Uncertainty of the characteristics of electrical devices based on the measurements of time-current characteristics of MV fuses

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Abstract. Functional properties of some electrical devices are expressed in the form of a dependence between parameters defining a given aspect of device duty or work circumstance (e.g. breakdown voltage): contacts distance for a circuit breaker, current (voltage for voltage limiters or reaction time), and load current for over current protection. Such characteristics are obtained experimentally, usually in a set of test series, each performed with a fixed independent parameter. Results of each series generate sets of data for estimation of statistical properties of a dependent parameter: distribution, expected value, variance and confidence interval. These statistics concern one point of the tested characteristic, so to get data of it as a whole, that would satisfy the needs for designing an electrical system, a large number of tests can be necessary. The way to reduce the number of tests may consists in: defining the characteristic not as a series of points, but as an analytical function with some specific parameters. This can be combined with aggregation of results of all tests in one set of data for estimation of statistical properties of the mentioned parameter. This paper presents an application of the above approach to tests of time-current characteristics of fuses.

Key words: time-current characteristics, MV fuse, approximation of measurement results, fuse element, statistics.

1. Introduction

Medium voltage fuse-links are designed to provide stable and reliable time-current characteristics. Their body is composed of a porcelain, glazed tube with very high mechanical and thermal resistance. Along with a temperature-sensitive striker that prevents damage to the switchgear due to high temperatures, high-voltage fuses allow for correct and cost-effective protection of distribution transformers with fuse disconnector sets. The process leading to the short-circuit and fuse-link ignition is random, due to the scattered characteristics of these electrical devices. Fuse-links manufacturers take this into account by providing an inscription that contains the actual go-off times for fuses. The randomness of the process that leads to fuse burnout causes difficulties in determining the performance characteristics of the fuse. Statistical methods are essential for providing a cost-effective manner of performing fuse-links research. The use of such methods can reduce the time between testing and implementation of the mentioned device.

The current-time characteristic provide information about the time that elapses from the moment the power is switched on until the fuse-link burns out. Standards define this characteristic as the dependence of the pre-arc time on the value of the current causing the fuse to melt. The characteristic is presented in the form of a band.

In order to obtain comparability of fuse-link characteristics, the standards require that the fuses ought to be prepared in the cold initial state and as the function of the RMS current. \( I_{\text{ogr}} \) (fuse limiting current) is the highest instantaneous short-circuit current in the circuit when the electric arc is ignited in the fuse-link [1–4]. The characteristic of limiting current is shown in Fig. 1. Examination of the fuse-links characteristics requires many attempts and a vast amount of devices. In full tests, 41 samples are used and additional 12 in order to repeat the entire test series. In view of the above, it should be considered that the

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Fig. 1. Characteristic of limiting current for fuses
fuse-links current-time characteristic testing is expensive [5–7]. This is caused not only by the number of the attempts, but also by the testing system itself. According to the requirements of the given regulations, the parameters R and L of the circuit can be of any value, but for pre-arching times below 0.1 s, the testing system should be practically induction-free [8, 9].

The authors of research and publications determine these characteristics based on empirical research. They focus on the assessment of the designated results based on empirical tests in accordance with standards [10, 11]. Hence the idea to use a mathematical and statistical apparatus to determine the current-time characteristics of medium voltage fuses proves relevant.

2. Principle of aggregation for fuse time measurement results

The time-current characteristic $t(I)$ of a fuse insert presents the dependence between the current charging the fuse and the time that elapsed between current rise and break – Fig. 2. This reaction time is a random variable defined by an expected value $t_0(I)$ and probability distribution (1) with assumed $\eta$, typically $\eta = 0.025$.

$$P(t_0 - \Delta t_c \leq t \leq t_0 + \Delta t_c) < \eta.$$  

Significant economic savings, while maintaining statistical correctness of determining the characteristic and its distribution, can be achieved by combining time measurement results for current values distributed in the full range of characteristic into one set. For proper statistical analysis of results, it is possible to get the mathematical expression of a characteristic and objectively determined band of uncertainty of the time of fuse operation [12].

The value $\Delta t_c$ at which $P(t_0 - \Delta t_c) = \eta$ and $P(t_0 + \Delta t_c) = \eta$ defines the confidence interval $\pm \Delta t_c$.

In the conventional approach, dependences $t(I)$ and $\Delta t_c(I)$ are determined – as mentioned earlier – in a set of series of experiments performed with fixed values of the current. On the other hand, standards tend to find current value for a given time: 10 s, 1 s and 0.01 s. In this case, the “up-and-down” method can prove useful. However both methods need at least 100 tests to be statistically reliable. A number of methods and laboratory setups that meet this demand were created [13, 14].

The proposed approach assumes that the subject of the research is a given type of inserts, defined in terms of construction, technology and rated current. The rated time-current characteristic $t_0(I)$ is assumed to be unknown.

The experiment consists of a series of $m$ tests of operation of a fuse with a set of test current values $\{I_i\}$ and results of times to melt down the fuse element $\{t_i\}$. It should be assumed that these values are measured correctly, with negligible error.

To aggregate the time measurements for different values of the current, it is to assumed that the fuse reaction time depends on the overcurrent (2).

$$J = I - I_N$$

where $I_N$ is the rated current of a fuse

$$t_0(J) = A_0 e^{\alpha_0}$$

where $A_0$, $\alpha_0$ are nominal parameters of a fuse, being the research subject.

Another assumption made is the random deviation of the real time of operation of a fuse from a nominal value to be proportional to nominal (depending on current) rated operating time $t_0$ (4).

$$t = t_0(J)e^\varepsilon$$

where: $\varepsilon$ – random variable with zero mean value and normal distribution of standard deviation $\sigma$ (5).

$$\varepsilon: N(0, \sigma).$$

The correctness of this assumption is hypothetical. It gives the natural connection of absolute values of time scatter with its deterministic part.

3. Laboratory setup

All measurements were carried out at the Short Circuit Laboratory of the Institute of Electrical Power Engineering. Test benches and specialized control and measuring equipment allow for conducting tests of real phenomena accompanying the operation of an electrical apparatus.

The main areas of research include:
- examination of switching processes,
- long-term (up to 10 kA) and short-circuit (up to 100 kA) load tests,
- electromechanical diagnostics of high voltage circuit breakers,
- diagnosing the vacuum condition of vacuum switches.

In this laboratory, numerous tests of power equipment are carried out on behalf of or in cooperation with research centers or industrial plants, and as part of own work financed from public funds.
The test setup is shown in Fig. 3. It consisted of:

- MV grid;
- Current-limiting reactor (R);
- MV switch (S);
- MV circuit breaker (CB);
- Current and voltage transformers;
- Making switches (MS);
- Stepping down – 15/0.44 kV – transformer (TR);
- MV fuses (F).

![Fig. 3. Laboratory setup for testing MV fuses located at the Short Circuit Laboratory of the Institute of Electrical Power Engineering](image)

4. Analysis of measurement results

The main goal of testing fuses with nominal current \( I_N \) is to find estimates \( A_n \) and \( \alpha_n \) of coefficients \( A_0 \) and \( \alpha_0 \) and to determine statistical properties of \( \varepsilon \).

The estimation is based on a set of reaction times of the fuse \( \{ t_i \} \) measured at current value sets \( \{ I_i \} \) for \( i = 1, \ldots, m \). Measured values are gathered in vectors (6) and (7).

\[
J = [I_1 - I_N \ldots I_m - I_N]^T, \quad (6)
\]

\[
t = [t_1 \ldots t_m]^T. \quad (7)
\]

According to approximation (2) and assumption (5), operating time in the \( i \)-th test equals (8).

\[
t_i = A_0 I_i^{\alpha_0} e^{\alpha_i} \quad (8)
\]

Logarithm of this function is

\[
y_i = \ln(A_0) + \alpha_0 x_i + \varepsilon_i \quad (9)
\]

where \( x_i = \ln(I_i) \) and \( y_i = \ln(t_i) \).

Estimates \( A_n \) and \( \alpha_n \) of coefficients \( A_0 \) and \( \alpha_0 \) are obtained by LSQ approximation (10) for \( i = 1, \ldots, m \) [15].

\[
y_i \cong \hat{y}_i = \ln(A_n) + \alpha_n x_i. \quad (10)
\]

With denotations (11)–(14)

\[
x = [x_1 \ldots x_m]^T, \quad (11)
\]

\[
y = [y_1 \ldots y_m]^T, \quad (12)
\]

\[
b = [\ln(A_n) \; \alpha_n]^T \equiv [b_1 \; b_2]^T, \quad (13)
\]

\[
B = [1 \; x]^T. \quad (14)
\]

The normal equation of LSQ fitting yields (15).

\[
b = (B^T B)^{-1} y, \quad (15)
\]

The estimated nominal characteristic \( t_n(I) \) takes the form of (16) or – according to (13) – the form of (17).

\[
t_n = e^{b_1} I_{n}^{b_2} = e^{b_1} (I_n - I_N)^{b_2}, \quad (16)
\]

\[
t_n = A_n I_{n}^{\alpha_0} = A_n (I_n - I_N)_{\alpha_0}. \quad (17)
\]

Note that residue of approximation (18) can be treated as the estimate of deviations of operation time (19).

\[
r = y - \hat{y} = y - B x, \quad (18)
\]

\[
r = \hat{\varepsilon} = [\hat{\varepsilon}_1 \ldots \hat{\varepsilon}_m]^T. \quad (19)
\]

5. Statistical properties and estimation errors

The proposed estimation has statistical justification if vector \( \varepsilon \) from model (9) has zero expected value and is uncorrelated with the conditions and measurement results.

In such a case, the estimate vector (14) resulting from (15) is associated with true coefficient values (20), with relation (17) giving (21) [16].

\[
\beta = [\ln(A_0) \; \alpha_0]^T, \quad (20)
\]

\[
b = \beta + (B^T B)^{-1} B^T \varepsilon. \quad (21)
\]

The estimator (15):

- is unbiased,
- has a covariance matrix of (22),

\[
cov(b) = E \left\{ [b - \beta] [b - \beta]^T \right\} = \sigma^2 (B^T B)^{-1} \quad (22)
\]

where \( \sigma \) is the standard deviation of the random variable – see (5), estimated by means of residues (18).

The estimate \( \sigma \) is the square root of the variance from the sample of deviation (23), here vector \( r \).

\[
\sigma \cong s = \sqrt{\frac{r^T r}{m - p}}, \quad (23)
\]

where \( p \) is the number of estimated parameters – in this case \( p = 2 \).

Formula (21) can be used to determine the distribution of estimate \( b \) (24).

\[
b : N(\beta, C) \quad (24)
\]

with the covariance matrix of (22).

On this basis, confidence intervals for estimates \( b = [b_1 \; b_2]^T \) are considered separately for \( k = 1, 2 \) (25).

\[
b_k - t_{kryt} \sigma \sqrt{C_{kk}} < b_k < b_k + t_{kryt} \sigma \sqrt{C_{kk}} \quad (25)
\]

where \( t_{kryt} \) is a critical – for a given level of confidence – value of Student’s t-statistics depending on the number of degrees of freedom \( m - p \).
6. Results of analysis of operation tests of fuses with rated current of 40 A

Figure 4 shows the results of measuring the operating times of 15 fuses with rated current of 40 A. The samples were grouped into threes around five current values: 225 A, 280 A, 330 A, 420 A and 500 A.

Approximation of the above relationships $t(J)$ with function (1) brought the results presented parametrically in Table 1 as well as Fig. 5 and 6.

Approximation residue (19) and the values resulting from the found coefficients (17) gave the results shown in Fig. 6. They allow for the assumption that the uncertainty model of action expressed by (3) is justified.

![Fig. 4. Primary data – results of measurements of operating time of 15 fuses with rated current of 40 A](image)

![Fig. 5. Results of approximation](image)

![Fig. 6. Distribution of $r$ – rest of approximation](image)

<table>
<thead>
<tr>
<th>Estimate value</th>
<th>Uncertainty interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b(1) = \log(A_n)$</td>
<td>23.626</td>
</tr>
<tr>
<td>$b(2) = \alpha_n$</td>
<td>-4.487</td>
</tr>
</tbody>
</table>
7. Uncertainty of estimate of current-time characteristics

The uncertainty of the estimate of current-time characteristics \( t_e(J) = A_n J_0^p - (16), (17) \) – results from the uncertainty of the estimates of coefficients \( A_n \) and \( \alpha_n \), whose uncertainty is in turn determined by the uncertainty of coefficient \( b \). Simple calculation of the uncertainty band using limits of confidence intervals for both parameters may be misleading, as shown in Fig. 7.

\[
(b - \beta)^T D (b - \beta) \leq \chi^2(p + 1) F_{kryt}(26)
\]

where: \( D = B^T B \); \( \chi^2 \) – variance from the sample, \( p \) – number of model parameters, \( F_{kryt} \) – critical value of the F-Snedecor distribution with \( pn \) and \( n \) degrees of freedom (27).

\[
F_{kryt} : \ P \left( F_{p} + 1, n - p - 1 \leq F_{kryt} \right).
\]

Relation (27) states that for a given estimation result \( b \), possible true \( \beta \) must satisfy this inequality.

The confidence ellipsoid for the results given in Table 1, on the level of 0.95, are shown in Fig. 8 in bold lines. Inside its area, dots show results of estimation performed using the Monte Carlo method for 1000 series of tests with statistical parameters found in the estimation [19, 20].

These results are also depicted in Fig. 9 on the current-time characteristic.

The uncertainty band of the nominal performance characteristics can now be determined as a set of characteristics calculated with parameter \( b \) found in simulated tests – dots in Fig. 9. Effects of such a procedure are shown in Fig. 9 as a green band.

Uncertainty of the current-time characteristic can be found analytically taking account of the dependence of the fuse reaction time on parameters \( \beta \) and statistical properties of their estimate \( b \).

It is useful to introduce random deviation (28), which has covariance (23) and distribution (29).

\[
\Delta b = b - \beta,
\]

\[
\Delta b : \ N \left( 0, \sigma^2 (B^T B)^{-1} \right).
\]
Denoting (30)–(34) it can be stated that the true current-time characteristic, expressed in logarithmic terms, is (34).

\[ \tau_0 = \ln(t_0), \]  
\[ \xi = \ln(J), \]  
\[ \beta_1, \beta_2: \beta = [\beta_1 \ beta_2]^T, \]  
\[ \Delta b_1, \Delta b_2: \Delta b = [\Delta b_1 \ Delta b_2]^T, \]  
\[ \tau_0(\xi; \beta_1, \beta_2) = \tau_n(\xi; b_1, b_2) - \frac{\partial \tau_0}{\partial \beta_1} \Delta b_1 - \frac{\partial \tau_0}{\partial \beta_2} \Delta b_1. \]  

The coefficients used in (34) now are (35), (36).

\[ \tau_0(\xi; \beta_1, \beta_2) = \beta_1 + \beta_2 \xi, \]  
\[ \frac{\partial \tau_0}{\partial \beta_1} = 1, \]  
\[ \frac{\partial \tau_0}{\partial \beta_2} = \xi. \]  

In this context, the characteristic \( \tau_0(\xi) \) is random, depending on random \( \Delta b_1 \) and \( \Delta b_2 \). Its random component is (37).

\[ \Delta \tau_0(\xi) = -\frac{\partial \tau_0}{\partial \beta_1} \Delta b_1 - \frac{\partial \tau_0}{\partial \beta_2} \Delta b_1. \]  

Formula (37) can be evaluated taking account of statistical properties \( \Delta b_1 \) and \( \Delta b_2 \). According to (36), in matrix form it can be expressed as (38) with variance (39).

\[ \Delta \tau_0(\xi) = [1 \ \xi] \Delta b, \]  
\[ \sigma_\xi^2(\xi) = \text{var}(\Delta \tau_0) = \sigma^2 [1 \ \xi] (B^T B)^{-1} [1 \ \xi]^T. \]  

Then the estimate of the confidence interval for the nominal current-time characteristic \( \tau_0(\xi) \) can be established as:

\[ \tau_0(\xi) - \lambda_n(\xi) \leq \tau_0(\xi) \leq \tau_0(\xi) + \lambda_n(\xi) \]  

where \( \lambda_n(\xi) \) is formulated as (41).

\[ \lambda_n(\xi) = t_{k,\xi} \sigma_\xi(\xi) \]  

where: \( t_{k,\xi} \) is the critical value of Student’s t-statistics, the same as in (25).

In natural coordinates \( I - I_0 \), uncertainty of the nominal current-time characteristic is expressed by inequality (42).

\[ A_n(I - I_n)^{\alpha_n} e^{-\lambda_\alpha(\xi)} \leq t_0(I) \leq A_n(I - I_n)^{\alpha_n} e^{\lambda_\alpha(\xi)} \]  

with \( \xi = \ln(I - I_N) \).

Expression (41) was used to draw the boundary characteristic \( t_0(I) \) in Fig. 9.
The results of calculations made on the basis of the presented measurements are shown in Fig. 10. It contains two pairs of graphs:

- upper and lower limits of inequality (42), i.e. the confidence interval of the nominal characteristics,
- the upper and lower limits of inequality (46), i.e. the confidence interval for real fuse reaction time.

9. Conclusions

The presented measurement results and their analysis confirm the proposed model with tested devices properties. This was further confirmed by relatively small and random residues of the approximation, whose distribution at this stage of the research can be considered normal.

The proposed method of statistical analysis of measurement results allows to plan an experiment, especially in terms of cost-effectiveness optimization of ratings. Improvement of the method requires increasing the number of tested specimens for more credible evaluation of the operating times spread. It would be helpful – not only to improve the method, but also to improve the technology of fuses – to create a mathematical model of the melting process in a fuse, taking random factors into account.

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REFERENCES


