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An observer-based controller design for nonlinear discrete-time switched systems with time-delay and affine parametric uncertainty

N.A. BALEGHI and M.H. SHAFIEI

This paper proposes a design procedure for observer-based controllers of discrete-time switched systems, in the presence of state's time-delay, nonlinear terms, arbitrary switching signals, and affine parametric uncertainties. The proposed switched observer and the state-feedback controller are designed simultaneously using a set of linear matrix inequalities (LMIs). The stability analysis is performed based on an appropriate Lyapunov–Krasovskii functional with one switched expression, and in the meantime, the sufficient conditions for observer-based stabilization are developed. These conditions are formulated in the form of a feasibility test of a proposed bilinear matrix inequality (BMI) which is a non-convex problem. To make the problem easy to solve, the BMI is transformed into a set of LMIs using the singular value decomposition of output matrices. An important advantage of the proposed method is that the established sufficient conditions depend only on the upper bound of uncertain parameters. Furthermore, in the proposed method, an admissible upper bound for unknown nonlinear terms of the switched system may be calculated using a simple search algorithm. Finally, the efficiency of the proposed controller and the validity of the theoretical results are illustrated through a simulation example.

Key words: discrete-time switched systems, time-delay, affine parametric uncertainty, observer-based controller

1. Introduction

Analysis and synthesis of switched systems have been of interest in the literature of control theory [1–4]. An important category in this field is related to the control of discrete-time switched systems in the presence of time-delay [5–7]. The proposed controllers in the existing studies are in both forms of state-feedback [8, 9] and output-feedback controllers [10, 11].

N.A. Baleghi and M.H. Shafiei (corresponding Author – E-mail: shafiei@sutech.ac.ir) are with Department of Electrical and Electronics Engineering, Shiraz University of Technology, Modares Blvd., Shiraz, Iran.

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Although state-feedback controllers have considerable advantages with respect to output-feedback types, in practical situations, all the state variables of a system are not measurable to be used as a feedback signal. This problem is more serious in the case of switched systems with several subsystems. Therefore, the design of state observers for switched systems has attained great importance in recent years. The main challenge of classical observers is to overcome the effect of uncertain parameters of the system. To do that, several design procedures have been proposed for continuous-time and discrete-time switched systems [12–17]. For the discrete-time case, in [15] a non-fragile observer has been designed for a time-delay nonlinear switched system, based on the average dwell-time approach. Moreover, [16] has presented an observer-based controller of a discrete-time switched system with time-varying delay, based on the switched Lyapunov function method. Furthermore, in [17], the problem of observer-based L_2–L_∞ control has been studied for a class of discrete-time switched systems. However, in the aforementioned papers, the type of uncertainty is different from what is considered in this paper, i.e. affine parametric uncertainty.

Two types of uncertainty have been further investigated in the literature; norm-bounded and polytopic uncertainties. Another type of uncertainty, which is more compatible with the variation of system parameters, is affine parametric uncertainty [8, 10, 18–22]. Although there are many existing results on the switched systems with polytopic or norm-bounded uncertainties, the problem of controller design, especially observer-based control, have not been addressed for discrete-time switched delay systems with affine parametric uncertainties.

In this paper, an observer-based controller is designed for discrete-time switched systems under arbitrary switching signals, in the presence of state's time-delay, affine parametric uncertainties and nonlinear terms satisfying the Lipschitz condition. The proposed observer is a switched system, which depends only on the nominal values of the uncertain parameters. Besides, when the control loop is closed, this observer can reduce the undesirable effects of nonlinear terms in the dynamics of the switched system. Moreover, the observer-based controller is designed simultaneously through solving a set of linear matrix inequalities (LMIs) which is summarized as an algorithmic procedure. Another contribution of this paper is to derive sufficient conditions to ensure the robust stabilization of the switched system using the state variables of the observer. Then, based on the approach of switched Lyapunov functions, an appropriate Lyapunov–Krasovskii functional is constructed to guarantee the asymptotic stability of the closed-loop system. The proposed sufficient conditions are formulated as a feasibility test of a non-convex problem that is a bilinear matrix inequality (BMI). In this paper, using some mathematical manipulations, the BMI is transformed into its equivalent form (an LMI) using a singular value decomposition of output matrices. Consequently, the state feedback and observer gain matrices are achieved through the solution of the proposed LMIs. An important advantage of the proposed method is that the proposed conditions depend only on the upper bounds of uncertain parameters and the Lipschitz coefficients of nonlinear functions. Finally, the efficiency of the proposed observer-based controller is illustrated through a simulation example.

Notation: In this paper $\mathbb{R}^n$ represents the set of all n-dimensional real-valued vectors. The notation for a positive-definite (negative-definite) matrix A is standard as $A > 0$ $(A < 0)$. Moreover, the Euclidean norm of a vector is denoted by $\|\,.\,\|$ and I is an identity matrix of appropriate dimensions. Finally, the set of positive integers is denoted by $\mathbb{Z}$.

2. Preliminaries and problem formulation

Consider the following discrete-time nonlinear switched system with a state-delay:

$$
\begin{aligned}
S_{\sigma(k)}:\quad x(k+1) &= \left(A_{\sigma(k)} + \Delta A_{\sigma(k)}\right) x(k) + \left(B_{\sigma(k)} + \Delta B_{\sigma(k)}\right) x(k-\tau) \\
&\quad + \left(D_{\sigma(k)} + \Delta D_{\sigma(k)}\right) u(k) + f_{\sigma(k)}\left(k, x(k)\right), \\
y(k) &= C_{\sigma(k)} x(k), \\
x(l) &= \phi(l), \qquad l = -\tau,\ -\tau+1,\ \ldots, 0,
\end{aligned}
\tag{1}
$$

where $x(k) \in \mathbb{R}^{n_x}$, $u(k) \in \mathbb{R}^{n_u}$, and $y(k) \in \mathbb{R}^{n_y}$ are respectively, the state vector, control input, and the measured output. Moreover, the switching signal has been denoted by $\sigma(k)\colon \mathbb{Z} \to M = \{1, \ldots, m\}$, where m is the number of subsystems in the switched system ($\sigma(k) = i$, means activation of i-th subsystem at the instant k). Also, $\phi(l)$ is a vector-valued initial function and τ is the time delay of the system. It is assumed that the matrices A_i, B_i and D_i for $i = 1, \ldots, m$ are constant and known with appropriate dimensions and ΔA_i, ΔB_i, ΔD_i are uncertain matrices, which are defined as

$$
\begin{aligned}
\Delta A_i &= \sum_{s=1}^{r} \delta p_s E_s^i, \\
\Delta B_i &= \sum_{s=1}^{r} \delta p_s F_s^i, \\
\Delta D_i &= \sum_{s=1}^{r} \delta p_s G_s^i,
\end{aligned}
\tag{2}
$$

where r is the number of uncertain parameters and $\delta p_s \in [-e_s, e_s]$, for $s = 1, \ldots, r$ are the uncertain parameters. The matrices E_s^i, F_s^i and G_s^i are uncertainty structure matrices with known real parameters, to show the dependency of A_i, B_i and D_i on the uncertain parameter δp_s. If an uncertain parameter does not appear in the i-th subsystem, the corresponding E_s^i, F_s^i or G_s^i will be a zero matrix. It is assumed that $f_i(\,.,.\,)$, for $i = 1,\ldots,m$ are known nonlinear functions satisfying the following Lipschitz condition:

$$\|f_i\left(k, x_1(k)\right) - f_i\left(k, x_2(k)\right)\| \leqslant \gamma \left\|H_i\left(x_1(k) - x_2(k)\right)\right\|, \tag{3}$$

where H_i's are known weighting matrices and γ is a constant value.

The problem considered in this paper is to design an observer-based state-feedback controller such that the closed-loop system is stable. More specifically, the following observer-based controller is proposed to simultaneously estimate the state variables of the system (1) and to construct the control signal:

$$\begin{aligned} \hat{S}_{\sigma(k)}: \quad \hat{x}(k+1) &= A_{\sigma(k)}\hat{x}(k) + B_{\sigma(k)}\hat{x}(k-\tau) + D_{\sigma(k)}u(k) \\ &\quad + f_{\sigma(k)}\left(k, \hat{x}(k)\right) + L_{\sigma(k)}\left(y(k) - \hat{y}(k)\right), \\ \hat{y}(k) &= C_{\sigma(k)}\hat{x}(k), \\ u(k) &= K_{\sigma(k)}\hat{x}(k), \\ \hat{x}(l) &= 0, \qquad l = -\tau,\ -\tau{+}1,\ \ldots,\ 0, \end{aligned} \tag{4}$$

where $\hat{x}(k) \in \mathbb{R}^{n_x}$ is the estimated value of the state vector $x(k)$, $\hat{y}(k) \in \mathbb{R}^{n_y}$ is the observer output vector, $K_i \in \mathbb{R}^{n_u \times n_x}$ and $L_i \in \mathbb{R}^{n_x \times n_y}$ for $i \in M$, are the state-feedback and the observer gains, respectively, which will be evaluated later.

Define $e(k) = x(k) - \hat{x}(k)$ as the estimation error; then, the augmented switched system is as follows:

$$\begin{aligned} S_{e_{\sigma(k)}}: \quad \widetilde{x}(k+1) &= \widetilde{A}_{\sigma(k)}\widetilde{x}(k) + \widetilde{B}_{\sigma(k)}\widetilde{x}(k-\tau) + \widetilde{f}_{\sigma(k)}(k), \\ \widetilde{x}(l) &= \begin{bmatrix} \phi^T(l) & \phi^T(l) \end{bmatrix}^T, \qquad l = -\tau,\ -\tau{+}1,\ \ldots,\ 0, \end{aligned} \tag{5}$$

where $\widetilde{x}(k) = \begin{bmatrix} x^T(k) & e^T(k) \end{bmatrix}^T$ and

$$\widetilde{A}_{\sigma(k)} = \begin{bmatrix} \Xi_{11} & \Xi_{12} \\ \Xi_{21} & \Xi_{22} \end{bmatrix},$$

$$\widetilde{B}_{\sigma(k)} = \begin{bmatrix} \left(B_{\sigma(k)} + \Delta B_{\sigma(k)}\right) & 0 \\ \Delta B_{\sigma(k)} & B_{\sigma(k)} \end{bmatrix},$$

$$\widetilde{f}_{\sigma(k)}(k) = \begin{bmatrix} f_{\sigma(k)}\left(k, x(k)\right) \\ f_{\sigma(k)}\left(k, x(k)\right) - f_{\sigma(k)}\left(k, \hat{x}(k)\right) \end{bmatrix},$$

$$\Xi_{11} = \left(A_{\sigma(k)} + \Delta A_{\sigma(k)}\right) + \left(D_{\sigma(k)} + \Delta D_{\sigma(k)}\right) K_{\sigma(k)},$$

$$\Xi_{12} = -\left(D_{\sigma(k)} + \Delta D_{\sigma(k)}\right) K_{\sigma(k)} ,$$
$$\Xi_{21} = \Delta A_{\sigma(k)} + \Delta D_{\sigma(k)} K_{\sigma(k)} ,$$
$$\Xi_{22} = A_{\sigma(k)} - L_{\sigma(k)} C_{\sigma(k)} - \Delta D_{\sigma(k)} K_{\sigma(k)} .$$

Before proceeding to study the problem, the following assumption and lemmas should be introduced:

Assumption 1 *The matrices $C_i \in \mathbb{R}^{n_y \times n_x}$ $\left(n_y \leqslant n_x\right)$, for $i = 1, \ldots, m$ is of full-row rank, i.e., rank$(C_i) = n_y$.*

According to this assumption, the singular value decomposition of C_i could be considered as

$$C_i = U_i \begin{bmatrix} C_{0i} & 0 \end{bmatrix} V_i^T , \tag{6}$$

where $U_i \in \mathbb{R}^{n_y \times n_y}$ and $V_i \in \mathbb{R}^{n_x \times n_x}$ are unitary matrices and $C_{0i} \in \mathbb{R}^{n_y \times n_y}$ is a diagonal matrix.

Lemma 1 [23]: *For a given matrix $C \in \mathbb{R}^{n_y \times n_x}$ with rank$(C) = n_y$, assume that $X \in \mathbb{R}^{n_x \times n_x}$ is a symmetric matrix, then there exists a matrix $R \in \mathbb{R}^{n_y \times n_y}$ such that $CX = RC$, if and only if*

$$X = V \begin{bmatrix} X_1 & \\ & X_2 \end{bmatrix} V^T, \tag{7}$$

where $X_1 \in \mathbb{R}^{n_y \times n_y}$, $X_2 \in \mathbb{R}^{(n_x - n_y) \times (n_x - n_y)}$ and the matrix V is specified using the singular value decomposition of C.

Lemma 2 [21]: *For any scalar $\varepsilon > 0$, matrices $X_i \in \mathbb{R}^{n \times m}$, $i = 1, \ldots, s$ and any positive definite matrix $P \in \mathbb{R}^{n \times n}$, the following inequalities hold:*

$$X_1^T P X_2 + X_2^T P X_1 \leqslant \varepsilon X_1^T P X_1 + \varepsilon^{-1} X_2^T P X_2 \tag{8}$$

and

$$\left(X_1 + X_2\right)^T P \left(X_1 + X_2\right) \leqslant (1 + \varepsilon) X_1^T P X_1 + \left(1 + \varepsilon^{-1}\right) X_2^T P X_2 \tag{9}$$

and

$$\left(X_1 + \ldots + X_s\right)^T P \left(X_1 + \ldots + X_s\right) \leqslant s \left(X_1^T P X_1 + \ldots + X_s^T P X_s\right) . \tag{10}$$

3. Main results

In the sequel, the goal is to design an observer-based switched state-feedback controller for the uncertain switched system (1), to guarantee the stability of the closed-loop system. The switching signal $\sigma(k)$ is assumed to be available in real-time. This assumption is compatible with many practical implementations. Moreover, it is considered that the switching signal activates the subsystem i at instant k. The following theorem provides conditions to stabilize the switched system (1):

Theorem 1 *The switched system (1) with parameters $\delta p_s \in [-e_s, e_s]$, for $s = 1, \ldots, r$ is stabilized via the nonlinear observer-based controller (4), if there exist positive scalars ε, α_i, β_i and matrices $X_{xi} > 0$, $X_{ei} > 0$, $T_{xi} > 0$, $T_{ei} > 0$, S_i, Z_{xi}, Z_{ei} for $i = 1, \ldots, m$, with appropriate dimensions such that the following matrix inequalities hold:*

$$\begin{bmatrix} \overline{\Psi}_{11} & \overline{\Psi}_{12} & \overline{\Psi}_{13} & \overline{\Psi}_{14} & \overline{\Psi}_{15} & \overline{\Psi}_{16} & \overline{\Psi}_{17} & \overline{\Psi}_{18} & \overline{\Psi}_{19} & \overline{\Psi}_{10} \\ * & \overline{\Psi}_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -X_x & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -X_e & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -X_x & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -X_e & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -X_x & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -X_e & 0 & 0 \\ * & * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & * & -I \end{bmatrix} < 0 \quad \forall\, i, j \in M, \tag{11}$$

$$X_{xi} > \alpha_i^{-1} I, \tag{12}$$

$$X_{ei} > \beta_i^{-1} I, \tag{13}$$

where

$$\overline{\Psi}_{11} = \begin{bmatrix} -X_{xi} + T_{xi} & 0 & 0 & 0 \\ * & -T_{xi} & 0 & 0 \\ * & * & -X_{ei} + T_{ei} & 0 \\ * & * & * & -T_{ei} \end{bmatrix},$$

$$\overline{\Psi}_{12} = \begin{bmatrix} m_1\left(X_{xi}A_i^T + Z_{xi}^T D_i^T\right) & 0 \\ m_1 X_{xi} B_i^T & 0 \\ -m_1 Z_{xi}^T D_i^T & m_1\left(X_{ei}A_i^T - C_i^T Z_{ei}^T\right) \\ 0 & m_1 X_{ei} B_i^T \end{bmatrix},$$

$$\overline{\Psi}_{13} = \begin{bmatrix} m_2\left(X_{xi}\left(\bar{E}_1^i\right)^T + Z_{xi}^T\left(\bar{G}_1^i\right)^T\right) & \dots & m_2\left(X_{xi}\left(\bar{E}_s^i\right)^T + Z_{1i}^T\left(\bar{G}_s^i\right)^T\right) \\ 0 & \dots & 0 \\ 0 & \dots & 0 \\ 0 & \dots & 0 \end{bmatrix},$$

$$\overline{\Psi}_{14} = \overline{\Psi}_{13}, \qquad \overline{\Psi}_{15} = \begin{bmatrix} 0 & \dots & 0 \\ m_2 X_{xi}\left(\bar{F}_1^i\right)^T & \dots & m_2 X_{xi}\left(\bar{F}_s^i\right)^T \\ 0 & \dots & 0 \\ 0 & \dots & 0 \end{bmatrix},$$

$$\overline{\Psi}_{16} = \overline{\Psi}_{15}, \qquad \overline{\Psi}_{17} = \begin{bmatrix} 0 & \dots & 0 \\ 0 & \dots & 0 \\ m_2(S_i)^T\left(\bar{G}_1^i\right)^T & \dots & m_2(S_i)^T\left(\bar{G}_s^i\right)^T \\ 0 & \dots & 0 \end{bmatrix},$$

$$\overline{\Psi}_{18} = \overline{\Psi}_{17}, \qquad \overline{\Psi}_{19} = \begin{bmatrix} \gamma\sqrt{m_2\alpha_j} X_{xi} H_i^T \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$\overline{\Psi}_{10} = \begin{bmatrix} 0 \\ 0 \\ \gamma\sqrt{m_2\beta_j} X_{ei} H_i^T \\ 0 \end{bmatrix}, \qquad \overline{\Psi}_{22} = \begin{bmatrix} -X_{xj} & 0 \\ 0 & -X_{ej} \end{bmatrix},$$

$$X_{ei} = V_i \begin{bmatrix} X_{ei1} & \\ & X_{ei2} \end{bmatrix} V_i^T, \qquad X_x = diag\left\{X_{xj}, \ \dots, \ X_{xj}\right\},$$

$$X_e = diag\left\{X_{ej}, \ \dots, \ X_{ej}\right\}$$

and $m_1 = (1+\varepsilon)$, $m_2 = (3r+1)(1+\varepsilon^{-1})$, $\bar{E}_s^i = e_s E_s^i$, $\bar{F}_s^i = e_s F_s^i$, $\bar{G}_s^i = e_s G_s^i$.

Consequently, the controller and observer gains are given as $K_i = Z_{xi} X_{xi}^{-1}$ and $L_i = Z_{ei} U_i C_{0i} X_{ei1}^{-1} C_{0i}^{-1} U_i^T$, where V_i, U_i, C_{0i} have been defined in Assumption 1.

Proof. Consider the following switched Lyapunov–Krasovskii functional for the augmented system (5):

$$V(k) = V_1(k) + V_2(k), \tag{14}$$

where

$$V_1(k) = \widetilde{x}^T(k)P_{\sigma(k)}\widetilde{x}(k), \qquad V_2(k) = \sum_{s=k-\tau}^{k-1} \widetilde{x}^T(s)Q\widetilde{x}(s)$$

and $P_i = diag\{P_{xi}, P_{ei}\}$ and $Q = diag\{Q_x, Q_e\}$ are the symmetric positive definite weighting matrices of appropriate dimensions. As the switched Lyapunov functional has to be switched under arbitrary switching laws, without loss of generality, it is assumed that $\sigma(k+1) = j$, $\sigma(k) = i$, for all $i, j \in M$. Then, taking the forward difference along the trajectories of the system (1) yields,

$$\begin{aligned}\Delta V_1(k) &= V_1(k+1) - V_1(k)\\ &= \widetilde{x}^T(k+1)P_j\widetilde{x}(k+1) - \widetilde{x}^T(k)P_i\widetilde{x}(k).\end{aligned} \tag{15}$$

Using (2) and (5), one has,

$$\begin{aligned}\Delta V_1(k) = &\left(\left(A_i + D_iK_i + \sum_{s=1}^{r} \delta p_s(E_s^i + G_s^iK_i)\right)x(k) - \left(D_iK_i + \sum_{s=1}^{r} \delta p_s G_s^i K_i\right)e(k)\right.\\ &\left. + \left(B_i + \sum_{s=1}^{r} \delta p_s F_s^i\right)x(k-\tau) + f_i(k, x(k))\right)^T\\ &\cdot P_{xj}\left(\left(A_i + D_iK_i + \sum_{s=1}^{r} \delta p_s(E_s^i + G_s^iK_i)\right)x(k)\right.\\ &\left. - \left(D_iK_i + \sum_{s=1}^{r} \delta p_s G_s^i K_i\right)e(k) + \left(B_i + \sum_{s=1}^{r} \delta p_s F_s^i\right)x(k-\tau) + f_i(k, x(k))\right)\\ &+ \left(\left(\sum_{s=1}^{r} \delta p_s(E_s^i + G_s^iK_i)\right)x(k) + \left(A_i - L_iC_i - \sum_{s=1}^{r} \delta p_s G_s^i K_i\right)e(k)\right.\\ &\left. + \left(\sum_{s=1}^{r} \delta p_s F_s^i\right)x(k-\tau) + B_ie(k-\tau) + f_i(k, x(k)) - f_i\left(k, \hat{x}(k)\right)\right)^T\\ &\cdot P_{ej}\left(\left(\sum_{s=1}^{r} \delta p_s(E_s^i + G_s^iK_i)\right)x(k) + \left(A_i - L_iC_i - \sum_{s=1}^{r} \delta p_s G_s^i K_i\right)e(k)\right.\\ &\left. + \left(\sum_{s=1}^{r} \delta p_s F_s^i\right)x(k-\tau) + B_ie(k-\tau) + f_i(k, x(k)) - f_i\left(k, \hat{x}(k)\right)\right)\\ &- x^T(k)P_{xi}x(k) - e^T(k)P_{ei}e(k).\end{aligned} \tag{16}$$

By applying Lemma 2 twice, one has,

$$
\begin{aligned}
\Delta V_1(k) \leqslant{} & (1+\varepsilon)\Big\{\left((A_i + D_iK_i)x(k) - D_iK_ie(k) + B_ix(k-\tau)\right)^T \\
& \cdot P_{xj}\left((A_i + D_iK_i)\,x(k) - D_iK_ie(k) + B_ix(k-\tau)\right) \\
& +\left((A_i - L_iC_i)\,e(k) + B_ie(k-\tau)\right)^T P_{ej}\left((A_i - L_iC_i)\,e(k) + B_ie(k-\tau)\right)\Big\} \\
& + \left(1+\varepsilon^{-1}\right)\left\{\left(\left(\sum_{s=1}^{r}\delta p_s(E_s^i + G_s^iK_i)\right)x(k) - \left(\sum_{s=1}^{r}\delta p_sG_s^iK_i\right)e(k)\right.\right. \\
& \left. + \left(\sum_{s=1}^{r}\delta p_sF_s^i\right)x(k-\tau) + f_i(k,x(k))\right)^T P_{xj}\left(\left(\sum_{s=1}^{r}\delta p_s(E_s^i + G_s^iK_i)\right)x(k)\right. \\
& \left. - \left(\sum_{s=1}^{r}\delta p_sG_s^iK_i\right)e(k) + \left(\sum_{s=1}^{r}\delta p_sF_s^i\right)x(k-\tau) + f_i(k,x(k))\right) \\
& + \left(\left(\sum_{s=1}^{r}\delta p_s(E_s^i + G_s^iK_i)\right)x(k) - \left(\sum_{s=1}^{r}\delta p_sG_s^iK_i\right)e(k)\right. \\
& \left. + \left(\sum_{s=1}^{r}\delta p_sF_s^i\right)x(k-\tau) + f_i(k,x(k)) - f_i\left(k,\hat{x}(k)\right)\right)^T \\
& \cdot P_{ej}\left(\left(\sum_{s=1}^{r}\delta p_s(E_s^i + G_s^iK_i)\right)x(k) - \left(\sum_{s=1}^{r}\delta p_sG_s^iK_i\right)e(k)\right. \\
& \left.\left. + \left(\sum_{s=1}^{r}\delta p_sF_s^i\right)x(k-\tau) + f_i(k,x(k)) - f_i\left(k,\hat{x}(k)\right)\right)\right\} \\
& - x^T(k)P_{xi}x(k) - e^T(k)P_{ei}e(k).
\end{aligned}
\tag{17}
$$

Using (10) and considering the intervals $\delta p_s \in [-e_s,\ e_s]$, for $s = 1,\dots,r$, one has,

$$
\begin{aligned}
\Delta V_1(k) \leqslant{} & (1+\varepsilon)\Big\{\left((A_i + D_iK_i)x(k) - D_iK_ie(k) + B_ix(k-\tau)\right)^T \\
& \cdot P_{xj}\left((A_i + D_iK_i)\,x(k) - D_iK_ie(k) + B_ix(k-\tau)\right) \\
& +\left((A_i - L_iC_i)\,e(k) + B_ie(k-\tau)\right)^T P_{ej}\left((A_i - L_iC_i)\,e(k) + B_ie(k-\tau)\right)\Big\} \\
& + \left(1+\varepsilon^{-1}\right)(3r+1)\left\{x(k)^T\left(\sum_{s=1}^{r}e_s^2\left(E_s^i + G_s^iK_i\right)^T P_{xj}(E_s^i + G_s^iK_i)\right)x(k)\right. \\
& + x(k-\tau)^T\left(\sum_{s=1}^{r}e_s^2(F_s^i)^TP_{xj}F_s^i\right)x(k-\tau)
\end{aligned}
$$

$$
\begin{aligned}
&+\ e(k)^T\left(\sum_{s=1}^{r} e_s^2 (G_s^i K_i)^T P_{xj} G_s^i K_i\right) e(k) + f_i(k, x(k))^T P_{xj} f_i(k, x(k)) \\
&+\ x(k)^T\left(\sum_{s=1}^{r} e_s^2 \left(E_s^i + G_s^i K_i\right)^T P_{ej} \left(E_s^i + G_s^i K_i\right)\right) x(k) \\
&+\ x(k-\tau)^T\left(\sum_{s=1}^{r} e_s^2 (F_s^i)^T P_{ej} F_s^i\right) x(k-\tau) \\
&+\ e(k)^T\left(\sum_{s=1}^{r} e_s^2 (G_s^i K_i)^T P_{ej} G_s^i K_i\right) e(k) \\
&+\ \left(f_i(k, x(k)) - f_i\left(k, \hat{x}(k)\right)\right)^T P_{ej} \left(f_i(k, x(k)) - f_i\left(k, \hat{x}(k)\right)\right)\Big\} \\
&-\ x^T(k) P_{xi} x(k) - e^T(k) P_{ei} e(k).
\end{aligned} \tag{18}
$$

Moreover, from (3), (12) and (13), we have,

$$
f_i\left(k, x(k)\right)^T P_{xj} f_i\left(k, x(k)\right) \leqslant \alpha_j \gamma^2 x(k)^T H_i^T H_i x(k) \tag{19}
$$

and

$$
\begin{aligned}
&\left(f_i\left(k, x(k)\right) - f_i\left(k, \hat{x}(k)\right)\right)^T P_{ej} \left(f_i(k, x(k)) - f_i\left(k, \hat{x}(k)\right)\right) \\
&\qquad\qquad\qquad\qquad \leqslant \beta_j \gamma^2 e(k)^T H_i^T H_i e(k).
\end{aligned} \tag{20}
$$

Therefore,

$$
\begin{aligned}
\Delta V_1(k) \leqslant (1+\varepsilon)\eta(k)^T &\left(\begin{bmatrix} (A_i + D_i K_i)^T \\ B_i^T \\ (-D_i K_i)^T \\ 0 \end{bmatrix} P_{xj} \begin{bmatrix} (A_i + D_i K_i) & B_i & (-D_i K_i) & 0 \end{bmatrix}\right. \\
&\left. + \begin{bmatrix} 0 \\ 0 \\ (A_i - L_i C_i)^T \\ B_i^T \end{bmatrix} P_{ej} \begin{bmatrix} 0 & 0 & (A_i - L_i C_i) & B_i \end{bmatrix}\right) \eta(k) \\
&+ (3r+1)\left(1+\varepsilon^{-1}\right)\eta(k)^T \begin{bmatrix} \Omega_1 - P_{xi} & 0 & 0 & 0 \\ 0 & \Omega_2 & 0 & 0 \\ 0 & 0 & \Omega_3 - P_{ei} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \eta(k),
\end{aligned} \tag{21}
$$

where $\bar{E}_s^i = e_s E_s^i$, $\bar{F}_s^i = e_s F_s^i$, $\bar{G}_s^i = e_s G_s^i$ and

$$
\eta(k) = \left[x^T(k), x^T(k-\tau), e^T(k), e^T(k-\tau)\right]^T,
$$

$$\Omega_1 = \sum_{s=1}^{r} \left(\bar{E}_s^i + \bar{G}_s^i K_i\right)^T P_{xj} \left(\bar{E}_s^i + \bar{G}_s^i K_i\right) + \sum_{s=1}^{r} \left(\bar{E}_s^i + \bar{G}_s^i K_i\right)^T P_{ej} \left(\bar{E}_s^i + \bar{G}_s^i K_i\right) + \alpha_j \gamma^2 H_i^T H_i ,$$

$$\Omega_2 = \sum_{s=1}^{r} \left(\bar{F}_s^i\right)^T P_{xj} \bar{F}_s^i + \sum_{s=1}^{r} \left(\bar{F}_s^i\right)^T P_{ej} \bar{F}_s^i ,$$

$$\Omega_3 = \sum_{s=1}^{r} \left(\bar{G}_s^i K_i\right)^T P_{xj} \bar{G}_s^i K_i + \sum_{s=1}^{r} \left(\bar{G}_s^i K_i\right)^T P_{ej} \bar{G}_s^i K_i + \beta_j \gamma^2 H_i^T H_i .$$

Moreover, the expression $\Delta V_2(k)$ is given by,

$$\begin{aligned} \Delta V_2(k) &= \sum_{s=k+1-\tau}^{k} \widetilde{x}^T(s) Q \widetilde{x}(s) - \sum_{s=k-\tau}^{k-1} \widetilde{x}^T(s) Q \widetilde{x}(s) \\ &= \widetilde{x}^T(k) Q \widetilde{x}(k) - \widetilde{x}^T(k-\tau) Q \widetilde{x}(k-\tau). \end{aligned} \tag{22}$$

Now, from (21), (22), one has,

$$\Delta V(k) \leqslant \eta(k)^T \Omega(ij) \eta(k),$$

where

$$\Omega(i,j) = (1+\varepsilon) \left\{ \begin{bmatrix} (A_i + D_i K_i)^T \\ B_i^T \\ (-D_i K_i)^T \\ 0 \end{bmatrix} P_{xj} \begin{bmatrix} (A_i + D_i K_i) & B_i & (-D_i K_i) & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ (A_i - L_i C_i)^T \\ B_i^T \end{bmatrix} P_{ej} \begin{bmatrix} 0 & 0 & (A_i - L_i C_i) & B_i \end{bmatrix} \right\} + \begin{bmatrix} \overline{\Omega}_1 & 0 & 0 & 0 \\ 0 & \overline{\Omega}_2 & 0 & 0 \\ 0 & 0 & \overline{\Omega}_3 & 0 \\ 0 & 0 & 0 & -Q_e \end{bmatrix},$$

$$\overline{\Omega}_1 = (3r+1)\left(1+\varepsilon^{-1}\right) \left\{ \sum_{s=1}^{r} \left(\bar{E}_s^i + \bar{G}_s^i K_i\right)^T P_{xj} \left(\bar{E}_s^i + \bar{G}_s^i K_i\right) + \sum_{s=1}^{r} \left(\bar{E}_s^i + \bar{G}_s^i K_i\right)^T P_{ej} \left(\bar{E}_s^i + \bar{G}_s^i K_i\right) + \alpha_j \gamma^2 H_i^T H_i \right\} - P_{xi} + Q_x ,$$

$$\overline{\Omega}_2 = (3r+1)\left(1+\varepsilon^{-1}\right) \left\{ \sum_{s=1}^{r} \left(\bar{F}_s^i\right)^T P_{xj} \bar{F}_s^i + \sum_{s=1}^{r} \left(\bar{F}_s^i\right)^T P_{ej} \bar{F}_s^i \right\} - Q_x ,$$

$$\overline{\Omega}_3 = (3r+1)\left(1+\varepsilon^{-1}\right)\left\{\sum_{s=1}^{r}\left(\bar{G}_s^i K_i\right)^T P_{xj}\bar{G}_s^i K_i \right.$$
$$\left. + \sum_{s=1}^{r}\left(\bar{G}_s^i K_i\right)^T P_{ej}\bar{G}_s^i K_i + \beta_j \gamma^2 H_i^T H_i\right\} - P_{ei} + Q_e .$$

Inequalities $\Omega(i,j) < 0$, $\forall\, i, j \in M$, guarantee $\Delta V(k) < 0$ for all nonzero $\eta(k)$. Therefore, according to the Lyapunov–Krasovskii stability theorem, the augmented switched system (5) is stable.

Now, using the Schur complement, $\Omega(i,j) < 0$ is equivalent to

$$\begin{bmatrix}
\Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} & \Psi_{15} & \Psi_{16} & \Psi_{17} & \Psi_{18} & \Psi_{19} & \Psi_{10} \\
* & \Psi_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & -P_x & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & -P_e & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & -P_x & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -P_e & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -P_x & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -P_e & 0 & 0 \\
* & * & * & * & * & * & * & * & -I & 0 \\
* & * & * & * & * & * & * & * & * & -I
\end{bmatrix} < 0, \tag{23}$$

where $m_1 = (1+\varepsilon)$, $m_2 = (3r+1)(1+\varepsilon^{-1})$ and

$$\Psi_{11} = \begin{bmatrix}
-P_{xi} + Q_x & 0 & 0 & 0 \\
* & -Q_x & 0 & 0 \\
* & * & -P_{ei} + Q_e & 0 \\
* & * & * & -Q_e
\end{bmatrix},$$

$$\Psi_{12} = \begin{bmatrix}
m_1 (A_i + D_i K_i)^T P_{xj} & 0 \\
m_1 B_i^T P_{xj} & 0 \\
m_1 (-D_i K_i)^T P_{xj} & m_1 (A_i - L_i C_i)^T P_{ej} \\
0 & m_1 B_i^T P_{ej}
\end{bmatrix},$$

$$\Psi_{13} = \begin{bmatrix}
m_2 \left(\bar{E}_1^i + \bar{G}_1^i K_i\right)^T P_{xj} & \ldots & m_2 \left(\bar{E}_s^i + \bar{G}_s^i K_i\right)^T P_{xj} \\
0 & \ldots & 0 \\
0 & \ldots & 0 \\
0 & \ldots & 0
\end{bmatrix},$$

$$\Psi_{14} = \begin{bmatrix} m_2\left(\bar{E}_1^i + \bar{G}_1^i K_i\right)^T P_{ej} & \ldots & m_2\left(\bar{E}_s^i + \bar{G}_s^i K_i\right)^T P_{ej} \\ 0 & \ldots & 0 \\ 0 & \ldots & 0 \\ 0 & \ldots & 0 \end{bmatrix},$$

$$\Psi_{15} = \begin{bmatrix} 0 & \ldots & 0 \\ m_2\left(\bar{F}_1^i\right)^T P_{xj} & \ldots & m_2\left(\bar{F}_s^i\right)^T P_{xj} \\ 0 & \ldots & 0 \\ 0 & \ldots & 0 \end{bmatrix},$$

$$\Psi_{16} = \begin{bmatrix} 0 & \ldots & 0 \\ m_2\left(\bar{F}_1^i\right)^T P_{ej} & \ldots & m_2\left(\bar{F}_s^i\right)^T P_{ej} \\ 0 & \ldots & 0 \\ 0 & \ldots & 0 \end{bmatrix},$$

$$\Psi_{17} = \begin{bmatrix} 0 & \ldots & 0 \\ 0 & \ldots & 0 \\ m_2\left(\bar{G}_1^i K_i\right)^T P_{xj} & \ldots & m_2\left(\bar{G}_s^i K_i\right)^T P_{xj} \\ 0 & \ldots & 0 \end{bmatrix},$$

$$\Psi_{18} = \begin{bmatrix} 0 & \ldots & 0 \\ 0 & \ldots & 0 \\ m_2(\bar{G}_1^i K_i)^T P_{ej} & \ldots & m_2(\bar{G}_s^i K_i)^T P_{ej} \\ 0 & \ldots & 0 \end{bmatrix},$$

$$\Psi_{19} = \begin{bmatrix} \gamma\sqrt{m_2\alpha_j} H_i^T \\ 0 \\ 0 \\ 0 \end{bmatrix}, \qquad \Psi_{10} = \begin{bmatrix} 0 \\ 0 \\ \gamma\sqrt{m_2\beta_j} H_i^T \\ 0 \end{bmatrix},$$

$$\Psi_{22} = \begin{bmatrix} -P_{xj} & 0 \\ 0 & -P_{ej} \end{bmatrix},$$

$$P_x = diag\left\{P_{xj}, \ldots, P_{xj}\right\},$$

$$P_e = diag\left\{P_{ej}, \ldots, P_{ej}\right\}.$$

In view of Lemma 1, if there exist matrices V_i such that $X_{ei} = V_i \begin{bmatrix} X_{ei1} & \\ & X_{ei2} \end{bmatrix} V_i^T$ is established, then the condition $C_i X_{ei} = R_i C_i$ holds. Let $X_{xi} = P_{xi}^{-1}$, $X_{ei} = P_{ei}^{-1}$, applying the congruent transformation,

$$diag\{X_{xi},\, X_{xi},\, X_{ei},\, X_{ei},\, X_{xj},\, X_{ej},\, X_{xj}, \ldots,\quad X_{xj},\, X_{ej},\, \ldots,\, X_{ej},\, X_{xj},\, \ldots,\, X_{xj},\\ X_{ej},\, \ldots,\, X_{ej},\, X_{xj},\, \ldots,\, X_{xj},\, X_{ej},\, \ldots,\, X_{ej},\, I,\, I\}$$

we obtain (11) where $Z_{xi} = K_i X_{xi}$, $Z_{ei} = L_i R_i$ and

$$\overline{\Psi}_{11} = \begin{bmatrix} -X_{xi} + X_{xi} Q_x X_{xi} & 0 & 0 & 0 \\ * & -X_{xi} Q_x X_{xi} & 0 & 0 \\ * & * & -X_{ei} + X_{ei} Q_e X_{ei} & 0 \\ * & * & * & -X_{ei} Q_e X_{ei} \end{bmatrix},$$

$$\overline{\Psi}_{12} = \begin{bmatrix} m_1 (X_{xi} A_i^T + Z_{xi}^T D_i^T) & 0 \\ m_1 X_{xi} B_i^T & 0 \\ -m_1 Z_{xi}^T D_i^T & m_1 (X_{ei} A_i^T - C_i^T Z_{ei}^T) \\ 0 & m_1 X_{ei} B_i^T \end{bmatrix},$$

$$\overline{\Psi}_{13} = \begin{bmatrix} m_2 \left(X_{xi} \left(\bar{E}_1^i \right)^T + Z_{xi}^T \left(\bar{G}_1^i \right)^T \right) & \ldots & m_2 \left(X_{xi} \left(\bar{E}_s^i \right)^T + Z_{1i}^T \left(\bar{G}_s^i \right)^T \right) \\ 0 & \ldots & 0 \\ 0 & \ldots & 0 \\ 0 & \ldots & 0 \end{bmatrix},$$

$$\overline{\Psi}_{14} = \overline{\Psi}_{13},$$

$$\overline{\Psi}_{15} = \begin{bmatrix} 0 & \ldots & 0 \\ m_2 X_{xi} \left(\bar{F}_1^i \right)^T & \ldots & m_2 X_{xi} \left(\bar{F}_s^i \right)^T \\ 0 & \ldots & 0 \\ 0 & \ldots & 0 \end{bmatrix}, \qquad \overline{\Psi}_{16} = \overline{\Psi}_{15},$$

$$\overline{\Psi}_{17} = \begin{bmatrix} 0 & \ldots & 0 \\ 0 & \ldots & 0 \\ m_2 \left(Z_{xi} P_{xi} X_{ei} \right)^T \left(\bar{G}_1^i \right)^T & \ldots & m_2 \left(Z_{xi} P_{xi} X_{ei} \right)^T \left(\bar{G}_s^i \right)^T \\ 0 & \ldots & 0 \end{bmatrix},$$

$$\overline{\Psi}_{18} = \overline{\Psi}_{17}, \qquad \overline{\Psi}_{19} = \begin{bmatrix} \gamma\sqrt{m_2\alpha_j}X_{xi}H_i^T \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$\overline{\Psi}_{10} = \begin{bmatrix} 0 \\ 0 \\ \gamma\sqrt{m_2\beta_j}X_{ei}H_i^T \\ 0 \end{bmatrix}, \qquad \overline{\Psi}_{22} = \begin{bmatrix} -X_{xj} & 0 \\ 0 & -X_{ej} \end{bmatrix},$$

$$X_x = diag\left\{X_{xj}, \ldots, X_{xj}\right\}, \qquad X_e = diag\left\{X_{ej}, \ldots, X_{ej}\right\}.$$

Finally, according to the equation $Z_{xi} = K_iX_{xi}$, one has $K_i = Z_{xi}X_{xi}^{-1}$. On the other hand, substituting (6) into the equation $C_iX_{ei} = R_iC_i$, leads to $U_i\left[C_{0i} \;\; 0\right]V_i^TX_{ei} = R_iU_i\left[C_{0i} \;\; 0\right]V_i^T$, that is $U_iC_{0i}X_{ei1} = R_iU_iC_{0i}$. Using this equation and $Z_{ei} = L_iR_i$, we can conclude that $L_i = Z_{ei}U_iC_{0i}X_{ei1}^{-1}C_{0i}^{-1}U_i^T$. Therefore, the proof is completed. □

Remark 1 *In Theorem 1, the maximal bound for nonlinear term γ which guarantees the stability of the uncertain switched system (1) can be found with a search algorithm. First, set an arbitrary initial value for γ and solve the LMIs. Then, increase γ until the LMIs are feasible.*

Remark 2 *Instead of the Lipschitz condition (3), a more general condition called one-sided Lipschitz condition may be used [24, 25]. This increases the applicability of the proposed method.*

Based on Theorem 1, the algorithmic procedure for the design of the proposed observer-based controller is presented as follows:

Algorithm 1

Step 1. Check the conditions in Assumption 1 to obtain V_i, U_i, C_{0i} for every C_i.

Step 2. Set arbitrary initial values for ε, α_i and β_i.

Step 3. Check the feasibility of LMIs (11)–(13) to obtain $X_{xi}, X_{zi}, T_{xi}, T_{ei}, S_i, Z_{xi}$, Z_{ei} for $i = 1, \ldots, m$ (where m is the number of subsystems in the switched system). If the LMIs were not feasible, set another value for ε.

Step 4. Calculate the controller and observer gains K_i, L_i for the observer-based controller (4).

4. Numerical example

Now, to demonstrate the effectiveness of the proposed observer-based controller a computer simulation is presented. Consider the discrete-time switched system (1) with two subsystems with the state's time-delays. The matrices of subsystems are as follow:

$$
\begin{aligned}
&A_1 = \begin{bmatrix} 0.7 & 0.2 \\ 0.4 & 1.01 \end{bmatrix} + \begin{bmatrix} \delta p_1 & 0 \\ 0 & \delta p_1 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 0.6 & 0.1 \\ 0.3 & 1.03 \end{bmatrix} + \begin{bmatrix} \delta p_2 & 0 \\ 0 & \delta p_2 \end{bmatrix}, \\
&B_1 = \begin{bmatrix} -0.2 & 0.5 \\ 0.2 & -0.1 \end{bmatrix} + \begin{bmatrix} \delta p_1 & 0 \\ 0 & \delta p_1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.3 & -0.1 \\ 0.5 & 0.2 \end{bmatrix} + \begin{bmatrix} \delta p_2 & 0 \\ 0 & \delta p_2 \end{bmatrix}, \\
&C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \\
&D_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} \delta p_1 & 0 \\ 0 & \delta p_1 \end{bmatrix}, \qquad D_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} \delta p_2 & 0 \\ 0 & \delta p_2 \end{bmatrix}, \\
&\quad f_1(k, x(k)) = \begin{bmatrix} 0.3x_1(k)\sin(x_2(k)) \\ 0.3x_1(k)\cos(x_2(k)) \end{bmatrix}, \\
&\quad f_2(k, x(k)) = \begin{bmatrix} 0.5x_2(k)\cos(x_1(k)) \\ 0.5x_2(k)\sin(x_1(k)) \end{bmatrix},
\end{aligned}
\tag{24}
$$

where the uncertain parameters are considered as $\delta p_1 \in [-0.01,\ 0.01]$ and $\delta p_2 \in [-0.02,\ 0.02]$. It is worth noting that the subsystems 1 and 2 are both unstable. Their nominal matrices are as,

$$
\begin{aligned}
&A_1^0 = \begin{bmatrix} 0.7 & 0.2 \\ 0.4 & 1.01 \end{bmatrix}, \qquad A_2^0 = \begin{bmatrix} 0.6 & 0.1 \\ 0.3 & 1.03 \end{bmatrix}, \\
&B_1^0 = \begin{bmatrix} -0.2 & 0.5 \\ 0.2 & -0.1 \end{bmatrix}, \qquad B_2^0 = \begin{bmatrix} 0.3 & -0.1 \\ 0.5 & 0.2 \end{bmatrix}, \\
&D_1^0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad D_2^0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.
\end{aligned}
\tag{25}
$$

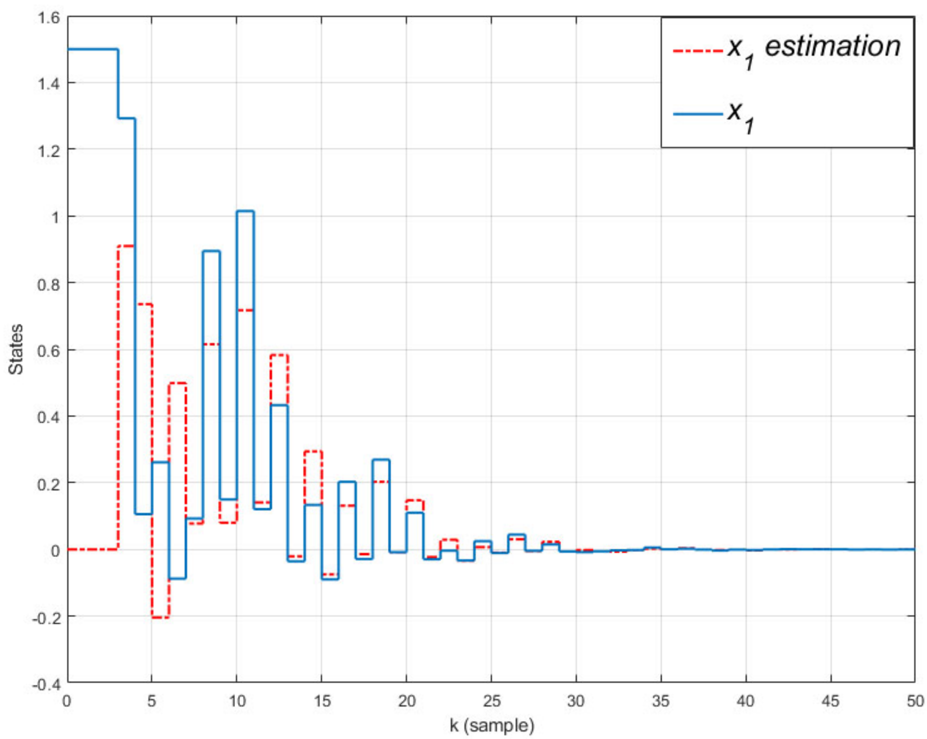
Besides, the uncertainty structure matrices are $E_1^1 = E_1^2 = F_1^1 = F_1^2 = G_1^1 = G_1^2 = I$, where I denote the identity matrix and for nonlinear functions, we have

$$
H_1 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix}, \qquad H_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}. \tag{26}
$$

Now, according to Theorem 1, an observer-based controller is designed to stabilize the closed-loop system. For the nominal matrices (25), the feasible solutions of (11)–(13) may be obtained using the LMI toolbox of MATLAB for $\varepsilon = 0.5$, $\gamma = 1$, $\alpha_i = 1$, $\beta_i = 1$. The observer gain matrices for the controller (4) are as,

$$\begin{aligned} K_1 &= \begin{bmatrix} -0.3877 & -0.1278 \\ -0.1158 & -0.4724 \end{bmatrix}, \qquad K_2 = \begin{bmatrix} -0.3994 & -0.1318 \\ -0.1270 & -0.5029 \end{bmatrix}, \\ L_1 &= \begin{bmatrix} 0.6707 & 0.2924 \\ 0.2491 & 1.0430 \end{bmatrix}, \qquad L_2 = \begin{bmatrix} 0.6853 & 0.2944 \\ 0.2810 & 1.0254 \end{bmatrix}. \end{aligned} \tag{27}$$

Therefore, the system (1) with the parameters (24) can be stabilized for the uncertain parameter belonging to the mentioned interval. In the simulation, it has been supposed that $\delta p_1 = 0.01\sin(k)$, $\delta p_2 = 0.02\sin(k)$ and two subsystems are activated in the sequence S_1, S_2, S_1, S_2, ... The simulation results for $\tau = 3$ are shown in Fig. 1, where depicts the trajectories of the state variables. It is observed that the closed-loop switched system is stable.



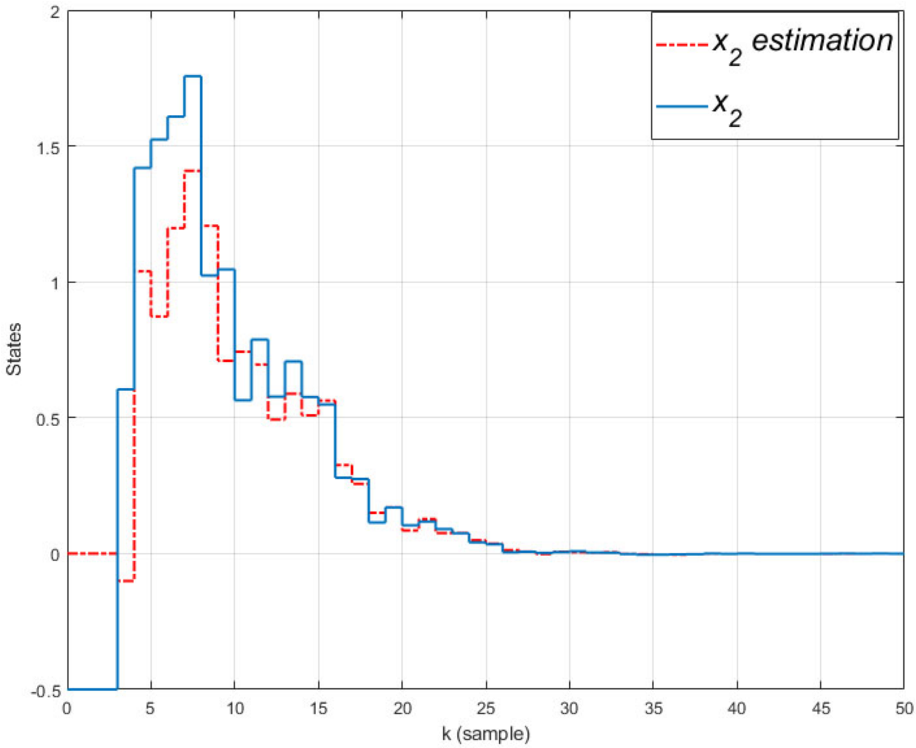


Figure 1: The time history of the state variables of the closed-loop system and their corresponding estimates

5. Conclusion

In this paper, the stabilization problem for a discrete-time switched system with state time-delay and parametric uncertainties was investigated. Consequently, an observer-based controller was constructed to stabilize the closed-loop system. Based on a switched Lyapunov function approach, an appropriate Lyapunov–Krasovskii functional was constructed to determine the state feedback and the observer gains simultaneously based on linear matrix inequalities. An important advantage of the proposed method is that the conditions only depend on the upper bound of uncertain parameters and Lipschitz coefficients of nonlinear functions of the switched system.

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