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AN INFLUENCE OF THE COVARIANCE BETWEEN SINGLE ORBIT PARAMETERS ON THE ACCURACY OF OBSERVATIONS OF THE PSEUDO-RANGES AND PHASE DIFFERENCES

Jonas Skeivalas, Eimuntas Paršeliūnas, Raimundas Putrimas, Dominykas Šlikas

Vilnius Gediminas Technical University, Institute of Geodesy, Vilnius, Lithuania (*jonas.skeivalas@vgtu.lt, eimis@vgtu.lt, raimundas.putrimas@vgtu.lt,* ✉ *dominykas.slikas@vgtu.lt,* +370 6 983 8198)

Abstract

The possibilities to improve values of the satellite orbit elements by employing the pseudo-ranges and differences of carrier phase frequencies measured at many reference GPS stations are analysed. An improvement of orbit ephemeris is achieved by solving an equation system of corrections of the pseudo-ranges and phase differences with the least-squares method. Also, equations of space coordinates of satellite orbit points expressed by ephemeris at fixed moments are used. The relation between the accuracy of the pseudo-ranges and phase differences and the accuracy of the satellite ephemeris is analysed. Formulae for estimation of the influence of the ephemeris on the measured pseudo-ranges and phase differences and for prediction of the accuracy of the pseudo-ranges and phase differences were obtained. An influence of the covariance between single orbit parameters on the accuracy of the pseudo-ranges and phase differences is detected.

Keywords: Global Positioning System, ephemeris, Doppler effect, dispersion, geodesy.

1. Introduction

The modernization and improvement of the *Global Navigation Satellite Systems* (GNSS) like GPS, GLONASS, upcoming of COMPASS (BeiDou) and GALILEO, probably QZSS, create the excellent presumptions to improve performance, safety and precision for geodetic and navigation purposes in the near future [1–3]. Without any doubts the quality of the satellite ephemeris and clock parameters plays a vital role ensuring positioning accuracy and integrity [4–11]. Some authors performed wide practical analysis of signal-in-space ranging errors for all current satellite navigation systems [12–18]. The satellites orbit errors and clock biases are the keys to precise point positioning [19–26]. So increasing the positioning precision is the primary goal of GNSS users.

In this paper the influence of the accuracy of the GPS satellites ephemeris on the accuracy of the pseudo-range and phase observations from both theoretical and practical perspective will be analysed. Usually ephemerides are corrected according to pseudo-range, phase and Doppler observations [27, 28]. These observations are processed by the least squares method solving a system of equations of the pseudo-range and phase observations, when the coordinates of the points of satellite orbit are expressed by the orbit parameters. Therefore, a correlation between the orbit parameters occurs, and its influence should be taken into account in performing measurements and estimating the accuracy of the pseudo-range and phase difference measurements [29–32]. The stress will be put on the analysis of the influence of the satellite orbit parameters on the accuracy of the pseudo-range and phase observations. Because different satellite orbit parameters influence the accuracy of the pseudo-range and phase observations in different degree, it is purposeful to choose optimal ratios of appropriate parameters. This is advisable in creating linear models of GPS measurements [28, 33, 34].

2. Theoretical principles

Parametric equation systems based on the measurements of the pseudo-range and phase differences of carrier oscillations and the number of Doppler cycles will be applied. The obtained values are functions of the space coordinates of satellites and the points on the earth surface. The coordinates of satellites at any fixed moment t can be expressed by orbit parameters, so they are functions of the ephemeris [27, 28]. Following [28] the equations of the pseudo-ranges, phase differences of carrier oscillations and the number of Doppler cycles can be written:

$$R_j^k(t) = \sqrt{\left(X^k(t) - X_j\right)^2 + \left(Y^k(t) - Y_j\right)^2 + \left(Z^k(t) - Z_j\right)^2} + c\delta t, \tag{1}$$

$$\lambda_j \Phi_j^k(t) = \sqrt{\left(X^k(t) - X_j\right)^2 + \left(Y^k(t) - Y_j\right)^2 + \left(Z^k(t) - Z_j\right)^2} - \lambda_j N_j^k + c\Delta\delta(t), \tag{2}$$

$$\begin{aligned}\lambda_j N_{12} = {} & \sqrt{\left(X^k(t_2) - X_j\right)^2 + \left(Y^k(t_2) - Y_j\right)^2 + \left(Z^k(t_2) - Z_j\right)^2} - \\ & - \sqrt{\left(X^k(t_1) - X_j\right)^2 + \left(Y^k(t_1) - Y_j\right)^2 + \left(Z^k(t_1) - Z_j\right)^2} + \left(f_j - f^k\right)(t_2 - t_1)\,\lambda_j\,,\end{aligned} \tag{3}$$

where $R_j^k(t)$ – the pseudo-range between satellite k at a fixed moment t and a GPS receiver on an earth point j; $X^k(t)$, $Y^k(t)$, $Z^k(t)$ – coordinates of the satellite in the rectangular geocentric coordinate system; X_j, Y_j, Z_j – geocentric coordinates of the GPS receiver; c – the speed of electromagnetic oscillations in vacuum; δt – correction of the GPS receiver clock, $\Delta\delta(t) = \delta_j(t) - \delta^k(t)$, $\Phi_j^k(t)$ – a phase difference at moment t of oscillations transmitted from satellite k and received by GPS receiver on point j; N_j^k – the initial number of round cycles; λ – wavelength L_1 or L_2 of the carrier oscillations; N_{12} – the number of Doppler cycles during a period $(t_2 - t_1)$. Both equations should be defined for L_1 and L_2.

The coordinates of a satellite in the rectangular geocentric coordinate system can be expressed through orbital parameters as follows [34]:

$$\left.\begin{aligned} X^k(t) &= r\,(\cos u \cos L - \sin u \sin L \cos i) = r\phi_x \\ Y^k(t) &= r\,(\cos u \sin L + \sin u \cos L \cos i) = r\phi_y \\ Z^k(t) &= r \sin u \sin i = r\phi_z \end{aligned}\right\}, \tag{4}$$

where r – the geocentric satellite distance; $u = \omega + v$ – the argument of latitude; ω – the argument of perigee; v – the true anomaly; $L = \Omega - S$ – the longitude of the orbit ascension node; Ω – the

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rectascense of the ascension node; S – the Greenwich solar time; i – the angle between the planes of the orbit and the Earth equator.

The geocentric satellite distance could be expressed as follows [28]:

$$r = a\,(1 - e\cos E)\,, \tag{5}$$

where a – the major semi-axis of the orbit; e – the eccentricity of the orbit; E – the eccentric anomaly.

The eccentric anomaly E can be calculated from the Kepler equation, which is transcendent and can be solved by the method of iterations:

$$E - e\sin E = \overline{M}, \tag{6}$$

and further $E_1 = \overline{M}$, $E_2 = \overline{M} + \sin E_1$, $E_3 = \overline{M} + e\sin E_2$, where $\overline{M}$ – the average anomaly.

The average anomaly can be determined from the equation:

$$\overline{M} = \frac{2\pi}{U_0}(t - t_0) = \sqrt{\frac{GM_0}{a^3}}(t - t_0), \tag{7}$$

where U_0 – the period of making a single orbit by the satellite; t_0 – the moment of the satellite crossing its perigee; G – the gravitational constant; M_0 – the Earth mass.

Having pseudo-ranges, phase differences and numbers of Doppler cycles measured from a number of GPS stations we can write the following system of correction equations:

$$\left.\begin{array}{l} V\left\{R_j^k(t)\right\} = \left(\dfrac{\partial R_j^k}{\partial u}\right)_0 \tau u + \left(\dfrac{\partial R_j^k}{\partial \lambda}\right)_0 \tau\lambda + \left(\dfrac{\partial R_j^k}{\partial i}\right)_0 \tau i + \\ \quad + \left(\dfrac{\partial R_j^k}{\partial a}\right)_0 \tau a + \left(\dfrac{\partial R_j^k}{\partial e}\right)_0 \tau e + \left(\dfrac{\partial R_j^k}{\partial E}\right)_0 \tau E + l_{R,j}^k(t) \\ \qquad j = 1, 2, \ldots n \end{array}\right\}, \tag{8}$$

$$\left.\begin{array}{l} V\left\{\lambda\Phi_j^k(t)\right\} = \left(\dfrac{\partial \lambda\Phi_j^k}{\partial u}\right)_0 \tau u + \left(\dfrac{\partial \lambda\Phi_j^k}{\partial \lambda}\right)_0 \tau\lambda + \left(\dfrac{\partial \lambda\Phi_j^k}{\partial i}\right)_0 \tau i + \\ \quad + \left(\dfrac{\partial \lambda\Phi_j^k}{\partial a}\right)_0 \tau a + \left(\dfrac{\partial \lambda\Phi_j^k}{\partial e}\right)_0 \tau e + \left(\dfrac{\partial \lambda\Phi_j^k}{\partial E}\right)_0 \tau E + l_{\Phi,j}^k(t) \\ \qquad j = 1, 2, \ldots n \end{array}\right\}, \tag{9}$$

$$\left.\begin{array}{l} V\left\{\lambda N_{12}\right\} = \left(\dfrac{\partial \lambda N_{12}}{\partial u}\right)_0 \tau u + \left(\dfrac{\partial \lambda N_{12}}{\partial \lambda}\right)_0 \tau\lambda + \left(\dfrac{\partial \lambda N_{12}}{\partial i}\right)_0 \tau i + \\ \quad + \left(\dfrac{\partial \lambda N_{12}}{\partial a}\right)_0 \tau a + \left(\dfrac{\partial \lambda N_{12}}{\partial e}\right)_0 \tau e + \left(\dfrac{\partial \lambda N_{12}}{\partial E}\right)_0 \tau E + l_{N,j}^k(t) \\ \qquad j = 1, 2, \ldots n \end{array}\right\}, \tag{10}$$

where $V\left\{R_j^k(t)\right\}$, $V\left\{\lambda\Phi_j^k(t)\right\}$, $V\{\lambda N_{12}\}$, τu – the corrections to the pseudo-ranges, phase differences, numbers of Doppler cycles and corresponding parameters; $l_{R,j}^k(t)$, $l_{\Phi,j}^k(t)$, $l_{N,j}^k(t)$ – the free members; $\left(\dfrac{\partial R_j^k}{\partial u}\right)$, $\left(\dfrac{\partial \lambda\Phi_j^k}{\partial u}\right)$, $\left(\dfrac{\partial \lambda N_{12}}{\partial u}\right)$ – the coefficients of correction equations or values of the partial derivatives, calculated for a fixed moment t, when the transmitted values of parameters are known; n – the number of GPS stations. The correction equations are written for a fixed moment t and for an appropriate satellite k.

In each orbit 4 GPS satellites are flying, what leads to the condition that even in the best case we can observe two satellites only. The correction equations of the pseudo-ranges and phase differences of each satellite for a corresponding moment t can be solved by the least square method independently from the measurements of the pseudo-ranges and phase differences of other satellites and applying the data of observations obtained from many GPS sites.

The free members can be calculated according to the formulae:

$$\begin{aligned} l_{R,j}^k(t) &= R_j^k(t) - R_{j,0}^k(t) \\ &= R_j^k(t) - \left\{\sqrt{\left(X_0^k - X_{j,0}\right)^2 + \left(Y_0^k - Y_{j,0}\right)^2 + \left(Z_0^k - Z_{j,0}\right)^2} + c\delta t\right\}, \end{aligned} \tag{11}$$

$$\begin{aligned} l_{\Phi,j}^k(t) &= \lambda_j \Phi_j^k(t) - \lambda_j \Phi_{j,0}(t) \\ &= \lambda_j \Phi_j^k(t) - \left\{\sqrt{\left(X_0^k - X_{j,0}\right)^2 + \left(Y_0^k - Y_{j,0}\right)^2 + \left(Z_0^k - Z_{j,0}\right)^2} - \lambda_j N_j^k + c\Delta\delta(t)\right\}, \end{aligned} \tag{12}$$

$$\begin{aligned} l_{N,j}^k(t) &= \lambda_j N_{12} - \lambda_j N_{12,0} \\ &= \lambda_j N_{12} - \sqrt{\left(X_0^k(t_2) - X_{j,0}\right)^2 + \left(Y_0^k(t_2) - Y_{j,0}\right)^2 + \left(Z_0^k(t_2) - Z_{j,0}\right)^2} - \\ &- \sqrt{\left(X_0^k(t_1) - X_{j,0}\right)^2 + \left(Y_0^k(t_1) - Y_{j,0}\right)^2 + \left(Z_0^k(t_1) - Z_{j,0}\right)^2} + \left(f_j - f^k\right)(t_2 - t_1)\,\lambda_j, \end{aligned} \tag{13}$$

where X_0^k, Y_0^k, Z_0^k – geodetic geocentric coordinates of a satellite, calculated according to the transmitted ephemeris; $X_{j,0}$, $Y_{j,0}$, $Z_{j,0}$ – geodetic geocentric coordinates of a GPS station; δt – the correction of GPS receiver clock, calculated from the measured pseudo-ranges.

3. Method

Applying formulae (1), (2), (4), the expressions to calculate the partial derivatives of the pseudo-ranges and phase differences according to orbit parameters can be written. These calculations are executed for all moments. So we have:

$$\begin{aligned} \left(\frac{\partial R_j^k}{\partial u}\right)_0 = \left(\frac{\partial \lambda\Phi_j^k}{\partial u}\right)_0 = a_{j1} = \frac{r}{R_j^k}\Big\{&-\left(X^k - X_j\right)(\sin u \cos L + \cos u \sin L \cos i) + \\ &+ \left(Y^k - Y_j\right)(-\sin u \sin\lambda + \cos u \cos\lambda \cos i) + \left(Z^k - Z_j\right)(\cos u \sin i)\Big\}, \end{aligned} \tag{14}$$

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$$\left(\frac{\partial R_j^k}{\partial L}\right)_0 = \left(\frac{\partial \lambda \Phi_j^k}{\partial L}\right)_0 = a_{j2} = \frac{r}{R_j^k}\left\{-\left(X^k - X_j\right)(\cos u \sin \lambda + \sin u \cos L \cos i) + \right. \\ \left. + \left(Y^k - Y_j\right)(\cos u \cos L - \sin u \sin L \cos i)\right\}, \tag{15}$$

$$\left(\frac{\partial R_j^k}{\partial i}\right)_0 = \left(\frac{\partial \lambda \Phi_j^k}{\partial i}\right)_0 = a_{j3} = \frac{r}{R_j^k}\left\{\left(X^k - X_j\right)\sin u \sin \lambda \sin i + \right. \\ \left. + \left(Y^k - Y_j\right)(-\sin u \cos \lambda \sin i) + \left(Z^k - Z_j\right)\sin u \cos i\right\}, \tag{16}$$

$$\left(\frac{\partial R_j^k}{\partial a}\right)_0 = \left(\frac{\partial \lambda \Phi_j^k}{\partial a}\right)_0 = a_{j4} = \frac{1}{R_j^k}\left\{\phi_x\left(X^k - X_j\right)(1 - e\cos E) \right. \\ \left. + \phi_y\left(Y^k - Y_j\right)(1 - e\cos E) + \phi_z\left(Z^k - Z_j\right)(1 - e\cos E)\right\}, \tag{17}$$

$$\left(\frac{\partial R_j^k}{\partial e}\right)_0 = \left(\frac{\partial \lambda \Phi_j^k}{\partial e}\right)_0 = a_{j5} = \frac{-1}{R_j^k}\left\{\phi_x\left(X^k - X_j\right)a\cos E + \right. \\ \left. + \phi_y\left(Y^k - Y_j\right)a\cos E + \phi_z\left(Z^k - Z_j\right)a\cos E\right\}, \tag{18}$$

$$\left(\frac{\partial R_j^k}{\partial E}\right)_0 = \left(\frac{\partial \lambda \Phi_j^k}{\partial E}\right)_0 = a_{j6} = \frac{1}{R_j^k}\left\{\phi_x\left(X^k - X_j\right)ae\sin E + \right. \\ \left. + \phi_y\left(Y^k - Y_j\right)ae\sin E + \phi_z\left(Z^k - Z_j\right)ae\sin E\right\}. \tag{19}$$

The values of partial derivatives of Doppler cycles according to orbit parameters can be obtained from the formulae (for corresponding moments):

$$\left(\frac{\partial \lambda N_{12}}{\partial u}\right)_0 = a_{j1}(t_2) - a_{j1}(t_1), \tag{20}$$

$$\left(\frac{\partial \lambda N_{12}}{\partial L}\right)_0 = a_{j2}(t_2) - a_{j2}(t_1), \tag{21}$$

$$\left(\frac{\partial \lambda N_{12}}{\partial i}\right)_0 = a_{j3}(t_2) - a_{j3}(t_1), \tag{22}$$

$$\left(\frac{\partial \lambda N_{12}}{\partial a}\right)_0 = a_{j4}(t_2) - a_{j4}(t_1), \tag{23}$$

$$\left(\frac{\partial \lambda N_{12}}{\partial e}\right)_0 = a_{j5}(t_2) - a_{j5}(t_1), \tag{24}$$

$$\left(\frac{\partial \lambda N_{12}}{\partial E}\right)_0 = a_{j6}(t_2) - a_{j6}(t_1), \tag{25}$$

where the values of coefficients $a_{j1}(t_i)$, $a_{j2}(t_i)$ for corresponding moments t_i are obtained from formulae (14)–(19). For these calculations approximate values of coordinates of the satellites and GPS stations can be applied.

The system of correction equations of the pseudo-ranges, phase differences and numbers of Doppler cycles can be written In the form of matrixes as follows:

$$v = A\tau + b, \tag{26}$$

$$N\tau + \omega = 0, \tag{27}$$

$$\tau = -N^{-1}\omega, \tag{28}$$

$$K_{\widetilde{T}} = K_\tau = \sigma_0^2 N^{-1}, \tag{29}$$

$$K_{\widetilde{R}} = \sigma_0^2 A N^{-1} A^T, \tag{30}$$

where A – a matrix of correction coefficients of corresponding systems of equations; $N = A^T P A$ – a matrix of coefficients of normal equations; P – a matrix of weights of corresponding members; τb – vectors of corrections and free parameters; $\omega = A^T P b$; $K_{\widetilde{T}}$, $K_{\widetilde{R}}$ – matrixes of covariance of the adjusted orbit parameters $\widetilde{T}$ and corresponding adjusted members $\widetilde{R}$; σ_0 – standard deviation of the measurement result, which weight is equal to unit.

Let us estimate the accuracy of the measured pseudo-ranges and phase differences in dependence on the accuracy of the orbit parameters. The expression of the pseudo-ranges and phase differences looks like:

$$\begin{aligned} \sigma_R^2 = \sigma_{\lambda\Phi}^2 = a_{j1}^2\sigma_u^2 + a_{j2}^2\sigma_L^2 + a_{j3}^2\sigma_i^2 + a_{j4}^2\sigma_a^2 + a_{j5}^2\sigma_e^2 + \\ + a_{j6}^2\sigma_E^2 + 2a_{j4}a_{j5}K(e,a) + 2a_{j5}a_{j6}K(e,E) + 2a_{j4}a_{j6}K(a,E), \end{aligned} \tag{31}$$

$$\begin{aligned} \sigma_{\lambda N_{12}}^2 = \sigma_{j1}^2(t_{12})\sigma_u^2 + a_{j2}^2(t_{12})\sigma_L^2 + a_{j3}^2(t_{12})\sigma_i^2 + a_{j4}^2(t_{12})\sigma_a^2 + a_{j5}^2(t_{12})\sigma_e^2 + \\ + a_{j6}^2(t_{12})\sigma_E^2 + 2a_{j4}(t_{12})a_{j5}(t_{12})K(e,a) + 2a_{j4}(t_{12})a_{j6}(t_{12})K(a,E) + \\ + 2a_{j5}(t_{12})a_{j6}(t_{12})K(e,E), \end{aligned} \tag{32}$$

where σ_R^2, σ_u^2 – symbols of dispersions of corresponding parameters; $K(e,a)$, $K(e,E)$, $K(a,E)$ – symbols of covariance between corresponding parameters.

There is no correlation between parameters u, λ and i, whereas between parameters a, e, E a functioning correlation exists. Let us determine the expressions for the covariance values. A covariance $K(e,a)$:

$$K(e,a) = M\left\{(e - Me)(a - Ma)\right\} = M(\delta e \cdot \delta a) = M\left\{\left(\frac{\partial e}{\partial a}\delta a\right)\delta a\right\} = \frac{(1-e^2)}{ae}\sigma_a^2, \tag{33}$$

where M – a symbol of mean (expected value), δe, δa – random errors.

A covariance $K(e,E)$:

$$\begin{aligned} K(e,E) = M\left\{(e - Me)(E - ME)\right\} = M(\delta e, \delta E) = \\ = M\left\{\left(\frac{\partial e}{\partial E}\delta E\right)\delta E\right\} = \frac{\sigma_E^2}{\sin E}(1 - e\cos E). \end{aligned} \tag{34}$$

The standard deviations of eccentricity e and eccentric anomaly E can be calculated from the formulae:

$$\sigma_e = \left(\frac{\partial e}{\partial a}\right)_0 \sigma_a = \frac{1-e^2}{ae}\sigma_a, \tag{35}$$

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$$\sigma_E = \left(\frac{\partial E}{\partial a}\right)\sigma_e = \frac{\sin E}{1 - e\cos E}\sigma_e\,, \tag{36}$$

The covariance $K(a, E)$ can be calculated according to the formula:

$$\begin{aligned} K(a,E) &= M\left(\delta a \cdot \delta E\right) = M\left\{\left(\frac{\partial a}{\partial E}\delta E\right)(\delta E)\right\} = \\ &= M\left\{\left(\frac{\partial a}{\partial e}\frac{\partial e}{\partial E}\right)\cdot(\delta E)^2\right\} = \frac{ae(1 - e\cos E)}{(1 - e^2)\sin E}\sigma_E^2\,. \end{aligned} \tag{37}$$

4. Results

As an example we will use the data of a GPS station VLNS of the Europe Permanent Network [35, 36]. The illustration of some data in the form of time series of geodetic geocentric coordinates is shown in Fig. 1.

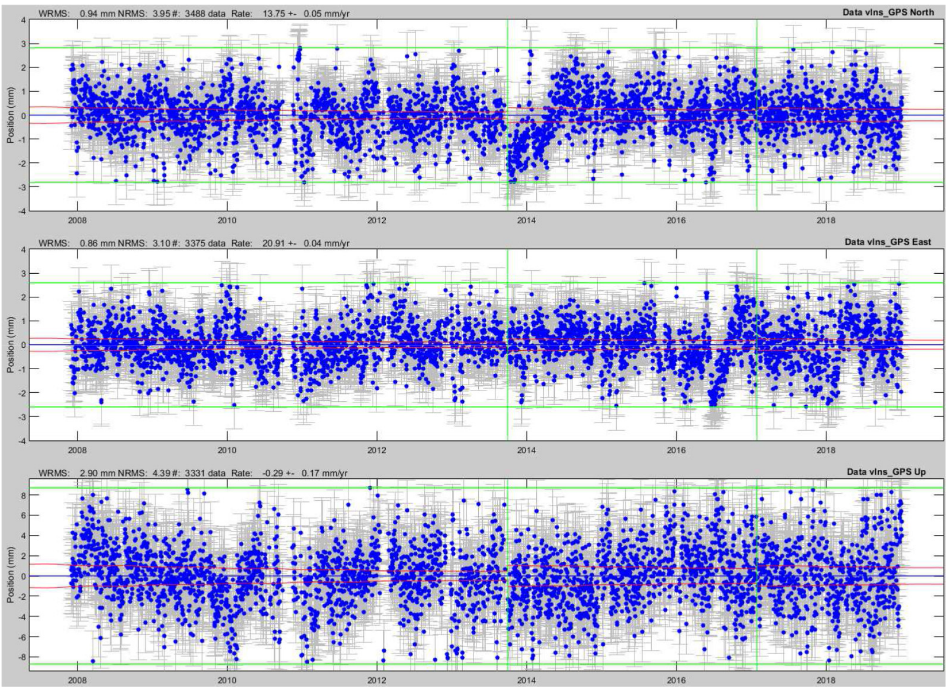

Fig. 1. Time series of geodetic geocentric coordinates during the last decade.

For calculations it was assumed that the geodetic geocentric coordinates of the VLNS station are:

$$\begin{aligned} X_{j,0} &= 3343600.974 \text{ m}, \\ Y_{j,0} &= 1580417.579 \text{ m}, \\ Z_{j,0} &= 5179337.091 \text{ m}. \end{aligned}$$

Applying the fixed coordinates of a GPS satellite for a single moment according to formulae (31)–(37) the covariance values will be:

$$K(e,a) = 8.08 \cdot 10^{-10},$$
$$K(e,E) = 4.38 \cdot 10^{-15},$$
$$K(a,E) = 5.42 \cdot 10^{-10}$$

and the standard deviations

$$\sigma_e = 8.08 \cdot 10^{-8},$$
$$\sigma_E = 5.42 \cdot 10^{-8},$$

where:

$$\sigma_a = 0.01 \text{ m},$$
$$\sigma_u = \sigma_L = \sigma_i = 0.5 \cdot 10^{-5} \text{ rad}.$$

The values of correlation coefficients between orbit elements will be equal to unit, *i.e.* $r(e,a) = r(e,E) = r(a,E) = 1.0$.

The influence of errors of the single-orbit parameters u, L and i on the accuracy of the pseudo-ranges R and phase differences $\lambda\Phi$ is presented in Table 1.

Table 1. Standard deviations of the pseudo-ranges and phase differences due to the influence of the errors of orbit elements.

Standard deviations of the pseudo-ranges and phase differences due to the influence of the errors of orbit elements	**Value, m**
Due to the argument of latitude – σ_{Ru}	0.0002031
Due to the longitude of the orbit ascension node – σ_{RL}	0.0003130
Due to the angle between planes of the orbit and the Earth equator – σ_{Ri}	0.0000030
Due to the orbit major semi-axis – σ_{Ra}	0.0051905
Due to the orbit eccentricity –σ_{Re}	0.0000082
Due to the orbit eccentric anomaly σ_{RE}	0.0000000

The influence of the sum of errors of orbit parameters u, L and i on the accuracy of the pseudo-ranges R, phase differences $\lambda\Phi$ and numbers of Doppler cycles λN_{12} is presented in Table 2.

Table 2. Standard deviations of the pseudo-ranges, phase differences and numbers of Doppler cycles due to the influence of the sum of errors of orbit elements.

Standard deviations of the pseudo-ranges, phase differences and numbers of Doppler cycles due to the influence of the sum of errors of orbit elements	**Value, m**
Due to the sum of orbit element errors (without covariance values) on the pseudo-range – σ_{Rp}	0.00520
Due to the sum of covariance values on the pseudo-range – σ_{Rk}	0.00029
Due to the sum of all errors and covariance values on the pseudo-range – σ_R	0.00521
Due to the sum of orbit element errors (without covariance values) on the number of Doppler cycles – σ_{Np}	0.01101
Due to the sum of covariance values on the number of Doppler cycles – σ_{Nk}	0.00066
Due to the sum of all errors and covariance values on the number of Doppler cycles – σ_N	0.01103

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Further, having at the same time the measurement results of the pseudo-ranges and phase differences of the same satellite and many GPS stations, we can construct a system of correction equations. Solving this system by the least squares method we will receive the most reliable values of the pseudo-ranges, phase differences and orbit elements.

5. Conclusions

The accuracy of the measured pseudo-ranges and phase differences significantly depends on the precision of the satellite ephemeris. To estimate this accuracy the formulae (31) to (37) were derived.

It was proved that the covariance between single-satellite orbit elements has an insignificant influence on the accuracy of the measured pseudo-ranges, phase differences and numbers of Doppler cycles.

The biggest influence on the accuracy of the measured pseudo-ranges, phase differences and numbers of Doppler cycles comes from the errors of the orbit semi-axis a, approaching $\sigma_{Ra} = 0.0052$ m. The influence of the errors of the argument of latitude u and the longitude of the orbit ascension node L is by about one order less, equalling to $\sigma_{Ru} = 0.0002$ m and $\sigma_{RL} = 0.003$ m. The optimal set of standard deviations of the orbit parameters is as follows: $\sigma_u = \sigma_\lambda = \sigma_i = 0.25 \cdot 10^{-5}$ rad ($\approx 0.5''$), $\sigma_a = 0.001$ m, $\sigma_E = 10^{-8}$ rad ($\approx 0.002''$).

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