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Balancing reactive compensation at three-phase four-wire systems with a sinusoidal and asymmetrical voltage source

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Abstract. The article presents the essentials of reactance compensation of unbalanced loads in three-phase four-wire systems powered by a sinusoidal and asymmetrical voltage source. The whole of compensation and symmetrization is based on the Currents' Physical Components (CPC) theory. Reactance compensation, i.e. compensation based solely on inductors and capacitors, in four-wire systems requires the device to be included in a star (Y) structure in order to compensate for the reactive current (reactive power) and the current at the neutral conductor caused by zero sequence asymmetry, and for the device in a delta (Δ) structure to allow compensation of the reactive current (reactive power) and current, causing asymmetry of the negative sequence.

Key words: Currents' Physical Components (CPC), power theory, asymmetrical voltage supply, balancing reactive compensation.

1. Introduction

In industrial plants as well as commercial and residential buildings, four-wire circuits are the dominant power systems. A characteristic property of such systems is the existence of reactive power and currents asymmetry, which also affects imbalance (asymmetry) of the supply voltage.

The imbalance of a load and power source along with reactive power contribute to an increase in energy losses, which in turn leads to a decrease in its quality [15].

Two approaches can be used to improve power quality. The first approach entails using switching compensators, known also as active power filters. Such an approach cannot be used in systems with a high value of the load currents since the transistors used for shaping the compensation currents have limited switching power. The second approach involves using reactive elements to build the so-called passive filters. Passive filters are usually designed to compensate for reactive power, and thus do not significantly affect the balancing of the load.

The best solution is to use balancing compensators, which were first developed by Steinmetz in 1917 [2] for three-wire systems with sinusoidal voltage. In later years, based on the work of Steinmetz, many authors presented their research on the construction of such compensators [3–5].

The increase in the number of four-wire systems initiated development of mathematical description of power and currents [6, 8, 9], and allowed to come up with methods of designing compensators in such circuits [9–14].

Works [1, 19, 20] present a description of the balancing compensators based on the Currents' Physical Components (CPC) theory for both three-wire and four-wire circuits with symmetrical sinusoidal or nonsinusoidal voltage supply.

Moreover, in publications [16–18] the method of determining the parameters of the balancing compensator in a delta structure, supplied with asymmetric sinusoidal voltage, is also presented. This method is based on the CPC theory.

The article presents a method for calculating the reactance parameters of the balancing compensator in the star structure along with a short description of the balancing compensator in the delta structure. Those are widely described in [16–18]. Parameters of both balancing compensators are determined for four-wire systems, where the voltage source is asymmetric. Moreover, because the reader should be familiarized with the description of the CPC theory in four-wire systems supplied with asymmetrical sinusoidal voltage, the authors recommend article [8]. Because it is written in Polish, this paper includes a summary of this work in Section 2.

2. Currents' Physical Components (CPC) theory in three-phase four-wire systems at sinusoidal and asymmetric voltage source

An unbalanced linear time-invariant (LTI) load supplied by a sinusoidal but asymmetric voltage source is shown in Fig. 1.

The sinusoidal voltage source that supplies the LTI unbalanced load can be presented in the form of:

$$\boldsymbol{u}(t) = \begin{bmatrix} u_{\mathrm{R}}(t) \\ u_{\mathrm{S}}(t) \\ u_{\mathrm{T}}(t) \end{bmatrix} = \sqrt{2}\mathrm{Re}\begin{bmatrix} \boldsymbol{U}_{\mathrm{R}} \\ \boldsymbol{U}_{\mathrm{S}} \\ \boldsymbol{U}_{\mathrm{T}} \end{bmatrix} e^{j\omega t} = \sqrt{2}\mathrm{Re}\left\{\boldsymbol{U} e^{j\omega t}\right\} \quad (1)$$

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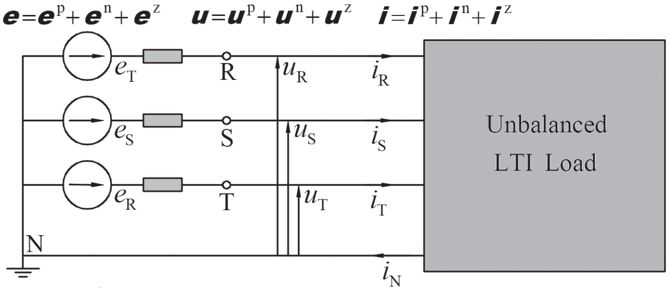

Fig. 1. LTI unbalanced load supplied by a four-wire line

The supply voltage $\boldsymbol{u}$ can be asymmetrical – Fig. 1 – therefore it is possible to present it as the sum of voltages of positive, negative and zero sequences:

$$\begin{aligned}\boldsymbol{u} &= \boldsymbol{u}^{\mathrm{p}} + \boldsymbol{u}^{\mathrm{n}} + \boldsymbol{u}^{\mathrm{z}} = \sqrt{2}\mathrm{Re}\left\{(\boldsymbol{U}^{\mathrm{p}} + \boldsymbol{U}^{\mathrm{n}} + \boldsymbol{U}^{\mathrm{z}})\, e^{j\omega t}\right\} \\ &= \sqrt{2}\mathrm{Re}\left\{(\boldsymbol{1}^{\mathrm{p}} U^{\mathrm{p}} + \boldsymbol{1}^{\mathrm{n}} U^{\mathrm{n}} + \boldsymbol{1}^{\mathrm{z}} U^{\mathrm{z}})\, e^{j\omega t}\right\}\end{aligned} \tag{2}$$

where U^{p}, U^{n} and U^{z} are the complex rms (crms) values of the symmetrical components of positive, negative and zero sequences, described by the Fortescue transformation [1, 7, 21] and $\boldsymbol{1}^{\mathrm{p}}$, $\boldsymbol{1}^{\mathrm{n}}$, $\boldsymbol{1}^{\mathrm{z}}$ are the unit symmetrical vectors.

The line current can be presented identically:

$$\boldsymbol{i}(t) = \begin{bmatrix} i_{\mathrm{R}}(t) \\ i_{\mathrm{S}}(t) \\ i_{\mathrm{T}}(t) \end{bmatrix} = \sqrt{2}\mathrm{Re} \begin{bmatrix} I_{\mathrm{R}} \\ I_{\mathrm{S}} \\ I_{\mathrm{T}} \end{bmatrix} e^{j\omega t} = \sqrt{2}\mathrm{Re}\left\{\boldsymbol{I} e^{j\omega t}\right\} \tag{3}$$

which, as a result of the imbalance and asymmetry of the voltage supply, also has components of positive, negative and zero sequences:

$$\begin{aligned}\boldsymbol{i} &= \boldsymbol{i}^{\mathrm{p}} + \boldsymbol{i}^{\mathrm{n}} + \boldsymbol{i}^{\mathrm{z}} = \sqrt{2}\mathrm{Re}\left\{(\boldsymbol{I}^{\mathrm{p}} + \boldsymbol{I}^{\mathrm{n}} + \boldsymbol{I}^{\mathrm{z}})\, e^{j\omega t}\right\} \\ &= \sqrt{2}\mathrm{Re}\left\{(\boldsymbol{1}^{\mathrm{p}} I^{\mathrm{p}} + \boldsymbol{1}^{\mathrm{n}} I^{\mathrm{n}} + \boldsymbol{1}^{\mathrm{z}} I^{\mathrm{z}})\, e^{j\omega t}\right\}.\end{aligned} \tag{4}$$

The load current (3) can be decomposed into five components described in the CPC theory for three-phase four-wire systems supplied from an asymmetrical voltage source [24].

This paper presents solely the values of the component waveforms.

The active current $\boldsymbol{i}_{\mathrm{a}}$ is defined as follows:

$$\begin{aligned}\boldsymbol{i}_{\mathrm{a}} &= G_{\mathrm{b}} \boldsymbol{u} = \sqrt{2}\mathrm{Re}\left\{G_{\mathrm{b}}\,(\boldsymbol{U}^{\mathrm{p}} + \boldsymbol{U}^{\mathrm{n}} + \boldsymbol{U}^{\mathrm{z}})\, e^{j\omega t}\right\} \\ &= \sqrt{2}\mathrm{Re}\left\{G_{\mathrm{b}}\,(\boldsymbol{1}^{\mathrm{p}} U^{\mathrm{p}} + \boldsymbol{1}^{\mathrm{n}} U^{\mathrm{n}} + \boldsymbol{1}^{\mathrm{z}} U^{\mathrm{z}})\, e^{j\omega t}\right\}\end{aligned} \tag{5}$$

where G_{b} is the equivalent conductance of the balanced load.

The reactive current $\boldsymbol{i}_{\mathrm{r}}$ is described as follows:

$$\begin{aligned}\boldsymbol{i}_{\mathrm{r}} &= B_{\mathrm{b}} \boldsymbol{u}(t \pm T/4) = \sqrt{2}\mathrm{Re}\left\{jB_{\mathrm{b}}\,(\boldsymbol{U}^{\mathrm{p}} + \boldsymbol{U}^{\mathrm{n}} + \boldsymbol{U}^{\mathrm{z}})\, e^{j\omega t}\right\} \\ &= \sqrt{2}\mathrm{Re}\left\{jB_{\mathrm{b}}\,(\boldsymbol{1}^{\mathrm{p}} U^{\mathrm{p}} + \boldsymbol{1}^{\mathrm{n}} U^{\mathrm{n}} + \boldsymbol{1}^{\mathrm{z}} U^{\mathrm{z}})\, e^{j\omega t}\right\}\end{aligned} \tag{6}$$

where B_{b} is the equivalent susceptance of the balanced load.

In the CPC theory, there is the unbalanced current $\boldsymbol{i}_{\mathrm{u}}$, which is expressed as follows:

$$\begin{aligned}\boldsymbol{I}_{\mathrm{u}} = (\boldsymbol{Y}_{\mathrm{d}})\,\boldsymbol{U} + \boldsymbol{1}^{\mathrm{p}}\,(\boldsymbol{A}^{\mathrm{n}} \boldsymbol{U}^{\mathrm{z}} + \boldsymbol{A}^{\mathrm{z}} \boldsymbol{U}^{\mathrm{n}}) \\ + \boldsymbol{1}^{\mathrm{n}}\,(\boldsymbol{A}^{\mathrm{n}} \boldsymbol{U}^{\mathrm{p}} + \boldsymbol{A}^{\mathrm{z}} \boldsymbol{U}^{\mathrm{z}}) + \boldsymbol{1}^{\mathrm{z}}\,(\boldsymbol{A}^{\mathrm{n}} \boldsymbol{U}^{\mathrm{n}} + \boldsymbol{A}^{\mathrm{z}} \boldsymbol{U}^{\mathrm{p}})\end{aligned} \tag{7}$$

and can include three components, namely:

- the unbalanced current of the positive sequence $\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}$:

$$\begin{aligned}\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}} &= \sqrt{2}\mathrm{Re}\left\{\boldsymbol{I}_{\mathrm{u}}^{\mathrm{p}} e^{j\omega t}\right\} \\ &= \sqrt{2}\mathrm{Re}\left\{\boldsymbol{1}^{\mathrm{p}}\,(\boldsymbol{Y}_{\mathrm{d}} \boldsymbol{U}^{\mathrm{p}} + \boldsymbol{A}^{\mathrm{n}} \boldsymbol{U}^{\mathrm{z}} + \boldsymbol{A}^{\mathrm{z}} \boldsymbol{U}^{\mathrm{n}})\, e^{j\omega t}\right\}\end{aligned} \tag{8}$$

- the unbalanced current of the negative sequence $\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}$:

$$\begin{aligned}\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}} &= \sqrt{2}\mathrm{Re}\left\{\boldsymbol{I}_{\mathrm{u}}^{\mathrm{n}} e^{j\omega t}\right\} \\ &= \sqrt{2}\mathrm{Re}\left\{\boldsymbol{1}^{\mathrm{n}}\,(\boldsymbol{Y}_{\mathrm{d}} \boldsymbol{U}^{\mathrm{n}} + \boldsymbol{A}^{\mathrm{n}} \boldsymbol{U}^{\mathrm{p}} + \boldsymbol{A}^{\mathrm{z}} \boldsymbol{U}^{\mathrm{z}})\, e^{j\omega t}\right\}\end{aligned} \tag{9}$$

- the unbalanced current of the zero sequence $\boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}}$:

$$\begin{aligned}\boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}} &= \sqrt{2}\mathrm{Re}\left\{\boldsymbol{I}_{\mathrm{u}}^{\mathrm{z}} e^{j\omega t}\right\} \\ &= \sqrt{2}\mathrm{Re}\left\{\boldsymbol{1}^{\mathrm{z}}\,(\boldsymbol{Y}_{\mathrm{d}} \boldsymbol{U}^{\mathrm{z}} + \boldsymbol{A}^{\mathrm{n}} \boldsymbol{U}^{\mathrm{n}} + \boldsymbol{A}^{\mathrm{z}} \boldsymbol{U}^{\mathrm{p}})\, e^{j\omega t}\right\}\end{aligned} \tag{10}$$

In formulas (7)–(10), individual admittances denote:

- the voltage asymmetry dependent admittance Y_{d}, expressed as:

$$\begin{aligned}\boldsymbol{Y}_{\mathrm{d}} &= G_{\mathrm{d}} + jB_{\mathrm{d}} = \boldsymbol{Y}_{\mathrm{e}} - \boldsymbol{Y}_{\mathrm{b}} \\ &= \frac{1}{3}(\boldsymbol{Y}_{\mathrm{R}} + \boldsymbol{Y}_{\mathrm{S}} + \boldsymbol{Y}_{\mathrm{T}}) - \frac{\boldsymbol{Y}_{\mathrm{R}} U_{\mathrm{R}}^2 + \boldsymbol{Y}_{\mathrm{S}} U_{\mathrm{S}}^2 + \boldsymbol{Y}_{\mathrm{T}} U_{\mathrm{T}}^2}{\|\boldsymbol{u}\|^2}\end{aligned} \tag{11}$$

- the unbalanced admittance of the negative sequence A^{n} [22, 23], equal to:

$$\boldsymbol{A}^{\mathrm{n}} = \frac{1}{3}(\boldsymbol{Y}_{\mathrm{R}} + \alpha \boldsymbol{Y}_{\mathrm{S}} + \alpha^* \boldsymbol{Y}_{\mathrm{T}}) \tag{12}$$

- the unbalanced admittance of the zero sequence A^{z} [22, 23], which is defined as:

$$\boldsymbol{A}^{\mathrm{z}} = \frac{1}{3}(\boldsymbol{Y}_{\mathrm{R}} + \alpha^* \boldsymbol{Y}_{\mathrm{S}} + \alpha \boldsymbol{Y}_{\mathrm{T}}) \tag{13}$$

It should be mentioned that the equivalent conductance and its susceptance of the balanced load, used in (5) and (6), respectively, together produce the admittance of the balanced load responsible for the active power P and reactive power Q of the load. The formula for this admittance is as follows:

$$\boldsymbol{Y}_{\mathrm{b}} = G_{\mathrm{b}} + jB_{\mathrm{b}} = \frac{P - jQ}{\|\boldsymbol{u}\|^2} \tag{14}$$

If the load is supplied from a symmetrical voltage source, the equivalent admittance of the balanced load is:

$$\boldsymbol{Y}_{\mathrm{b}} = G_{\mathrm{b}} + jB_{\mathrm{b}} = \frac{P - jQ}{\|\boldsymbol{u}\|^2} = \frac{1}{3}(\boldsymbol{Y}_{\mathrm{R}} + \boldsymbol{Y}_{\mathrm{S}} + \boldsymbol{Y}_{\mathrm{T}}) = \boldsymbol{Y}_{\mathrm{e}} \tag{15}$$

The difference between (15) and (14) results in equation (11).

3. Parameters of the balancing reactive compensator

The only current component necessary for the load to draw active power P is the active current $\boldsymbol{i}_{\mathrm{a}}$. The other currents, i.e. reactive current $\boldsymbol{i}_{\mathrm{r}}$ and the unbalanced current $\boldsymbol{i}_{\mathrm{u}}$, increase the three-phase rms value $\|\boldsymbol{i}\|$ of the current of the load, and contribute to energy loss, proportional to the square of this value.

The load is described by four admittances i.e. the equivalent admittance of the balanced load $\boldsymbol{Y}_{\mathrm{b}}$, the voltage asymmetry dependent admittance $\boldsymbol{Y}_{\mathrm{d}}$, the unbalanced admittance of the negative sequence $\boldsymbol{A}^{\mathrm{n}}$, and the unbalanced admittance of the zero sequence $\boldsymbol{A}^{\mathrm{z}}$.

In an attempt to demonstrate the possibility of reactance compensation in four-wire systems with an asymmetrical voltage source, additional decomposition of the voltage asymmetry dependent admittance $\boldsymbol{Y}_{\mathrm{d}}$ should be accomplished. After decomposition, relationship (11) is expressed by three components:

- the voltage asymmetry dependent admittance of the negative sequence $\boldsymbol{Y}_{\mathrm{d}}^{\mathrm{n}}$, described as:

$$\boldsymbol{Y}_{\mathrm{d}}^{\mathrm{n}}=\frac{-2n}{3\left[1+n^{2}+z^{2}\right]} \cdot\left[\boldsymbol{Y}_{\mathrm{R}}\cos(\psi_{1})+\boldsymbol{Y}_{\mathrm{S}}\cos\left(\psi_{1}-\frac{2\pi}{3}\right)+\boldsymbol{Y}_{\mathrm{T}}\cos\left(\psi_{1}+\frac{2\pi}{3}\right)\right] \tag{16}$$

where n is a complex asymmetry coefficient of the supply voltage of the negative sequence, namely:

$$\boldsymbol{n}=ne^{j\psi_{1}}=\frac{\boldsymbol{U}^{\mathrm{n}}}{\boldsymbol{U}^{\mathrm{p}}}=\frac{U^{\mathrm{n}}e^{j\delta}}{U^{\mathrm{p}}e^{j\varphi}}=\frac{U^{\mathrm{n}}}{U^{\mathrm{p}}}e^{j(\delta-\varphi)} \tag{17}$$

and z is a complex asymmetry coefficient of the supply voltage of the zero sequence:

$$\boldsymbol{z}=ze^{j\psi_{2}}=\frac{\boldsymbol{U}^{\mathrm{z}}}{\boldsymbol{U}^{\mathrm{p}}}=\frac{U^{\mathrm{z}}e^{j\varepsilon}}{U^{\mathrm{p}}e^{j\varphi}}=\frac{U^{\mathrm{z}}}{U^{\mathrm{p}}}e^{j(\varepsilon-\varphi)} \tag{18}$$

- the voltage asymmetry dependent admittance of the zero sequence $Y_{\mathrm{d}}^{\mathrm{z}}$, defined as follows:

$$Y_{\mathrm{d}}^{\mathrm{z}}=\frac{-2z}{3\left[1+n^{2}+z^{2}\right]} \cdot\left[Y_{\mathrm{R}}\cos(\psi_{2})+Y_{\mathrm{S}}\cos\left(\psi_{2}+\frac{2\pi}{3}\right)+Y_{\mathrm{T}}\cos\left(\psi_{2}-\frac{2\pi}{3}\right)\right] \tag{19}$$

and

- the voltage asymmetry dependent admittance of the "mixed" sequence $Y_{\mathrm{d}}^{\mathrm{nz}}$, equal to:

$$Y_{\mathrm{d}}^{\mathrm{nz}}=\frac{-2m_{1}}{3\left[1+m_{1}^{2}+m_{2}^{2}\right]} \cdot\left[Y_{\mathrm{R}}\cos(\psi_{3})+Y_{\mathrm{S}}\cos\left(\psi_{3}-\frac{2\pi}{3}\right)+Y_{\mathrm{T}}\cos\left(\psi_{3}+\frac{2\pi}{3}\right)\right] \tag{20}$$

where m_1 denotes a complex asymmetry coefficient of the supply voltage of the "mixed" sequence between the zero sequence component of the voltage related to the negative sequence component of the voltage, expressed as:

$$\boldsymbol{m}_{1}=m_{1}e^{j\psi_{3}}=\frac{\boldsymbol{U}^{\mathrm{z}}}{\boldsymbol{U}^{\mathrm{n}}}=\frac{U^{\mathrm{z}}e^{j\varepsilon}}{U^{\mathrm{n}}e^{j\delta}}=\frac{U^{\mathrm{z}}}{U^{\mathrm{n}}}e^{j(\varepsilon-\delta)} \tag{21}$$

and m_2 is a complex asymmetry coefficient of the supply voltage of the "mixed" sequence between the positive sequence component of the voltage related to the negative sequence component of the voltage, described as:

$$\boldsymbol{m}_{2}=m_{2}e^{j\psi_{4}}=\frac{\boldsymbol{U}^{\mathrm{p}}}{\boldsymbol{U}^{\mathrm{n}}}=\frac{U^{\mathrm{p}}e^{j\varphi}}{U^{\mathrm{n}}e^{j\delta}}=\frac{U^{\mathrm{p}}}{U^{\mathrm{n}}}e^{j(\varphi-\delta)} \tag{22}$$

In line with this, we get a description of the load, consisting of six admittances contributing to the unbalanced and reactive currents of the load.

The balancing compensation in four-wire systems, regardless of whether the voltage supply is symmetrical or asymmetrical, requires two structures, i.e. the compensation device connected in the Y-configuration – it is thus possible to compensate for the reactive current (reactive power) and the balancing of the load as a result of compensation of the zero sequence unbalanced current. The second structure is a device connected in the Δ-structure – it allows for compensation of the reactive current (reactive power) and for the balancing of the load as a result of the compensation of the negative sequence unbalanced current (in systems with symmetrical voltage supply) and the positive sequence unbalanced current (in systems with asymmetrical voltage supply).

When determining the parameters of the device compensating for reactive power and balancing the unbalanced components of the currents, we assume that it is a lossless device. In the Y-structure, wires' susceptances are marked with symbols T_{R}, T_{S}, and T_{T}. In the Δ-structure, susceptances line-to-line are marked with symbols T_{RS}, T_{ST}, and T_{TR}. In addition, the symbols of the compensator's admittances are complemented with the C_{Y} index for the star configuration and the C_{Δ} index for the delta configuration.

The equivalent admittances of the load in Y-structure can be expressed by compensator's susceptances, namely:

- the susceptance responsible for compensation of the reactive current (reactive power):

$$\boldsymbol{Y}_{\mathrm{CYb}}=jB_{\mathrm{CYb}}=j\frac{T_{\mathrm{R}}U_{\mathrm{R}}^{2}+T_{\mathrm{S}}U_{\mathrm{S}}^{2}+T_{\mathrm{T}}U_{\mathrm{T}}^{2}}{\|\boldsymbol{u}\|^{2}} \tag{23}$$

- the susceptance responsible for compensation of the currents associated with the admittances of the voltage asymmetry dependent admittance:

$$\boldsymbol{Y}_{\mathrm{CYd}}^{\mathrm{n}}=-j\frac{2n}{3\left[1+n^{2}+z^{2}\right]} \cdot\left[T_{\mathrm{R}}\cos(\psi_{1})+T_{\mathrm{S}}\cos\left(\psi_{1}-\frac{2\pi}{3}\right)+T_{\mathrm{T}}\cos\left(\psi_{1}+\frac{2\pi}{3}\right)\right] \tag{24}$$

$$\boldsymbol{Y}_{\mathrm{CYd}}^{\mathrm{z}} = -j\frac{2z}{3\left[1+n^2+z^2\right]} \cdot \left[T_{\mathrm{R}}\cos(\psi_2)+T_{\mathrm{S}}\cos\left(\psi_2+\frac{2\pi}{3}\right)+T_{\mathrm{T}}\cos\left(\psi_2-\frac{2\pi}{3}\right)\right] \tag{25}$$

$$\boldsymbol{Y}_{\mathrm{CYd}}^{\mathrm{nz}} = -j\frac{2m_1}{3\left[1+m_1^2+m_2^2\right]} \cdot \left[T_{\mathrm{R}}\cos(\psi_3)+T_{\mathrm{S}}\cos\left(\psi_3-\frac{2\pi}{3}\right)+T_{\mathrm{T}}\cos\left(\psi_3+\frac{2\pi}{3}\right)\right] \tag{26}$$

- the susceptance responsible for compensation of the currents related to the unbalanced admittance of the negative sequence:

$$\boldsymbol{A}_{\mathrm{C_Y}}^{\mathrm{n}} = j\frac{1}{3}\left(T_{\mathrm{R}}+\alpha T_{\mathrm{S}}+\alpha^* T_{\mathrm{T}}\right) \tag{27}$$

- the susceptance responsible for compensation of the currents related to the unbalanced admittance of the zero sequence:

$$\boldsymbol{A}_{\mathrm{C_Y}}^{\mathrm{z}} = j\frac{1}{3}\left(T_{\mathrm{R}}+\alpha^* T_{\mathrm{S}}+\alpha T_{\mathrm{T}}\right) \tag{28}$$

In order to define the parameters of the compensator that will compensate for the reactive power and the unbalanced current, while simultaneously not interfering with the active power of the system, the following conditions should be met:

- the reactive current of the voltage source is zero if:

$$B_{\mathrm{CYb}}+B_{\mathrm{b}} = 0 \tag{29}$$

- the unbalanced current (7) equals to zero if:

$$\begin{aligned}(\boldsymbol{Y}_{\mathrm{CYd}}+\boldsymbol{Y}_{\mathrm{d}})\boldsymbol{U} &+ \left(\boldsymbol{1}^{\mathrm{n}}\boldsymbol{A}^{\mathrm{n}}+\boldsymbol{1}^{\mathrm{n}}\boldsymbol{A}_{\mathrm{C_Y}}^{\mathrm{n}}+\boldsymbol{1}^{\mathrm{z}}\boldsymbol{A}^{\mathrm{z}}+\boldsymbol{1}^{\mathrm{z}}\boldsymbol{A}_{\mathrm{C_Y}}^{\mathrm{z}}\right)\boldsymbol{U}^{\mathrm{p}}\\ &+\left(\boldsymbol{1}^{\mathrm{z}}\boldsymbol{A}^{\mathrm{n}}+\boldsymbol{1}^{\mathrm{z}}\boldsymbol{A}_{\mathrm{C_Y}}^{\mathrm{n}}+\boldsymbol{1}^{\mathrm{p}}\boldsymbol{A}^{\mathrm{z}}+\boldsymbol{1}^{\mathrm{p}}\boldsymbol{A}_{\mathrm{C_Y}}^{\mathrm{z}}\right)\boldsymbol{U}^{\mathrm{n}}\\ &+\left(\boldsymbol{1}^{\mathrm{p}}\boldsymbol{A}^{\mathrm{n}}+\boldsymbol{1}^{\mathrm{p}}\boldsymbol{A}_{\mathrm{C_Y}}^{\mathrm{n}}+\boldsymbol{1}^{\mathrm{n}}\boldsymbol{A}^{\mathrm{z}}+\boldsymbol{1}^{\mathrm{n}}\boldsymbol{A}_{\mathrm{C_Y}}^{\mathrm{z}}\right)\boldsymbol{U}^{\mathrm{z}} = 0.\end{aligned} \tag{30}$$

The $(\boldsymbol{Y}_{\mathrm{CYd}}+\boldsymbol{Y}_{\mathrm{d}})\boldsymbol{U}$ quantity can be expressed as:

$$\left(\boldsymbol{Y}_{\mathrm{CYd}}^{\mathrm{n}}+\boldsymbol{Y}_{\mathrm{CYd}}^{\mathrm{z}}+\boldsymbol{Y}_{\mathrm{CYd}}^{\mathrm{nz}}+\boldsymbol{Y}_{\mathrm{d}}^{\mathrm{n}}+\boldsymbol{Y}_{\mathrm{d}}^{\mathrm{z}}+\boldsymbol{Y}_{\mathrm{d}}^{\mathrm{nz}}\right)\boldsymbol{U}. \tag{31}$$

The coefficients in (30) are identical for each transmission line. Therefore it is sufficient to satisfy the dependence for one selected line, and in particular for the R-line:

$$\begin{aligned}&\left(\boldsymbol{Y}_{\mathrm{CYd}}^{\mathrm{n}}+\boldsymbol{Y}_{\mathrm{CYd}}^{\mathrm{z}}+\boldsymbol{Y}_{\mathrm{CYd}}^{\mathrm{nz}}+\boldsymbol{Y}_{\mathrm{d}}^{\mathrm{n}}+\boldsymbol{Y}_{\mathrm{d}}^{\mathrm{z}}+\boldsymbol{Y}_{\mathrm{d}}^{\mathrm{nz}}\right)\boldsymbol{U}_{\mathrm{R}}\\ &\quad+\left(\boldsymbol{A}^{\mathrm{n}}+\boldsymbol{A}_{\mathrm{C_Y}}^{\mathrm{n}}+\boldsymbol{A}^{\mathrm{z}}+\boldsymbol{A}_{\mathrm{C_Y}}^{\mathrm{z}}\right)\boldsymbol{U}^{\mathrm{p}}\\ &\quad+\left(\boldsymbol{A}^{\mathrm{n}}+\boldsymbol{A}_{\mathrm{C_Y}}^{\mathrm{n}}+\boldsymbol{A}^{\mathrm{z}}+\boldsymbol{A}_{\mathrm{C_Y}}^{\mathrm{z}}\right)\boldsymbol{U}^{\mathrm{n}}\\ &\quad+\left(\boldsymbol{A}^{\mathrm{n}}+\boldsymbol{A}_{\mathrm{C_Y}}^{\mathrm{n}}+\boldsymbol{A}^{\mathrm{z}}+\boldsymbol{A}_{\mathrm{C_Y}}^{\mathrm{z}}\right)\boldsymbol{U}^{\mathrm{z}} = 0.\end{aligned} \tag{32}$$

With regard to the fact that the description in (32) is submitted in the form of complex numbers, this equation must be fulfilled separately for the real part and the imaginary part, thus:

$$\begin{aligned}\mathrm{Re}\Big\{&\left(\boldsymbol{Y}_{\mathrm{CYd}}^{\mathrm{n}}+\boldsymbol{Y}_{\mathrm{CYd}}^{\mathrm{z}}+\boldsymbol{Y}_{\mathrm{CYd}}^{\mathrm{nz}}+\boldsymbol{Y}_{\mathrm{d}}^{\mathrm{n}}+\boldsymbol{Y}_{\mathrm{d}}^{\mathrm{z}}+\boldsymbol{Y}_{\mathrm{d}}^{\mathrm{nz}}\right)\boldsymbol{U}_{\mathrm{R}}\\ &+\left(\boldsymbol{A}^{\mathrm{n}}+\boldsymbol{A}_{\mathrm{C_Y}}^{\mathrm{n}}+\boldsymbol{A}^{\mathrm{z}}+\boldsymbol{A}_{\mathrm{C_Y}}^{\mathrm{z}}\right)\boldsymbol{U}^{\mathrm{p}}\\ &+\left(\boldsymbol{A}^{\mathrm{n}}+\boldsymbol{A}_{\mathrm{C_Y}}^{\mathrm{n}}+\boldsymbol{A}^{\mathrm{z}}+\boldsymbol{A}_{\mathrm{C_Y}}^{\mathrm{z}}\right)\boldsymbol{U}^{\mathrm{n}}\\ &+\left(\boldsymbol{A}^{\mathrm{n}}+\boldsymbol{A}_{\mathrm{C_Y}}^{\mathrm{n}}+\boldsymbol{A}^{\mathrm{z}}+\boldsymbol{A}_{\mathrm{C_Y}}^{\mathrm{z}}\right)\boldsymbol{U}^{\mathrm{z}}\Big\} = 0\end{aligned} \tag{33}$$

and

$$\begin{aligned}\mathrm{Im}\Big\{&\left(\boldsymbol{Y}_{\mathrm{CYd}}^{\mathrm{n}}+\boldsymbol{Y}_{\mathrm{CYd}}^{\mathrm{z}}+\boldsymbol{Y}_{\mathrm{CYd}}^{\mathrm{nz}}+\boldsymbol{Y}_{\mathrm{d}}^{\mathrm{n}}+\boldsymbol{Y}_{\mathrm{d}}^{\mathrm{z}}+\boldsymbol{Y}_{\mathrm{d}}^{\mathrm{nz}}\right)\boldsymbol{U}_{\mathrm{R}}\\ &+\left(\boldsymbol{A}^{\mathrm{n}}+\boldsymbol{A}_{\mathrm{C_Y}}^{\mathrm{n}}+\boldsymbol{A}^{\mathrm{z}}+\boldsymbol{A}_{\mathrm{C_Y}}^{\mathrm{z}}\right)\boldsymbol{U}^{\mathrm{p}}\\ &+\left(\boldsymbol{A}^{\mathrm{n}}+\boldsymbol{A}_{\mathrm{C_Y}}^{\mathrm{n}}+\boldsymbol{A}^{\mathrm{z}}+\boldsymbol{A}_{\mathrm{C_Y}}^{\mathrm{z}}\right)\boldsymbol{U}^{\mathrm{n}}\\ &+\left(\boldsymbol{A}^{\mathrm{n}}+\boldsymbol{A}_{\mathrm{C_Y}}^{\mathrm{n}}+\boldsymbol{A}^{\mathrm{z}}+\boldsymbol{A}_{\mathrm{C_Y}}^{\mathrm{z}}\right)\boldsymbol{U}^{\mathrm{z}}\Big\} = 0.\end{aligned} \tag{34}$$

As can be seen from (32), it describes the unbalanced current consisting of three components of the relevant sequences, i.e. positive, negative and zero, and therefore, it is not possible to compensate for the unbalanced current with only one configuration of the compensator.

From relationships (33) and (34) only parameters (admittances) describing the unbalanced current of the zero sequence must remain, and thus:

$$\begin{aligned}\mathrm{Re}\Big\{&\left(\boldsymbol{Y}_{\mathrm{CYd}}^{\mathrm{n}}+\boldsymbol{Y}_{\mathrm{CYd}}^{\mathrm{z}}+\boldsymbol{Y}_{\mathrm{CYd}}^{\mathrm{nz}}+\boldsymbol{Y}_{\mathrm{d}}^{\mathrm{n}}+\boldsymbol{Y}_{\mathrm{d}}^{\mathrm{z}}+\boldsymbol{Y}_{\mathrm{d}}^{\mathrm{nz}}\right)\boldsymbol{U}_{\mathrm{R}}^{\mathrm{z}}\\ &+\left(\boldsymbol{A}^{\mathrm{z}}+\boldsymbol{A}_{\mathrm{C_Y}}^{\mathrm{z}}\right)\boldsymbol{U}^{\mathrm{p}}+\left(\boldsymbol{A}^{\mathrm{n}}+\boldsymbol{A}_{\mathrm{C_Y}}^{\mathrm{n}}\right)\boldsymbol{U}^{\mathrm{n}}\Big\} = 0\end{aligned} \tag{35}$$

and

$$\begin{aligned}\mathrm{Im}\Big\{&\left(\boldsymbol{Y}_{\mathrm{CYd}}^{\mathrm{n}}+\boldsymbol{Y}_{\mathrm{CYd}}^{\mathrm{z}}+\boldsymbol{Y}_{\mathrm{CYd}}^{\mathrm{nz}}+\boldsymbol{Y}_{\mathrm{d}}^{\mathrm{n}}+\boldsymbol{Y}_{\mathrm{d}}^{\mathrm{z}}+\boldsymbol{Y}_{\mathrm{d}}^{\mathrm{nz}}\right)\boldsymbol{U}_{\mathrm{R}}^{\mathrm{z}}\\ &+\left(\boldsymbol{A}^{\mathrm{z}}+\boldsymbol{A}_{\mathrm{C_Y}}^{\mathrm{z}}\right)\boldsymbol{U}^{\mathrm{p}}+\left(\boldsymbol{A}^{\mathrm{n}}+\boldsymbol{A}_{\mathrm{C_Y}}^{\mathrm{n}}\right)\boldsymbol{U}^{\mathrm{n}}\Big\} = 0\end{aligned} \tag{36}$$

Equations (29), (35) and (36) form a system of three equations with three unknowns T_{R}, T_{S}, and T_{T}. After the transformation, they can be presented in the form of the matrix equation of the compensator in the Y-configuration:

$$\begin{bmatrix} U_{\mathrm{R}}^2 & U_{\mathrm{S}}^2 & U_{\mathrm{T}}^2\\ \mathrm{Re}\boldsymbol{K}_1 & \mathrm{Re}\boldsymbol{K}_2 & \mathrm{Re}\boldsymbol{K}_3\\ \mathrm{Im}\boldsymbol{K}_1 & \mathrm{Im}\boldsymbol{K}_2 & \mathrm{Im}\boldsymbol{K}_2\end{bmatrix}\begin{bmatrix}T_{\mathrm{R}}\\ T_{\mathrm{S}}\\ T_{\mathrm{T}}\end{bmatrix} = \begin{bmatrix}-B_{\mathrm{b}}\|\boldsymbol{u}\|^2\\ -\mathrm{Re}\boldsymbol{K}_4\\ -\mathrm{Im}\boldsymbol{K}_4\end{bmatrix} \tag{37}$$

where parameters from $\boldsymbol{K}_1$ to $\boldsymbol{K}_4$ are equal to:

$$\begin{aligned}\boldsymbol{K}_1 &= (\boldsymbol{c}_1+\boldsymbol{c}_4+\boldsymbol{c}_7)\cdot ze^{j\psi_2}+j\frac{1}{3}\left(1+ne^{j\psi_1}\right),\\ \boldsymbol{K}_2 &= (\boldsymbol{c}_2+\boldsymbol{c}_5+\boldsymbol{c}_8)\cdot ze^{j\psi_2}+j\frac{1}{3}\left(\alpha^*+\alpha ne^{j\psi_1}\right),\\ \boldsymbol{K}_3 &= (\boldsymbol{c}_3+\boldsymbol{c}_6+\boldsymbol{c}_9)\cdot ze^{j\psi_2}+j\frac{1}{3}\left(\alpha+\alpha^* ne^{j\psi_1}\right),\\ \boldsymbol{K}_4 &= \left(\boldsymbol{Y}_{\mathrm{d}}^{\mathrm{n}}+\boldsymbol{Y}_{\mathrm{d}}^{\mathrm{z}}+\boldsymbol{Y}_{\mathrm{d}}^{\mathrm{nz}}\right)\cdot ze^{j\psi_2}+\boldsymbol{A}^{\mathrm{z}}+\boldsymbol{A}^{\mathrm{n}}ne^{j\psi_1}\end{aligned} \tag{38}$$

and the coefficients from $\boldsymbol{c}_1$ to $\boldsymbol{c}_9$ are:

$$\begin{aligned}&\boldsymbol{c}_1 = -j\frac{2n\cos(\psi_1)}{3\left(1+n^2+z^2\right)}, \quad \boldsymbol{c}_2 = -j\frac{2n\cos(\psi_1-120^\circ)}{3\left(1+n^2+z^2\right)},\\ &\boldsymbol{c}_3 = -j\frac{2n\cos(\psi_1+120^\circ)}{3\left(1+n^2+|z|^2\right)}, \quad \boldsymbol{c}_4 = -j\frac{2z\cos(\psi_2)}{3\left(1+n^2+z^2\right)},\\ &\boldsymbol{c}_5 = -j\frac{2z\cos(\psi_2+120^\circ)}{3\left(1+n^2+z^2\right)}, \quad \boldsymbol{c}_6 = -j\frac{2z\cos(\psi_2-120^\circ)}{3\left(1+n^2+z^2\right)},\\ &\boldsymbol{c}_7 = -j\frac{2m_1\cos(\psi_3)}{3\left(1+m_1^2+m_2^2\right)}, \quad \boldsymbol{c}_8 = -j\frac{2m_1\cos(\psi_3-120^\circ)}{3\left(1+m_1^2+m_2^2\right)},\\ &\boldsymbol{c}_9 = -j\frac{2m_1\cos(\psi_3+120^\circ)}{3\left(1+m_1^2+m_2^2\right)}.\end{aligned} \tag{39}$$

After substituting the data and solving the matrix equation of the compensator (37), we obtain parameters T_R, T_S, and T_T. If any of the parameters takes the value with a "minus" sign, then its quantity is calculated from the following relationship:

$$L = \frac{-1}{\omega T_X} \tag{40}$$

where T_X denotes the value of the calculated susceptance for a particular transmission line. Alternatively, the following relationship can be used:

$$C = \frac{T_X}{\omega}. \tag{41}$$

In (40) L denotes that the calculated susceptance should be inductive, while in (41) C means that the obtained susceptance should be capacitive.

Correct calculation of the compensator parameters causes the value of the reactive power seen from the side of the voltage source to be equal to 0, while the value of the unbalanced current of the zero sequence is also equal to 0, although the current will still flow in the neutral conductor on account of asymmetry of the active currents resulting from asymmetry of the supply voltages. Therefore:

$$\boldsymbol{I}_{\mathrm{aR}} + \boldsymbol{I}_{\mathrm{aS}} + \boldsymbol{I}_{\mathrm{aT}} \neq 0. \tag{42}$$

In addition, the inclusion of the Y-structure compensator results in a change of the value of the positive sequence unbalanced current and the value of the negative sequence unbalanced current because it changes the value of the unbalanced admittance of the negative sequence (12) and the value of the unbalanced admittance of the zero sequence (13).

On the basis of [16–18], the equivalent admittances of the load in the Δ-structure can be described by the compensator susceptance as follows:

- the susceptance responsible for compensation of the reactive current (reactive power):

$$\boldsymbol{Y}_{\mathrm{C_\Delta b}} = jB_{\mathrm{C_\Delta b}} = j\frac{T_{\mathrm{RS}}U_{\mathrm{RS}}^2 + T_{\mathrm{ST}}U_{\mathrm{ST}}^2 + T_{\mathrm{TR}}U_{\mathrm{TR}}^2}{\|\boldsymbol{u}_\Delta\|^2} \tag{43}$$

where $\|\boldsymbol{u}_\Delta\|$ denotes the three-phase rms value of the voltage source described in [19–21].

- the susceptance responsible for compensation of the currents (positive and negative sequences) associated with the voltage asymmetry dependent admittance:

$$\begin{aligned}\boldsymbol{Y}_{\mathrm{C_\Delta d}} &= j\frac{2n}{1+n^2}\\ &\cdot\left[T_{\mathrm{ST}}\cos(\psi_1) + T_{\mathrm{TR}}\cos\left(\psi_1 - \frac{2\pi}{3}\right) + T_{\mathrm{RS}}\cos\left(\psi_1 + \frac{2\pi}{3}\right)\right]\end{aligned} \tag{44}$$

- the susceptance responsible for compensation of the currents related to the unbalanced admittance of the positive sequence:

$$\boldsymbol{A}_{\mathrm{C_\Delta}}^{\mathrm{p}} = -j\left(T_{\mathrm{ST}} + \alpha T_{\mathrm{TR}} + \alpha^* T_{\mathrm{RS}}\right) \tag{45}$$

- the susceptance responsible for compensation of the currents linked with the unbalanced admittance of the negative sequence:

$$\boldsymbol{A}_{\mathrm{C_\Delta}}^{\mathrm{n}} = -j\left(T_{\mathrm{ST}} + \alpha^* T_{\mathrm{TR}} + \alpha T_{\mathrm{RS}}\right) \tag{46}$$

For determining the parameters of the compensator in the Δ configuration, which will compensate for the unbalanced current of the positive sequence and the unbalanced current of the negative sequence, and for unchanged the active power of the load, we have to meet the following conditions:

- the currents associated with the voltage asymmetry dependent admittance and the unbalanced currents of the positive and negative sequences are equal to 0 if:

$$\begin{aligned}\left(\boldsymbol{Y}_{\mathrm{C_\Delta d}} + \boldsymbol{Y}_{\mathrm{d}}\right)\boldsymbol{U} + \boldsymbol{1}^{\mathrm{n}}\left(\boldsymbol{A}^{\mathrm{p}} + \boldsymbol{A}_{\mathrm{C_\Delta}}^{\mathrm{p}}\right)\boldsymbol{U}^{\mathrm{p}}&\\ + \boldsymbol{1}^{\mathrm{p}}\left(\boldsymbol{A}^{\mathrm{n}} + \boldsymbol{A}_{\mathrm{C_\Delta}}^{\mathrm{n}}\right)\boldsymbol{U}^{\mathrm{n}} &= 0.\end{aligned} \tag{47}$$

Identically as in (32), parameters can be determined for the R-line, namely:

$$\begin{aligned}\left(\boldsymbol{Y}_{\mathrm{C_\Delta d}} + \boldsymbol{Y}_{\mathrm{d}}\right)\left(\boldsymbol{U}_{\mathrm{R}}^{\mathrm{p}} + \boldsymbol{U}_{\mathrm{R}}^{\mathrm{n}}\right) + \left(\boldsymbol{A}^{\mathrm{p}} + \boldsymbol{A}_{\mathrm{C_\Delta}}^{\mathrm{p}}\right)\boldsymbol{U}^{\mathrm{p}}&\\ + \left(\boldsymbol{A}^{\mathrm{n}} + \boldsymbol{A}_{\mathrm{C_\Delta}}^{\mathrm{n}}\right)\boldsymbol{U}^{\mathrm{n}} &= 0.\end{aligned} \tag{48}$$

Analogously to (33) and (34), equation (48) must be fulfilled separately for the real part and the imaginary part:

$$\begin{aligned}\mathrm{Re}\Big\{\left(\boldsymbol{Y}_{\mathrm{C_\Delta d}} + \boldsymbol{Y}_{\mathrm{d}}\right)\left(\boldsymbol{U}_{\mathrm{R}}^{\mathrm{p}} + \boldsymbol{U}_{\mathrm{R}}^{\mathrm{n}}\right) + \left(\boldsymbol{A}^{\mathrm{p}} + \boldsymbol{A}_{\mathrm{C_\Delta}}^{\mathrm{p}}\right)\boldsymbol{U}^{\mathrm{p}}&\\ + \left(\boldsymbol{A}^{\mathrm{n}} + \boldsymbol{A}_{\mathrm{C_\Delta}}^{\mathrm{n}}\right)\boldsymbol{U}^{\mathrm{n}}\Big\} &= 0\end{aligned} \tag{49}$$

and

$$\begin{aligned}\mathrm{Im}\Big\{\left(\boldsymbol{Y}_{\mathrm{C_\Delta d}} + \boldsymbol{Y}_{\mathrm{d}}\right)\left(\boldsymbol{U}_{\mathrm{R}}^{\mathrm{p}} + \boldsymbol{U}_{\mathrm{R}}^{\mathrm{n}}\right) + \left(\boldsymbol{A}^{\mathrm{p}} + \boldsymbol{A}_{\mathrm{C_\Delta}}^{\mathrm{p}}\right)\boldsymbol{U}^{\mathrm{p}}&\\ + \left(\boldsymbol{A}^{\mathrm{n}} + \boldsymbol{A}_{\mathrm{C_\Delta}}^{\mathrm{n}}\right)\boldsymbol{U}^{\mathrm{n}}\Big\} &= 0.\end{aligned} \tag{50}$$

In order to determine the parameters of a balancing compensator with a Δ-structure, use expression (48), whose solution for three-phase three-wire circuits supplied with sinusoidal asymmetrical voltage with a load connected in a delta configuration is presented in [19–21]. Despite the rightness of the solution given by the author in some works, for three-phase four-wire systems supplied from an asymmetrical sinusoidal voltage source, relationship (48) must be subjected to additional modification as follows:

$$\begin{aligned}&\left[\left(\boldsymbol{Y}_{\mathrm{C_\Delta d}} + \boldsymbol{Y}_{\mathrm{d}}^{\mathrm{n\#}} + \boldsymbol{Y}_{\mathrm{d}}^{\mathrm{z\#}} + \boldsymbol{Y}_{\mathrm{d}}^{\mathrm{nz\#}}\right)\left(\boldsymbol{U}_{\mathrm{R}}^{\mathrm{p}} + \boldsymbol{U}_{\mathrm{R}}^{\mathrm{n}}\right)\right]\\ &+ \left[\boldsymbol{A}_{\mathrm{C_\Delta}}^{\mathrm{p}}\boldsymbol{U}^{\mathrm{p}} + \boldsymbol{A}^{\mathrm{n\#}}\left(\boldsymbol{U}^{\mathrm{p}} + \boldsymbol{U}^{\mathrm{z}}\right) + \boldsymbol{A}_{\mathrm{C_\Delta}}^{\mathrm{n}}\boldsymbol{U}^{\mathrm{n}} + \boldsymbol{A}^{\mathrm{z\#}}\left(\boldsymbol{U}^{\mathrm{n}} + \boldsymbol{U}^{\mathrm{z}}\right)\right]\\ &= 0\end{aligned} \tag{51}$$

where admittances with the "#" symbol denote that they were calculated as a result of a parallel connection of the phase admittance of the load and the phase susceptances of the balancing reactive compensator with the star structure.

Additionally, in order to determine the parameters of the compensator in the Δ-structure, the condition for the reactive power compensation should be included:

$$B_{C_{\Delta b}} + B_b = 0 \tag{52}$$

even though it has already been used in calculations for the Y-structure.

According to (49)–(52), they form a system of three equations with three unknowns T_{RS}, T_{ST} and T_{TR}. Following the transformations, the system in the form of the matrix equation of the compensator in the Δ-structure looks as follow:

$$\begin{bmatrix} U_{RS}^2 & U_{ST}^2 & U_{TR}^2 \\ \mathrm{Re}\boldsymbol{K}_5 & \mathrm{Re}\boldsymbol{K}_6 & \mathrm{Re}\boldsymbol{K}_7 \\ \mathrm{Im}\boldsymbol{K}_5 & \mathrm{Im}\boldsymbol{K}_6 & \mathrm{Im}\boldsymbol{K}_7 \end{bmatrix} \begin{bmatrix} T_{RS} \\ T_{ST} \\ T_{TR} \end{bmatrix} = \begin{bmatrix} -B_b \|\boldsymbol{u}_\Delta\|^2 \\ -\mathrm{Re}\boldsymbol{K}_8 \\ -\mathrm{Im}\boldsymbol{K}_8 \end{bmatrix} \tag{53}$$

also, it should be remembered that the reactive power has been already compensated for in (37):

$$\begin{bmatrix} \boldsymbol{U}_{RS}^2 & \boldsymbol{U}_{ST}^2 & U_{TR}^2 \\ \mathrm{Re}\boldsymbol{K}_5 & \mathrm{Re}\boldsymbol{K}_6 & \mathrm{Re}\boldsymbol{K}_7 \\ \mathrm{Im}\boldsymbol{K}_5 & \mathrm{Im}\boldsymbol{K}_6 & \mathrm{Im}\boldsymbol{K}_7 \end{bmatrix} \begin{bmatrix} T_{RS} \\ T_{ST} \\ T_{TR} \end{bmatrix} = \begin{bmatrix} 0 \\ -\mathrm{Re}\boldsymbol{K}_8 \\ -\mathrm{Im}\boldsymbol{K}_8 \end{bmatrix} \tag{54}$$

where parameters from $\boldsymbol{K}_5$ to $\boldsymbol{K}_8$ are equal to:

$$\begin{aligned} \boldsymbol{K}_5 &= \boldsymbol{c}_{12}\left(1 + ne^{j\psi_1}\right) - j\left(\alpha^* + \alpha ne^{j\psi_1}\right), \\ \boldsymbol{K}_6 &= \boldsymbol{c}_{10}\left(1 + ne^{j\psi_1}\right) - j\left(1 + ne^{j\psi_1}\right), \\ \boldsymbol{K}_7 &= \boldsymbol{c}_{11}\left(1 + ne^{j\psi_1}\right) - j\left(\alpha + \alpha^* ne^{j\psi_1}\right), \\ \boldsymbol{K}_8 &= \left(\boldsymbol{Y}_d^{n\#} + \boldsymbol{Y}_d^{z\#} + \boldsymbol{Y}_d^{nz\#}\right)\left(1 + ne^{j\psi_1}\right) \\ &\quad + \boldsymbol{A}^{n\#}\left(1 + ze^{j\psi_2}\right) + \boldsymbol{A}^{z\#}\left(ne^{j\psi_1} + ze^{j\psi_2}\right). \end{aligned} \tag{55}$$

and the coefficients from $\boldsymbol{c}_{10}$ to $\boldsymbol{c}_{12}$ are:

$$\begin{aligned} &\boldsymbol{c}_{10} = j\frac{2n\cos(\psi_1)}{1+n^2}, \quad \boldsymbol{c}_{11} = j\frac{2n\cos(\psi_1 - 120°)}{1+n^2}, \\ &\boldsymbol{c}_{12} = j\frac{2n\cos(\psi_1 + 120°)}{1+n^2}. \end{aligned} \tag{56}$$

After activating the compensator in the Y-structure and the compensator in the Δ-structure, assuming the losslessness of such systems, fixed load parameters over time and no change in the internal impedance of the voltage source, we obtain only the active current described in (5).

It should be mentioned that it is not possible to completely, ideally compensate for the reactive power with the balancing compensator with the delta structure because in the system it is not possible to limit the reactive current associated with the symmetrical component of the zero sequence.

4. Theoretical calculations

For theoretical calculations, the three-phase four-wire system powered with the asymmetrical sinusoidal voltage source is shown in Fig. 2. All calculations are based on the assumption of LTI unbalanced load. In addition, the supply voltage includes internal impedance (unchanged over time) and is, therefore, an asymmetrical source that only generates the fundamental harmonic whose frequency is 50 Hz.

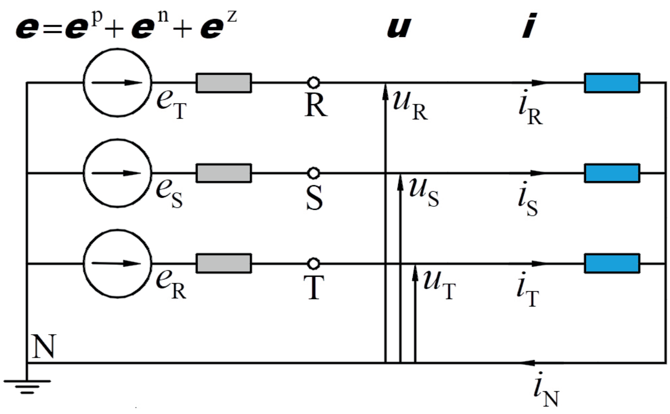

Fig. 2. Three-phase four-wire circuit diagram chosen for theoretical verification

The active power P of the system from Fig. 2 is 53053 W, while the consumed reactive power Q assumes the value 20898 var and it is a capacitive reactive power.

Configurations and values of the phase impedances of the load, shown in Fig. 2, are compiled in Table 1.

Table 1
List of phase impedances in ohms with division into resistance, inductive and capacitive reactance values

Line	R	X_L	X_C
R	4	–	2
S	1	1	–
T	0.5	–	1

Table 2 presents the phase values of the voltage supply as well as the values of the line current of the load, and the values of the line current of the components obtained in the CPC theory.

Table 2
List of the values of the phase voltages and the values of the line currents

Quan.	Phase R	Phase S	Phase T
$\boldsymbol{U}$	$230e^{j5°}$	$215e^{-j110°}$	$220e^{j99°}$
$\boldsymbol{I}$	$51.43e^{j31.6°}$	$152.03e^{-j155°}$	$196.77e^{j162.4°}$
$\boldsymbol{I}_a$	$82.71e^{j5°}$	$77.32e^{-j110°}$	$79.12e^{j99°}$
$\boldsymbol{I}_r$	$32.58e^{j95°}$	$30.46e^{-j20°}$	$31.16e^{-j171°}$
$\boldsymbol{I}_u^p$	$26.32e^{-j8.3°}$	$26.32e^{-j128.3°}$	$26.32e^{j111.7°}$
$\boldsymbol{I}_u^n$	$45.38e^{-j8.1°}$	$45.38e^{j111.9°}$	$45.38e^{-j128.1°}$
$\boldsymbol{I}_u^z$	$106.75e^{-j178.6°}$	$106.75e^{-j178.6°}$	$106.75e^{-j178.6°}$

The waveform of the instantaneous value of the phase voltages at the load's terminals is shown in Fig. 3.

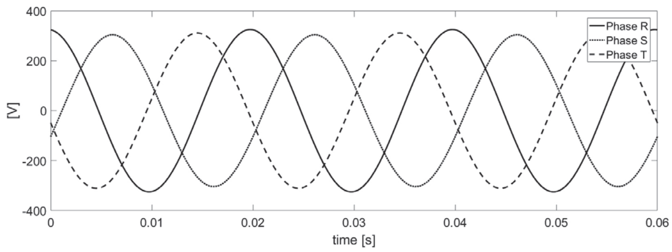

Fig. 3. The waveform of the instantaneous value of the phase voltages at the terminals of the load

The three-phase rms value $\|\boldsymbol{u}\|$ of supplying voltage (1) is equal to:

$$\|\boldsymbol{u}\| = 384.09 \text{ V}.$$

Using the Fortescue system [1, 7], the symmetrical components of the supplying voltage are:

$$\begin{bmatrix} \boldsymbol{U}^{\mathrm{p}} \\ \boldsymbol{U}^{\mathrm{n}} \\ \boldsymbol{U}^{\mathrm{z}} \end{bmatrix} = \begin{bmatrix} 215.53e^{-j1.9^\circ} \\ 30.82e^{j149.9^\circ} \\ 42.07e^{j16.2^\circ} \end{bmatrix} \text{ V}.$$

The waveform of the instantaneous value of the load's line currents is presented in Fig. 4.

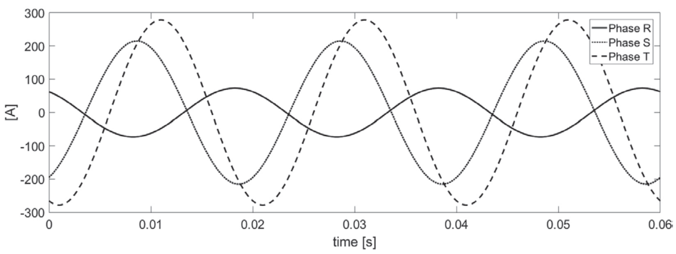

Fig. 4. The waveform of the instantaneous value of the line currents of the load

The three-phase rms value of the load current is:

$$\|\boldsymbol{i}\| = 253.92 \text{ A}.$$

Table 3 compiles values of the individual admittances and complex asymmetry coefficients.

Table 4 presents values of the coefficients from $\boldsymbol{c}_1$ to $\boldsymbol{c}_9$ (39).

The values of the coefficients from $\boldsymbol{c}_1$ to $\boldsymbol{c}_9$ and the parameters of the balancing compensator from $\boldsymbol{K}_1$ to $\boldsymbol{K}_4$ are compiled in Table 5.

The solution of matrix system (37) is the vector of the phase admittances values necessary for the compensation of the reactive power and symmetrization of the unbalanced current of the zero sequence. The values of the calculated admittances for individual line are equal to: $T_{\mathrm{R}} = -0.01$, $T_{\mathrm{S}} = 0.57$ and $T_{\mathrm{R}} = -0.97$.

Table 3
List of the individual admittances and asymmetry coefficients

Quan.	Value
$\boldsymbol{A}^{\mathrm{n}}$	$0.29e^{j2.4^\circ}$ S
$\boldsymbol{A}^{\mathrm{z}}$	$0.46e^{-j174.3^\circ}$ S
$\boldsymbol{Y}_{\mathrm{d}}^{\mathrm{n}}$	$0.05e^{j118.2^\circ}$ S
$\boldsymbol{Y}_{\mathrm{d}}^{\mathrm{z}}$	$0.05e^{-j48.9^\circ}$ S
$\boldsymbol{Y}_{\mathrm{d}}^{\mathrm{nz}}$	$0.02e^{-j97.4^\circ}$ S
$\boldsymbol{n}$	$0.14e^{j151.9^\circ}$
$\boldsymbol{z}$	$0.19e^{j18.2^\circ}$
$\boldsymbol{m}_1$	$1.37e^{-j133.7^\circ}$
$\boldsymbol{m}_2$	$6.99e^{-j151.9^\circ}$

Table 4
List of the coefficients' value from $\boldsymbol{c}_1$ to $\boldsymbol{c}_9$

Coefficient	Value
$\boldsymbol{c}_1$	$j0.01$
$\boldsymbol{c}_2$	$-j0.01$
$\boldsymbol{c}_3$	$-j0.01$
$\boldsymbol{c}_4$	$-j0.12$
$\boldsymbol{c}_5$	$j0.09$
$\boldsymbol{c}_6$	$j0.03$
$\boldsymbol{c}_7$	$j0.01$
$\boldsymbol{c}_8$	$j0.01$
$\boldsymbol{c}_9$	$-j0.02$

Table 5
List of the parameters of the balancing compensator with the star structure

Parameter	Value
$\boldsymbol{K}_1$	$0.3e^{j94.2^\circ}$
$\boldsymbol{K}_2$	$0.4e^{-j25.7^\circ}$
$\boldsymbol{K}_3$	$0.3e^{-j158.3^\circ}$
$\boldsymbol{K}_4$	$0.5e^{-j176.7^\circ}$

To calculate inductance or capacitance, we use expressions (40) or (41), depending on the sign that is at the given admittances in the vector of calculated values. The inductance value in the R-line is equal to 733.1 mH and in the T-line it is equal to 3.3 mH. In S-line we obtained the capacity value, which is equal to 1.8 mF.

Figure 5 presents the analyzed system with an added balancing reactive compensator with a star structure.

In order to perform full balancing compensation in four-wire systems supplied with asymmetrical sinusoidal voltage, besides

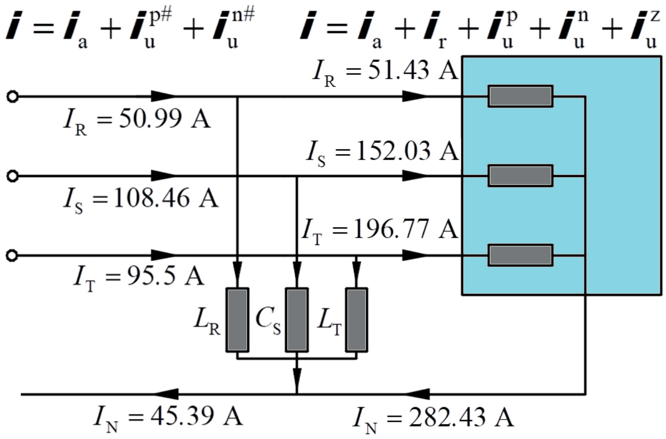

Fig. 5. Equivalent scheme of the analyzed circuit with an added balancing compensator with a star configuration

a balancing compensator with a star structure, a balancing compensator with a delta structure should be aggregated, which balances the unbalanced currents of the positive and negative sequences.

Table 6 presents the recalculated values of the unbalanced admittance and voltage asymmetry dependent admittances.

Table 6
List of the individual admittances

Quan.	Value
$\boldsymbol{A}^{n\#}$	$0.17e^{j152.8^\circ}$ S
$\boldsymbol{A}^{z\#}$	$0.03e^{j127.5^\circ}$ S
$\boldsymbol{Y}_d^{n\#}$	$0.02e^{j172.8^\circ}$ S
$\boldsymbol{Y}_d^{z\#}$	$0.03e^{-j15.9^\circ}$ S
$\boldsymbol{Y}_d^{nz\#}$	$0.005e^{j113.9^\circ}$ S

Additionally [19–21], phase-to-phase voltages should be calculated of the components of the positive and negative sequences of the Fortescue transformation:

- the phase-to-phase voltage (R to S) is:

$$\boldsymbol{U}_{RS} = (\boldsymbol{U}^p + \boldsymbol{U}^n) - (\alpha^* \boldsymbol{U}^p + \alpha \boldsymbol{U}^n) = 375.40e^{j36.3^\circ} \text{ V}$$

- the phase-to-phase voltage (S to T) is:

$$\boldsymbol{U}_{ST} = (\alpha^* \boldsymbol{U}^p + \alpha \boldsymbol{U}^n) - (\alpha \boldsymbol{U}^p + \alpha^* \boldsymbol{U}^n) = 421.15e^{-j95.3^\circ} \text{ V}$$

- the phase-to-phase voltage (T to R) is:

$$\boldsymbol{U}_{TR} = (\alpha \boldsymbol{U}^p + \alpha^* \boldsymbol{U}^n) - (\boldsymbol{U}^p + \boldsymbol{U}^n) = 329.18e^{j143.2^\circ} \text{ V}.$$

Table 7 compiles values of the coefficients from $\boldsymbol{c}_{10}$ to $\boldsymbol{c}_{12}$, and values of the parameters from $\boldsymbol{K}_5$ to $\boldsymbol{K}_8$ of the balancing compensator with the delta structure.

The solution of matrix system (54) is the vector of the line-to-line admittance values necessary for the symmetrization of the unbalanced currents of the positive and negative sequences. The values of the calculated admittances for an individual branch are equal to: $T_{RS} = -0.12$, $T_{ST} = 0.05$ and $T_{TR} = 0.06$.

Table 7
List of balancing compensator parameters and coefficients in the delta structure

Param./Coeff.	Value
$\boldsymbol{c}_{10}$	$-j0.25$
$\boldsymbol{c}_{11}$	$j0.24$
$\boldsymbol{c}_{12}$	$j0.01$
$\boldsymbol{K}_5$	$1.1e^{j153.5^\circ}$
$\boldsymbol{K}_6$	$1.1e^{-j85.6^\circ}$
$\boldsymbol{K}_7$	$1.1e^{j32.4^\circ}$
$\boldsymbol{K}_8$	$0.2e^{j155.9^\circ}$

To calculate the inductance or capacitance, we use expressions (40) or (41), depending on the sign that is at the given admittances in the vector of calculated values. The inductance value in the RS-line is equal to 27.2 mH. The capacity value in the ST-line is equal to 172.7 μF and in TR-line it is equal to 201.7 μF.

Figure 6 presents the analyzed system with added balancing compensators with the Y-structure and with the Δ-structure.

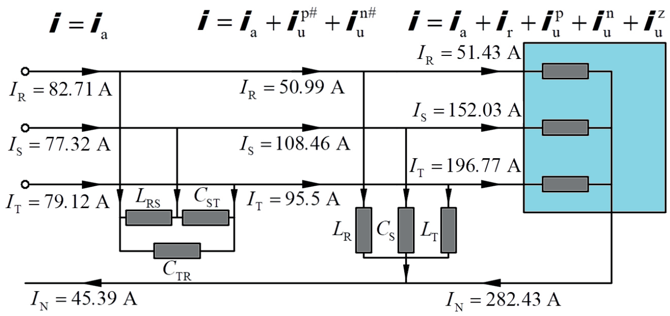

Fig. 6. Equivalent scheme of the analyzed circuit with added balancing compensators with the structures of the star and the delta

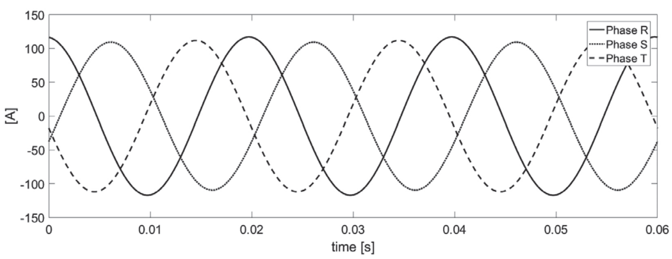

Fig. 7. The waveform of the instantaneous value of the line current after connecting the balancing compensators

The waveform of the instantaneous value of the line currents, after compensation of the balancing compensators with the Y-structure and Δ-structure, is equal to the waveform of the active currents expressed by formula (5).

5. Conclusion

The paper shows that it is possible to calculate the balancing compensator parameters that completely compensate for the reactive current (reactive power) and unbalanced current, even when the supply voltage is asymmetric. This increases the power factor to unity.

Moreover, the article shows that as a result of the asymmetry of supply voltages, the current will still flow in the neutral conductor, resulting from the asymmetry of line active currents.

The method of defining the balancing compensator parameters presented in the paper can be extended to the ideal balancing compensation or minimization of respective currents' components in nonsinusoidal systems.

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