all three profile lines.

Along the profile lines, a supporting gravimetric network was created with 25 supporting gravimetric points, which served as the basis for linking the ordinary gravimetric points, that is, the points of staying of leveling rods. The distances between ordinary gravimetric points were on average 15÷20 m, and in more flat areas (profile line 5.9 km) −50÷100 m.

The measured values of gravity were calculated in conditional system, where $g_O = 980000.00$ mGal (point O). The mean square error of the measured value of gravitational acceleration at the reference gravimetric points was on average m = 0.15 mGal, and at the ordinary gravimetric points was m = 0.24 mGal. Significant values of mean square errors of gravimetric measurements are due to the complexity of the physical and geographical conditions of the highland study area and the imperfect method of measurement.

On the basis of the experimental data, it became possible to obtain reduction corrections for each point of setting the leveling rods to reduce (to project) the measured values of gravitational acceleration to the level surface of the reference point O. For this purpose, using formula (18), a fragment of correction V_i for 17.3 km profile line at the section of leveling stations 145÷150 is shown in Table 1.

Table 1. The values of corrections V_i

No	g_i, mGal	H_i, m	V_i, mGal	g_{ip}, mGal
145	979 944.17	267.543	56.63	980 000.80
146	979 943.48	270.222	57.44	980 000.92
147	979 942.87	272.770	58.11	980 000.98
148	979 942.29	275.229	58.75	980 001.04
149	979 941.71	277.802	59.36	980 001.07
150	979 941.11	280.443	60.00	980 001.11

In this table g_i are calculated based on the field measurements of gravitational acceleration, g_{ip} – gravitational acceleration at point i corrected by the reduction.

The analysis of the results presented in Table 1 confirms the practical feasibility of implementation of this methodological approach and allow to solve an extremely difficult problem of reducing these measurements to a level surface of relativity, which is an important element of scientific and practical research on technogenic and geodynamic polygons.

5. Conclusions

The considered methodological approach of determining the reduction corrections in the measured values of gravitational acceleration on geodynamic and technogenic polygons allows to obtain them without using data on the inner density of the intermediate layer of the earth's surface and the influence of topographic masses.

The results of experimental studies have shown that this methodological approach can be effectively used for solving the extremely difficult problem of reducing gravimetric measurements.

The prospect of further research is to study and develop a methodology for using this method in different geographical conditions on the basis of a modern variety of geodetic and gravimetric equipment.

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