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# A TWO-STAGE STOCHASTIC PROGRAMMING APPROACH FOR PRODUCTION PLANNING SYSTEM WITH SEASONAL DEMAND

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Abstract
Seasonality is a function of a time series in which the data experiences regular and predictable changes that repeat each calendar year. Two-stage stochastic programming model for real industrial systems at the case of a seasonal demand is presented. Sampling average approximation (SAA) method was applied to solve a stochastic model which gave a productive structure for distinguishing and statistically testing a different production plan. Lingo tool is developed to obtain the optimal solution for the proposed model which is validated by Math works Matlab. The actual data of the industrial system; from the General Manufacturing Company, was applied to examine the proposed model. Seasonal future demand is then estimated using the multiplicative seasonal method, the effect of seasonality was presented and discussed. One might say that the proposed model is viewed as a moderately accurate tool for industrial systems in case of seasonal demand. The current research may be considered a significant tool in case of seasonal demand. To illustrate the applicability of the proposed model a numerical example is solved using the proposed technique. ANOVA analysis is applied using MINITAB 17 statistical software to validate the obtained results.

Keywords
Process manufacturing system, two-stage stochastic programming, sampling average approximation.

## Introduction

In the case of manufacturing planning, a more possible approach is to identify and address one or some uncertainties within a model which can be used to derive optimal solutions. This is the focus of this study, and the objective is to reduce variable production costs, in the case of seasonal demand.

Uncertainty; of one or more of the principal attributes, often exists in discrete manufacturing systems. Consequently, the precision of production relies to a significant degree on the accuracy of estimating these uncertainties. Uncertainties are incorporated in the four elements of manufacturing systems: demand, processing, failure and maintenance time and cost [1, 2]. These uncertainties increase the required practical production planning decisions. The incorporation of stochastic parameters in a system increases the complexity of the system modeling and solution. Hence, it is more appropriate to address the problem using a stochastic programming approach [3].

A review of some existing literature on production planning under uncertainty has been provided by [4]. Escudero et al. [5] presented a multi-stage stochastic programming approach that was used for addressing a multi-period multi-product production planning model with random demand. The authors introduced two scenario-based models for formalizing implementable policies. Similarly, Bakir and Byrne [6] developed a linear programming (LP) model that depended on the two-stage deterministic equal problem including demand uncertainty in a multi-period multi-product (MPMP) production planning model. In 2005, Alfieri and Brandimarte [7] revised multi-stage stochastic models carried out in multi-period production and capacity planning for manufacturing systems. They had stressed the importance of proper model formulation from two points of view: the first was building strong mixed-integer formulations; while the second was generating scenario trees to represent uncertainty. Deterministic models are analyzed using optimization techniques that are generally based on linear programming (LP) or other mathematical programming approaches (e.g. mixed-integer linear programming and Non-linear programming). While uncertain models include probabilistic approaches [8, 9]. Depending on a two-stage approach in case of uncertain multi-period production planning Mahmutogullar et al. [10] investigated a two-stage approach, the plan for the whole multi-period planning horizon is determined before the uncertainty is determined, then an only restricted variety of resource changes may then be taken into consideration. Alternatively, a multi-stage approach permits revising the planning decisions whenever further information regarding the uncertainties is revealed. So, the multi-stage model is a better description of the dynamic planning process and supplies more flexibility than does the two-stage model. Reviewing the previously mentioned research work, stochastic programming models are classified and exhibited in Table 1.

The major contributions of this paper that distinguish it from the above-mentioned literature can be outlined as follows:

- The current work takes into consideration demand seasonality and various trends at each stage of the demand scenario tree as recommended by Kazemi et al. [16].
- Manufacturing set up costs; that was ignored in the models in previously developed models, is considered in the current proposed model.
- "Math works Matlab" R2015a is used to validate the mathematical model while the results are verified using ANOVA statistical analysis.
- Focusing on the validation of the obtained results is applied.

Table 1
Classification of stochastic Programming Models.

| Researchers | Year | Stochastic Programming Models Types | Solution method | Software used |
|---|---|---|---|---|
| Bakir, Byrne [6] | 1998 | Two-Stage Stochastic Programming | Discrete approximation | SIMAN |
| Kazemi et al. [11] | 2007 | Two-Stage Stochastic Programming | Sample Average Approximation | OPL 3.7.1 |
| Khor et al. [12] | 2008 | Two-Stage Stochastic Programming | Markowitz's MV approach | GAMS |
| Kazemi et al. [13] | 2008a | Two-Stage Stochastic Programming | Sample Average Approximation | CPLEX 9 |
| Kazemi et al. [14] | 2008b | Multi-Stage Stochastic Programming | Sample Average Approximation | CPLEX 10 |
| Huang K., Shabbir A. [15] | 2009 | Multi-Stage Stochastic Programming | L-Shaped Decomposition | CPLEX 9.0 |
| Kazemi et al. [16] | 2010 | Multi-Stage Stochastic Programming | Sample Average Approximation | CPLEX 10 OPL 5.1 |
| Kazemi et al. [17] | 2011 | Two-Stage Stochastic Programming | Sample Average Approximation | CPLEX 9 |
| Kazemi et al. [18] | 2013 | Multi-Stage Stochastic Programming | Sample Average Approximation | CPLEX 11 |
| Zhengyang, Guiping [19] | 2016 | Two-Stage Stochastic Programming | ———— | GAMS |
| Hasany, Shafahi [20] | 2017 | Two-Stage Stochastic Programming | L-Shaped Decomposition | CPLEX 12.2 |
| Damghania et al. [21] | 2017 | Two-Stage Stochastic Programming | STEM | LINGO |
| Maria et al. [22] | 2017 | Two-Stage Stochastic programming | L-Shaped Decomposition | GNU/Linux CPLEX12.5.0.1 |
| Mahmutogullarıet al. [10] | 2019 | Multi-Stage Stochastic Programming | RH Approach | CPLEX 12.6 |

## Model development

In actual problems, some important steps should be considered in the designing phase of the model. Firstly, the collection of accurate data and various parameters is highly important for demand forecasts in the TSP model. Furthermore, appropriate seasonal forecasting methods that will provide minimum error between forecasted values and actual data should be selected after that the TSP model can be formulated considering: obtained data, assumptions, objectives, parameters and constraints. Finally, the model can be applied in a real industrial system. Figure 1 shows the steps of the proposed model development.

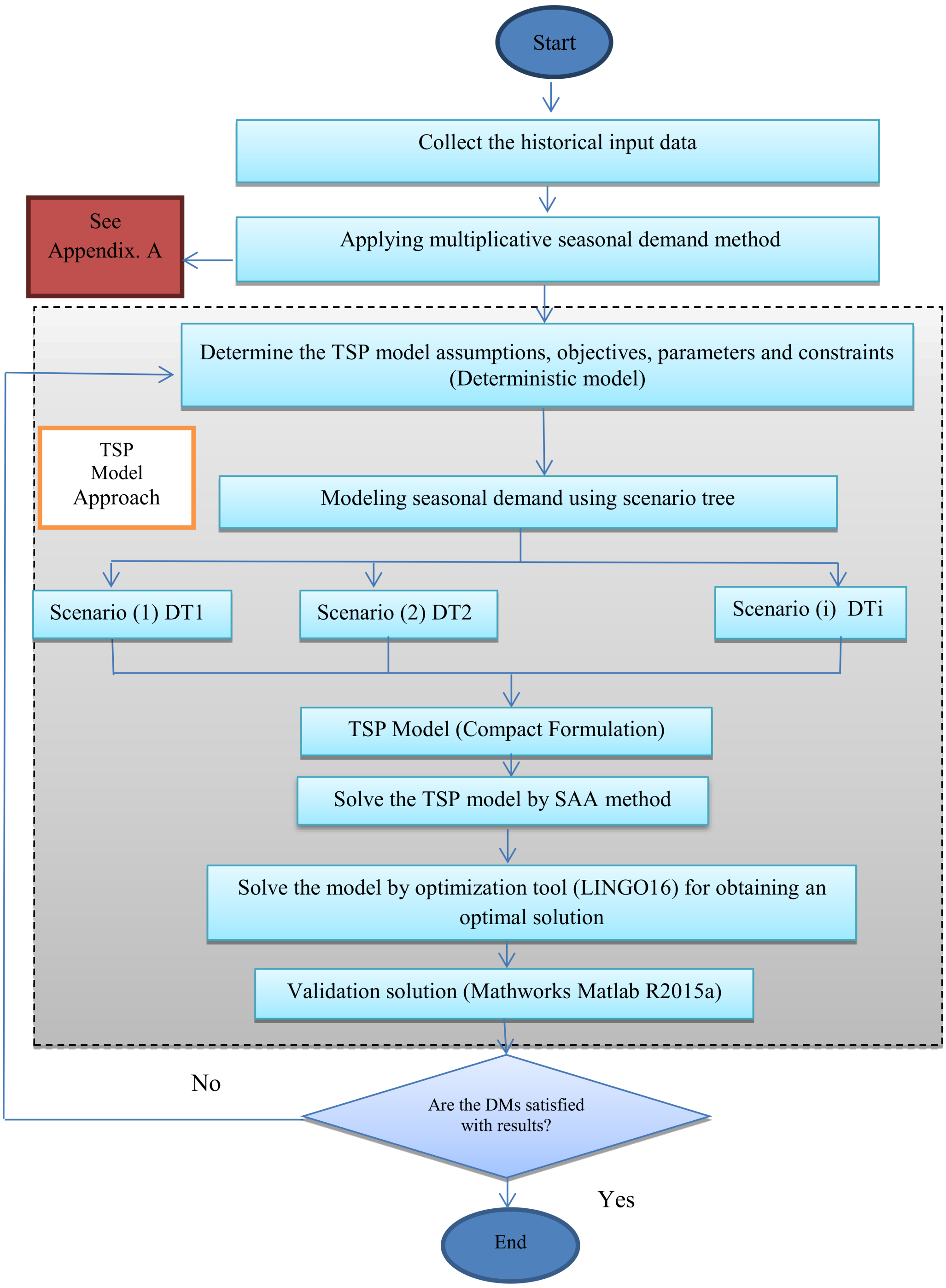



Fig. 1. Schematic flow chart of model development.

## Data collection and application of multiplicative seasonal demand method

This phase of the model developed includes determining required data such as unit costs of regular time production, overtime production, and space requirement for per unit product inventory, backorder, hired/laid-off workers, time requirements for labor and machine hours per unit product. Some required data for planning periods can be gathered easily, but some of them are difficult. In this model, the demand values are not known as there are seasonality and forecast by using the multiplicative demand method [23, 24], see Appendix.

## Two-Stag stochastic programming (deterministic model)

In this study, the notations in the formulation will be introduced. It has to be stated that all cost parameters are indicated in the Egyptian pound (L.E). A summary of different indices, parameters, and decision variables is presented in Tables 2a and 2b.

Table 2a
Notation summary of dependent variables.

| Dependent variables | |
|---|---|
| $X_{ijt}$ | The number of units of a family $i$ produced by department $j$ in period $t$. |
| $IF_{it}$ | Inventory of family $i$ at the end of period $t$. |
| $R_{jt}$ | Regular time used by department $j$ in period $t$. |
| $O_{jt}$ | Overtime used by department $j$ in period $t$. |
| $W_{it}$ | Workforce level family $i$ at the end of period $t$. |
| $B_{it}$ | Backorder of family $i$ at the end of period $t$. |
| $Z_{kitw}$ | A number of units of item $k$ produced by department $j$ in sub-period $w$ of period $t$. |
| $I_{kitw}$ | Inventory of item $k$ at the end sub-period $w$ of period $t$. |
| $RR_{lt}$ | Regular time used by resource $l$ in period $t$. |
| $OR_{lt}$ | Overtime used by resource $l$ in period $t$. |
| $C_{ijt}$ | Average Unit cost for producing one unit of family $i$ by department $j$ in period $t$. |
| $W_{it}$ | Workforce level for family $i$ in period $t$. |
| $H_{it}$ | A worker hired of family $i$ at the end of period $t$. |
| $L_{it}$ | Worker lay-off of family $i$ at the end of period $t$. |
| $r_{jt}$ | The average cost of one regular time unit for department $j$ during period $t$. |
| $o_{jt}$ | The average cost of one overtime unit for department $j$ during period $t$. |
| $w_{it}$ | The average labor cost of family $i$ at the end of period $t$. |
| $h_{it}$ | The average holding cost for family $i$ in period $t$. |
| $d_{it}$ | Demand for family $i$ in period $t$. |
| $b_{it}$ | Average backorder cost for family $i$ in period $t$. |
| $a_{ij}$ | The average total time required to produce one unit of family $i$ at department $j$. |
| $BP_{ij}$ | Setup cost for a family $i$ in primary department $j$. |
| $BS_{ij}$ | Setup cost for a family $i$ in secondary department $j$. |
| $bp_{ij}$ | Setup time for a family $i$ in primary department $j$. |
| $bs_{ij}$ | Setup time for a family $i$ in secondary department $j$. |
| $Q_{ijt}$ | Initial estimation of the lot of size of family $i$ in department $j$ in period $t$. |
| $d_{ktw}$ | Demand for item $k$ in sub-period $w$ of period $t$. |
| $PR_{kl}$ | The average total time required to produce one unit of item $k$ using resource $l$. |
| $\alpha_{lt}$ | The average proportion of time resource $l$ is down in period $t$. |

Table 2b
Notation summary of independent variables.

| Independent variables | |
|---|---|
| $P_j$ | Set of families in which their primary department is $j$. |
| $FS(j)$ | A feasible set of families assignable to department $j$. |
| $TI(i)$ | A set of items belonging to family $i$. |
| $t$ | $n * w$, where $n$ is an integer multiple. |
| $R(j)$ | Set of resources belonging to department $j$. |
| $J$ | A number of departments. |
| $N$ | A number of families. |
| $K$ | A number of items. |
| $L$ | A number of resources. |

The problem is presented in an analytic model (Non-linear, two-Stage stochastic programming as appropriate). The objective function is to minimize production cost (i.e. the sum of all costs; production, cell setup, inventory holding, and regular capacity). The formulation of the deterministic mathematical model for the system is as follows:

The objective function is presented in Eq. (1). The three classes of the suggested model constraints for department loading are; families and items production constraints, families and items inventory constraints, and production capacity and resource capacity constraints through the planning stage. The constraints (2) and (6) are families and items inventory constraints, both the amount of inventory left in the stock at the end of each period and the amount of inventory of the last period. The backorder is not allowed. Constraint (7) shows the link between the item and family inventory. Constraints (3) and (8) are capacity feasibility constraints department and resources.

Objective Function

$$\begin{aligned}\text{Min Cost} = \sum_{t=1}^{T}\sum_{j=1}^{J}\Bigg(&\sum_{i=FS(j)} C_{ijt}X_{ijt}\\ +\sum_{i\in P_j}(BP_{ij}/Q_{ijt})\cdot X_{ijt}&+\sum_{i\in S_j}(BS_{ij}/Q_{ijt})\cdot X_{ijt}\\ +o_{jt}\cdot O_{jt}+r_{jt}\cdot R_{jt}\Bigg)&+\sum_{t=1}^{T}\sum_{i=1}^{N} h_{it}\cdot IF_{it}\\ +\sum_{t=1}^{T}\sum_{i=1}^{N} b_{it}\cdot B_{it}&+\sum_{t=1}^{T}\sum_{i=1}^{N} W_{it}\cdot w_{it}.\end{aligned} \tag{1}$$

Subject to:
production, inventory and backorder equations for each family

$$\sum_{j=1}^{J} X_{ijt} + IF_{i,t-1} + B_{i,t-1} - IF_{it} - B_{it} = d_{it}, \quad \text{for } i = 1,2,...,N, \quad t = 1,2,...,T \tag{2}$$

capacity restrictions for each cell

$$\begin{aligned}\sum_{i\in FS(j)} a_{ij}X_{ijt} &+ \sum_{i\in P_j}(bp_{ij}/Q_{ijt})\cdot X_{ijt}\\ &+\sum_{i\in S_j}(bs_{ij}/Q_{ijt})\cdot X_{ijt} - O_{jt} = R_{jt},\\ &\text{for } j = 1,2,...J, \quad t = 1,2,...,T,\end{aligned} \tag{3}$$

$$0 \le R_{jt} \le \text{(upper limit)} \quad \forall\, j,t, \tag{4}$$

$$0 \le O_{jt} \le \text{(upper limit)} \tag{5}$$

production and inventory balance equations for each item $t$

$$\sum_{j=1}^{J} Z_{kjtw} + I_{kt,w-1} - I_{ktw} = d_{ktw} \quad \forall\, k\in TI(i), w, t \tag{6}$$

inventory consistency equations

$$\sum_{k\in TI(i)}\sum_{w=I}^{n} I_{ktw} - IF_{it} = 0 \quad \forall\, i,t, \tag{7}$$

capacity restrictions for each resource

$$\sum_{i\in FS(j)}\sum_{k\in TI(i)}\left(PR_{kl}\sum_{w=I}^{n} Z_{kjtw}\right) - OR_{LT} = RR_{LT} \quad \forall\, L\in LR(j), j, t \tag{8}$$

$$0 \le RR_{Lt} \le \text{(upper limit)}\ (1-\alpha_{Lt}) \quad \forall\, L,t \tag{9}$$

$$0 \le OR_{LT} \le \text{(upper limit)} \tag{10}$$

resource consistency relations:

$$\sum_{l\in LR(j)} RR_{Lt} - R_{jt} = 0 \quad \forall\, j,t \tag{11}$$

$$\sum_{l\in LR(j)} OR_{Lt} - O_{jt} = 0 \tag{12}$$

workforce balance:

$$W_{it} = W_{i,t-1} + H_{it} - L_{it} \quad \text{for } i = 1,2,...,M, \quad t = 1,2,...,T \tag{13}$$

non-negativity restrictions:

$$\begin{aligned}X_{ijt}, IF_{it}, R_{jt}, Z_{kjtw}, I_{ktw}, RR_{Lt}, O_{jt}, OR_{Lt},\\ W_{it}, H_{it}, L_{it}, B_{it} \ge 0\\ \forall\, i,j,k,L,t \text{ and } w.\end{aligned} \tag{14}$$

### Scenario tree

A scenario tree is a computationally feasible method of discretizing the underlying dynamic stochastic records through dynamic stochastic data overtime in a problem [25, 26]. An illustration of a scenario tree is presented in Fig. 2. In a scenario tree, each stage denotes a period. Stages would possibly consist of some periods in the planning horizon. The scenario tree consists of many nodes and arcs at each stage. Each node n in the scenario tree introduces a possible state concerned with a set of data (stochastic demand, stochastic cost, and stochastic yield. etc) in the identical stage. The root node of the tree represents the present state. The arcs (branches) in the scenario tree indicate the scenarios for the following stage. A probability is associated with each arc of the scenario tree, which marks the probability of the corresponding scenario to that arc. It ought to be stated that the probability of each node in the scenario tree is calculated as the product of the probability of the arcs from the root node to that particular node. Moreover, the sum of the probabilities of nodes at each stage must be equal to 1. A direction from the root node to a given node $n$ describes one scenario of the stochastic process from the current time to the period where node $n$ appears. A complete direction of the stochastic process over the whole planning horizon is the direction from the root node to a leaf node is called a scenario. Figure 2 introduces 2 stages, 9 scenarios for 3 products ($p$). The probabilities are (0.2, 0.3, and 0.5).

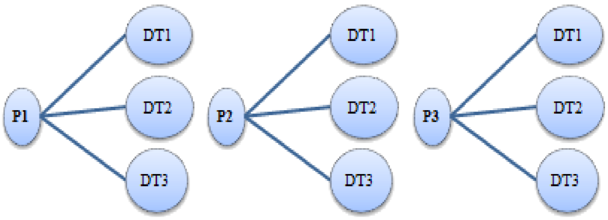



Fig. 2. Scenario tree notations.

### Two-stag stochastic programming model (compact formulation)

TSP model (compact formulation) means the combination between the deterministic model and the result of the scenario tree. The main variables of the deterministic model (inventory, backorder, regular time, and overtime variables) are affected by seasonal demand. Hence, there are several assumptions of the deterministic model to avoid demand uncertainty, shown in Table 3. Applying the assumptions; in Table 3, on the deterministic model the TSP model (compact formulation) is as follows:

Objective function

$$\text{Min Cost} = \sum_{n\in\text{Tree}} p(n)\Bigg(\sum_{t=1}^{\text{tn}}\sum_{j=1}^{J}\Bigg(\sum_{i=FS(j)} C_{ijt}X_{ijt} + \sum_{i\in P_j}(BP_{ij}/Q_{ijt})\cdot X_{ijt} + \sum_{i\in S_j}(BS_{ij}/Q_{ijt})\cdot X_{ijt} + o_{jt}\cdot O_{jt}(n) + r_{jt}\cdot R_{jt}(n)\Bigg)\Bigg) + \sum_{n\in\text{Tree}} p(n)(\sum_{t=1}^{\text{tn}}\sum_{i=1}^{N} h_{it}\cdot IF_{it}(n) + \sum_{t=1}^{\text{tn}}\sum_{i=1}^{N} b_{it}\cdot B_{it}(n)) + \sum_{t=1}^{\text{tn}}\sum_{i=1}^{N} W_{it}\cdot w_{it}. \tag{15}$$

Subject to
production, inventory and backorder equations for each family

$$\sum_{j=1}^{J} X_{ijt} + IF_{i,t-1}(m) + B_{i,t-1}(m) - IF_{it}(n) - B_{it}(n) = d_{it}(n),$$
$$\text{for} \quad i = 1,2,...,N, \quad t = 1,2,...,t_n, \quad n \in \text{Tree}, \tag{16}$$

Table 3
TSP model (compact formulation) assumptions.

| | |
|---|---|
| Indices | |
| Tree | Scenario tree. |
| $n$, $m$ | The node of scenario tree. |
| $a(n)$ | Definition of node $n$ in the scenario tree. |
| $t_n$ | Set of periods corresponding to node $n$ in the scenario tree. |
| Parameter | |
| $d_{it}(n)$ | Demand for family $i$ in period $t$ at node $n$ of the scenario tree. |
| $d_{ktw}(n)$ | Demand for item $k$ in sub-period $w$ of period $t$ at node $n$ of the scenario tree. |
| $P(n)$ | Probability of node $n$ of the scenario tree. |
| Decision variables | |
| $IF_{it}(n)$ | Inventory of family $i$ at the end of period $t$ at node $n$ of the scenario tree. |
| $I_{kitw}(n)$ | Inventory of item $k$ at the end sub-period $w$ of period $t$ at node $n$ of the scenario tree. |
| $B_{it}(n)$ | Backorder of family $i$ at the end of period $t$ at node $n$ of the scenario tree. |
| $O_{jt}(n)$ | Overtime used by department $j$ in period $t$ at node $n$ of the scenario tree. |
| $R_{jt}(n)$ | Regular time used by department $j$ in period $t$ at node $n$ of the scenario tree. |

capacity restrictions for each cell

$$\sum_{i\in FS(j)} a_{ij}X_{ijt} + \sum_{i\in P_j} (bp_{ij}/Q_{ijt})\cdot X_{ijt} + \sum_{i\in S_j} (bs_{ij}/Q_{ijt})\cdot X_{ijt} - O_{jt}(n) = R_{jt}(n), \quad \text{for} \quad j=1,2,...,J, \quad t=1,2,...,t_n, \tag{17}$$

$$m = \begin{cases} a(n), & t-1 \notin t_n \\ n, & t-1 \in_n \end{cases}$$

$$0 \le R_{jt}(n) \le \text{ (upper limit)} \quad \forall\, j,t,n \in \text{Tree}, \tag{18}$$

$$0 \le O_{jt}(n) \le \text{ (upper limit)} \tag{19}$$

production and inventory balance equations for each item $t$

$$\sum_{j=1}^{J} Z_{kjtw} + I_{kt,w-1}(m) - I_{ktw}(n) = d_{ktw}(n) \quad \forall\, k \in TI(i), w, t, \quad n \in \text{Tree}, \tag{20}$$

inventory consistency equations

$$\sum_{k\in TI(i)} \sum_{w=I}^{n} I_{ktw}(n) - IF_{it}(n) = 0 \quad \forall\, i,t,n \in \text{Tree}, \tag{21}$$

capacity restrictions for each resource

$$\sum_{i\in FS(j)} \sum_{k\in TI(i)} \left( PR_{kl} \sum_{w=I}^{n} Z_{kjtw} \right) - OR_{LT} = RR_{LT} \quad \forall\, L \in LR(j), j, t \tag{22}$$

$$0 \le RR_{Lt} \le \text{ (upper limit) } (1-\alpha_{Lt}) \quad \forall\, L,t \tag{23}$$

$$0 \le OR_{LT} \le \text{ (upper limit)} \tag{24}$$

resource consistency relations:

$$\sum_{l\in LR(j)} RR_{Lt} - R_{jt}(n) = 0 \quad \forall\, j,t,n \in \text{Tree}, \tag{25}$$

$$\sum_{l\in LR(j)} OR_{Lt} - O_{jt}(n) = 0 \tag{26}$$

workforce balance:

$$W_{it} = W_{i,t-1} + H_{it} - L_{it}, \quad \text{for} \quad i=1,2,...,N, \quad t=1,2,...,t_n \tag{27}$$

non-negativity restrictions:

$$X_{ijt}, IF_{it}(n), R_{jt}(n), Z_{kjtw}, I_{ktw}(n), RR_{Lt}, O_{jt}(n), OR_{Lt}, W_{it}, H_{it}, L_{it}, B_{it}(n) \ge 0 \quad \forall\, i,j,k,L,t_n,w, \quad n \in \text{Tree}. \tag{28}$$

## Solution procedure using sampling average approximation

SAA [27–29] is a sampling-based method that can be used to solve the TSP. The SAA method is applied to control and solve the stochastic model which presents an effective scope for identifying and statistically testing a set of selected production plans. The resulting SAA problem has then solved the usage of deterministic optimization techniques.

In the SAA method, a random sample of n scenarios of the random vector $\zeta$ and the anticipation is as follows:

$$\sum_{n\in \text{Tree}} p(n) \left( \sum_{t=1}^{tn} \sum_{i=1}^{N} h_{it} IF_{it}(n) + \sum_{t=1}^{tn} \sum_{i=1}^{N} b_{it} B_{it}(n) \right).$$

This is approximated by the following sample average function:

$$\frac{1}{n} \left[ \sum_{n\in \text{Tree}} p(n) \left( \sum_{t=1}^{tn} \sum_{i=1}^{N} h_{it} IF_{it}(n) + \sum_{t=1}^{tn} \sum_{i=1}^{N} b_{it} B_{it}(n) \right) \right].$$

Using SAA, the true "objective function" is given by Eq. (29):

$$\begin{aligned} \text{Min}\widehat{Z} = {} & \sum_{n\in \text{Tree}} p(n) \Bigg( \sum_{t=1}^{tn} \sum_{j=1}^{J} \sum_{i=FS(j)} C_{ijt} X_{ijt} + \sum_{i\in P_j} (BP_{ij}/Q_{ijt})\, X_{ijt} + \sum_{i\in S_j} (BS_{ij}/Q_{ijt})\, X_{ijt} \\ & + o_{jt}\cdot O_{jt}(n) + r_{jt}\cdot R_{jt}(n) \Bigg) + \frac{1}{n} \left[ \sum_{n\in \text{Tree}} p(n) \left( \sum_{t=1}^{tn} \sum_{i=1}^{N} h_{it} IF_{it}(n) + \sum_{t=1}^{tn} \sum_{i=1}^{N} b_{it} B_{it}(n) \right) \right] + \sum_{t=1}^{tn} \sum_{i=1}^{N} W_{it} w_{it}. \end{aligned} \tag{29}$$

Using SAA method, the solution of the mathematical model must be repeated with independent samples. Solution quality relies on the statistical confidence interval. Using SAA method, there could be an optimality gap (OG); that is defined as the difference between right objective value and true optimal solution. In this study, normal distribution is used to obtain the optimality gap (OG) confidence interval which gives the sample mean and variance. This approach is presented as follows:

Step (1): Create $n_g$ consider as independent identically distributed of samples batches of size $n$ from the normal distribution of $\zeta$ie. $\{\zeta_{1y}, \zeta_{2y}, \zeta_{3y}, ..., \zeta_{ny}\}$, for $y = 1, ..., n_g$ for each sample. Solve the SAA Eq. (29). Let $\widehat{Z}_n^y$ optimal objective value.

Step (2): Calculate

$$\bar{\bar{Z}}_{n,n_g} = \frac{1}{n_g} \sum_{y=1}^{n_g} \widehat{Z}_n^y, \tag{30}$$

$$S^2_{\bar{\widehat{Z}}_{n,n_g}} = \frac{1}{n_g(n_g-1)} \sum_{y=1}^{n_g} \left( \widehat{Z}_n^y - \bar{Z}_{n,n_g} \right). \tag{31}$$

In this approach, it should be regarded that the value of $\widehat{Z}_n$ is less than or equal to the optimal value of $Z^*$ the problem as $E\left[\bar{\bar{Z}}_{n,n_g}\right] \leq Z^*$. Therefore, $\bar{\bar{Z}}_{n,n_g}$ introduces a lower statistical bound value for the optimal value $Z^*$ of the problem, while $S^2_{\bar{\bar{Z}}_{n,n_g}}$ is the variance of the estimator $E\left[\bar{\bar{Z}}_{n,n_g}\right]$.

## Implementation of the proposed model

### Case study data

An explanatory real case study is introduced to demonstrate the model application. The model is applied in General Manufacturing Company (GMC) which produces three main product groups: fully-automatic washing machine, gas cooker and electrical water heater (EWH) with volumes of (30, 40, 50, 80, 100 Liter). In this study different types of cost, parameters are used in the computational study. EWH family was selected for the case study and relevant data is collected. The company produces three families of the EWH; 50L, 80L and 100L. The data comprise; seven manufacturing departments, three planning horizon period (12 weeks), five production resources (machine, manpower, material, maintenance (spare parts) and energy), 65 (parts) items. The included assumptions are:

P1 is a set of families at the primary department (1), $FS(1), ..., FS(7) = \{\text{family}1, 2, 3\}$, where TI(1) is a set of items to family (1) $LR(1) = LR(2), ..., LR(7) = \{\text{resource}1, 2, 3, 4, 5\}$, where LR(1) is a set of resources belonging to department (1).

Overtime is allowed while the backorder is not.

Variable parameters of the system are summarized in Table 5, where UN$\sim$ [a, b] represents a uniformly distributed random variable in the interval [a, b]. It must be mentioned that most of the values of these parameters are constant during the planning horizon. Also, the processing time of each item at each feasible resource ($PR_{kl}$) and the number of operations for each item is fixed.

Table 5
Data collection (fixed parameters).

| Parameters | Set of values |
|---|---|
| Total number of items, $k$ | 65 |
| Total number of resources, $L$ | 5 |
| No of periods, $T$ | 12 weeks |
| Probability distribution $p(n)$ | (0.5, 0.3, 0.2) |
| No of sub periods, $w$ | 4 weeks |
| Cost of production, $C_{ijt}$ | [2000,2250] |
| Cost of regular time, $r_{jt}$ | [320,400] |
| Inventory Holding Cost, $h_{it}$ | [40,48] |
| Setup cost for the families, $BP_{ij}$ | [200,260] |
| Processing time, $PR_{kl}$ | [0.07,0.075] |
| No of operation per part | 130 |
| Setup time, $bp_{ij}$ | [0.361,0.331] |
| Total time required to produce one unit of family $i$ at department $j$, $a_{ij}$ | [3.37,4.5] |

### Computational results

The mathematical model (Eqs (15)–(28)) that represents the SAA method of the TSP is solved optimally with the commercial Non-linear programming solver LINGO 16.0 (LINDO Systems Inc.). All experimental runs were conducted using an Intel Core (TM) i5-2450M CPU (2.50 GHz) and 6.0 GB RAM, working under Windows. The numerical results from LINGO 16.0 are validated using Mathworks Matlab R2015a (64-Bit), Appendix.

To examine the stability and robustness of the proposed stochastic model considering seasonality, an analysis of the investigated scenarios are introduced. In this analysis, three typical demand values of each product (per period) with different probability distributions (0.5, 0.3, and 0.2) are introduced. Therefore, three interest trees (DT1, DT2, and DT3) and an aggregate of three test problems are considered as shown in Table 6, and Fig. 3.

Table 6
Total product cost $Z$ with different probability distribution.

| Total product cost $Z$ (L.E/Month) | DT1(P=0.2) | DT2(P=0.3) | DT3(P=0.5) |
|---|---|---|---|
| EWH (50L) | 0.9122E+10 | 1.08E+10 | 1.47E+10 |
| EWH (80L) | 1.10E+10 | 1.98E+09 | 3.40E+09 |
| EWH (100L) | 3.09E+09 | 3.25E+09 | 3.52E+09 |

From Fig. 3, it is clear that the minimum production cost of the three EWH sizes (50, 80, and 100L) is related to DT2 of probability distribution P = 0.3. While the maximum cost for 100L, EWH is that of DT3 (P = 0.5) and for the 80L EWH the maximum cost is related to DT1 (P = 0.2). The production cost of the EWH 100L is almost the same for the three investigated probability distributions.

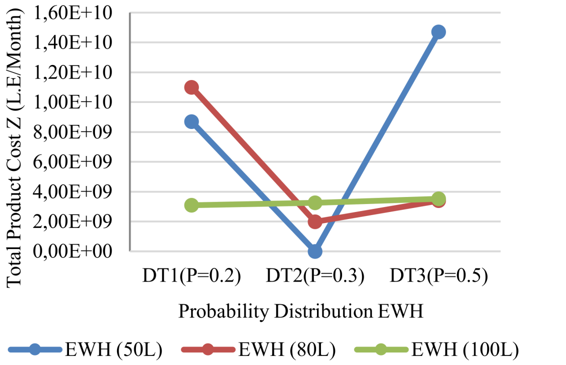



Fig. 3. Scenarios of the total product cost with different probability distributions of EWH.

Generally, a mathematical model states a system with a set of variables and a set of equations that express the relationships between the variables. In this work, using MINITAB 17 statistical software, a mathematical model was introduced using multiple regression methods. ANOVA test is usually used to analyze the results obtained from the experimentation of a full set of system configurations to reach significant observations. Interaction plots are also very indicative illustrations [30]. Hence, ANOVA analysis specializing in statistical validation of the acquired outcomes has been applied.

Table 7 exhibits data used for ANOVA Analysis [30–32] the main variables (inventory $IF$, regular time $R$, overtime $O$, and probability distribution values $p(n)$), that are affected by seasonal demand are analyzed. Table 8 exhibits an analysis of variance results for the min total production cost.

It could be noticed in table 8 which the analysis is considered for a level of significance of 5%, with 95%confidence levels that there is a significant effect of $P(n)$, $IF$, $R$ and $O$ but $IF$, and $O$ are significant factors than $P(n)$ as $P$-values less than 0.0500. $F$ test was calculated for that factor analysis ($IF$, $O$) equals 3812.91 and 819.51 respectively. It was noted that higher value than other factors, this result shows also that factors have a significant effect on $z$ (min total production cost).

Table 7
Data used for ANOVA analysis.

| Factor | Type | Levels | Values |
|---|---|---|---|
| $P(n)$ | Fixed | 3 | 0.2,0.3,0.5 |
| $IF$ | Fixed | 3 | 1.73,1.9,2.0 |
| $R$ | Fixed | 3 | 2000,2100,2500 |
| $O$ | Fixed | 3 | 1.1,1.5,2.0 |

Figure 4 illustrates the probability plot diagram to visualize the normality of the data, while Fig. 5 represents the main effects plot of data means on $Z$.

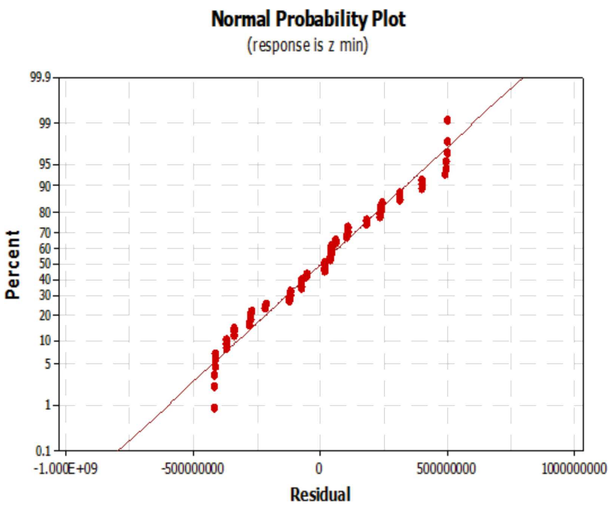



Fig. 4. Normal probability plot diagram for $Z$ (min total production cost).

Table 8
Analysis of variance for $Z$ (min total production cost).

| Source | DF | SS (L.E ) | MS (L.E ) | F | P |
|---|---|---|---|---|---|
| P(n) | 2 | 5.61602E+20 | 2.80801E+20 | 0.01 | 0.995 |
| IF (Ratio) | 2 | 5.37363E+17 | 2.68681E+17 | 819.51 | 0.000 |
| R (Min) | 2 | 1.20705E+20 | 6.03524E+19 | 3.65 | 0.031 |
| O (Hr) | 2 | 7.88888E+14 | 3.94444E+14 | 3812.91 | 0.000 |
| Error | 72 | 5.30242E+18 | 7.36447E+16 | | |
| Total | 80 | 6.88147E+20 | | | |
| S=271375590 L.E | | R-Sq = 99.23% | R-Sq (adj) = 99.14% | | |

Notes: SS is the Sum of squares, MS is the Mean of squares, DF is the Degrees of freedom, F and P-value: test statistics.

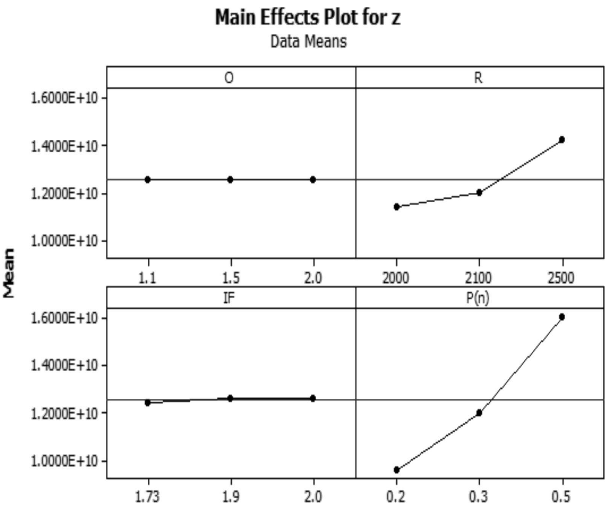



Fig. 5. The main effects plot of data means on $Z$ (min total production cost).

In Fig. 4, the adequacy of the model is also investigated by the examination of residuals. The residuals, which are the difference between the respective, observe responses are examined using the normal probability plots of the residuals. If the model is adequate, the points on the normal probability plots of the residuals should form a straight line. Figure 4 illustrates the normal probability plot for testing Z (min total production cost) with the cumulative distributions of the residuals. The error distributions seem to be normal. The residual is around $\pm 1.0$ E+09. From Fig. 5 it can be noted that $IF$, and $O$ are the most effective factors on the optimal solution of $z$.

## Conclusion

This model successfully solved the problems of production planning for the following purposes:

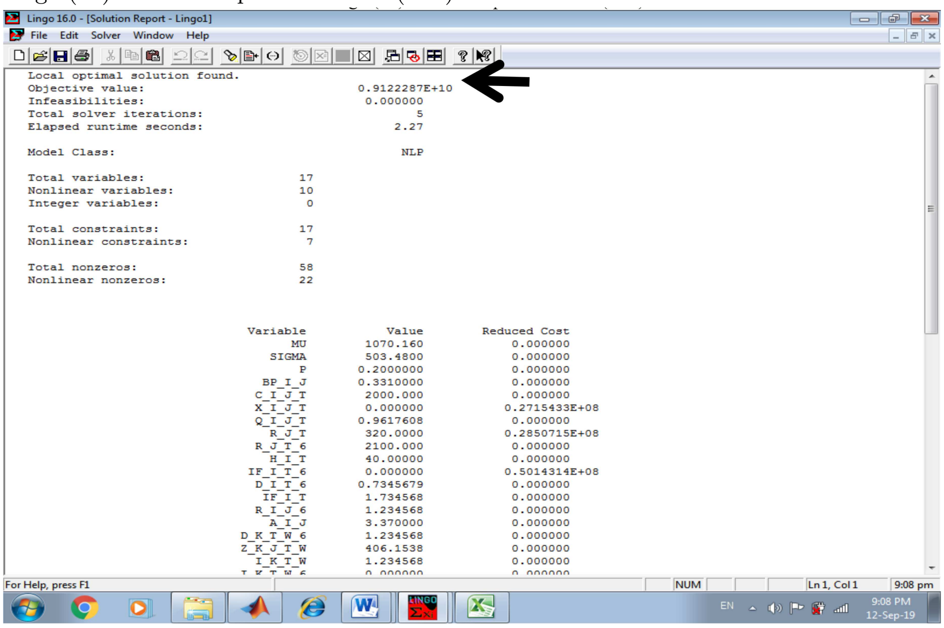
1) the model can solve problems in case of seasonal demand as no previous research was included, where it's recommendations of Kazemi et al. [16],
2) the developed analytical model (Non-linear, two-Stage stochastic programming) to into consideration manufacturing set up costs ignored in previously developed models,
3) an advantage of the proposed model is that it is verified using Mathworks Matlab R2015a,
4) using ANOVA indicated the most significant factors affected by seasonal demand that are; inventory, and overtime.

## Appendix

We are used for solving proposed model LINGO 16.0 software and validation through Matlab, and the result is the same sound as illustrated in Appendix. A program screen (code) and table presented this validation. It introduced the results (Min Z) of LINGO 16.0 and Matlab is 9.1*10E+9 L.E/Month, 9.8 *10E+9 L.E/Month, respectively with probability distributions (0.2) for EWH (50L). As there is no difference gap between results.

| | | |
|---|---|---|
| LINGO 16 | EWH (50L), p(0.2) | Min z=9.1*10E9 L.E/Month |
| Matlab | EWH (50L), p(0.2) | Min z=9.8*10E9 L.E/Month |

Lingo (16) Solution report for EWE (50L)



Matlab Solution report for EWE (50L)

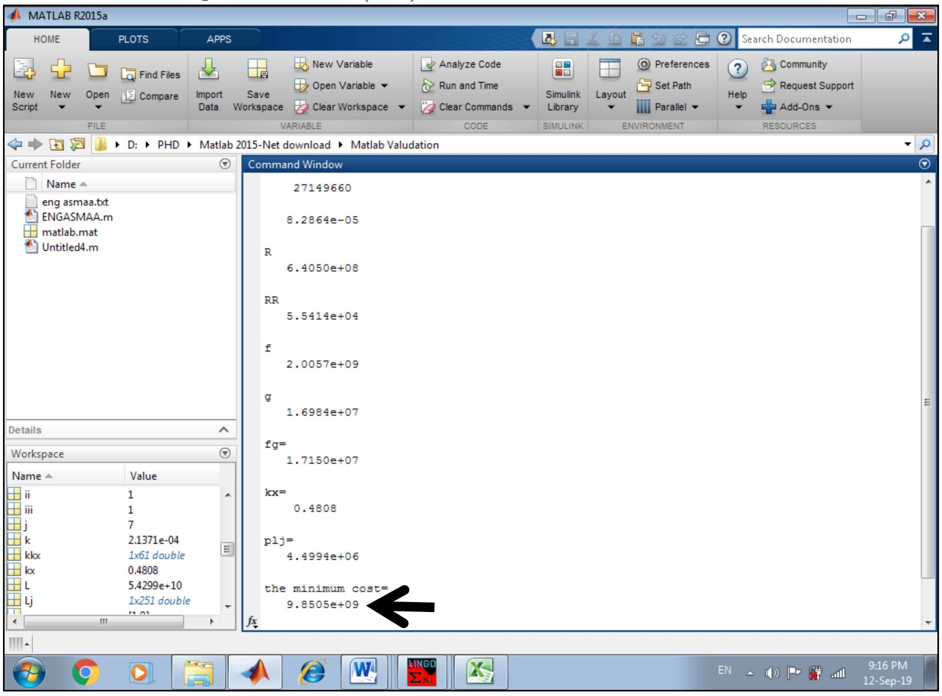



Table
Multiplicative seasonal method EWH (50L).

| t week | 2016 | Seasonal factor (1) | t week | 2017 | Seasonal factor (2) | t week | 2018 | Seasonal factor (3) | t week | 2019 | Seasonal factor (4) | t week | Average seasonal factor | **Estimation of 2020** | Weekly forecasted (product) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1350 | 0.99 | 1 | 2055 | 1.54 | 1 | 1485 | 1.39 | 1 | 1200 | 0.78 | 1 | 1.17 | | 1637 |
| 2 | 760 | 0.56 | 2 | 1988 | 1.49 | 2 | 2358 | 2.20 | 2 | 999 | 0.65 | 2 | 1.22 | | 2041 |
| 3 | 666 | 0.49 | 3 | 1485 | 1.11 | 3 | 605 | 0.57 | 3 | 789 | 0.51 | 3 | 0.67 | | 1116 |
| 4 | 889 | 0.65 | 4 | 2358 | 1.77 | 4 | 1374 | 1.28 | 4 | 1012 | 0.66 | 4 | 1.09 | | 1816 |
| 5 | 2000 | 1.47 | 5 | 1374 | 1.03 | 5 | 950 | 0.89 | 5 | 2555 | 1.66 | 5 | 1.26 | | 2100 |
| 6 | 2003 | 1.47 | 6 | 950 | 0.71 | 6 | 814 | 0.76 | 6 | 3005 | 1.95 | 6 | 1.22 | | 2037 |
| 7 | 1350 | 0.99 | 7 | 814 | 0.61 | 7 | 654 | 0.61 | 7 | 2100 | 1.36 | 7 | 0.89 | | 1489 |
| 8 | 1450 | 1.06 | 8 | 1386 | 1.04 | 8 | 1386 | 1.30 | 8 | 835 | 0.54 | 8 | 0.98 | | 1641 |
| 9 | 1878 | 1.38 | 9 | 520 | 0.39 | 9 | 762 | 0.71 | 9 | 2055 | 1.33 | 9 | 0.95 | | 1588 |
| 10 | 1999 | 1.46 | 10 | 890 | 0.67 | 10 | 810 | 0.76 | 10 | 1566 | 1.02 | 10 | 0.98 | | 1627 |
| 11 | 1050 | 0.77 | 11 | 990 | 0.74 | 11 | 750 | 0.70 | 11 | 1988 | 1.29 | 11 | 0.88 | | 1459 |
| 12 | 985 | 0.72 | 12 | 1200 | 0.90 | 12 | 894 | 0.84 | 12 | 396 | 0.26 | 12 | 0.68 | | 1131 |
| Total | 16380 | | Total | 16010 | | Total | 12842 | | Total | 18500 | | Total | | **16731** | |

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