THE ARCHIVE OF MECHANICAL ENGINEERING

VOL. LIV 2007 Number 4

DOI: 10.24425/ame.2007.131564

Key words: crushing, kinetic friction and static friction, friction factor, fed material, product

FELIKS CHWARŚCIANEK [*]

THE CONSTRUCTIONAL INCREASE OF CRUMBLING EFFECTIVENESS

The study presents an attempt at increasing the effectiveness of the crushing process through the application of a new original crumbling system. In the process of crushing materials, friction is present in many crushers as extremely significant or even dominating factor. The proposed construction solution is characterized by the occurrence – always on one of the working surfaces – of the static friction factor, and thus a friction that is greater than the kinetic friction.

1. Introduction

The crushing machines used in industrial practice to crush solid materials are characterized by small technical efficiency (usually a few to several per cent) in the realized process and, what is connected with that, a big energy consumption – which is particularly important if there are large quantities of material to be crushed (e.g. in the power engineering industry, grain mills).

The crushing method and the machinery used for this purpose should be adapted to the type of the material being crushed and, in particular, to its mechanical properties [4], [5], [6], [7]. The main differentiation can be observed between the organic materials (of plant and animal origin) and mineral matter (aggregates, minerals).

The most popular crushing machines equipped with mechanical working (crushing) elements include the following:

- impact crushers – universal, applied to almost all types of materials, characterized by small technical efficiency, usually of <7%,

---

[*] *University of Technology and Life Sciences in Bydgoszcz, Poland Faculty of Mechanical Engineering, Department of Special Purpose Machines and Environmental Protection; 85-789 Bydgoszcz, Al. Prof. S. Kaliskiego 7; E-mail: felixc@utp.edu.pl*

- roll crushers (with two or more rolls) – usually applied to brittle grainy materials of plant and mineral origin, usually characterized by a relatively high technical efficiency of even >20%.

The common use of crushing processes in the industrial processing of minerals and organic materials [1], [3], [6], [7], [8], as well as the corresponding variety and multitude of crusher constructions, result in permanent search for better ways and methods of crushing. It is particularly a search for optimal conditions for the crushing processes [5], [6], [7] and development of constructions which would be increasingly effective in terms of volume processing yield, efficiency and energy saving, and design solutions [2].

2. Lead-in

In the most general understanding, the crushing of any solid material is the breaking up of its mass into parts. The breaking up of a material into parts of required dimensions is made after overcoming the internal material cohesion forces. So we can define crushing as discretization of the material, because the dimensions of broken-up pieces (of the product or fed material) differ "in steps" – discontinuously. Crushing usually does not refer to the breaking up of a big uniform mass of material, but a material which is already broken-up into parts (in pieces) either because of (natural or forced) coarse crushing – e.g. mineral aggregates, mines, or because the material occurs broken-up (in pieces) in its natural environment – e.g. organic (plant) grainy materials.

The measure (factor) of the breaking up (disintegration) of the material into parts is the so-called degree of fineness (degree of discretization) [3], [4], [7], which is generally defined as quotient of dimensions of quantities characterizing a material piece before breaking up (A) and after breaking up (a), which is defined by the general relationship:

$$\lambda = \frac{\mathrm{A}}{\mathrm{a}}. \tag{1}$$

The quantities which characterize the particles of broken up material may be: mass, volume, surface, linear dimensions (e.g. diameter, length, width), both actual ones or the so-called substitute ones. As a result, there may be quantities referring to the discretization of a material in the form of the degree of fineness: mass fineness, surface fineness, linear fineness [2], [3], [7], [8].

The effectiveness of material crushing depends mainly on the generally understood susceptibility of the material to crushing, and it (the effectiveness) can be measured through the amount of consumed energy or the work necessary to realize the crushing process [4], [6], [7]. The susceptibility of a material to crushing is, in particular, connected with a loss of its cohesion.

The amount of energy used for crushing of a given material depends on:

- Properties of the material
- Degree of fineness
- Method of crushing
- Conditions of crushing.

Each of the factors affecting the amount of energy used for crushing is characterized by the following quantities and properties:

a) Properties of the material
   Strength of the material (values of boundary stresses and forces) referring to:
   - cohesion (decohesion, destruction, disintegration),
   - plasticity (plastic strains),
   - elasticity (elastic/reversible strains),

   the value of boundary stresses is connected with the types of properties concerning the strength of the material being broken up, e.g.:
   - tearing, squeezing,
   - shearing,
   - surface pressures,
   - impact resistance,
   - bending, breaking,
   - grindability (from external to internal friction).

b) Degree of fineness
   The sizes of material particles before and after crushing (mass, volume, surface, linear dimension).
   The value of energy of the free surface of material. Resistance of the material to decohesion.

c) Method (process) of crushing
   Taking account and making use of the phenomena accompanying and arising during crushing (e.g. breaking, abrasion, crushing, cutting, striking), and referring, in particular, to energy dissipation (e.g. heat, friction).
   A construction solution for a crushing machine making use of the physical phenomena involved in crushing (type of machines and devices).

d) Conditions of crushing
   The interaction and influence of the environment (natural conditions); temperature, moisture (primary conditions).
   The effects of the crushing process; temperature, moisture, friction, acoustic waves, vibrations (secondary conditions).

The crushing process is described by numerous hypotheses [4,6] concerning the relations between the energy (work) needed for crushing and relevant quantities characterizing crushing, in particular, referring to the degree of fineness (usually described by means of geometrical quantities) and material properties. The hypotheses (theories) applied generally do not make allowances for the methods and conditions of crushing.

The subject of this study is an innovative concept of crushing proposed in the roll-plate crusher. The concept increases the effectiveness of the crushing process through its construction, making special use of the friction phenomenon occurring during crushing – it determines the occurrence of the static friction.

3. Structure and principle of operation

With the configuration of working crushing elements as presented in Figures 1 and 2 and the described method of feeding the working space with the material [1], [2], the roll-plate crusher consists of a rotary-driven roll (1) and a plate, immovable during operation (2), attached at one side to a rotary joint and supported by an overload system on the other (4).

The system of working elements – roll (1) and a plate located askew (usually within the angle value of β = 0 – 45°) in relation to the plate's vertical (2), as presented in Figure 1, is a more general case. The configuration presented in Figure 2 results from Figure 1 for a special case in which the angle of deflection of the plate from the vertical equals β = 0.

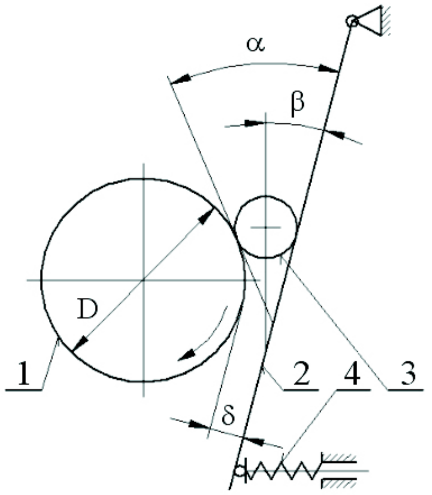

Fig. 1. Crusher with a deflecting slanting plate

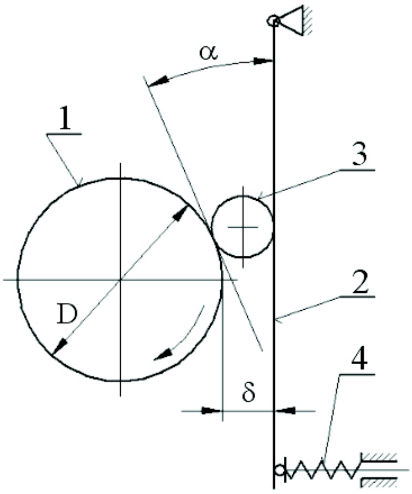

Fig. 2. Crusher with a deflecting vertical plate

According to Figures 1 and 2, the working and crushing elements are:

- driven roll (1), whose working surface being its generating line, may be even or profiled (e.g. grooved),
- immovable plate (2), whose working surface can be even or profiled.

The crushing of the fed material is performed in the proposed solution mainly through the use of two phenomena:

- friction of particles against the surfaces of working elements,
- crushing – the crushing of fed material particles within the working space gap.

The adjustment of the size of the working gap δ (Fig. 1 and Fig. 2) between the working elements can be performed with the mechanism (4) through a change of the position of the non-driven element, i.e. the working plate (e.g. through turning – as presented in Figures, or through shifting). The system (4) of changing the position of the plate should also ensure its deflection under the conditions overloading the crushing (e.g. when there is foreign matter in the fed material).

The feeding of the working space (between the roll and the plate) with material (3), can be carried out at a pre-set speed, e.g. gravitationally or at a set speed from the previous crushing section.

We can distinguish two different, specific cases of the material feeding and crushing, with the occurrence of dominating effect of forced static friction [1]:

A) the particles of the fed material (3) move in relation to the working plate (2) at a speed synchronized with the circumferential velocity of the roll (1) (equal to the velocity) – the friction of the particles of fed material against the plate surface (2) is a kinetic friction,
  - the particles of fed material are motionless in relation to the roll surface – so the friction of the particles of fed material against the roll surface is a static friction,

B) the particles of fed material in relation to the working plate (2) are motionless) (the fed material fills the space over the working gap) – so the friction of the particles of fed material against the plate surface is a static friction,
  - the working surface of the roll moves in relation to the particles of fed material at its circumferential velocity – the friction of the particles of fed material against the roll surface is a kinetic friction.

In practice, there may be cases when the speed of the particles of fed material in relation to the rotating roll (material feeding case A) or in relation to the plate (material feeding case B) is of only very small value (close to zero), which consequently means that the friction factor has a value close to the static friction.

The newly designed construction concept of the crusher [2], in which the working elements are the driven and rotating roll with an even or profiled (e.g. grooved) surface and a deflecting plate with a corresponding surface – even or profiled, enables crushing grainy materials of organic origin (crops grains) or mineral origin (aggregates).

To increase the degree of fineness of fed material we can use systems with a few stages of crushing [1], [2] as presented in Figures 3 and 4. The working rolls (1) may work in a skew system, Figure 3 or a vertical one, Figure 4. The working plates (2) are coupled with one another by means of, for example, rotary joints (5) – “hinges”, which allow the plates to make the deflecting movement in order to adjust the working gap ( ). The required value of the gap is set with the mechanism (4) which locks the plates in the working position. The mechanism has the option of reacting in case of overloading the working elements of the crusher.

In the discussion on the description of the influence on the elements of the working couple, it is necessary to consider the influence of the kinematics of relative motion of fed material particles and the working surfaces on the value of the friction factor over each working surface and next on the value of the friction forces on respective surfaces.

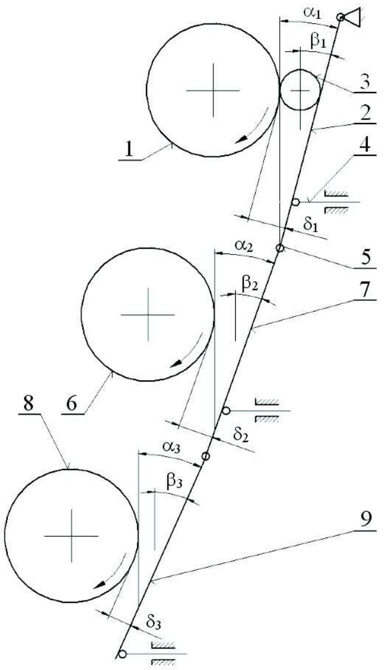

Fig. 3. Crusher with a few stages of crushing – plates in a skew system

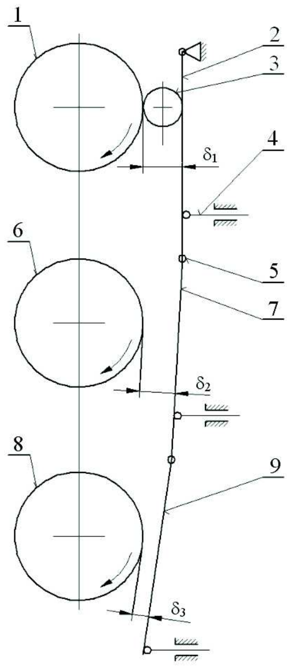

Fig. 4. Crusher with a few stages of crushing – plates in a vertical system

4. Kinematic and dynamic relationships

For both cases A) and B) of feeding the working space of the crusher, there is an identical relation between the value of the friction factor and the movement of the working surface or fed material. Its principle is that if on one working surface (of the roll or the plate) there is static friction, then on the other working surface there is kinetic friction.

A different relative velocity of each working surface in relation to the fed material particles and an adequately changing friction factor lead to an increase in the level and effectiveness of crushing. The friction factor μ of surfaces (e.g. the particles of fed material and the roll) against each other is expressed by the relationship:

$$\mu = \mathrm{tg}\rho, \tag{2}$$

where: ρ – so-called friction angle.

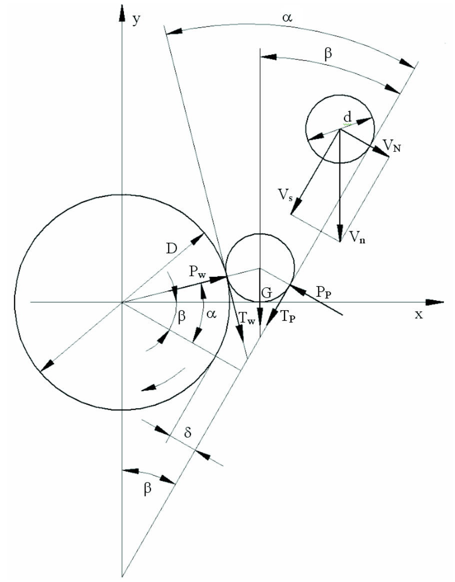

Fig. 5. Impact of the working elements and fed material in the plate crusher

The friction factor takes on its maximum value for the case of static friction when the friction angle reaches a value corresponding to the self-locking angle. The relation between the kinetic friction factor μ_k and the static friction factor μ_s is described by an approximate relationship:

$$\mu_k = a\mu_s \tag{3}$$

where: a = 0.3 – 0.8 (approximately).

The basic configuration of the working elements, i.e. the roll and the plate [1] is shown in Figure 5. The working plate is deflected from the vertical by the angle β and makes it possible (through the deflection) to adjust the size of the working gap δ.

The determination of relationships containing the dynamic and kinematic quantities in the description of crushing should correspond to the occurring cases of feeding with material. Feeding with material can be carried out the way as described for case A) (which is shown in Figure 5) or for case B), in which the fed material fills the space over the working gap and its particles stay in relative motion in relation to the roll surface.

1) *Case A*

The crushing machine is fed with material with a pre-set initial velocity $\mathbf{v}_n$, e.g. caused by its gravity fall. The velocity of fed material can be expressed through the components (subscript "w" refers to the roll, "p" refers to the plate) as:

$$\mathbf{v}_n = \mathbf{v}_{ps} + \mathbf{v}_{pn} \tag{4}$$

where:
$\mathbf{v}_{ps}$ – velocity along the plate surface (tangential velocity)
$\mathbf{v}_{pn}$ – velocity perpendicular to the plate surface (normal velocity).

The velocity of fed material, and, more precisely, its tangential component, is synchronized with the circumferential velocity of the roll $\mathbf{v}_w$, according to this relationship:

$$\mathbf{v}_{ps} = \mathbf{v}_w \tag{5}$$

and

$$\mathbf{v}_{ps} - \mathbf{v}_w = 0. \tag{6}$$

The tangential component of the velocity of fed material is determined by the following relationship:

$$\mathbf{v}_{ps} = \mathbf{v}_n \mathrm{ctg}\beta. \tag{7}$$

Due to the presence of friction, the actual velocity $\mathbf{v}_p$ of fed material particles will be:

$$\mathbf{v}_p < \mathbf{v}_{ps} \qquad \text{or} \qquad \mathbf{v}_p \approx \mathbf{v}_{ps}. \tag{8}$$

The normal component of the velocity of fed material is determined by the following dependence:

$$\mathbf{v}_{pn} = \mathbf{v}_n \mathrm{tg}\beta. \tag{9}$$

The normal component causes the formation of pressure on the plate surface due to the piling up of the stream of fed material (slowing down the velocity). The pressure can be determined as a function of the dynamic pressure of fed material stream in the following form:

$$p = \frac{\gamma}{2g} v_{pn}^2, \tag{10}$$

where:
γ – specific gravity of particles of fed material
g – acceleration of gravity
p – static pressure.

For the configuration of the crushing machine as in Fig. 2 (vertical plate, i.e. $\beta = 0$), the following relationships occur:

$$\mathbf{v}_n = \mathbf{v}_{ps} = \mathbf{v}_w, \tag{11}$$

$$\mathbf{v}_{pn} = 0, \tag{12}$$

$$p = 0. \tag{13}$$

2) ***Case B***

For the geometrical configuration as in Fig. 5, when the fed material does not have an initial velocity of feeding, the fed material fills the space over the working gap:

$$\mathbf{v}_n = 0, \tag{14}$$

that is, there are no velocity components of the fed material on the working plate surface:

$$\mathbf{v}_{ps} = 0 \quad \text{and} \quad \mathbf{v}_{pn} = 0. \tag{15}$$

There is only the circumferential velocity $\mathbf{v}_w$ on the working surface of the roll, so:

$$\mathbf{v}_w - \mathbf{v}_{ps} \neq 0 \quad \text{or} \quad \mathbf{v}_w - \mathbf{v}_n \neq 0. \tag{16}$$

For the case as in Fig. 2 (for $\mathbf{v}_n = 0$), there is an analogous relationship, i.e.:

$$\mathbf{v}_{\mathrm{w}} - \mathbf{v}_{\mathrm{n}} \neq 0. \tag{17}$$

The description of dynamic quantities associated with the crushing process will be made for cases of feeding with material, analogous with the kinematic description.

I. ***General case*** *(crusher with a deflected plate, as in Fig. 5)*
Within the working space of the crusher, there act the following forces:

- **P** – crushing (reaction) force of the fed material,
- **T** – force of friction of the fed material against the working surface,
- **G** – force of gravity of fed material particle,
- **C** – force of pressure of fed material particles.

The equilibrium of forces is determined by the general relationship:

$$\mathbf{P} + \mathbf{T} + \mathbf{G} + \mathbf{C} = 0. \tag{18}$$

in case of not considering the force of pressure on the plate:

$$\mathbf{P} + \mathbf{T} + \mathbf{G} = 0, \tag{19}$$

a) Normal force for the roll surface (crushing):

$$\mathbf{P}_{\mathrm{w}} = \mathbf{P}_{\mathrm{wx}} + \mathbf{P}_{\mathrm{wy}}, \tag{20}$$

$$\mathrm{P}_{\mathrm{w}} = \sqrt{\mathrm{P}_{\mathrm{wx}}^2 + \mathrm{P}_{\mathrm{wy}}^2}. \tag{21}$$

Components of the resultant force of the normal force:

$$\mathbf{P}_{\mathrm{wx}} = \mathbf{P}_{\mathrm{w}} \cos(\alpha - \beta) \tag{22}$$

$$\mathbf{P}_{\mathrm{wy}} = \mathbf{P}_{\mathrm{w}} \sin(\alpha - \beta) \tag{23}$$

b) Friction force on the roll surface:

$$\mathbf{T}_{\mathrm{w}} = \mathbf{T}_{\mathrm{wx}} + \mathbf{T}_{\mathrm{wy}} \tag{24}$$

$$\mathrm{T}_{\mathrm{w}} = \mu_{\mathrm{w}} \mathrm{P}_{\mathrm{w}} \tag{25}$$

where: μ_{w} – friction factor of fed material particles against the roll working surface
– in case of static friction, i.e. for $\mathbf{v}_{\mathrm{psw}} - \mathbf{v}_{\mathrm{w}} = 0$, the friction factor will take on the value of the static factor, i.e. $\mu_{\mathrm{w}} = \mu_{\mathrm{ws}}$,

– in care of kinetic friction, i.e. for $\mathbf{v}_{ps} - \mathbf{v}_w \neq 0$ and $\mathbf{v}_{ps} = 0$, the friction factor will equal to the kinetic factor, i.e. $\mu_w = \mu_{wk}$.

The components of the friction force along the x-axis and y-axis:

$$T_{wx} = T_w \sin(\alpha - \beta) = \mu_w P_w \sin(\alpha - \beta), \tag{26}$$

$$T_{wy} = T_w \cos(\alpha - \beta) = \mu_w P_w \cos(\alpha - \beta). \tag{27}$$

c) Normal force for the plate surface

If the particles of fed material have the velocity of $\mathbf{v}_{ps} = \mathbf{v}_w$, the plate surface is subjected to the force of pressure from particles of fed material (piling up of the stream of particles); the pressure is determined by the relationship (10). The force of pressure C against a unitary surface of the plate may be expressed in the following relationship:

$$\mathbf{C} = pf_p, \tag{28}$$

where: p – static pressure resulting from the piling up of the stream of fed material particles,

f_p – unitary surface of the plate.

The resultant force on the plate surface including the force of pressure, generally:

$$\mathbf{P}_c = \mathbf{P}_p + C. \tag{29}$$

The components of the resultant force as above, will be:

$$\mathbf{P}_{cx} = \mathbf{P}_{px} + \mathbf{C}_x, \tag{30}$$

$$\mathbf{P}_{cy} = \mathbf{P}_{py} + \mathbf{C}_y. \tag{31}$$

The resultant force on the plate surface may be presented through its components for a case including the force of pressure (subscript c) or without the force of pressure (subscript p):

– including the force of pressure

$$\mathbf{P}_c = \mathbf{P}_{cx} + \mathbf{P}_{cy}, \tag{32}$$

– without the force of pressure

$$\mathbf{P}_p = \mathbf{P}_{px} + \mathbf{P}_{py} \tag{33}$$

and respectively:

$$P_c = \sqrt{P_{cx}^2 + P_{cy}^2}, \tag{34}$$

$$P_p = \sqrt{P_{px}^2 + P_{py}^2}. \tag{35}$$

The components of the resultant force on the plate surface corresponding to (32) and (33) will be:

$$P_{cx} = P_c \operatorname{ctg} \beta, \tag{36}$$

$$P_{cy} = P_c \operatorname{tg} \beta, \tag{37}$$

$$P_{px} = P_p \operatorname{ctg} \beta, \tag{38}$$

$$P_{py} = P_p \operatorname{tg} \beta. \tag{39}$$

d) The friction force of the fed material against plate surface
For both cases (32) and (33), the friction force can be determined with the already known relationship:

$$T_c = \mu_p P_c \quad \text{or} \quad T_p = \mu_p P_p, \tag{40}$$

where: μ_p – fraction factor of fed material particles against the plate surface (for friction of crops grains against polished cast iron $\mu = 0.213 - 0.384$ and $\alpha \leq 15°$),
– in case of static friction, the friction factor takes on the value of the static friction factor, i.e. $\mu_p = \mu_{ps}$,
– in case of kinetic friction, the friction factor takes on the value of the kinetic fraction factor, i.e. $\mu_p = \mu_{pk}$.

The resultant friction force is expressed by the following relationships:

$$\mathbf{T}_p = \mathbf{T}_{px} + \mathbf{T}_{py} \quad \text{or} \quad \mathbf{T}_c = \mathbf{T}_{cx} + \mathbf{T}_{cy}, \tag{41}$$

and

$$T_c = \sqrt{T_{cx}^2 + T_{cy}^2} \quad \text{or} \quad T_p = \sqrt{T_{px}^2 + T_{py}^2}, \tag{42}$$

The components of the resultant friction force:

$$T_{cx} = T_c \operatorname{tg} \beta = \mu_p P_c \operatorname{tg} \beta \quad \text{or} \quad T_{px} = \mu_p P_p \operatorname{tg} \beta \tag{43}$$

$$T_{cy} = T_c \operatorname{ctg} \beta = \mu_p P_c \operatorname{ctg} \beta \quad \text{or} \quad T_{py} = \mu_p P_p \operatorname{ctg} \beta \tag{44}$$

e) The conditions for an equilibrium of forces
Based on the relationships (18) and (19) concerning the general equilibrium of forces and considering the action of forces on both working elements (the roll and the plate), we can determine the following relationships:

$$\mathbf{P}_w + \mathbf{P}_c + \mathbf{T}_w + \mathbf{T}_c + \mathbf{G} = 0, \tag{45}$$

$$\mathbf{P}_w + \mathbf{P}_p + \mathbf{T}_w + \mathbf{T}_p + \mathbf{G} = 0. \tag{46}$$

Determining the equilibrium of forces through its components, we receive:
against the x-axis:

$$P_{wx} - P_{cx} + T_{wx} - T_{cx} = 0 \tag{47}$$

or

$$P_{wx} - P_{px} + T_{wx} - T_{px} = 0, \tag{48}$$

against the y-axis:

$$P_{wy} + P_{cy} - T_{wy} - T_{cy} - G = 0 \tag{49}$$

or

$$P_{wy} + P_{py} - T_{wy} - T_{py} - G = 0. \tag{50}$$

f) The condition for pulling in the fed material
Based on the condition of the equilibrium of active forces, we can determine the condition for pulling the fed material particle into the working space of the crusher, in the following form:

$$T_{wy} + T_{cy} + G \geqslant P_{wy} + P_{cy} \tag{51}$$

or

$$T_{wy} + T_{py} + G \geqslant P_{wy} + P_{py}. \tag{52}$$

After including respective expressions for the components in the relationships (51) or (52), we receive the relationships:

$$\mu_w P_w \cos(\alpha - \beta) + \mu_p P_c \operatorname{ctg}\beta + G \geqslant P_w \sin(\alpha - \beta) + P_c \operatorname{tg}\beta, \tag{53}$$

or

$$\mu_w P_w \cos(\alpha - \beta) + \mu_p P_p \operatorname{ctg}\beta + G \geqslant P_w \sin(\alpha - \beta) + P_p \operatorname{tg}\beta. \tag{54}$$

In order to determine the geometrical condition for pulling in the fed material, both cases of feeding with material will be considered.

Case I. A – the particles of fed material are given an initial velocity, i.e. when $\mathbf{v}_{ps} = \mathbf{v}_w$, the friction on the roll surface is static whereas on the plate it is kinetic.

If the material of the roll and the plate is the same or if we assume equal factors of static friction for both working surfaces, we receive:

$$\mu_{ws} = \mu_{ps} = \mu_s. \tag{55}$$

Expressing kinetic friction with the relationship (3), a more general expression (53) will, after transformations, take on the following form:

$$\mu_s P_w \cos(\alpha - \beta) + a\mu_s P_c \operatorname{ctg}\beta \geqslant P_w \sin(\alpha - \beta) + P_c \operatorname{tg}\beta - G. \tag{56}$$

Relationship (56) allows us to determine a function describing the relation between the friction angle ρ, and the angle α which defines the possibility of capturing a fed material particle, as follows:

$$\mu_s = \operatorname{tg}\rho \geqslant \frac{P_w \sin(\alpha - \beta) + P_c \operatorname{tg}\beta - G}{P_w \cos(\alpha - \beta) + aP_c \operatorname{ctg}\beta} \tag{57}$$

and further

$$\rho \geqslant \operatorname{arctg}\left[\frac{P_w \sin(\alpha - \beta) + P_c \operatorname{tg}\beta - G}{P_w \cos(\alpha - \beta) + aP_c \operatorname{ctg}\beta}\right]. \tag{58}$$

Case I. B – the particles of fed material are not given any initial velocity, i.e. when $\mathbf{v}_n = 0$ and $\mathbf{v}_{ps} = 0$, the friction on the roll surface is kinetic, whereas on the plate surface it is static. Analogously with (56) it will be:

$$a\mu_s P_w \cos(\alpha - \beta) + \mu_s P_p \operatorname{ctg}\beta \geqslant P_w \sin(\alpha - \beta) + P_p \operatorname{tg}\beta - G. \tag{59}$$

Respectively to (57) and (58), we receive:

$$\mu_s = \operatorname{tg}\rho \geqslant \frac{P_w \sin(\alpha-\beta) + P_p \operatorname{tg}\beta - G}{aP_w \cos(\alpha-\beta) + P_p \operatorname{ctg}\beta}, \tag{60}$$

and

$$\rho \geqslant \operatorname{arctg}\left[\frac{P_w \sin(\alpha-\beta) + P_p \operatorname{tg}\beta - G}{aP_w \cos(\alpha-\beta) + P_p \operatorname{ctg}\beta}\right]. \tag{61}$$

II. ***Special case*** *(crusher as in Fig. 2)*
Case II. A of feeding with material
Based on the general condition of equilibrium of forces against the x-axis, we obtain:

$$P_{px} = P_{wx} + T_{wx} - T_{px}. \tag{62}$$

The equilibrium condition in the adopted (Fig. 2) geometrical configuration, in which $T_{px} = 0$, will take on the following form:

$$P_{px} = P_p = P_{wx} + T_{wx}. \tag{63}$$

After substituting the relationships (22) and (26) to (63), we obtain:

$$P_p = P_w \cos\alpha + \mu_w P_w \sin\alpha. \tag{64}$$

Using the relationship (56) and incorporating into it the relationship (64), we will have:

$$\mu_w P_w \cos\alpha + \mu_p(P_w \cos\alpha + \mu_w P_w \sin\alpha) + G \geqslant P_w \sin\alpha. \tag{65}$$

After suitable transformations, we receive the following relationship:

$$\operatorname{tg}\alpha \leqslant \frac{\mu_w + \mu_p + \frac{G}{P_w}\frac{1}{\cos\alpha}}{1 - \mu_p\mu_w}. \tag{66}$$

Assuming that for small values of the angle α (usually up to 15°) $\cos\alpha \approx 1$, we receive from the relationship (66) the following formula:

$$\operatorname{tg}\alpha \leqslant \frac{\mu_w + \mu_p + \frac{G}{P_w}}{1 - \mu_p\mu_w}. \tag{67}$$

Another special case is the omission of the mass of the fed material particle in the relationship (67), which is justified by small dimensions of material particles with a great degree of fineness, thus:

$$\mathrm{tg}\,\alpha \leqslant \frac{\mu_W + \mu_P}{1 - \mu_P\mu_W}. \tag{68}$$

After inclusion of assumptions made in the relationship (55) and adoption of the relationship (3), we obtain:

$$\mathrm{tg}\,\alpha \leqslant \frac{\mu_s(1 + a)}{1 - a\mu_s^2}. \tag{69}$$

The expressions (68) and (69) can be obtained directly from the relationship (57) after incorporation of the formula (64). The boundary value of the angle α, defining the possibility of capturing a particle of fed material in the working space, will be:

– for the relationship (68)

$$\alpha = \mathrm{arctg}\left(\frac{\mu_W + \mu_P}{1 - a\mu_s^2}\right) \tag{70}$$

– for the relationship (69)

$$\alpha = \mathrm{arctg}\left[\frac{(1 + a)\,\mu_s}{1 - a\mu_s^2}\right]. \tag{71}$$

Case II. B of feeding with material

In this case, from the relationship (54) and after incorporating (64) we receive relationships the same as in the item a), i.e. (68), (69), (70), (71).

5. Conclusions

1) The construction of the crushing machine ensures and forces always the obtainment of the static friction factor on one of the working surfaces, for each case A) and B).
2) A greater value of the static friction factor as compared with the kinetic friction enables to increase the angle of capturing the particle of fed material and pull a more sizable particle into the working space, which leads to an increase in effectiveness and efficiency of crushing.
3) The gravity feed with the material allows to adjust the velocity of feeding with particles to the circumferential velocity of the working roll.
4) In a crusher with a few stages of crushing it is possible to couple the circumferential velocity of rolls of the next stage with the velocity of throwing the material out of the previous stage. This enables one to obtain working conditions as for the crushing case A).

Manuscript received by Editorial Board, March 27, 2007;
final version, October 15, 2007.

REFERENCES

[1] Chwarścianek F.: Roll-Plate Crusher (original title in Polish: Rozdrabniacz walcowo-płytowy), Inżynieria i Aparatura Chemiczna, No. 3s/2005, Gliwice, SIGMA NOT, Warsaw, 2005, pp. 14÷15.

[2] Chwarścianek F.: Application No. W 114968 of utility pattern entitled "Grinding System" (original in Polish: Zgłoszenie Nr W 114968 wzoru użytkowego pt. "Układ rozdrabniający"), Patent Office of the Republic of Poland, 2004.

[3] Chwiej M.: Food Industry Equipment. Machines and Apparatuses (original title in Polish: Aparatura przemysłu spożywczego. Maszyny i aparaty), PWN, Warsaw, 1984.

[4] Górecka A., Otwinowski H.: Theories of Crushing and Their Classification, Parts 1 and 2 (original title in Polish: Teorie rozdrabniania i ich klasyfikacja, Część 1 i 2), Cement Wapno Beton, R. 6/68, 2001, pp. 96÷98, pp. 148÷150.

[5] Flizikowski J.: The Crushing of Plastics (original title in Polish: Rozdrabnianie tworzyw sztucznych), Wydawnictwa Uczelniane ATR, Bydgoszcz, 1998.

[6] Flizikowski J.: Construction of Food Crushing Machines (original title in Polish: Konstrukcja rozdrabniaczy żywności), Wydawnictwa Uczelniane ATR, Bydgoszcz, 2005.

[7] Koch R.: Mechanical Processes in Chemical Engineering (original title in Polish: Procesy mechaniczne w inżynierii chemicznej), WNT, Warsaw, 1998.

[8] Warych J.: Chemical and Process Equipment (original title in Polish: Aparatura chemiczna i procesowa), Publishing House of the Warsaw Polytechnic, 1996.

Konstrukcyjne zwiększenie efektywności rozdrabniania

Streszczenie

W opracowaniu przedstawiono próbę zwiększenia efektywności procesu rozdrabniania poprzez zastosowanie nowego oryginalnego układu rozdrabniającego. W wielu rozdrabniaczach jako niezwykle istotne, a nawet dominujące w procesie rozdrabniania materiałów, występuje zjawisko tarcia. Proponowany układ konstrukcyjny cechuje występowanie, zawsze na jednej z powierzchni roboczych, spoczynkowego współczynnika tarcia i tym samym większego tarcia od tarcia ruchowego.