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Multiple Attribute Decision Making method based on intuitionistic Dombi operators and its application in mutual fund evaluation

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In this paper, a new set of intuitionistic fuzzy aggregation operators have been introduced under the environment of intuitionistic fuzzy sets (IFSs). For this, firstly focused on some existing aggregation operators and then new operational rules known as Dombi operation have been proposed which make the advancement of flexibility behavior with the parameter. Based on Dombi operation laws, some new averaging and geometric aggregation operators namely, intuitionistic fuzzy Dombi weighted averaging, ordered weighted averaging and hybrid weighted averaging operator, classified as IFDWA, IFDOWA and IFDHWA operators respectively and intuitionistic fuzzy Dombi geometric, ordered weighted geometric and hybrid weighted geometric operators, labeled as IFDWG, IFDOWG and IFDHWG operators respectively have been proposed. Further, some properties such as idempotency, boundedness, monotonicity and commutative are investigated. Finally, a multi-attribute decision-making model has been developed for the proposed operators to select the best mutual fund for investment. The execution of the comparative study has been examined with the existing operators in this environment.

Key words: intuitionistic fuzzy elements, Dombi operations, averaging aggregation operators, geometric aggregation operators, multiple attribute decision making

1. Introduction

It is unfavourable to consider real attribute values as complexity appear significant level in decision science. In 1965, theory of fuzzy sets (FS) was published

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by Zadeh [52], a modern mathematical systems to handle multi-attribute decision making (MADM) and multi-attribute group decision making (MADGM) problems. Although, FS is a powerful frame but have a deficiency in broad mathematical frame. In that situation, Atnassov [1] proposed theory of intuitionistic fuzzy sets, which easily handle complex fuzzy information. IFS addressed an object in the universe by expressing membership function as well as non-membership function. A few years ago, researchers have been more concentrated about IFS [1] and IVIFS [1] because of these speculations have been effectively connected to many sensible applications. Recently, many tremendous works have been developed in IFS environment such as follows De et al. [4] defined operation on IFSs, Szmidt and Kacprzyk [35] studied similarity measure between IFSs, Guo and Song [12] proposed entropy between IFSs. It is generally seen that theory of IFSs is used to deal MAGDM problems, Kou et al. [27] used to develop algorithms based financial risk analysis MCDM model, Li et al. [28] introduced MADM method using Hassdrof's distance measure utilize generalize fuzzy numbers, Garg [7] proposed a generalized improved score function of IVIFSs and applied it in expert systems, Chen and Chiou [3] solved MADM problems based on IVIFSs, using PSO techniques and evidential reasoning methodology. Kumar and Garg [26] utilized TOPSIS method based on set pair analysis under IVIFSs environment. In [34], Lourenzuttia and Krohlingb studied TODIM problems based methodology in intuitionistic fuzzy and random environment.

Now a days information aggregation operators is a paramount research topic in MADM environment and become a concentration of the researchers to this areas, developed some important works (See [14, 18–20, 29, 31, 44–50]). Some traditional works [37,38,41,42] have been develop based on aggregation operators can aggregate a set of real values. Present time, some papers have been developed using extended aggregation operators, for example, Liu and Yu [32] focused on density aggregation functions for IFNs or IVIFNs, respectively, which containing the density of attributes values using density weights, Wu and Su [51] used to study prioritized relation based aggregation function in IFNs (IVIFNs) environment. In [5, 36, 39], presented some traditional decision-making problems using triangular norms in intuitionistic fuzzy environment. In [30], Hamacher aggregation operators are defined on an IVIFNs arguments and developed MAGDM methods. Instance, Garg and Kumar [11] presented possibility measure of IVIFNs, while Yu [43] presented Choquet aggregation operator to aggregate different intuitionistic fuzzy informations. The Dombi [6] norm and conorms which have good advantage of flexibility to handle the operational parameters. Using Dombi norms and others, many researchers addressed the problems of MCDM and MADM in diffrent fuzzy uncertain environments [2, 10, 15, 17–25, 33]. The fact that IFS have a powerful ability to model the uncertain information which arises in real world problems. Aprat from, traditional decision-making problems [37–42] and decision-making problems using Dombi norms [2, 6, 15, 17, 21, 22, 24, 25, 33] in different fuzzy environment makes us enough motivation to study our present paper. The object of this paper is to interpret some new aggregation operators using Dombi norms in the environment of IFS for aggregating the different preferences of the alternatives during the data analysis.

The remainder of this paper is reviewed as follows: In the next section, briefly review some basic concepts of the IFSs and its operations. In section 3, intuitionistic fuzzy Dombi weighted, ordered weighted and hybrid weighted averaging operators are defined. In section 4, intuitionistic fuzzy Dombi weighted geometric, order weighted, and hybrid weighted geometric operators are proposed. In section 5, using these operators, we solved intuitionistic fuzzy MADM problems. An illustrative example is given for the selection of best mutual fund is given in section 6. In the section 7, some remarks are given to the paper.

2. Preliminaries

2.1. Intuitionistic fuzzy sets

Definition 1 [40] *A IFS is defined over the universe of discourse Z as*

$$\widetilde{I} = \{\langle z, \mu(z), \nu(z)\rangle | z \in Z\}, \tag{1}$$

where $\mu(z) : Z \to [0,1]$ and $\nu(z) : Z \to [0,1]$ respectively denotes membership, non-membership degrees of an element $z \in X$ to a IFS. The $\pi(z) = 1-\mu(z)-\nu(z)$, is called indeterminacy degree of the element z to the set $\widetilde{I}$. The set $\langle(\mu,\nu)\rangle$ denotes intuitionistic fuzzy numbers (IFNs) or intuitionistic fuzzy elements (IFEs) or intuitionistic fuzzy values (IFVs).

Xu [40] provided some operations on IFEs as follows:

Definition 2 [40] *Let $\widetilde{I}_1 = (\langle \mu_{I_1}(z), \nu_{I_1}(z)\rangle)$ and $\widetilde{I}_2 = (\langle \mu_{I_2}(z), \nu_{I_2}(z)\rangle)$ be two IFEs. Then operations on IFEs are defined as:*

(i) $\widetilde{I}_1 \subseteq \widetilde{I}_2$, *if* $\mu_{I_1}(z) \leqslant \mu_{I_2}(z)$, $\nu_{I_1}(z) \geqslant \nu_{I_2}(z)$ *for all* $z \in Z$,

(ii) $\widetilde{I}_1 \cup \widetilde{I}_2 = \{\langle z, \max\{\mu_{I_1}(z), \mu_{I_2}(z)\}, \min\{\nu_{I_1}(z), \nu_{I_2}\}\rangle | z \in Z\}$,

(iii) $\widetilde{I}_1 \cap \widetilde{I}_2 = \{\langle z, \min\{\mu_{I_1}(z), \mu_{I_2}(z)\}, \max\{\nu_{I_1}(z), \nu_{I_2}\}\rangle | z \in Z\}$,

(iv) $\overline{I_1} = \{\langle z, \nu_{I_1}(z), \mu_{I_1}(z)\rangle | z \in Z\}$.

Based on score E and accuracy L functions given in [16], we proposed score and accuracy functions:

Definition 3 *Let $\widetilde{I}_1 = (\mu_{I_1}, \nu_{I_1})$ be an IFEs, then score function E and accuracy function L for IFEs is provided as:*

$$E(\widetilde{I}_1) = \frac{1 + \mu_{I_1} - \nu_{I_1}}{2}, \qquad E(\widetilde{I}_1) \in [0, 1] \tag{2}$$

and accuracy function is introduced as:

$$L(\widetilde{I}_1) = \frac{\mu_{I_1} + \nu_{I_1}}{2}, \qquad L(\widetilde{I}_1) \in [0, 1]. \tag{3}$$

Based on $E(\widetilde{I}_1)$ and $L(\widetilde{I}_1)$, we defined order relation on two IFEs $\widetilde{I}_1 = (\langle \mu_{I_1}(z), \nu_{I_1}(z) \rangle)$ and $\widetilde{I}_2 = (\langle \mu_{I_2}(z), \nu_{I_2}(z) \rangle)$ as follows:

Definition 4

(i) If $E(\widetilde{I}_1) < E(\widetilde{I}_2)$, then $\widetilde{I}_1 \prec \widetilde{I}_2$,

(ii) If $E(\widetilde{I}_1) > E(\widetilde{I}_2)$, then $\widetilde{I}_1 \succ \widetilde{I}_2$,

(iii) If $E(\widetilde{I}_1) = E(\widetilde{I}_2)$, then

(1) If $L(\widetilde{I}_1) < L(\widetilde{I}_2)$, then $\widetilde{I}_1 \prec \widetilde{I}_2$.
(2) If $L(\widetilde{I}_1) > L(\widetilde{I}_2)$, then $\widetilde{I}_1 \succ \widetilde{I}_2$.
(3) If $L(\widetilde{I}_1) = L(\widetilde{I}_2)$, then $\widetilde{I}_1 \sim \widetilde{I}_2$.

On IFEs, Xu [40] defined some operations are presented below:

Definition 5 [40] *Let $\widetilde{I}_1 = (\langle \mu_{I_1}(z), \nu_{I_1}(z) \rangle)$ and $\widetilde{I}_2 = (\langle \mu_{I_2}(z), \nu_{I_2}(z) \rangle)$ be two IFEs, then:*

(i) $\widetilde{I}_1 \wedge \widetilde{I}_2 = \{\langle z, \min\{\mu_{I_1}(z), \mu_{I_2}(z)\}, \max\{\nu_{I_1}(z), \nu_{I_2}(z)\}\rangle | z \in Z\}$,

(ii) $\widetilde{I}_1 \vee \widetilde{I}_2 = \{\langle z, \max\{\mu_{I_1}(z), \mu_{I_2}(z)\}, \min\{\nu_{I_1}(z), \nu_{I_2}(z)\}\rangle | z \in Z\}$,

(iii) $\widetilde{I}_1 \oplus \widetilde{I}_2 = (\langle \mu_{I_1}(z) + \mu_{I_2}(z) - \mu_{I_1}(z)\mu_{I_2}(z), \nu_{I_1}(z)\nu_{I_2}(z) \rangle)$,

(iv) $\widetilde{I}_1 \otimes \widetilde{I}_2 = (\langle \mu_{I_1}(z)\mu_{I_2}(z), \nu_{I_1}(z) + \nu_{I_2}(z) - \nu_{I_1}(z)\nu_{I_2}(z) \rangle)$,

(v) $\lambda \widetilde{I}_1 = (1 - (1 - \mu_{I_1}(z))^{\lambda}, \nu_{I_1}(z)^{\lambda})$,

(vi) $\widetilde{I}_1^{\lambda} = (\mu_{I_1})^{\lambda}(z), (1 - \nu_{I_1}(z))^{\lambda})$.

Xu [37] introduced intuitionistic fuzzy weighted averaging, ordered weighted averaging, and hybrid weighted averaging operators, labeled them IFWA, IFOWA and IFHWA operators as follows:

Definition 6 [37] *Let* $\widetilde{I}_q = (\mu_q, \nu_q)$ $(q = 1, 2, \ldots, b)$ *be a set of IFEs. IFWA operator of dimension* b *is a mapping* $\widetilde{I}FE^b \to \widetilde{I}FE$ *with the associated weight vector* $\omega = (\omega_1, \omega_2, \ldots, \omega_b)^T$ *such that* $\omega > 0$ *and* $\sum_{q=1}^{b} \omega_q = 1$*, then*

$$IFWA_w\left(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_b\right) = \bigoplus_{q=1}^{b} \left(\omega_q \widetilde{I}_q\right) = \left(1 - \prod_{q=1}^{b} (1 - \mu_q)^{\omega_q}, \prod_{q=1}^{b} {\nu_q}^{\omega_q}\right). \tag{4}$$

Xu [37] defined IFOWA operator in the next definition.

Definition 7 [37] *Let* $\widetilde{I}_q = (\mu_q, \nu_q)$ $(q = 1, 2, \ldots, b)$ *be a set of IFEs. An IFOWA operator of dimension* b *is a function* $IFE^b \to IFE$ *which have associated weight vector* $w = (w_1, w_2, \ldots, w_b)^T$ *for which* $w > 0$ *and* $\sum_{q=1}^{b} w_q = 1$*. Furthermore,*

$$\begin{aligned} IFOWA_w\left(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_n\right) &= \bigoplus_{q=1}^{b} \left(w_q \widetilde{I}_{\sigma(q)}\right) \\ &= \left(1 - \prod_{q=1}^{b} \left(1 - \mu_{\sigma(q)}\right)^{w_q}, \prod_{q=1}^{b} \nu_{\sigma(q)}^{w_q}\right), \end{aligned} \tag{5}$$

where $(\sigma(1), \sigma(2), \ldots, \sigma(b))$ is a permutation of $(1, 2, \ldots, b)$, such that $\widetilde{I}_{\sigma(q-1)} \geqslant \widetilde{I}_{\sigma(q)}$ for all $q = 1, 2, \ldots, b$.

In [37], Xu proposed IFHA operator.

Definition 8 [37] *Let* $\widetilde{I}_q = (\mu_q, \nu_q)$ $(q = 1, 2, \ldots, b)$ *be a set of IFEs. An IFHWA operator is a function* $\widetilde{I}FE^b \to \widetilde{I}FE$ *which has associated weight vector* $w = (\omega_1, \omega_2, \ldots, \omega_b)^T$ *with* $\omega > 0$ *and* $\sum_{q=1}^{b} \omega_q = 1$*. Furthermore,*

$$\begin{aligned} IFHWA_w\left(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_b\right) &= \bigoplus_{q=1}^{b} \left(\omega_q \widetilde{I}_{\sigma(q)}\right) \\ &= \left(1 - \prod_{q=1}^{b} \left(1 - \dot{\mu}_{\sigma(q)}\right)^{\omega_q}, \prod_{q=1}^{b} \dot{\nu}_{\sigma(q)}^{\omega_q}\right), \end{aligned} \tag{6}$$

where $\widetilde{I}_{\sigma(q)}$ is the q^{th} largest weighted IFV $\widetilde{I}_q$ ($\widetilde{I}_q = b\omega_t\widetilde{I}_q$, $q = 1, 2, \ldots, b$), and $\omega = (\omega_1, \omega_2, \ldots, \omega_b)^T$ be the weight vector of $\widetilde{I}_q$ with $\omega_q > 0$ and $\sum_{q=1}^{b} \omega_q = 1$, where b is the balancing coefficient. When $\omega = (1/b, 1/b, \ldots, 1/b)$, then IFWA operator is marked as particular case of IFHA operator. Let $\omega = (1/b, 1/b, \ldots, 1/b)$, then IFOWA is marked as particular case of IFHA operator. Thus, IFHA operator is a generalization of IFWA and IFOWA operators, which throw back the degrees of the applied arguments and their ordered positions.

In [38], Xu, similarly developed IFWG, IFOWG, IFHWG operators the weighted intuitionistic fuzzy geometric aggregation operators as.

2.2. Dombi operations of IFEs

Dombi defined Dombi triangular norms given below.

Definition 9 [6] *Let x and y be any two real numbers. The Dombi norm and Dombi conorm are expressed as:*

$$Dom(x, y) = \frac{1}{1 + \left\{\left(\frac{1-x}{x}\right)^{\kappa} + \left(\frac{1-y}{y}\right)^{\kappa}\right\}^{1/\kappa}}, \tag{7}$$

$$Dom^*(x, y) = 1 - \frac{1}{1 + \left\{\left(\frac{x}{1-x}\right)^{\kappa} + \left(\frac{y}{1-y}\right)^{\kappa}\right\}^{1/\kappa}}, \tag{8}$$

where, $\kappa \geqslant 1$ and $(x, y) \in [0, 1] \times [0, 1]$.

Based on the Dombi triangular norms, we defined Dombi operations on two IFEs.

Definition 10 *The Dombi operations on two IFEs $\widetilde{I}_1 = (\mu_1, \nu_1)$, $\widetilde{I}_2 = (\mu_2, \nu_2)$ are*

$$(i)\ \ \widetilde{I}_1 \oplus \widetilde{I}_2 = \left\langle 1 - \frac{1}{1 + \left\{\left(\frac{\mu_1}{1-\mu_1}\right)^{\kappa} + \left(\frac{\mu_2}{1-\mu_2}\right)^{\kappa}\right\}^{1/\kappa}}, \frac{1}{1 + \left\{\left(\frac{1-\nu_1}{\nu_1}\right)^{\kappa} + \left(\frac{1-\nu_2}{\nu_2}\right)^{\kappa}\right\}^{1/\kappa}} \right\rangle,$$

$$(ii)\ \ \widetilde{I}_1 \otimes \widetilde{I}_2 = \left\langle \frac{1}{1 + \left\{\left(\frac{1-\mu_1}{\mu_1}\right)^{\kappa} + \left(\frac{1-\mu_2}{\mu_2}\right)^{\kappa}\right\}^{1/\kappa}}, 1 - \frac{1}{1 + \left\{\left(\frac{\nu_1}{1-\nu_1}\right)^{\kappa} + \left(\frac{\nu_2}{1-\nu_2}\right)^{k}\right\}^{1/\kappa}} \right\rangle,$$

$$(iii)\quad \lambda . \widetilde{I}_1 = \left\langle 1 - \frac{1}{1 + \left\{\lambda \left(\frac{\mu_1}{1-\mu_1}\right)^{\kappa}\right\}^{1/\kappa}},\ \frac{1}{1 + \left\{\lambda \left(\frac{1-\nu_1}{\nu_1}\right)^{\kappa}\right\}^{1/\kappa}} \right\rangle,$$

$$(iv)\quad (\widetilde{I}_1)^{\lambda}_1 = \left\langle \frac{1}{1 + \left\{\lambda \left(\frac{1-\mu_1}{\mu_1}\right)^{\kappa}\right\}^{1/\kappa}},\ 1 - \frac{1}{1 + \left\{\lambda \left(\frac{\nu_1}{1-\nu_1}\right)^{\kappa}\right\}^{1/\kappa}} \right\rangle.$$

3. Intuitionistic fuzzy Dombi arithmetic aggregation operators

In this section, we introduce intuitionistic fuzzy Dombi weighted averaging (IFDWA) operator, order weighted averaging (IFDOWA) operator and hybrid weighted averaging (IFDHWA) operator and their corresponding properties.

In the following theorem, we prove some properties using Dombi norms.

Theorem 1 *Let $\widetilde{I} = (\mu, \nu)$, $\widetilde{I}_1 = (\mu_1, \nu_1)$, $\widetilde{I}_2 = (\mu_2, \nu_2)$ be three IFEs, then*

(1) $\widetilde{I}_1 \oplus \widetilde{I}_2 = \widetilde{I}_2 \oplus \widetilde{I}_1,$

(2) $\widetilde{I}_1 \otimes \widetilde{I}_2 = \widetilde{I}_2 \otimes \widetilde{I}_1,$

(3) $\lambda(\widetilde{I}_1 \oplus \widetilde{I}_2) = \lambda\widetilde{I}_1 \oplus \lambda\widetilde{I}_2, \quad \lambda > 0,$

(4) $(\lambda_1 + \lambda_2)\widetilde{I} = \lambda_1\widetilde{I} \oplus \lambda_2\widetilde{I}, \quad \lambda_1, \lambda_2 > 0,$

(5) $(\widetilde{I}_1 \otimes \widetilde{I}_2)^{\lambda} = \widetilde{I}_1^{\lambda} \otimes \widetilde{I}_2^{\lambda}, \quad \lambda > 0,$

(6) $\widetilde{I}^{\lambda_1} \otimes \widetilde{I}^{\lambda_2} = \widetilde{I}^{(\lambda_1+\lambda_2)}, \quad \lambda_1, \lambda_2 > 0.$

Proof. For the three IFEs $\widetilde{I}$, $\widetilde{I}_1$ and $\widetilde{I}_2$, and $\lambda, \lambda_1, \lambda_2 > 0$, then by Definition 10, we can obtain

$$\begin{aligned}
(1)\quad \widetilde{I}_1 \oplus \widetilde{I}_2 &= \left\langle 1 - \frac{1}{1 + \left\{\left(\frac{\mu_1}{1-\mu_1}\right)^{\kappa} + \left(\frac{\mu_2}{1-\mu_2}\right)^{\kappa}\right\}^{1/\kappa}},\ \frac{1}{1 + \left\{\left(\frac{1-\nu_1}{\nu_1}\right)^{\kappa} + \left(\frac{1-\hat{\nu}_2}{\nu_2}\right)^{\kappa}\right\}^{1/\kappa}} \right\rangle \\
&= \left\langle 1 - \frac{1}{1 + \left\{\left(\frac{\mu_2}{1-\mu_2}\right)^{\kappa} + \left(\frac{\mu_1}{1-\mu_1}\right)^{\kappa}\right\}^{1/\kappa}},\ \frac{1}{1 + \left\{\left(\frac{1-\nu_2}{\nu_2}\right)^{\kappa} + \left(\frac{1-\nu_1}{\nu_1}\right)^{\kappa}\right\}^{1/\kappa}} \right\rangle \\
&= \widetilde{I}_2 \oplus \widetilde{I}_1 .
\end{aligned}$$

(2) The proof of this result is obvious.

(3) Let

$$t = 1 - \frac{1}{1 + \left\{ \left(\frac{\mu_1}{1-\mu_1} \right)^{\kappa} + \left(\frac{\mu_2}{1-\mu_2} \right)^{\kappa} \right\}^{1/\kappa}} .$$

Then, we have

$$\frac{t}{1-t} = \left\{ \left(\frac{\hat{\mu}_1}{1-\mu_1} \right)^{\kappa} + \left(\frac{\mu_2}{1-\mu_2} \right)^{\kappa} \right\}^{1/\kappa} .$$

Therefore

$$\left(\frac{t}{1-t} \right)^{\kappa} = \left(\frac{\mu_1}{1-\mu_1} \right)^{\kappa} + \left(\frac{\mu_2}{1-\mu_2} \right)^{\kappa} .$$

Using above results, we get

$$\lambda\left(\widetilde{I}_1 \oplus \widetilde{I}_2\right) = \lambda \left\langle 1 - \frac{1}{1+\left\{ \left(\frac{\mu_1}{1-\mu_1} \right)^{\kappa} + \left(\frac{\mu_2}{1-\mu_2} \right)^{\kappa} \right\}^{1/\kappa}}, \frac{1}{1+\left\{ \left(\frac{1-\nu_1}{\nu_1} \right)^{\kappa} + \left(\frac{1-\nu_2}{\nu_2} \right)^{\kappa} \right\}^{1/\kappa}} \right\rangle$$

$$= \left\langle 1 - \frac{1}{1+\left\{ \lambda\left(\frac{\mu_1}{1-\mu_1} \right)^{k} + \lambda\left(\frac{\mu_2}{1-\mu_2} \right)^{\kappa} \right\}^{1/\kappa}}, \frac{1}{1+\left\{ \lambda\left(\frac{1-\nu_1}{\nu_1} \right)^{k} + \lambda\left(\frac{1-\nu_2}{\nu_2} \right)^{\kappa} \right\}^{1/\kappa}} \right\rangle .$$

Now,

$$\lambda\widetilde{I}_1 \oplus \lambda\widetilde{I}_2 = \left\langle 1 - \frac{1}{1 + \left\{ \lambda\left(\frac{\mu_1}{1-\mu_1} \right)^{\kappa} \right\}^{1/\kappa}}, \frac{1}{1 + \left\{ \lambda\left(\frac{1-\nu_1}{\nu_1} \right)^{\kappa} \right\}^{1/\kappa}} \right\rangle$$

$$\oplus \left\langle 1 - \frac{1}{1 + \left\{ \lambda\left(\frac{\mu_2}{1-\mu_2} \right)^{\kappa} \right\}^{1/\kappa}}, \frac{1}{1 + \left\{ \lambda\left(\frac{1-\nu_2}{\nu_2} \right)^{\kappa} \right\}^{1/\kappa}} \right\rangle$$

$$= \left\langle 1 - \frac{1}{1+\left\{ \lambda\left(\frac{\mu_1}{1-\mu_1} \right)^{\kappa} + \lambda\left(\frac{\mu_2}{1-\mu_2} \right)^{\kappa} \right\}^{1/\kappa}}, \frac{1}{1+\left\{ \lambda\left(\frac{1-\nu_1}{\nu_1} \right)^{\kappa} + \lambda\left(\frac{1-\nu_2}{\nu_2} \right)^{\kappa} \right\}^{1/\kappa}} \right\rangle$$

$$= \lambda\left(\widetilde{I}_1 \oplus \widetilde{I}_2\right) .$$

$$(4)\ \lambda_1\widetilde{I}\oplus\lambda_2\widetilde{I}=\left\langle 1-\frac{1}{1+\left\{\lambda_1\left(\frac{\mu}{1-\mu}\right)^{\kappa}\right\}^{1/\kappa}},\ \frac{1}{1+\left\{\lambda_1\left(\frac{1-\nu}{\nu}\right)^{\kappa}\right\}^{1/\kappa}}\right\rangle$$

$$\oplus\left\langle 1-\frac{1}{1+\left\{\lambda_2\left(\frac{\mu}{1-\mu}\right)^{\kappa}\right\}^{1/\kappa}},\ \frac{1}{1+\left\{\lambda_2\left(\frac{1-\nu}{\nu}\right)^{\kappa}\right\}^{1/\kappa}}\right\rangle$$

$$=\left\langle 1-\frac{1}{1+\left\{(\lambda_1+\lambda_2)\left(\frac{\mu}{1-\mu}\right)^{\kappa}\right\}^{1/\kappa}},\ \frac{1}{1+\left\{(\lambda_1+\lambda_2)\left(\frac{1-\nu}{\nu}\right)^{\kappa}\right\}^{1/\kappa}}\right\rangle$$

$$=(\lambda_1+\lambda_2)\widetilde{I}.$$

$$(5)\ \left(\widetilde{I}_1\otimes\widetilde{I}_2\right)^{\lambda}=\left\langle\frac{1}{1+\left\{\left(\frac{1-\mu_1}{\mu_1}\right)^{\kappa}+\left(\frac{1-\mu_2}{\mu_2}\right)^{\kappa}\right\}^{1/\kappa}},\ 1-\frac{1}{1+\left\{\left(\frac{\nu_1}{1-\nu_1}\right)^{\kappa}+\left(\frac{\nu_2}{1-\nu_2}\right)^{\kappa}\right\}^{1/\kappa}}\right\rangle^{\lambda}$$

$$=\left\langle\frac{1}{1+\left\{\lambda\left(\frac{1-\mu_1}{\mu_1}\right)^{\kappa}+\lambda\left(\frac{1-\mu_2}{\mu_2}\right)^{\kappa}\right\}^{1/\kappa}},\ 1-\frac{1}{1+\left\{\lambda\left(\frac{\nu_1}{1-\nu_1}\right)^{k}+\lambda\left(\frac{\nu_2}{1-\nu_2}\right)^{\kappa}\right\}^{1/\kappa}}\right\rangle$$

$$=\left\langle\frac{1}{1+\left\{\lambda\left(\frac{1-\mu_1}{\mu_1}\right)^{\kappa}\right\}^{1/\kappa}},\ 1-\frac{1}{1+\left\{\lambda\left(\frac{\nu_1}{1-\nu_1}\right)^{\kappa}\right\}^{1/\kappa}}\right\rangle$$

$$\otimes\left\langle\frac{1}{1+\left\{\lambda\left(\frac{1-\mu_2}{\mu_2}\right)^{\kappa}\right\}^{1/\kappa}},\ 1-\frac{1}{1+\left\{\lambda\left(\frac{\nu_2}{1-\nu_2}\right)^{\kappa}\right\}^{1/\kappa}}\right\rangle=\widetilde{I}_1^{\lambda}\otimes\widetilde{I}_2^{\lambda}.$$

$$(6)\ \widetilde{I}^{\lambda_1}\otimes\widetilde{I}^{\lambda_2}=\left\langle\frac{1}{1+\left\{\lambda_1\left(\frac{1-\mu}{\mu}\right)^{\kappa}\right\}^{1/\kappa}},\ 1-\frac{1}{1+\left\{\lambda_1\left(\frac{\nu}{1-\nu}\right)^{\kappa}\right\}^{1/\kappa}}\right\rangle$$

$$\otimes\left\langle\frac{1}{1+\left\{\lambda_2\left(\frac{1-\mu}{\mu}\right)^{\kappa}\right\}^{1/\kappa}},\ 1-\frac{1}{1+\left\{\lambda_2\left(\frac{\nu}{1-\nu}\right)^{\kappa}\right\}^{1/\kappa}}\right\rangle$$

$$=\left\langle\frac{1}{1+\left\{(\lambda_1+\lambda_2)\left(\frac{1-\mu}{\mu}\right)^{\kappa}\right\}^{1/\kappa}},\ 1-\frac{1}{1+\left\{(\lambda_1+\lambda_2)\left(\frac{\nu}{1-\nu}\right)^{\kappa}\right\}^{1/\kappa}}\right\rangle$$

$$=\widetilde{I}^{(\lambda_1+\lambda_2)}.\qquad\square$$

Definition 11 *Let $\widetilde{I}_q = (\mu_q, \nu_q)$ $(q = 1, 2, \ldots, b)$ be a collection of IFEs. The IFDWA operator is a function $IFE^b \rightarrow IFE$ such that*

$$IFDWAA_{\omega}\left(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_b\right) = \bigoplus_{q=1}^{b}\left(\omega_q \widetilde{I}_q\right), \tag{9}$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_b)^T$ be the weight vector of $\widetilde{I}_q$ $(q = 1, 2, \ldots, b)$ with $\omega_j > 0$ and $\sum_{q=1}^{b} \omega_j = 1$.

We get the next theorem that follows the Dombi operations on IFEs.

Theorem 2 *Let $\widetilde{I}_q = (\mu_q, \nu_q)$ $(q = 1, 2, \ldots, b)$ be a set of IFEs, then aggregated value of IFEs using the IFDWA operator is also a IFE, and*

$$\begin{aligned} IFDWA_{\omega}\left(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_b\right) &= \bigoplus_{q=1}^{b}\left(\omega_q \widetilde{I}_q\right) \\ &= \left\langle 1 - \frac{1}{1 + \left\{\sum\limits_{q=1}^{b} \omega_q \left(\frac{\mu_q}{1-\mu_q}\right)^{\kappa}\right\}^{1/\kappa}}, \frac{1}{1 + \left\{\sum\limits_{q=1}^{b} \omega_q \left(\frac{1-\nu_q}{\nu_q}\right)^{\kappa}\right\}^{1/\kappa}} \right\rangle, \end{aligned} \tag{10}$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_b)$ as the weight vector of $\widetilde{I}_q$ $(q = 1, 2, \ldots, b)$ such that $\omega_q > 0$, and $\sum_{q=1}^{b} \omega_q = 1$.

Theorem 2 can be obtained by using mathematical induction.

Proof. (i) When $q = 2$, based on Dombi operations on IFEs computed the results $IFDWA_{\omega}\left(\widetilde{I}_1, \widetilde{I}_2\right) = \widetilde{I}_1 \oplus \widetilde{I}_2 = (\mu_1, \nu_1) \oplus (\mu_2, \nu_2)$ and for right side of (10), we have

$$\begin{aligned} &\left\langle 1 - \frac{1}{1 + \left\{\omega_1 \left(\frac{\mu_1}{1-\mu_1}\right)^{\kappa} + \omega_2 \left(\frac{\mu_2}{1-\mu_2}\right)^{\kappa}\right\}^{1/\kappa}}, \frac{1}{1 + \left\{\omega_1 \left(\frac{1-\nu_1}{\nu_1}\right)^{\kappa} + \omega_2 \left(\frac{1-\nu_2}{\nu_2}\right)^{\kappa}\right\}^{1/\kappa}} \right\rangle \\ &= \left\langle 1 - \frac{1}{1 + \left\{\sum\limits_{q=1}^{2} \omega_q \left(\frac{\mu_q}{1-\mu_q}\right)^{\kappa}\right\}^{1/\kappa}}, \frac{1}{1 + \left\{\sum\limits_{q=1}^{2} \omega_q \left(\frac{1-\nu_q}{\nu_q}\right)^{\kappa}\right\}^{1/\kappa}} \right\rangle. \end{aligned} \tag{11}$$

Therefore, (10) holds for $q \geqslant 2$.

(ii) Suppose (10) holds for $q = m$, then from Eq. (10), we have

$$IFDWA_{\omega}\left(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_m\right) = \bigoplus_{q=1}^{m}\left(\omega_q \widetilde{I}_q\right)$$

$$= \left\langle 1 - \frac{1}{1 + \left\{\sum\limits_{q=1}^{m} \omega_q \left(\frac{\mu_q}{1-\mu_q}\right)^{\kappa}\right\}^{1/\kappa}}, \frac{1}{1 + \left\{\sum\limits_{q=1}^{m} \omega_q \left(\frac{1-\nu_q}{\nu_q}\right)^{\kappa}\right\}^{1/\kappa}}\right\rangle. \tag{12}$$

Now for $q = m + 1$, we get

$$IFDWA_{\omega}\left(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_m, \widetilde{I}_{m+1}\right) = \bigoplus_{q=1}^{m}(\omega_q I_q) \oplus (\omega_{m+1} I_{m+1})$$

$$= \left\langle 1 - \frac{1}{1 + \left\{\sum\limits_{q=1}^{m} \omega_q \left(\frac{\mu_q}{1-\mu_q}\right)^{\kappa}\right\}^{1/\kappa}}, \frac{1}{1 + \left\{\sum\limits_{q=1}^{m} \omega_q \left(\frac{1-\nu_q}{\nu_q}\right)^{\kappa}\right\}^{1/\kappa}}\right\rangle$$

$$\oplus \left\langle 1 - \frac{1}{1 + \left\{\omega_{m+1} \left(\frac{\mu_{m+1}}{1-\mu_{m+1}}\right)^{\kappa}\right\}^{1/\kappa}}, \frac{1}{1 + \left\{\omega_{m+1} \left(\frac{1-\nu_{m+1}}{\nu_{m+1}}\right)^{\kappa}\right\}^{1/\kappa}}\right\rangle$$

$$= \left\langle 1 - \frac{1}{1 + \left\{\sum\limits_{q=1}^{m+1} \omega_q \left(\frac{\mu_q}{1-\mu_q}\right)^{\kappa}\right\}^{1/\kappa}}, \frac{1}{1 + \left\{\sum\limits_{q=1}^{m+1} \omega_q \left(\frac{1-\nu_q}{\nu_q}\right)^{\kappa}\right\}^{1/\kappa}}\right\rangle. \tag{13}$$

Thus, Eq. (10) holds for $q = m + 1$.

Thus, by euations (i) and (ii), we conclude that (10) is true for any $q \in N$. □

Example.

Suppose there are four IFEs,

$$I_1 = (0.6, 0.3), \quad I_2 = (0.5, 0.4), \quad I_3 = (0.7, 0.2), \quad I_4 = (0.2, 0.3),$$

and $\omega = (0.2, 0.1, 0.3, 0.4)$ is the weight vector.

Then

$$IFDWA_\omega\left(\widetilde{I}_1,\widetilde{I}_2,\ldots,\widetilde{I}_4\right)=\bigoplus_{q=1}^{4}\left(\omega_q\widetilde{I}_q\right)$$

$$=\left\langle 1-\frac{1}{1+\left\{\sum_{q=1}^{4}\omega_q\left(\frac{\mu_q}{1-\mu_q}\right)^\kappa\right\}^{1/\kappa}},\ \frac{1}{1+\left\{\sum_{q=1}^{4}\omega_q\left(\frac{1-\nu_q}{\nu_q}\right)^\kappa\right\}^{1/\kappa}}\right\rangle$$

$$=\left\langle 1-\frac{1}{1+\left\{0.2\left(\frac{0.6}{1-0.6}\right)^2+0.1\left(\frac{0.5}{1-0.5}\right)^2+0.3\left(\frac{0.7}{1-0.7}\right)^2+0.4\left(\frac{0.2}{1-0.2}\right)^2\right\}^{1/2}},\right.$$

$$\left.\frac{1}{1+\left\{0.2\left(\frac{1-0.3}{0.3}\right)^2+0.1\left(\frac{1-0.4}{0.4}\right)^2+0.3\left(\frac{1-0.2}{0.2}\right)^2+0.4\left(\frac{1-0.3}{0.3}\right)^2\right\}^{1/2}}\right\rangle$$

$$=\left\langle(0.5977,0.2578)\right\rangle.$$

The IFDWA operator follows the following properties.

Theorem 3 *(Idempotency) If $\widetilde{I}_q=(\mu_q,\nu_q)$ $(q=1,2,\ldots,b)$ be a set of equal IFEs, i.e., $\widetilde{I}_q=\widetilde{I}$ for all b, so*

$$IFDWA_\omega\left(\widetilde{I}_1,\widetilde{I}_2,\ldots,\widetilde{I}_b\right)=\widetilde{I}. \tag{14}$$

Proof. Since $\widetilde{I}_q=(\mu_q,\nu_q)=\widetilde{I}$ $(q=1,2,\ldots,b)$. Then from Eq. (10), we have

$$IFDWA_\omega\left(\widetilde{I}_1,\widetilde{I}_2,\ldots,\widetilde{I}_b\right)=\bigoplus_{q=1}^{b}\left(\omega_q\widetilde{I}_q\right)$$

$$=\left\langle 1-\frac{1}{1+\left\{\sum_{q=1}^{b}\omega_q\left(\frac{\mu_q}{1-\mu_q}\right)^\kappa\right\}^{1/\kappa}},\ \frac{1}{1+\left\{\sum_{q=1}^{b}\omega_q\left(\frac{1-\nu_q}{\nu_q}\right)^\kappa\right\}^{1/\kappa}}\right\rangle$$

$$=\left\langle 1-\frac{1}{1+\left\{\left(\frac{\hat{\mu}^+}{1-\mu}\right)^\kappa\right\}^{1/\kappa}},\frac{1}{1+\left\{\left(\frac{1-\nu}{\nu}\right)^\kappa\right\}^{1/\kappa}}\right\rangle$$

$$=\left\langle 1-\frac{1}{1+\frac{\mu}{1-\mu}},\ \frac{1}{1+\frac{1-\nu}{\nu}}\right\rangle=(\mu,\nu)=\widetilde{I}.$$

Hence, $IFDWA_\omega\left(\widetilde{I}_1,\widetilde{I}_2,\ldots,\widetilde{I}_b\right)=\widetilde{I}$ holds. □

Theorem 4 *(Boundedness) Let $\widetilde{I}_q = (\mu_q, \nu_q)$ $(q = 1, 2, \ldots, b)$ be a set of IFEs. Let*

$$\widetilde{I}^- = \min\left(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_b\right) = (\mu'^-, \nu'^-)$$

and

$$\widetilde{I}^+ = \max\left(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_b\right) = (\mu'^+, \nu'^+),$$

where,

$$\mu'^- = \min_q\{\mu_q\}, \quad \nu'^- = \max_q\{\nu_q\}, \quad \mu'^+ = \max_q\{\mu_q\}, \quad \text{and} \quad \nu'^+ = \min_q\{\nu_q\}.$$

Then, we find the inequalities,

$$1 - \frac{1}{1 + \left\{\sum\limits_{q=1}^{b} \omega_q \left(\frac{\mu_q'^-}{1-\mu_q'^-}\right)^\kappa\right\}^{1/\kappa}} \leqslant 1 - \frac{1}{1 + \left\{\sum\limits_{q=1}^{b} \omega_q \left(\frac{\mu_q}{1-\mu_q}\right)^\kappa\right\}^{1/\kappa}}$$
$$\leqslant 1 - \frac{1}{1 + \left\{\sum\limits_{q=1}^{b} \omega_q \left(\frac{\mu_q'^+}{1-\mu_q'^+}\right)^\kappa\right\}^{1/\kappa}}$$

$$\frac{1}{1+\left\{\sum\limits_{q=1}^{b} \omega_q \left(\frac{1-\nu_q'^-}{\nu_q'^-}\right)^\kappa\right\}^{1/\kappa}} \leqslant \frac{1}{1+\left\{\sum\limits_{q=1}^{b} \omega_q \left(\frac{1-\nu_q}{\nu_q}\right)^\kappa\right\}^{1/\kappa}} \leqslant \frac{1}{1+\left\{\sum\limits_{q=1}^{b} \omega_q \left(\frac{1-\nu_q'^+}{\nu_q'^+}\right)^\kappa\right\}^{1/\kappa}}.$$

Therefore,

$$\widetilde{I}^- \leqslant IFDWA_\omega\left(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_b\right) \leqslant \widetilde{I}^+. \tag{15}$$

Theorem 5 *(Monotonicity) Let $\widetilde{I}_q$ $(q = 1, 2, \ldots, b)$ and $\widetilde{I}_t'$ $(q = 1, 2, \ldots, b)$ be two sets of IFEs, if $\widetilde{I}_q \leqslant \widetilde{I}_q'$ for all q, then*

$$IFDWA_\omega\left(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_b\right) \leqslant IFDWA_\omega\left(\widetilde{I}_1', \widetilde{I}_2', \ldots, \widetilde{I}_b'\right). \tag{16}$$

Now, we introduce IFDOWA operator.

Definition 12 *Let $\widetilde{I}_q = (\mu_q, \nu_q)$ $(q = 1, 2, \ldots, b)$ be a set of IFEs. An IFDOWAA operator of dimension b is a function $IFDOWA : IFE^b \to IFE$ with associated vector $\omega = (\omega_1, \omega_2, \ldots, \omega_b)^T$ such that $\omega_q > 0$, and $\sum\limits_{q=1}^{b} \omega_q = 1$. Therefore,*

$$IFDOWA_\omega\left(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_b\right) = \bigoplus_{q=1}^{b} \left(\omega_q \widetilde{I}_{\sigma(q)}\right), \tag{17}$$

where $(\sigma(1), \sigma(2), \ldots, \sigma(b))$ *is the permutation of* $(q = 1, 2, \ldots, b)$, *for which* $\widetilde{I}_{\sigma(q-1)} \geqslant \widetilde{I}_{\sigma(q)}$ *for all* $q = 1, 2, \ldots, b$.

Above definition follows the next theorem on IFEs.

Theorem 6 *Let* $\widetilde{I}_q = (\mu_q, \nu_q)$ $(q = 1, 2, \ldots, b)$ *be a set of IFEs. IFDOWA operator of dimension* b *is a mapping* $IFDOWA : IFE^b \to IFE$ *with associated weight vector* $\omega = (\omega_1, \omega_2, \ldots, \omega_b)^T$ *such that* $\omega_q > 0$, *and* $\sum_{q=1}^{b} \omega_q = 1$. *Then,*

$$IFDOWA_\omega\left(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_b\right) = \bigoplus_{q=1}^{b}\left(\omega_q \widetilde{I}_{\sigma(q)}\right)$$

$$= \left\langle 1 - \frac{1}{1 + \left\{\sum\limits_{q=1}^{b} \omega_q \left(\frac{\mu_{\sigma(q)}}{1-\mu_{\sigma(q)}}\right)^{\kappa}\right\}^{1/\kappa}}, \frac{1}{1 + \left\{\sum\limits_{q=1}^{b} \omega_q \left(\frac{1-\nu_{\sigma(q)}}{\nu_{\sigma(q)}}\right)^{\kappa}\right\}^{1/\kappa}} \right\rangle, \quad (18)$$

where $(\sigma(1), \sigma(2), \ldots, \sigma(b))$ *is a permutation of* $(q = 1, 2, \ldots, b)$, *follows* $I_{\sigma(q-1)} \geqslant I_{\sigma(q)}$ *for all* $q = 1, 2, \ldots, b$.

Example. Let us considered four

$$I_1 = (0.5, 0.3), \quad I_2 = (0.6, 0.3), \quad I_3 = (0.7, 0.3) \quad \text{and} \quad I_4 = (0.2, 0.4)$$

be four IFEs, $\omega = (0.2, 0.1, 0.3, 0.4)^T$ is the weight vector of these IFEs. Then aggregated value of IFEs is for $\kappa = 3$, and by Definition 12, scores of I_q $(q = 1, 2, 3, 4)$ can be evaluated as:

$$E(I_1) = \frac{1 + 0.5 - 0.3}{2} = 0.6, \qquad E(I_2) = \frac{1 + 0.6 - 0.3}{2} = 0.65,$$

$$E(I_3) = \frac{1 + 0.7 - 0.3}{2} = 0.7, \qquad E(I_4) = \frac{1 + 0.2 - 0.4}{2} = 0.4.$$

Since,

$$E(I_3) > E(I_2) > E(I_1) > E(I_4),$$

then $I_{\sigma(1)} = I_3 = (0.7, 0.3)$, $I_{\sigma(2)} = I_2 = (0.6, 0.3)$, $I_{\sigma(3)} = I_1 = (0.5, 0.3)$ and $I_{\sigma(4)} = I_4 = (0.2, 0.4)$. Then, by IFDOWA operator, we have

$$
\begin{aligned}
IFDOWA_\omega\left(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_4\right) &= \bigoplus_{q=1}^{4}\left(\omega_q \widetilde{I}_{\sigma(q)}\right)\\
&= \left\langle 1 - \frac{1}{1+\left\{\sum\limits_{q=1}^{4}\omega_q\left(\frac{\mu_{\sigma(q)}}{1-\mu_{\sigma(q)}}\right)^3\right\}^{1/3}}, \frac{1}{1+\left\{\sum\limits_{q=1}^{4}\omega_q\left(\frac{1-\nu_{\sigma(q)}}{\nu_{\sigma(q)}}\right)^3\right\}^{1/3}}\right\rangle\\
&= \left\langle 1 - \frac{1}{1+\left\{0.2\left(\frac{0.7}{1-0.7}\right)^3 + 0.1\left(\frac{0.6}{1-0.6}\right)^3 + 0.3\left(\frac{0.5}{1-0.5}\right)^3 + 0.4\left(\frac{0.2}{1-0.2}\right)^3\right\}^{1/3}},\right.\\
&\qquad \left.\frac{1}{1+\left\{0.2\left(\frac{1-0.3}{0.3}\right)^3 + 0.1\left(\frac{1-0.3}{0.3}\right)^3 + 0.3\left(\frac{1-0.3}{0.3}\right)^3 + 0.4\left(\frac{1-0.4}{0.4}\right)^3\right\}^{1/3}}\right\rangle\\
&= \left\langle (0.5953, 0.3249)\right\rangle.
\end{aligned}
$$

The properties of IFDOWA operator can be proved easily.

Theorem 7 *(Idempotency) If $\widetilde{I}_q = (\mu_q, \nu_q)$ $q = 1, 2, \ldots, b$ are all equal, i.e. $\widetilde{I}_q = \widetilde{I}$ for all b, then $IFDOWA_\omega\left(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_b\right) = \widetilde{I}$.*

Theorem 8 *(Boundedness) Let $\widetilde{I}_q = (\mu_q, \nu_q)$ $(q = 1, 2, \ldots, b)$ be a set of IFEs. Let $I^- = \min\limits_q \widetilde{I}_q$, and $\widetilde{I}^+ = \max\limits_q \widetilde{I}_q$. Then*

$$\widetilde{I}^- \leqslant IFDOWA_\omega\left(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_b\right) \leqslant \widetilde{I}^+. \tag{19}$$

Theorem 9 *(Monotonicity) Let $\widetilde{I}_q = (\mu_q, \nu_q)$ $(q = 1, 2, \ldots, b)$ and $\widetilde{I}'_q$ $(q = 1, 2, \ldots, b)$ be sets of two IFEs, if $\widetilde{I}_q \leqslant \widetilde{I}'_q$ for all q, then*

$$IFDOWA_\omega\left(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_b\right) \leqslant IFDOWA_\omega\left(\widetilde{I}'_1, \widetilde{I}'_2, \ldots, \widetilde{I}'_b\right). \tag{20}$$

Theorem 10 *(Commutativity) Let $\widetilde{I}_q = (\mu_q, \nu_q)$ $(q = 1, 2, \ldots, b)$ and $\widetilde{I}'_t$ $(q = 1, 2, \ldots, b)$ be sets of two IFEs, then*

$$IFDOWA_\omega\left(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_b\right) = IFDOWA_\omega\left(\widetilde{I}'_1, \widetilde{I}'_2, \ldots, \widetilde{I}'_b\right),$$

where $\widetilde{I}'_q$ $(q = 1, 2, \ldots, b)$ is any permutation of $\widetilde{I}_q$ $(q = 1, 2, \ldots, b)$.

In Definition 11 and Definition 12, IFDWA operator considered weights of IFV, again IFDOWA operator weights implies the ordered position of IFV instead of weights of IFV themselves. Hence, weights in both the operators IFDWA and IFDOWA are follow in different aspects. But, they considered one time only. To overcome this difficulty, we introduce IFDHA operator.

Definition 13 *An IFDHA operator of dimension* b *is a function* $IFDHA$: $IFE^b \to IFE$, *with associated weight vector* $\omega = (\omega_1, \omega_2, \ldots, \omega_b)$ *where* $\omega_q > 0$, *and* $\sum_{q=1}^{b} \omega_q = 1$. *Therefore, IFDHWA operator can be computed as*

$$IFDHWA_\omega\left(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_b\right) = \bigoplus_{q=1}^{b} \left(\omega_q \dot{\widetilde{I}}_{\sigma(q)}\right)$$

$$= \left\langle 1 - \frac{1}{1 + \left\{\sum\limits_{q=1}^{b} \omega_q \left(\frac{\dot{\mu}_{\sigma(q)}}{1-\dot{\mu}_{\sigma(q)}}\right)^{\kappa}\right\}^{1/\kappa}}, \frac{1}{1 + \left\{\sum\limits_{q=1}^{b} \omega_q \left(\frac{1-\dot{\nu}_{\sigma(q)}}{\dot{\nu}_{\sigma(q)}}\right)^{\kappa}\right\}^{1/\kappa}} \right\rangle, \quad (21)$$

where $\dot{\widetilde{I}}_{\sigma(q)}$ *is the* q^{th} *largest weighted intuitionistic fuzzy values* $\dot{\widetilde{I}}_q$ $(\dot{\widetilde{I}}_q = b\omega_q \widetilde{I}_q), q = 1, 2, \ldots, b)$, *and* $\omega = (\omega_1, \omega_2, \ldots, \omega_b)^T$ *be the weight vector of* $\dot{\widetilde{I}}_t$ *with* $\omega_q > 0$ *and* $\sum_{q=1}^{b} \omega_q = 1$, *where* b *is the balancing coefficient. When* $w = (1/b, 1/b, \ldots, 1/b)$, *then IFDWA operator is marked as particular case of IFDHA operator. Let* $\omega = (1/b, 1/b, \ldots, 1/b)$, *then IFDOWA is a marked case of the operator IFDHA. Thus, IFDHA operator is a generalization of IFDWA and IFDOWA operators, which represents the degrees of the given arguments and their ordered positions.*

Example. There are four IFEs $I_1 = (0.5, 0.3)$, $I_2 = (0.6, 0.3)$, $I_3 = (0.7, 0.3)$ and $I_4 = (0.2, 0.4)$, $W = (0.20, 0.30, 30, 0.20)^T$ weight vector of these four IFEs and $\omega = (0.2, 0.1, 0.3, 0.4)^T$ is the associated weight vector. Then, by Definition 13 for aggregated of IFEs for $\kappa = 3$, by the way

$$\dot{\widetilde{I}}_1 = \left\langle\left(1 - \frac{1}{1 + \left\{4 \times 0.20 \times \left(\frac{0.5}{1-0.5}\right)^3\right\}^{1/3}}, \frac{1}{1 + \left\{4 \times 0.20 \times \left(\frac{1-0.3}{0.3}\right)^3\right\}^{1/3}}\right)\right\rangle$$

$$= \left\langle 0.4814, 0.3158 \right\rangle,$$

$$\dot{\tilde{I}}_2 = \left\langle \left(1 - \frac{1}{1+\left\{4\times 0.30\times\left(\frac{0.6}{1-0.6}\right)^3\right\}^{1/3}}, \frac{1}{1+\left\{4\times 0.30\times\left(\frac{1-0.3}{0.3}\right)^3\right\}^{1/3}} \right) \right\rangle$$
$$= \left\langle 0.6145, 0.2874 \right\rangle,$$

$$\dot{\tilde{I}}_3 = \left\langle \left(1 - \frac{1}{1+\left\{4\times 0.30\times\left(\frac{0.7}{1-0.7}\right)^3\right\}^{1/3}}, \frac{1}{1+\left\{4\times 0.30\times\left(\frac{1-0.3}{0.3}\right)^3\right\}^{1/3}} \right) \right\rangle$$
$$= \left\langle 0.7126, 0.2874 \right\rangle,$$

$$\dot{\tilde{I}}_4 = \left\langle \left(1 - \frac{1}{1+\left\{4\times 0.20\times\left(\frac{0.2}{1-0.2}\right)^3\right\}^{1/3}}, \frac{1}{1+\left\{4\times 0.20\times\left(\frac{1-0.4}{0.4}\right)^3\right\}^{1/3}} \right) \right\rangle$$
$$= \left\langle 0.1884, 0.4180 \right\rangle.$$

Scores of I_t (t=1,2,3,4) calculated as follows:

$$E\left(\dot{\tilde{I}}_1\right) = \frac{1+0.4814-0.3158}{2} = 0.5828,$$
$$E\left(\dot{\tilde{I}}_2\right) = \frac{1+0.6145-0.2874}{2} = 0.6636,$$
$$E\left(\dot{\tilde{I}}_3\right) = \frac{1+0.7126-0.2874}{2} = 0.7126,$$
$$E\left(\dot{\tilde{I}}_2\right) = \frac{1+0.1884-0.4180}{2} = 0.3852.$$

Since,

$$E\left(\dot{\tilde{I}}_3\right) > E\left(\dot{\tilde{I}}_2\right) > E\left(\dot{\tilde{I}}_1\right) > E\left(\dot{\tilde{I}}_4\right).$$

Then,

$$\dot{\tilde{I}}_{\sigma(1)} = \dot{\tilde{I}}_3 = (0.7126, 0.2874), \quad \dot{\tilde{I}}_{\sigma(2)} = \dot{\tilde{I}}_2 = (0.6145, 0.2874),$$

$$\dot{\tilde{I}}_{\sigma(3)} = \dot{\tilde{I}}_1 = (0.4814, 0.3158) \quad \text{and} \quad \dot{\tilde{I}}_{\sigma(4)} = \dot{\tilde{I}}_4 = (0.1884, 0.4180).$$

Therefore, aggregated values of IFEs, for $\kappa = 3$ by IFDHWA operator:

$$IFDHWA_{\omega}\left(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_4\right) = \bigoplus_{q=1}^{4}\left(\omega_q \dot{\widetilde{I}}_{\sigma(q)}\right)$$

$$= \left\langle 1 - \frac{1}{1 + \left\{\sum\limits_{q=1}^{4} \omega_q \left(\frac{\dot{\mu}_{\sigma(q)}}{1-\dot{\mu}_{\sigma(q)}}\right)^3\right\}^{1/3}}, \frac{1}{1 + \left\{\sum\limits_{q=1}^{4} \omega_q \left(\frac{1-\dot{\nu}_{\sigma(q)}}{\dot{\nu}_{\sigma(q)}}\right)^3\right\}^{1/3}} \right\rangle$$

$$= \left\langle 1 - \frac{1}{1+\left\{0.2\left(\frac{0.7126}{1-0.7126}\right)^3+0.1\left(\frac{0.6145}{1-0.6145}\right)^3+0.3\left(\frac{0.4814}{1-0.4814}\right)^3+0.4\left(\frac{0.1884}{1-0.1884}\right)^3\right\}^{1/3}}\right.$$

$$\left.\frac{1}{1+\left\{0.2\left(\frac{1-0.2874}{0.2874}\right)^3+0.1\left(\frac{1-0.2874}{0.2874}\right)^3+0.3\left(\frac{1-0.3158}{0.3158}\right)^3+0.4\left(\frac{1-0.4180}{0.4180}\right)^3\right\}^{1/3}}\right\rangle$$

$$= \left\langle (0.6073, 0.3271) \right\rangle. \tag{22}$$

4. Intuitionistic fuzzy Dombi geometric aggregation operators

In this section, we introduce intuitionistic fuzzy Dombi weighted geometic (IFDWG) operator, order weighted geometric (IFDOWG) operator and hybrid weighted geometric (IFDHWG) operator and their corresponding properties.

Definition 14 *Let $\widetilde{I}_q = (\mu_q, \nu_q)$ $(q = 1, 2, \ldots, b)$ be a set of IFEs. Then IFDWG operator is a function $IFE^b \rightarrow IFE$ such that*

$$IFDWG_{\omega}\left(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_b\right) = \bigotimes_{q=1}^{b}\left(\widetilde{I}_q\right)^{\omega_q}, \tag{23}$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_b)^T$ be the weight vector of $\widetilde{I}_q$ $(q = 1, 2, \ldots, b)$, such that $\omega_q > 0$ and $\sum\limits_{q=1}^{b} \omega_q = 1$.

Theorem 11 *$\widetilde{I}_q = (\mu_q, \nu_q)$ $(q = 1, 2, \ldots, b)$ be a set of IFEs, then aggregated value of $\widetilde{I}_q$ using the IFDWG operator is also a IFE, and*

$$IFDWG_{\omega}\left(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_b\right) = \bigotimes_{q=1}^{n} \left(\widetilde{I}_q\right)^{\omega_q}$$

$$= \left\langle \frac{1}{1 + \left\{\sum\limits_{q=1}^{b} \omega_q \left(\frac{1-\mu_q}{\mu_q}\right)^{\kappa}\right\}^{1/\kappa}}, 1 - \frac{1}{1 + \left\{\sum\limits_{q=1}^{b} \omega_q \left(\frac{\nu_q}{1-\nu_q}\right)^{\kappa}\right\}^{1/\kappa}} \right\rangle, \tag{24}$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_b)$ *be the weight vector of* $\widetilde{I}_q$ $(q = 1, 2, \ldots, b)$ *such that* $\omega_q > 0$*, and* $\sum\limits_{q=1}^{b} \omega_q = 1$.

Proof. Proof of the theorem follows from Theorem 2. □

Example. Let us considered four IFEs, $I_1 = (0.6, 0.3)$, $I_2 = (0.5, 0.4)$, $I_3 = (0.7, 0.2)$, $I_4 = (0.2, 0.3)$, and weight vector $\omega = (0.2, 0.1, 0.3, 0.4)$. Then for $\kappa = 3$, I_q $(q = 1, 2, 3, 4)$ by

$$IFDWG_{\omega}\left(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_4\right) = \bigoplus_{q=1}^{4} \left(\widetilde{I}_q\right)^{\omega_q}$$

$$= \left\langle 1 - \frac{1}{1 + \left\{\sum\limits_{q=1}^{4} \omega_q \left(\frac{1-\mu_q}{\mu_q}\right)^{\kappa}\right\}^{1/\kappa}}, 1 - \frac{1}{1 + \left\{\sum\limits_{q=1}^{4} \omega_q \left(\frac{\nu_q}{1-\nu_q}\right)^{\kappa}\right\}^{1/\kappa}} \right\rangle$$

$$= \left\langle \frac{1}{1 + \left\{0.2\left(\frac{1-0.6}{0.6}\right)^3 + 0.1\left(\frac{1-0.5}{0.5}\right)^3 + 0.3\left(\frac{1-0.7}{0.7}\right)^3 + 0.4\left(\frac{1-0.2}{0.2}\right)^3\right\}^{1/3}}, \right.$$

$$\left. 1 - \frac{1}{1 + \left\{0.2\left(\frac{0.3}{1-0.3}\right)^3 + 0.1\left(\frac{0.4}{1-0.4}\right)^3 + 0.3\left(\frac{0.2}{1-0.2}\right)^3 + 0.4\left(\frac{0.3}{1-0.3}\right)^3\right\}^{1/3}} \right\rangle$$

$$= \left\langle (0.2529, 0.3025) \right\rangle.$$

Following are the basic properties for IFDWG operator.

Theorem 12 *(Idempotency) If* $\widetilde{I}_q = (\mu_q, \nu_q)$ *for all* $q = 1, 2, \ldots, b$ *equal, i.e., then* $IFDWG_{\omega}\left(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_b\right) = \widetilde{I}$.

Theorem 13 *(Boundedness) Let* $\widetilde{I}_q = (\mu_q, \nu_q)$ $(q = 1, 2, \ldots, b)$ *be a set of IFEs. Let* $\widetilde{I}^- = \min_q \widetilde{I}_q$, $\widetilde{I}^+ = \max_q \widetilde{I}_q$. *Then*

$$\widetilde{I}^- \leqslant IFDWG_\omega \left(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_b\right) \leqslant \widetilde{I}^+. \tag{25}$$

Theorem 14 *(Monotonicity) Let* $\widetilde{I}_q = (\mu_q, \nu_q)$ $(q = 1, 2, \ldots, b)$ *and* $\widetilde{I}'_q = ({\mu'}_q, {\nu'}_q)$ $(q = 1, 2, \ldots, b)$ *be two sets of IFEs, if* $\widetilde{I}_q \leqslant \widetilde{I}'_q$ *for all q, then*

$$IFDWG_\omega \left(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_b\right) \leqslant IFDWG_\omega \left(\widetilde{I}'_1, \widetilde{I}'_2, \ldots, \widetilde{I}'_b\right). \tag{26}$$

Now, we introduce IFDOWG operator.

Definition 15 *Let* $\widetilde{I}_q = (\mu_q, \nu_q)$ $(q = 1, 2, \ldots, b)$ *be a set of IFEs. IFDOWG operator of dimension b is a function IFDOWG*: $IFE^b \rightarrow IFE$ *with associated vector* $\omega = (\omega_1, \omega_2, \ldots, \omega_b)^T$ *such that* $\omega_q > 0$, *and* $\sum_{q=1}^{b} \omega_q = 1$. *Therefore,*

$$IFDOWG_\omega \left(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_b\right) = \bigotimes_{q=1}^{b} \left(\widetilde{I}_{\sigma(q)}\right)^{\omega_q} \tag{27}$$

where $(\sigma(1), \sigma(2), \ldots, \sigma(b))$ *is a permutation of* $(q = 1, 2, \ldots, b)$, *for* $\widetilde{I}_{\sigma(q-1)} \geqslant \widetilde{I}_{\sigma(q)}$, *for* $q = 1, 2, \ldots, b$.

The above definition of IFDOWG operator follows next theorem.

Theorem 15 *Let* $\widetilde{I}_q = (\mu_q, \nu_q)$ $(q = 1, 2, \ldots, b)$ *be a set of IFEs. IFDOWG operator of dimension b is a function* $IFDOWG : IFE^b \rightarrow IFE$. *Furthermore,*

$$IFDOWG_\omega \left(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_b\right) = \bigotimes_{q=1}^{b} \left(\widetilde{I}_q\right)^{\omega_q}$$

$$= \left\langle \frac{1}{1 + \left\{\sum\limits_{q=1}^{b} \omega_q \left(\frac{1-\mu_{\sigma(q)}}{\mu_{\sigma(q)}}\right)^\kappa\right\}^{1/\kappa}}, 1 - \frac{1}{1 + \left\{\sum\limits_{q=1}^{b} \omega_q \left(\frac{\nu_{\sigma(q)}}{1-\nu_{\sigma(q)}}\right)^\kappa\right\}^{1/\kappa}} \right\rangle, \tag{28}$$

where $(\sigma(1), \sigma(2), \ldots, \sigma(b))$ *are the permutation of* $(q = 1, 2, \ldots, b)$, *for* $\widetilde{I}_{\sigma(q-1)} \geqslant \widetilde{I}_{\sigma(q)}$ *for all* $(q = 1, 2, \ldots, b)$, *with associated weight vector* $\omega = (\omega_1, \omega_2, \ldots, \omega_b)^T$ *such that* $\omega_q > 0$, *and* $\sum_{q=1}^{b} \omega_q = 1$.

Example. Let $\widetilde{I}_1 = (0.5, 0.1)$, $\widetilde{I}_2 = (0.6, 0.3)$, $\widetilde{I}_3 = (0.7, 0.1)$ and $\widetilde{I}_4 = (0.4, 0.2)$ be four IFEs with $\omega = (0.2, 0.1, 0.3, 0.4)^T$ is the weight vector of these IFEs. Then aggregated of IFEs is for $\kappa = 3$ and by Definition 15, scores of $\widetilde{I}_q$ $(q = 1, 2, 3, 4)$ computed as:

$$E\left(\widetilde{I}_1\right) = \frac{1 + 0.5 - 0.1}{2} = 0.70, \qquad E\left(\widetilde{I}_2\right) = \frac{1 + 0.6 - 0.3}{2} = 0.65,$$
$$E\left(\widetilde{I}_3\right) = \frac{1 + 0.7 - 0.1}{2} = 0.80, \qquad E\left(\widetilde{I}_4\right) = \frac{1 + 0.4 - 0.2}{2} = 0.60.$$

Since,

$$E\left(\widetilde{I}_3\right) > E\left(\widetilde{I}_1\right) > E\left(\widetilde{I}_2\right) > E\left(\widetilde{I}_4\right),$$

then $\widetilde{I}_{\sigma(1)} = \widetilde{I}_3 = (0.7, 0.1)$, $\widetilde{I}_{\sigma(2)} = \widetilde{I}_1 = (0.5, 0.1)$, $\widetilde{I}_{\sigma(3)} = \widetilde{I}_2 = (0.6, 0.3)$ and $\widetilde{I}_{\sigma(4)} = \widetilde{I}_4 = (0.4, 0.2)$. Then, by IFDOWG operator:

$$IFDOWG_\omega\left(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_4\right) = \bigoplus_{q=1}^{4}\left(\omega_q \widetilde{I}_{\sigma(q)}\right)$$
$$= \left\langle \frac{1}{1 + \left\{\sum\limits_{q=1}^{4} \omega_q \left(\frac{1-\mu_{\sigma(q)}}{\mu_{\sigma(q)}}\right)^3\right\}^{1/3}}, 1 - \frac{1}{1 + \left\{\sum\limits_{t=1}^{4} \omega_q \left(\frac{\nu_{\sigma(q)}}{1-\nu_{\sigma(q)}}\right)^3\right\}^{1/3}} \right\rangle$$
$$= \left\langle \frac{1}{1 + \left\{0.2\left(\frac{1-0.7}{0.7}\right)^3 + 0.1\left(\frac{1-0.5}{0.5}\right)^3 + 0.3\left(\frac{1-0.6}{0.6}\right)^3 + 0.4\left(\frac{1-0.4}{0.4}\right)^3\right\}^{1/3}}, \right.$$
$$\left. 1 - \frac{1}{1 + \left\{0.2\left(\frac{0.1}{1-0.1}\right)^3 + 0.1\left(\frac{0.1}{1-0.1}\right)^3 + 0.3\left(\frac{0.3}{1-0.3}\right)^3 + 0.4\left(\frac{0.2}{1-0.2}\right)^3\right\}^{1/3}} \right\rangle$$
$$= \left\langle (0.4633, 0.2376) \right\rangle.$$

The following properties can be easily proved for IFDOWG operator.

Theorem 16 *(Idempotency) If $\widetilde{I}_q = (\mu_q, \nu_q)$ $(q = 1, 2, \ldots, b)$ are equal IFEs, i.e., then $IFDOWG_\omega\left(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_b\right) = \widetilde{I}$, where, $\widetilde{I}_q = \widetilde{I}$.*

Theorem 17 *(Boundedness) Let $\widetilde{I}_q = (\mu_q, \nu_q)$ $(q = 1, 2, \ldots, b)$ be a set of IFEs. Let $\widetilde{I}^- = \min\limits_q \widetilde{I}_q$, and $\widetilde{I}^+ = \max\limits_q \widetilde{I}_q$. Then, $\widetilde{I}^- \leqslant IFDOWG_\omega\left(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_b\right) \leqslant \widetilde{I}^+$.*

Theorem 18 *(Monotonicity) Let* $\widetilde{I}_q = (\mu_q, \nu_q)$ $(q = 1, 2, \ldots, b)$ *and* $\widetilde{I}'_q = \left(\hat{\mu}'_t, \hat{\nu}'_q\right)$ $(q = 1, 2, \ldots, b)$ *be two sets of IFEs, if* $\widetilde{I}_q \leqslant \widetilde{I}'_q$ *for all q, then*

$$IFDOWG_\omega\left(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_b\right) \leqslant IFDOWG_\omega\left(\widetilde{I}'_1, \widetilde{I}'_2, \ldots, \widetilde{I}'_b\right). \tag{29}$$

Theorem 19 *(Commutativity) Let* $\widetilde{I}_q = (\mu_q, \nu_q)$ $(q = 1, 2, \ldots, b)$ *and* $\widetilde{I}'_q = \left(\hat{\mu}'_q, \hat{\nu}'_q\right)$ $(q = 1, 2, \ldots, b)$ *be two sets of IFEs, then*

$$IFDOWG_\omega\left(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_b\right) = IFDOWG_\omega\left(\widetilde{I}'_1, \widetilde{I}'_2, \ldots, \widetilde{I}'_b\right), \tag{30}$$

where $\widetilde{I}'_q$ $(q = 1, 2, \ldots, b)$ *is any permutation of* $\widetilde{I}_q$ $(q = 1, 2, \ldots, b)$.

In Definition 14 and Definition 15, the IFDWG operator considered the weights of only IFVs, on the other end the IFDOWG operator considered weights of only the ordered position of IFVs instead of weights of IFVs. Therefore, weights in both cases are in different aspects. But, they are considered only one of them. To overcome this drawback, we introduce intuitionistic fuzzy Dombi hybrid geometric (IFDHWG) operator.

Definition 16 *An IFDHG operator of dimension b is a function IFDHG:* $IFE^b \to IFE$, *with associated weight* $\omega = (\omega_1, \omega_2, \ldots, \omega_b)$ *such that* $\omega_q > 0$, *and* $\sum_{q=1}^{b} \omega_q = 1$. *Therefore, IFDHWG operator can be evaluated as*

$$IFDHWG_\omega\left(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_b\right) = \bigotimes_{q=1}^{b}\left(\dot{\widetilde{I}}_{\sigma(q)}\right)^{\omega_q}$$

$$= \left\langle \frac{1}{1 + \left\{\sum\limits_{q=1}^{b} \omega_q \left(\frac{1-\dot{\mu}_{\sigma(q)}}{\dot{\mu}_{\sigma(q)}}\right)^{\kappa}\right\}^{1/\kappa}},\ 1 - \frac{1}{1 + \left\{\sum\limits_{q=1}^{b} \omega_q \left(\frac{\dot{\nu}_{\sigma(q)}}{1-\dot{\nu}_{\sigma(q)}}\right)^{\kappa}\right\}^{1/\kappa}} \right\rangle, \tag{31}$$

where $\dot{\widetilde{I}}_{\sigma(q)}$ *is the* q^{th} *largest weighted IFVs* $\dot{\widetilde{I}}_t$ $(\dot{\widetilde{I}}_q = b\omega_q I_q, (q = 1, 2, \ldots, b)$, *and* $\omega = (\omega_1, \omega_2, \ldots, \omega_b)^T$ *be the weight vector of* $\dot{\widetilde{I}}_q$ *with* $\omega_q > 0$ *and* $\sum_{q=1}^{b} \omega_q = 1$, *where b is the balancing coefficient. When* $w = (1/b, 1/b, \ldots, 1/b)$, *then IFDWG is marked as particular case of IFDHG operator. Let* $\omega = (1/b, 1/b, \ldots, 1/b)$, *then IFDOWG is specify as particular case of IFDHG. Thus, IFDHG operator is a generalization of both IFDWG and IFDOWG operators, which flash back the degrees of the given arguments and their ordered positions.*

Example. There are four IFEs

$$I_1 = (0.5, 0.3), \quad I_2 = (0.6, 0.3), \quad I_3 = (0.7, 0.3), \quad I_4 = (0.2, 0.4),$$

and $W = (0.20, 0.30, 30, 0.20)^T$ weight vector of these four IFEs and $\omega = (0.2, 0.1, 0.3, 0.4)^T$ is the associated weight vector. Then, by Definition 16 for aggregated of IFEs for ($\kappa = 3$), by the way

$$\dot{\tilde{I}}_1 = \left\langle \left(\frac{1}{1 + \left\{4 \times 0.20 \times \left(\frac{1-0.5}{0.5}\right)^3\right\}^{1/3}}, \ 1 - \frac{1}{1 + \left\{4 \times 0.20 \times \left(\frac{0.3}{1-0.3}\right)^3\right\}^{1/3}} \right) \right\rangle$$
$$= \left\langle (0.5186, 0.2846) \right\rangle,$$

$$\dot{\tilde{I}}_2 = \left\langle \left(\frac{1}{1 + \left\{4 \times 0.30 \times \left(\frac{1-0.6}{0.6}\right)^3\right\}^{1/3}}, \ 1 - \frac{1}{1 + \left\{4 \times 0.30 \times \left(\frac{0.3}{1-0.3}\right)^3\right\}^{1/3}} \right) \right\rangle$$
$$= \left\langle (0.5853, 0.3129) \right\rangle,$$

$$\dot{\tilde{I}}_3 = \left\langle \left(\frac{1}{1 + \left\{4 \times 0.30 \times \left(\frac{1-0.7}{0.7}\right)^3\right\}^{1/3}}, \ 1 - \frac{1}{1 + \left\{4 \times 0.30 \times \left(\frac{0.3}{1-0.3}\right)^3\right\}^{1/3}} \right) \right\rangle$$
$$= \left\langle (0.6871, 0.3129) \right\rangle,$$

$$\dot{\tilde{I}}_4 = \left\langle \left(\frac{1}{1 + \left\{4 \times 0.20 \times \left(\frac{1-0.2}{0.2}\right)^3\right\}^{1/3}}, \ 1 - \frac{1}{1 + \left\{4 \times 0.20 \times \left(\frac{0.4}{1-0.4}\right)^3\right\}^{1/3}} \right) \right\rangle$$
$$= \left\langle (0.2122, 0.3823) \right\rangle.$$

Scores of I_t ($t = 1, 2, 3, 4$) calculated as follows:

$$E\left(\dot{\tilde{I}}_1\right) = \frac{1 + 0.5186 - 0.2846}{2} = 0.6170,$$
$$E\left(\dot{\tilde{I}}_2\right) = \frac{1 + 0.5853 - 0.3129}{2} = 0.6362,$$
$$E\left(\dot{\tilde{I}}_3\right) = \frac{1 + 0.6871 - 0.3129}{2} = 0.6871,$$
$$E\left(\dot{\tilde{I}}_4\right) = \frac{1 + 0.2122 - 0.3823}{2} = 0.4150.$$

Since,

$$E\left(\dot{\widetilde{I}}_3\right) > E\left(\dot{\widetilde{I}}_2\right) > E\left(\dot{\widetilde{I}}_1\right) > E\left(\dot{\widetilde{I}}_4\right).$$

Then, $\dot{\widetilde{I}}_{\sigma(1)} = \dot{\widetilde{I}}_3 = (0.6871, 0.3129)$, $\dot{\widetilde{I}}_{\sigma(2)} = \dot{\widetilde{I}}_2 = (0.5853, 0.3129)$, $\dot{\widetilde{I}}_{\sigma(3)} = \dot{\widetilde{I}}_1 = (0.5186, 0.2846)$ and $\dot{\widetilde{I}}_{\sigma(4)} = \dot{\widetilde{I}}_4 = (0.2122, 0.3823)$. Therefore, aggregated values of IFEs ($\kappa = 3$) by IFDHWG operator:

$$IFDHWG_{\omega}\left(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_4\right) = \bigoplus_{q=1}^{4}\left(\omega_q \dot{\widetilde{I}}_{\sigma(q)}\right)$$

$$= \left\langle \frac{1}{1 + \left\{\sum\limits_{q=1}^{4} \omega_q \left(\frac{1-\dot{\mu}_{\sigma(q)}}{\dot{\mu}_{\sigma(q)}}\right)^3\right\}^{1/3}},\ 1 - \frac{1}{1 + \left\{\sum\limits_{q=1}^{4} \omega_q \left(\frac{\dot{\nu}_{\sigma(q)}}{1-\dot{\nu}_{\sigma(q)}}\right)^3\right\}^{1/3}} \right\rangle$$

$$= \left\langle 1 - \frac{1}{1+\left\{0.2\left(\frac{1-0.6871}{0.6871}\right)^3+0.1\left(\frac{1-0.5853}{0.5853}\right)^3+0.3\left(\frac{1-0.5186}{0.5186}\right)^3+0.4\left(\frac{1-0.2122}{0.2122}\right)^3\right\}^{1/3}}, \right.$$

$$\left. \frac{1}{1+\left\{0.2\left(\frac{0.3129}{1-0.3129}\right)^3+0.1\left(\frac{0.3129}{1-0.3129}\right)^3+0.3\left(\frac{0.2846}{1-0.2846}\right)^3+0.4\left(\frac{0.3823}{1-0.3823}\right)^3\right\}^{1/3}} \right\rangle$$

$$= \left\langle (0.3582, 0.3429) \right\rangle.$$

5. Model for MADM using IFS information

In this section, we have develop MADM method using intuitionistic fuzzy weighted aggregation operators in which attribute weights are in real numbers and attribute values are in IFEs. Let $A = \{A_1, A_2, \ldots, A_a\}$ be a set of alternatives, $G = \{G_1, G_2, \ldots, G_b\}$ be a set of attributes. Let $\omega = (\omega_1, \omega_2, \ldots, \omega_b)$ be the weight vector of the attributes G_q $(q = 1, 2, \ldots, b)$ be known for decision-makers where $\omega_q > 0$ and $\sum\limits_{q=1}^{b} \omega_q = 1$. Suppose $D = (\mu_{pq}, \nu_{pq})_{a\times b}$ is the decision matrix, where μ_{ab} is denotes degree of membership for the alternative A_q satisfying the attribute G_q proposed by decision makers, and ν_{pq} says the degree that the alternative A_p does not agree with the attribute G_q by the decision maker, where $\mu_{pq} \in [0, 1]$, and $\nu_{pq} \in [0, 1]$ where $0 \leqslant \mu_{pq} + \nu_{pq} \leqslant 1$, $(p = 1, 2, \ldots, a)$ and $(q = 1, 2, \ldots, b)$.

We propose following algorithm to solve MADM problem using IFS arguments by IFDWA and IFDWG operators.

Step 1. Proposed information given in matrix D, and implemented IFDWA operator to compute the overall values of β_p $(p = 1, 2, \ldots, a)$ of the alternative A_p.

$$\beta_p = IFDWA(\beta_{p1}, \beta_{p2}, \ldots, \beta_{pb}) = \bigoplus_{q=1}^{b}(\omega_q \beta_{pq})$$
$$= \left\langle 1 - \frac{1}{1 + \left\{\sum\limits_{q=1}^{b} \omega_q \left(\frac{\mu_q}{1-\mu_q}\right)^{\kappa}\right\}^{1/\kappa}}, \frac{1}{1 + \left\{\sum\limits_{q=1}^{b} \omega_q \left(\frac{1-\nu_q}{\nu_q}\right)^{\kappa}\right\}^{1/\kappa}} \right\rangle \tag{32}$$

or

$$\beta_p = IFDWG(\beta_{p1}, \beta_{p2}, \ldots, \beta_{pb}) = \bigoplus_{q=1}^{b}(\beta_{pq})^{\omega_q}$$
$$= \left\langle \frac{1}{1 + \left\{\sum\limits_{q=1}^{b} \omega_q \left(\frac{1-\mu_q}{\mu_q}\right)^{\kappa}\right\}^{1/\kappa}}, 1 - \frac{1}{1 + \left\{\sum\limits_{q=1}^{b} \omega_q \left(\frac{\nu_q}{1-\nu_q}\right)^{\kappa}\right\}^{1/\kappa}} \right\rangle. \tag{33}$$

Step 2. Evaluate the score value of $E(\beta_p)$ $(p = 1, 2, \ldots, a)$ applying on overall IFVs β_p $(p = 1, 2, \ldots, a)$ to ranking A_p $(p = 1, 2, \ldots, a)$ for the selection of best A_p. If there is no difference between $E(\beta_p)$ and $E(\widetilde{\beta}_q)$, then proceed to calculate accuracy value $L(\beta_p)$ and $L(\beta_q)$ based on overall intuitionistic fuzzy information of β_p and β_q, and ranked the alternatives A_p depending on accuracy degrees of $L(\beta_p)$ and $L(\beta_q)$.

Step 3. Rank all A_p $(p = 1, 2, \ldots, a)$ to chose the desirable one(s) in accordance with $E(\beta_p)$ $(p = 1, 2, \ldots, a)$.

Step 4. Stop.

6. Numerical example and its comparative study

In the following, a multi-criteria decision-making method has been executed with a practical example concerning investment selection to fitness of the proposed MCDM problems. An investor wants to invest money in a mutual fund company. Before investment an investor seeing advise of an expert team. After analyzing the market by the five experts will give their judgement based on the basis of performance of four mutual fund companies A_1 : Pharma fund, A_2 : Liquid fund, A_3 : Blue chip fund, A_4 : Hybrid fund and A_5 : Tax sever fund. There is a expert team which select best mutual fund company among the five companies A_p $(p = 1, 2, \ldots, 5)$. They choose four attributes to assess five possible funds as follows:

G_1: Short term;

G_2: Mid term;

G_3: Long term;

G_4: Risk of the fund.

They have no dominance power to each other, decision-makers (DM) are required to exempted five possible mutual funds A_p $(p = 1, 2, \ldots, 5)$ under the mentioning attributes whose weights $(0.2, 0.1, 0.3, 0.4)$ addressed by DM, the decision matrix $D = (\beta_{pq})_{5\times4}$ which is provided in Table 1, where β_{pq} are in the form of IFEs.

Table 1: Intuitionistic fuzzy decision numbers

	A_1	A_2	A_3	A_4	A_5
G_1	(0.7,0.3)	(0.6,0.3)	(0.4,0.2)	(0.8,0.2)	(0.6,0.4)
G_2	(0.2,0.7)	(0.4,0.6)	(0.5,0.3)	(0.4,0.5)	(0.5,0.5)
G_3	(0.4,0.3)	(0.5,0.2)	(0.3,0.4)	(0.2,0.4)	(0.4,0.2)
G_4	(0.8,0.2)	(0.4,0.6)	(0.6,0.2)	(0.5,0.3)	(0.3,0.5)

In order to chose favourable mutual funds A_q $(q = 1, 2, \ldots, b)$, utilizing IFDWA and IFDWG operators to model MADM which can be computed as follows:

- **Step 1.** Let $\kappa = 1$, using the IFDWA operator overall preferences values β_p of A_p $(p = 1, 2, \ldots, 5)$ are
$\beta_1 = (0.6962, 0.2625)$, $\beta_2 = (0.4828, 0.3333)$, $\beta_3 = (0.4903, 0.2449)$, $\beta_4 = (0.5730, 0.3046)$, $\beta_5 = (0.4355, 0.3333)$.

- **Step 2.** Score values of $E(\beta_p)$ $(p = 1, 2, \ldots, 5)$ are as follows:
$E(\beta_1) = 0.7169$, $E(\beta_2) = 0.5748$, $E(\beta_3) = 0.6227$,
$E(\beta_4) = 0.6342$, $E(\beta_5) = 0.5511$.

- **Step 3.** Rank all mutual funds A_p $(p = 1, 2, \ldots, 5)$ in accordance with the score values of $E(\beta_p)$ $(p = 1, 2, \ldots, 5)$ of the overall IFEs as $A_1 \succ A_4 \succ A_3 \succ A_2 \succ A_5$.

- **Step 4.** A_1 is selected as most favourable mutual fund.

IFDWG operator if used instead, in similar manner problem is solved as.

- **Step 1.** Let $\kappa = 1$, using IFDWGA operator to compute the overall values $\widetilde{\beta}_p$ of insecticide companies A_p $(p = 1, 2, \ldots, 5)$
$\beta_1 = (0.4912, 0.3538)$, $\beta_2 = (0.4580, 0.4766)$, $\beta_3 = (0.4225, 0.2821)$,
$\beta_4 = (0.3571, 0.3427)$, $\beta_5 = (0.3822, 0.4146)$.

- **Step 2.** Compute score values $E(\beta_p)$ $(p = 1, 2, \ldots, 5)$ of the overall IFEs β_p $(p = 1, 2, \ldots, 5)$ as:
$E(\beta_1) = 0.5687$, $E(\beta_2) = 0.4907$, $E(\beta_3) = 0.5702$,
$E(\beta_4) = 0.5072$, $E(\beta_5) = 0.4838$.

- **Step 3.** Ranking all the selected funds A_p $(p = 1, 2, \ldots, 5)$ in the value of score functions $E(\beta_p)$ $(p = 1, 2, \ldots, 5)$ of the overall IFEs as $A_3 \succ A_1 \succ A_4 \succ A_2 \succ A_5$.

- **Step 4.** Return A_3 is selected as the most attractive mutual fund.

From the above computation, it shows from ranking order of the alternatives that A_1 is the most desirable mutual fund when IFDWA operator used whereas A_3 is favourable mutual fund when IFDWG operator used.

In order to diagnose sensitivity of the working parameter $\kappa \in [1, 10]$ on the ranking order of the alternatives A using IFDWA and IFDWG operators which are given in Tables 2 and 3.

Table 2: Effect of parameters on ranking orders using IFDWA operator

κ	$E(\beta_1)$, $E(\beta_2)$, $E(\beta_3)$, $E(\beta_4)$, $E(\beta_5)$	Ranking order
1	0.7169, 0.5747, 0.6227, 0.6342, 0.5511	$A_1 \succ A_4 \succ A_3 \succ A_2 \succ A_5$
2	0.7429, 0.6043, 0.6419, 0.6848, 0.5855	$A_1 \succ A_4 \succ A_3 \succ A_2 \succ A_5$
3	0.7566, 0.6232, 0.6551, 0.7152, 0.6114	$A_1 \succ A_4 \succ A_3 \succ A_2 \succ A_5$
4	0.7652, 0.6363, 0.6642, 0.7338, 0.6296	$A_1 \succ A_4 \succ A_3 \succ A_2 \succ A_5$
5	0.7711, 0.6460, 0.6706, 0.7462, 0.6422	$A_1 \succ A_4 \succ A_3 \succ A_2 \succ A_5$
6	0.7754, 0.6534, 0.6751, 0.7549, 0.6513	$A_1 \succ A_4 \succ A_3 \succ A_2 \succ A_5$
7	0.7787, 0.6592, 0.6785, 0.7613, 0.6581	$A_1 \succ A_4 \succ A_3 \succ A_2 \succ A_5$
8	0.7813, 0.6639, 0.6811, 0.7666, 0.6632	$A_1 \succ A_4 \succ A_3 \succ A_2 \succ A_5$
9	0.7834, 0.6677, 0.6832, 0.7701, 0.6673	$A_1 \succ A_4 \succ A_3 \succ A_2 \succ A_5$
10	0.7850, 0.6708, 0.6848, 0.7731, 0.6706	$A_1 \succ A_4 \succ A_3 \succ A_2 \succ A_5$

Table 3: Effect of parameters on ranking order using IFDWG operator

κ	$E(\beta_1),\ E(\beta_2),\ E(\beta_3),\ E(\beta_4),\ E(\beta_5)$	Ranking order
1	0.5687, 0.4907, 0.5702, 0.5072, 0.4838	$A_3 \succ A_1 \succ A_4 \succ A_2 \succ A_5$
2	0.4735, 0.4639, 0.5456, 0.4695, 0.4632	$A_3 \succ A_1 \succ A_4 \succ A_2 \succ A_5$
3	0.4080, 0.4481, 0.5262, 0.4458, 0.4497	$A_3 \succ A_5 \succ A_2 \succ A_4 \succ A_1$
4	0.3684, 0.4383, 0.5120, 0.4299, 0.4406	$A_3 \succ A_5 \succ A_2 \succ A_4 \succ A_1$
5	0.3437, 0.4317, 0.5016, 0.4182, 0.4341	$A_3 \succ A_5 \succ A_2 \succ A_4 \succ A_1$
6	0.3272, 0.4269, 0.4940, 0.4093, 0.4292	$A_3 \succ A_5 \succ A_2 \succ A_4 \succ A_1$
7	0.3155, 0.4234, 0.4881, 0.4022, 0.4255	$A_3 \succ A_5 \succ A_2 \succ A_4 \succ A_1$
8	0.3069, 0.4206, 0.4836, 0.3966, 0.4226	$A_3 \succ A_5 \succ A_2 \succ A_4 \succ A_1$
9	0.3002, 0.4184, 0.4800, 0.3919, 0.4203	$A_3 \succ A_5 \succ A_2 \succ A_4 \succ A_1$
10	0.2950, 0.4167, 0.4771, 0.3880, 0.4183	$A_3 \succ A_5 \succ A_2 \succ A_4 \succ A_1$

6.1. Analysis on the effect of parameter κ on decision making results

Here, the operational behavior of working parameter κ on MADM results, using different values of κ to rank the alternatives. The results of score function and ranking order of the alternatives A_q $(q = 1, 2 \ldots, 5)$ in $1 \leqslant \kappa \leqslant 10$ based on IFDWA and IFDWG operators are addressed in Table 2 and Table 3.

From Table 2, it shows that when κ is changed for IFDWA operator, and the corresponding favourable alternative is remain same. Thus, when $0 \leqslant \kappa \leqslant 10$, then ranking order of the alternatives remain identical as $A_1 \succ A_4 \succ A_3 \succ A_2 \succ A_5$, the best one is A_1. In Table 3, when κ is changed for IFDWG operator, the ranking orders are varies, and the corresponding best alternative is not identical. It shows that, when $1 \leqslant \kappa \leqslant 2$, then ranking orders are $A_3 \succ A_1 \succ A_4 \succ A_2 \succ A_5$, and best choice is A_3. When $3 \leqslant \kappa \leqslant 10$, then ranking order is $A_3 \succ A_5 \succ A_2 \succ A_4 \succ A_1$, then A_3 is best the selection. In both the cases, the best choice is A_3 though order sequence are different.

To these MADM problems based on IFDWA and IFDWG operators, showing for different values of working parameters κ can change corresponding ranking orders of the alternatives for IFDWG operator, which is more responsive to κ in this MADM process; while for various values of parameter κ could not changed raking forms corresponding to IFDWAA operator, which is less effective by κ in this MADM model.

For compare the effectiveness of proposed model with the existing methods in [8, 9, 13, 16, 37, 38] used intuitionistic weighted aggregation operators and their justification with the existing operator given in Table 4. It is noticed that the existing models can described fuzzy information without any difficulty but it is not comfortably make aggregation process of the data flexible by a parameter depicted in Table 5. Whereas our proposed method easily described fuzzy information as well as information aggregation process make more flexible by a parameter.

Table 4: Comparative analysis with some of the existing methods

methods	$E(\beta_1),\ E(\beta_2),\ E(\beta_3),\ E(\beta_4),\ E(\beta_5)$	Ranking order
Xu [37]	0.3320, 0.1240, 0.1994, 0.1756, 0.0193	$A_1 > A_3 > A_4 > A_2 > A_5$
Xu [38]	0.2732, 0.0881, 0.1849, 0.0907, 0.0321	$A_1 > A_3 > A_4 > A_2 > A_5$
Huang [16]	0.6756, 0.5408, 0.6037, 0.5827, 0.5216	$A_1 > A_3 > A_4 > A_2 > A_5$
Garg [8]	0.6354, 0.4729, 0.5987, 0.4862, 0.4148	$A_1 > A_3 > A_4 > A_2 > A_5$
Garg [9]	0.6882, 0.5624, 0.5971, 0.6668, 0.6029	$A_1 > A_4 > A_5 > A_3 > A_2$
He et al. [13]	0.6784, 0.5508, 0.5944, 0.6637, 0.5971	$A_1 > A_4 > A_5 > A_3 > A_2$
Proposed	0.7169, 0.5747, 0.6227, 0.6342, 0.5511	$A_1 > A_4 > A_3 > A_2 > A_5$
Proposed	0.5687, 0.4907, 0.5702, 0.5072, 0.4838	$A_3 > A_1 > A_4 > A_2 > A_5$

Table 5: Characteristic comparisons with some of the existing methods

Methods	Whether describe fuzzy information easier	Whether make information aggregation more flexible by a parameter
Garg [8]	√	×
Garg [9]	√	×
He [13]	√	×
Hua [16]	√	×
Xu [37]	√	×
Xu [38]	√	×
Proposed method	√	√

Therefore, our proposed MADM method for IFDWAA and IFDWGA operators investigated the improvement of its resilience in real utilizations. Thus, the advanced aggregation operators implements a new flexible measure for decision makers to control intuitionistic fuzzy MADM problems.

7. Conclusions

In this paper, we have studied MADM problem using intuitionistic fuzzy information. We have introduced weighted averaging and weighted geometric aggregation operators with intuitionistic fuzzy arguments for the development of new aggregation operators using Dombi norms such as intuitionistic fuzzy Dombi weighted averaging, order weighted averaging, hybrid weighted averaging, weighted geometric, order weighted geometric and hybrid weighted geometric operators are introduced. A multi-criteria decision-making problem has been constructed based on these aggregation operators and a comparative study of the proposed model can be done with some existing models. We think this proposed model can be applied to develop economic model, business and management areas, intelligent diagnosis, three-way decision-making and other environments with uncertainties.

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