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Structural Change in the Deterministic and Stochastic Part of VECM. I(1) and I(2) Case

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Abstract

The paper analyses the consequences of structural change in the presence of non-stationary stochastic processes I(1) or I(2). The structural change may concern the deterministic structure (in particular, the trend and the constant term) as well as the process generating the stochastic part. The focus of the paper is on the case of a discrete change in a regime for which the moment of switch is known. A change in the deterministic part does not alter the character of the cointegration relationships but its consequences for cotrending and cobreaking are interesting. The consequences of a change in the stochastic part are more complex, because then the stochastic process as well as the deterministic structure of the VECM are modified. The restrictions are analysed for both cases.

Keywords: structural change, DGP, cointegration, VAR model

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1 Introduction

Most economic variables are generated by integrated stochastic processes, meaning that only the first or even second increments are stationary. Among the wealth of studies on classical integration and cointegration analysis, relatively few deal with the cointegrating regression case of a quasi-switch character, and so when the parameters of long-run relationships are changed. Changes in the long-run equilibrium (cointegration) relationship may occur at random or have a structural character (partly planned by economic entities or macroeconomic decision makers). Accordingly, the analysis must take account of whether the stochastic process generating variable (stochastic part of the data generating process, DGP) changes, or whether the stochastic process is invariant, while the deterministic structure of the model describing the economic phenomenon is subject to modification. In both cases the vector error correction model (VECM) and the relevant parameters need to be appropriately modified. At the simplest case, the constant terms inside and/or outside the cointegrating space, as well as the trend parameters (of various orders) are modified. The problem become much more complicated when modifications should be performed in the stochastic part of the DGP rather than in its deterministic part. According to the widely accepted assumptions, changes in the systematic part do not require modifying the stochastic process. On the other hand, changes in the stochastic part of DGP are transferred to the deterministic part of the model. The work presents appropriate transformations to describe the mechanisms of this transfer. The focus of the research is on the presence of a change alone, assuming implicitly that the moment of its occurrence (switching) is known and identical across system processes, and for both potential types of change (in the constant term and in the slope of the linear trend). In our work, we limit ourselves to considering the case of cointegrated VAR models, hence we exclude extreme cases of jointly stationary systems modelling and the short-run VAR model for non-cointegrated processes.
The structure of the paper is as follows. Second section discusses general assumption made in this article. Its third part presents a VECM model for a structural change in its deterministic part (consisting of a linear trend and an intercept). The model is analysed in both I(1) and I(2) domains. Changes occurring in the constant term, the linear trend and in both these components simultaneously are considered. The fourth part analyses the consequences of a change in the stochastic part. In the last part, summaries, comparisons and conclusions are provided.

2 A Vector Error Correction Model with deterministic terms – general assumptions

Let us assume that the data generating process of the observed variables vector $\mathbf{v}_t$ contains a stochastic and a deterministic component (see Lütkepohl 2005, p. 256):

$$\mathbf{v}_t = \mathbf{y}_t + \mathbf{H}\mathbf{d}_t, \tag{1}$$

where:
$\mathbf{y}_t = \begin{bmatrix} y_{1t} & \dots & y_{Mt} \end{bmatrix}^T$ – a vector of M unobserved variables generated by processes integrated of order one or two, which are subject to the VAR process with zero mean (the deterministic component takes over the effect of non-zero expected value),
$\mathbf{d}_t = \begin{bmatrix} d_{1t} & \dots & d_{Zt} \end{bmatrix}^T$ – $Z \times 1$ vector of Z deterministic variables,
$\mathbf{H} = \begin{bmatrix} \mathbf{h}_1 & \vdots & \dots & \vdots & \mathbf{h}_Z \end{bmatrix}$ – $M \times Z$ matrix of parameters on the deterministic variables,
$\mathbf{h}_z = \begin{bmatrix} h_{1z} & \dots & h_{Mz} \end{bmatrix}^T$, $z = 1, ..., Z$.
The decomposition of observable variables $\mathbf{v}_t$ into non-zero deterministic components and unobservable stochastic components generated by nonstationary processes (in terms of second moments) with zero expected values indicates that the mean for the whole $\mathbf{v}_t$ is independent of the stochastic component parameters' distribution and is contained in the deterministic component. The parameters associated with deterministic components can be calculated without the knowledge of how the stochastic component parameters are distributed.
The appropriate VECM assuming (1) is as follows:

$$\begin{aligned}\Delta\mathbf{v}_t &= \mathbf{\Pi}\mathbf{y}_{t-1} + \sum_{s=1}^{S-1}\mathbf{\Gamma}_s\Delta\mathbf{y}_{t-s} + \mathbf{H}\Delta\mathbf{d}_t + \boldsymbol{\xi}_t = \mathbf{\Pi}(\mathbf{v}_{t-1} - \mathbf{H}\mathbf{d}_{t-1}) + \\ &+ \sum_{s=1}^{S-1}\mathbf{\Gamma}_s\Delta\mathbf{v}_{t-s} - \sum_{s=1}^{S-1}\mathbf{\Gamma}_s\mathbf{H}\Delta\mathbf{d}_{t-s} + \mathbf{H}\Delta\mathbf{d}_t + \boldsymbol{\xi}_t,\end{aligned} \tag{2}$$

where:
$\mathbf{y}_{t-s}$ – M - dimensional column vector of unobserved stochastic variables (in period $t-s$), the values of $\mathbf{y}_t$ for $t < 0$ are assumed to be non-random and predetermined;
$\mathbf{v}_t$ – M - dimensional column vector of observed variables in the model;
$\boldsymbol{\xi}_t = \begin{bmatrix} \xi_{1t} & \dots & \xi_{Mt} \end{bmatrix}^T$ – the vector of disturbances, each of which meets the classic Gauss-Markov assumptions.
Assuming that the cointegration rank (the number of linearly independent long-run dependencies) equals $R < M$, the long-run equilibrium (cointegrating) matrix $\mathbf{\Pi}$ can be decomposed into (Johansen 1988):

$$\mathbf{\Pi} = \mathbf{A}\mathbf{B}^T, \tag{3}$$

where:
$\mathbf{A}$ – $M \times R$ adjustment matrix,
$\mathbf{B}$ – $M \times R$ matrix of baseline cointegrating vectors.
It is notable that both matrices have a full column rank.
The assumption that the data generating process is the sum of the deterministic and stochastic components implies that deterministic trends in the levels of variables do not affect the cointegration rank R in the stochastic part. Accordingly, most cointegration analyses are robust to the selection of the deterministic structure of the VECM (for issues such as the dimension of cointegration space identification, the selection of the optimal testing strategy, modelling and the cointegration rank test critical values modification, see Pesaran, Shin and Smith 2000).
Because of (1), the following also holds true:

$$\Delta\mathbf{v}_t = \Delta\mathbf{y}_t + \mathbf{H}\Delta\mathbf{d}_t \tag{4a}$$

$$\Delta^2\mathbf{v}_t = \Delta^2\mathbf{y}_t + \mathbf{H}\Delta^2\mathbf{d}_t \tag{4b}$$

The representation of common stochastic trends assuming (1) has the additional term related to the deterministic part of data generating process.
Let us firstly consider the case of I(1) processes. The system is then proven to have a solution in the form of the common baseline stochastic trends I(1) representation (there is the multivariate version of Beveridge and Nelson 1981 decomposition). The stochastic process $\mathbf{v}_t$ can be decomposed into a non-stationary part I(1), a stationary part I(0) (Johansen 1995a) and additional deterministic term:

$$\mathbf{v}_t = \mathbf{C}\sum_{i=1}^{t}\xi_i + \mathbf{C}^*(L)\xi_t + \mathbf{H}\mathbf{d}_t + \mathbf{A}^{INI}, \tag{5}$$

where:
$\mathbf{A}^{INI}$ depends on the initial conditions,
$\mathbf{C}\sum_{i=1}^{t}\xi_i = \mathbf{B}_\perp(\mathbf{A}_\perp^T\mathbf{\Psi}\mathbf{B}_\perp)^{-1}\mathbf{A}_\perp^T\sum_{i=1}^{t}\xi_i$ contains long-acting shocks and their contribution to I(1) processes,
$\mathbf{B}_\perp$, $\mathbf{A}_\perp$ – $M\times(M-R)$ full column rank matrices such as: $\mathbf{B}^T\mathbf{B}_\perp = \mathbf{0}$ $(r(\mathbf{B}\ \mathbf{B}_\perp) = M)$ and
$\mathbf{A}^T\mathbf{A}_\perp = \mathbf{0}$ $(r(\mathbf{A}\ \mathbf{A}_\perp) = M)$,
$\mathbf{\Psi} = -\mathbf{\Gamma} = -\left(\sum_{s=1}^{S-1}\mathbf{\Gamma}_s - \mathbf{I}\right)$,
$\mathbf{C}^*(L)\xi_t = \sum_{j=0}^{\infty}\mathbf{C}_j^*\xi_{t-j}$ – covariance stationary process,
$\mathbf{C}_j^*$ – parameters' matrices measuring the declining effect of random shocks.
The rank of $\mathbf{C}$ is $M - R$, so that system has $M - R$ linearly independent common stochastic trends I(1).
The analysis becomes much more complex in the case of processes integrated of order two (see Johansen 1995b, Paruolo 1996, Haldrup 1999). For ease of interpretation,

let us replace original model (2) by its isomorphic transformation:

$$\begin{aligned}\Delta^2\mathbf{v}_t &= \mathbf{A}\mathbf{B}^T\mathbf{v}_{t-1} - \mathbf{A}\mathbf{B}^T\mathbf{H}\mathbf{d}_{t-1} + \mathbf{\Gamma}\Delta\mathbf{v}_{t-1} - \mathbf{\Gamma}\Delta\mathbf{H}\mathbf{d}_{t-1} + \\ &+ \sum_{s=1}^{S-2}\mathbf{\Psi}_s\Delta^2\mathbf{v}_{t-s} - \sum_{s=1}^{S-2}\mathbf{\Psi}_s\mathbf{H}\Delta^2\mathbf{d}_{t-s} + \mathbf{H}\Delta^2\mathbf{d}_t + \boldsymbol{\xi}_t,\end{aligned} \tag{6}$$

where:

$$\mathbf{\Psi}_s = -\sum_{j=s+1}^{S-1}\mathbf{\Gamma}_j. \tag{7}$$

The matrix $\mathbf{\Psi}_s$ is a $M \times M$ mean lag matrix (Juselius 2006). The presence of variables generated by I(2) processes in the model hinders interpretation of the cointegration space defined by the vectors forming matrix $\mathbf{B}$. The cointegration space consists of direct equilibrium relationships CI(2,2), which occur in the medium and long run, and of cointegrating relationships CI(2,1), which only occur in the long-run. Both types of relationships typically occur between stock categories. Cointegration CI(2,1) means that the equilibrium achievement between the first increments of system variables (they are flows) is relatively fast, but the levels (stocks) reach an equilibrium in a much longer perspective. In order to properly interpret these relationships, formulas enabling the projection of the general cointegration space onto subspaces CI(2,1) and CI(2,2) are necessary. Analogous projections should apply to the relevant adjustment matrix.

Because in the I(2) domain $(\mathbf{A}_\perp^T\mathbf{\Psi}\mathbf{B}_\perp)$ matrix is not invertible (has a reduced rank), solution (5) should be replaced by:

$$\mathbf{v}_t = \mathbf{C_1}\sum_{i=1}^{t}\xi_i + \mathbf{C}_2\sum_{j=1}^{t}\sum_{i=1}^{j}\xi_i + \mathbf{C}(L)\xi_t + \mathbf{H}\mathbf{d}_t + \mathbf{A}^{INI}, \tag{8}$$

where $\mathbf{C}_2$ is a matrix of parameters associated with the impact of autonomous stochastic trends I(2) $\sum_{j=1}^{t}\sum_{i=1}^{j}\xi_i$,

$\mathbf{A}^{INI}$ depends on the initial conditions.

Matrix $\mathbf{C}_2$ can be decomposed into:

$$\mathbf{C}_2 = \mathbf{B}_{2\perp}\left(\mathbf{A}_{2\perp}^T\left(\mathbf{\Psi}\mathbf{B}\left(\mathbf{B}^T\mathbf{B}\right)^{-1}\left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{\Psi} - \sum_{s=1}^{S-2}\mathbf{\Psi}_s\right)\mathbf{B}_{2\perp}\right)^{-1}\mathbf{A}_{2\perp}^T \tag{9}$$

whereas matrices $\mathbf{A}_{2\perp}^T$ (which can be interpreted as the matrix defining independent common stochastic trends I(2)) and $\mathbf{B}_{2\perp}$ are $M \times P_2$-dimensional, where P_2 is the number of common baseline stochastic trends I(2). It is assumed that $P_1 + P_2 = M - R$, where P_1 is the number of common baseline stochastic trends I(1) in the I(2) domain.

The projection of the cointegrating matrix and the matrix of weights (adjustment matrix) onto CI(2,2) and CI(2,1) spaces is enabled by the following formulas:

$$\mathbf{B}_1 = \mathbf{B}\boldsymbol{\Lambda}^T, \tag{10}$$

$$\mathbf{B}_0 = \mathbf{B}\boldsymbol{\Lambda}_\perp^T, \tag{11}$$

$$\mathbf{A}_1 = \mathbf{A}\overline{\mathbf{A}}^T\boldsymbol{\Psi}\overline{\mathbf{B}}_{2\perp}\mathbf{B}_{2\perp}^T\mathbf{K}(\mathbf{K}^T\mathbf{K})^{-1} = \mathbf{A}\overline{\mathbf{A}}^T\boldsymbol{\Psi}\overline{\mathbf{B}}_{2\perp}\mathbf{B}_{2\perp}^T\overline{\mathbf{K}}, \tag{12}$$

$$\mathbf{A}_0 = \mathbf{A}\overline{\mathbf{A}}^T\boldsymbol{\Psi}\overline{\mathbf{B}}_{2\perp}\mathbf{B}_{2\perp}^T\mathbf{K}_\perp(\mathbf{K}_\perp^T\mathbf{K}_\perp)^{-1} = \mathbf{A}\overline{\mathbf{A}}^T\boldsymbol{\Psi}\overline{\mathbf{B}}_{2\perp}\mathbf{B}_{2\perp}^T\overline{\mathbf{K}}_\perp, \tag{13}$$

where
$\mathbf{B}_1$ – $M \times R_1$-dimensional matrix of CI(2,1) cointegrating directions,
$\mathbf{B}_0$ – $M \times R_0$-dimensional matrix of CI(2,2) cointegrating directions, $R_0 + R_1 = R$,
$\mathbf{A}_1$ – $M \times R_1$-dimensional adjustment matrix to the CI(2,1) cointegrating relationships,
$\mathbf{A}_0$ – $M \times R_0$-dimensional adjustment matrix to the CI(2,2) cointegrating relationships and

$$\overline{\mathbf{A}}_{(M\times R)} = \mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1}, \tag{14}$$

$$\overline{\mathbf{K}}_{(M\times P_2)} = \mathbf{K}(\mathbf{K}^T\mathbf{K})^{-1}, \tag{15}$$

$$\mathbf{K}_{(M\times P_2)} = \mathbf{B}_{2\perp}\boldsymbol{\Lambda}\boldsymbol{\Lambda}^T, \tag{16}$$

$$\boldsymbol{\Lambda}^T = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\boldsymbol{\Psi}\mathbf{B}_{2\perp}(\mathbf{B}_{2\perp}^T\mathbf{B}_{2\perp})^{-1}, \tag{17}$$

$\boldsymbol{\Lambda}^T$ – $R \times M - R - P_1$-dimensional matrix,

$$\mathbf{A}_{2\perp} = \mathbf{A}_\perp\boldsymbol{\Xi}_\perp, \tag{18}$$

$\mathbf{A}_{2\perp}$ – $M \times P_2$-dimensional matrix defining I(2) stochastic shocks,

$$\mathbf{B}_{2\perp(M\times P_2)} = \mathbf{B}_\perp\mathbf{N}_\perp. \tag{19}$$

It should be stressed that I(2) processes should be additionally decomposed (the rank of $\mathbf{A}_\perp^T\boldsymbol{\Psi}\mathbf{B}_\perp$ is reduced):

$$\mathbf{A}_\perp^T\boldsymbol{\Psi}\mathbf{B}_\perp = \boldsymbol{\Xi}\mathbf{N}^T, \tag{20}$$

where $\boldsymbol{\Xi}$, $\mathbf{N}$ are $(M-R)\times P_1$ - dimensional matrices, $P_1 < M - R$:

$$\mathbf{A}_\perp^T\mathbf{A}_{1\perp} = \boldsymbol{\Xi}, \tag{21}$$

$\mathbf{A}_{1\perp}$ – $M \times P_1$ - dimensional matrix defining I(1) stochastic shocks,

$$\mathbf{B}_\perp^T\mathbf{B}_{1\perp} = \mathbf{N}, \tag{22}$$

$\mathbf{B}_{1\perp}$ – $M \times P_1$ - dimensional matrix.
It can be proved (Majsterek 2008) that the system is in equilibrium when $R_1 = M - R - P_1 = P_2$.
As a consequence of formulas (19)-(20), projection onto the space of common

stochastic I(1) trends, which are interpreted in the I(2) domain as stochastic cyclical (Juselius 2006), is obtained through (Paruolo 2000):

$$\overline{\mathbf{A}}_{\perp}\mathbf{\Xi} = \mathbf{A}_{\perp}\left(\mathbf{A}_{\perp}^{T}\mathbf{A}_{\perp}\right)^{-1}\mathbf{\Xi} = \mathbf{A}_{1\perp}, \tag{23}$$

$$\overline{\mathbf{B}}_{\perp}\mathbf{N} = \mathbf{B}_{\perp}\left(\mathbf{B}_{\perp}^{T}\mathbf{B}_{\perp}\right)^{-1}\mathbf{N} = \mathbf{B}_{1\perp}. \tag{24}$$

3 A Vector Error Correction Model with the structural break in the deterministic component

The inclusion of the constant term and the deterministic trend in vector $\mathbf{d}_t$ of model (2) leads to five different forms of the VECM, resulting from the restrictions imposed on the parameters related to deterministic variables (see Juselius 2006, pp. 99-100). Binary variables lying inside and/or outside the cointegrating space will be differently interpreted depending on whether their presence results from structural changes or outliers. Therefore three different vector error correction models with deterministic terms, which are present in DGP due to structural breaks are derived.

Firstly, let us consider a case of level break only where only the intercept changes. The deterministic component is represented by a constant term (the deterministic trend is absent). Analogously to (5), the solution of the model for $\mathbf{v}_t$ is then in I(1) domain as follows:

$$\begin{aligned}\mathbf{v}_t &= \mathbf{y}_t + \mathbf{h}_1 + \mathbf{h}_3 u_t + \mathbf{A}^{INI} = \mathbf{C}\sum_{i=1}^{t}\xi_i + \mathbf{C}^*(L)\xi_t + \mathbf{H}\mathbf{d}_t + \mathbf{A}^{INI} = \\ &= \mathbf{C}\sum_{i=1}^{t}\xi_i + \mathbf{C}^*(L)\xi_t + \mathbf{h}_1 + \mathbf{h}_3 u_t + \mathbf{A}^{INI},\end{aligned} \tag{25}$$

where $u_t = \begin{cases} 0 & \text{for } t < t_0 \\ 1 & \text{for } t \geq t_0 \end{cases}$,

$\mathbf{A}^{INI}$ depends on the initial conditions.

The above case should be interpreted as a discrete change in the expected value of data generation process.

Variable u_t is defined assuming that there is only one structural change in the known period t_0. The assumption can be generalized to allow for multiple structural changes (increasing the number of variables u_{ct}, in vector $\mathbf{d}_t$ where u_{ct} is a binary variable connected with c-th structural change).

The vector of parameters $\mathbf{h}_1$ associated with the constant term is responsible for non-zero expected value of the generating process $\mathbf{v}_t$. In practice, the considered model can be applied to the economic categories which are not trend-stationary, cointegrated and the structural break can be described as a discrete impulse change in the mean of all economic categories included in the system. Therefore $\mathbf{H} = [\mathbf{h}_1 \vdots \mathbf{h}_3]$ (due to the lack of deterministic trend $\mathbf{h}_2 = 0$), $\mathbf{d}_t = \begin{bmatrix} 1 & u_t \end{bmatrix}^T$ in the deterministic component of DGP and the respective VECM is given by (see formulas (2) and (25)):

$$\begin{aligned}\Delta\mathbf{v}_t &= \mathbf{\Pi}\mathbf{y}_{t-1} + \sum_{s=1}^{S-1}\mathbf{\Gamma}_s\Delta\mathbf{y}_{t-s} + \mathbf{H}\Delta\mathbf{d}_t + \boldsymbol{\xi}_t = \\ &= \mathbf{A}\mathbf{B}^T\mathbf{v}_{t-1} - \mathbf{A}\mathbf{B}^T\mathbf{h}_1 - \mathbf{A}\mathbf{B}^T\mathbf{h}_3 u_{t-1} + \sum_{s=1}^{S-1}\mathbf{\Gamma}_s\Delta\mathbf{y}_{t-s} + \mathbf{H}\Delta\mathbf{d}_t + \boldsymbol{\xi}_t\end{aligned} \tag{26}$$

because $\mathbf{A}\mathbf{B}^T\mathbf{v}_{t-1} = \mathbf{A}\mathbf{B}^T(\mathbf{y}_{t-1} + \mathbf{H}\mathbf{d}_t) = \mathbf{A}\mathbf{B}^T(\mathbf{y}_{t-1} + \mathbf{h}_1 + \mathbf{h}_3 u_{t-1})$.
Since model (26) still contains unobservable variables, it needs to be transformed as follows (for an alternative approach see Gosińska 2015):

$$\begin{aligned}\Delta\mathbf{v}_t &= \mathbf{A}\mathbf{B}^T\mathbf{v}_{t-1} - \mathbf{A}\mathbf{B}^T\mathbf{h}_1 - \mathbf{A}\mathbf{B}^T\mathbf{h}_3 u_{t-1} + \sum_{s=1}^{S-1}\mathbf{\Gamma}_s\Delta\mathbf{y}_{t-s} + \mathbf{H}\Delta\mathbf{d}_t + \boldsymbol{\xi}_t = \\ &= \mathbf{A}[\mathbf{B}^T\mathbf{v}_{t-1} + \mathbf{g}_1 + \mathbf{g}_3 u_{t-1}] + \sum_{s=1}^{S-1}\mathbf{\Gamma}_s\Delta\mathbf{v}_{t-s} + \sum_{s=0}^{S-1}\mathbf{f}_3^s\Delta u_{t-s} + \boldsymbol{\xi}_t\end{aligned} \tag{27}$$

for $t = S+1, S+2, ...$, where

$$\mathbf{g}_1 = -\mathbf{B}^T\mathbf{h}_1, \quad \mathbf{g}_3 = -\mathbf{B}^T\mathbf{h}_3, \quad \mathbf{f}_3^s = \begin{cases}\mathbf{h}_3 & \text{for } s = 0 \\ -\mathbf{\Gamma}_s\mathbf{h}_3 & \text{for } s = 1, 2, ..., S-1\end{cases}.$$

Depending on whether the structural change took place in the latest period or has already been fixed, its short-run impact on the variables used in the model will be measured by $\mathbf{h}_3$ or $-\mathbf{\Gamma}_s\mathbf{h}_3$ respectively. When the process generating variables contained in $\Delta\mathbf{v}_t$ is stationary (I(2) stochastic trends are excluded), then $-\mathbf{\Gamma}_s$ describe deceleration of the impact of a structural change (the effect of novelty decreases).
In the system of equations $\mathbf{g}_i = -\mathbf{B}^T\mathbf{h}_i$ $(i = 1, 3)$, the classical (invariant) cointegrating matrix may be treated as a restriction matrix. All these restrictions are the consequences of the nature of the considered processes. The condition $\mathbf{g}_1 = -\mathbf{B}^T\mathbf{h}_1$ prevents the constant term in the model from generating a linear trend (cotrending occurs). This is even better seen from the equivalent representation:

$$\Delta\mathbf{v}_t = \mathbf{A}[\mathbf{B}^T\mathbf{v}_{t-1} + \mathbf{g}_3 u_{t-1}] + \sum_{s=1}^{S-1}\mathbf{\Gamma}_s\Delta\mathbf{v}_{t-s} + \mathbf{f}_1 + \sum_{s=0}^{S-1}\mathbf{f}_3^s\Delta u_{t-s} + \boldsymbol{\xi}_t, \tag{28}$$

where $\mathbf{f}_1 = \mathbf{A}\mathbf{g}_1 = -\mathbf{A}\mathbf{B}^T\mathbf{h}_1$.

The constant term in equation (28) is not unlimited even if it is removed from the cointegration space (which is the goal of transition from (27) to (28)). This means that the intercept contained in the first increments will not generate a deterministic trend in the levels.
Restriction $\mathbf{g}_3 = -\mathbf{B}^T\mathbf{h}_3$ can be interpreted as cobreaking of processes levels which, by analogy to cointegration, means that a structural change immediately affects all variables in the system. Changes in the expected value of the DGP must relate to levels, not first increments, which is a logical consequence of data generation process and the previous assumption $\mathbf{g}_1 = -\mathbf{B}^T\mathbf{h}_1$. If the intercept changes for reasons that could not be predicted (e.g. cataclysms), then cobreaking is a special case of coincidence without economic interpretation in any longer perspective.
For $\mathbf{h}_3 = 0$, model (28) simplifies to a standard VECM with a deterministic component containing only a constant term ($\mathbf{v}_t = \mathbf{y}_t + \mathbf{h}_1$):

$$\Delta \mathbf{v}_t = \mathbf{A}\mathbf{B}^{*T}\mathbf{v}^*_{t-1} + \sum_{s=1}^{S-1} \mathbf{\Gamma}_s \Delta \mathbf{v}_{t-s} + \boldsymbol{\xi}_t, \tag{29}$$

where $\mathbf{v}^*_{t-1} = \begin{bmatrix} \mathbf{v}_{t-1} \\ 1 \end{bmatrix}$, $\mathbf{B}^{*T} = \begin{bmatrix} \mathbf{B}^T & \mathbf{g}_1 \end{bmatrix}$, $\mathbf{g}_1 = -\mathbf{B}^T\mathbf{h}_1$.
In the analysis with I(2) processes, all the above considerations remain valid because changes concern the deterministic part and not the stochastic part. For the interpretation purposes (see Majsterek 2008), it is more convenient to use representation (6) assuming that $\mathbf{d}_t = \begin{bmatrix} 1 & u_t \end{bmatrix}^T$:

$$\begin{aligned} \Delta^2 \mathbf{v}_t &= \mathbf{A}\left[\mathbf{B}^T\mathbf{v}_{t-1} + \mathbf{g}_1 + \mathbf{g}_3 u_{t-1}\right] + \mathbf{\Gamma}\Delta u_{t-1} - \mathbf{\Gamma}\mathbf{h}_3 \Delta u_{t-1} + \\ &+ \sum_{s=1}^{S-2} \mathbf{\Psi}_s \Delta^2 \mathbf{v}_{t-s} + \sum_{s=0}^{S-2} \mathbf{f}_3^{s,I(2)} \Delta^2 u_{t-s} + \boldsymbol{\xi}_t, \end{aligned} \tag{30}$$

where $\mathbf{f}_3^{s,I(2)} = \begin{cases} \mathbf{h}_3 & \text{for } s = 0 \\ -\mathbf{\Psi}_s\mathbf{h}_3 & \text{for } s = 1, 2, ..., S-2 \end{cases}$.
Because according to formula (30) a structural change only takes place in the deterministic part, the I(2) projections defined by formulas (10)–(24) will not apply in further analysis to the components of $\mathbf{f}_3^s$. In the case of long-term dependencies, the presence of deterministic variables and deterministic structural changes has no effect on the Johansen estimation procedure. Both in the simple I(1) case and in the two-stage approach applying in I(2) case all projection patterns remain robust to the deterministic structure of the model.
The interpretation of restriction $\mathbf{g}_1 = -\mathbf{B}^T\mathbf{h}_1$ for the I(2) domain is slightly different because of the accelerants on the left-hand side of model (30). In this case it becomes necessary to rule out the trend in increments formation but not necessarily in levels. Let us also note that $-\mathbf{\Psi}_s\mathbf{h}_3$ in the formulas for vector $\mathbf{f}_3^{s,I(2)}$ does not necessarily

implies the deceleration mechanism, because it only ensures the weakening of the novelty effect on the second increments. The effect of structural changes on the first increments is approximated by the matrix $\mathbf{\Gamma} = \sum_{s=1}^{S-1} \mathbf{\Gamma}_s - \mathbf{I}$, which means that this effect is cumulative and does not end as in the case of I(1) processes. This result seems to be fully acceptable from the interpretative perspective.
In the second case let us assume that the deterministic component contains the constant term, the change of this intercept and the trend. Then $\mathbf{d}_t = \left[\begin{array}{ccc} 1 & t & u_t \end{array}\right]^T$ and the DGP is written as follows:

$$\mathbf{v}_t = \mathbf{y}_t + \mathbf{h}_1 + \mathbf{h}_2 t + \mathbf{h}_3 u_t. \tag{31}$$

The above structural change still should be interpreted as a discrete change in the expected value of the DGP. The following VECM is being considered (an alternative derivation of this VECM representation is given by Gosińska 2015):

$$\begin{aligned} \Delta\mathbf{v}_t &= \mathbf{\Pi}\mathbf{y}_{t-1} + \sum_{s=1}^{S-1} \mathbf{\Gamma}_s \Delta\mathbf{y}_{t-s} + \mathbf{H}\Delta\mathbf{d}_t + \boldsymbol{\xi}_t = \quad (32) \\ &= \mathbf{A}\mathbf{B}^T\mathbf{v}_{t-1} - \mathbf{A}\mathbf{B}^T\mathbf{h}_1 - \mathbf{A}\mathbf{B}^T\mathbf{h}_2(t-1) - \mathbf{A}\mathbf{B}^T\mathbf{h}_3 u_{t-1} + \\ &\quad + \sum_{s=1}^{S-1} \mathbf{\Gamma}_s \Delta\mathbf{y}_{t-s} + \mathbf{H}\Delta\mathbf{d}_t + \boldsymbol{\xi}_t. \end{aligned}$$

After transformations, we obtain a VECM with a constant term, a trend and a change of intercept only for observable variables (see Saikkonen and Lütkepohl 2000):

$$\begin{aligned} \Delta\mathbf{v}_t &= \mathbf{A}\left[\mathbf{B}^T\mathbf{v}_{t-1} + \mathbf{g}_1 + \mathbf{g}_2(t-1) + \mathbf{g}_3 u_{t-1}\right] + \sum_{s=1}^{S-1} \mathbf{\Gamma}_s \Delta\mathbf{v}_{t-s} + \quad (33) \\ &\quad + \mathbf{f}_2\Delta t + \sum_{s=0}^{S-1} \mathbf{f}_3^s \Delta u_{t-s} + \boldsymbol{\xi}_t, \end{aligned}$$

for $t = S+1, S+2, ...$, where used symbols were explained before (cf. 27), additionally:

$$\mathbf{g}_2 = -\mathbf{B}^T\mathbf{h}_2, \quad \mathbf{f}_2 = \mathbf{h}_2 - \sum_{s=1}^{S-1} \mathbf{\Gamma}_s \mathbf{h}_2 = \mathbf{\Psi}\mathbf{h}_2.$$

Due to the linearity of a trend: $\mathbf{f}_2\Delta t = \mathbf{f}_2$. Model (33) is observationally equivalent to:

$$\begin{aligned} \Delta\mathbf{v}_t &= \mathbf{A}\left[\mathbf{B}^T\mathbf{v}_{t-1} + \mathbf{g}_2(t-1) + \mathbf{g}_3 u_{t-1}\right] + \sum_{s=1}^{S-1} \mathbf{\Gamma}_s \Delta\mathbf{v}_{t-s} + \quad (34) \\ &\quad + \mathbf{f}_2^* + \sum_{s=0}^{S-1} \mathbf{f}_3^s \Delta u_{t-s} + \boldsymbol{\xi}_t, \end{aligned}$$

where

$$\mathbf{f}_2^* = \mathbf{f}_2 + \mathbf{A}\mathbf{g}_1 = \mathbf{f}_2 - \mathbf{A}\mathbf{B}^T\mathbf{h}_1 = \mathbf{h}_2 - \sum_{s=1}^{S-1}\mathbf{\Gamma}_s\mathbf{h}_2 + \mathbf{A}\mathbf{g}_1 = \mathbf{\Psi}\mathbf{h}_2 + \mathbf{A}\mathbf{g}_1.$$

In the short-term part, in relation to the previously considered case without deterministic trends inherent in data generating processes two additional component are included: firstly, vector $\mathbf{h}_2$ which is multiplied by one (measure of growth in intercept) and secondly $-\sum_{s=1}^{S-1}\mathbf{\Gamma}_s\mathbf{h}_2$, which is an additional vector of $-\sum_{s=1}^{S-1}\mathbf{\Gamma}_s\mathbf{H}\Delta\mathbf{d}_{t-s}$ matrix after extension $\Delta\mathbf{d}_t = \begin{bmatrix} 0 & 1 & \Delta u_t \end{bmatrix}^T$. The latter plays the same role as $\sum_{s=0}^{S-1}\mathbf{f}_3^s\Delta u_{t-s}$ and can be treated as the measure of the decelerator of additional short-term benefits from structural changes, since constant growth originates from the higher base.

Because of the restriction $\mathbf{g}_1 = -\mathbf{B}^T\mathbf{h}_1$ in the model without the deterministic trend, the constant term in the model does not generate a linear trend in the cointegrating space, thus it is a blockade ensuring the fulfilment of this assumption. The restriction is also fulfilled in model (34), but the intercept in increments generates a trend in levels. It is apparent contradiction. However, subsequent transformations of (33) leading to (34) allow the constant term to be removed from the cointegrating space. As a result, the constant term in increments is related to the linear trend in the levels. In the models without a deterministic trend, the constant term is the only source of non-random factors, whereas in model (33), an additional linear trend appears in the variables.

The restriction $\mathbf{g}_2 = -\mathbf{B}^T\mathbf{h}_2$ prevents the linear trend included in the cointegration regression from generating a quadratic trend in the data. It is therefore a cotrending, but understood differently than in the case of a less expanded deterministic part. In model (33) there is a parameter vector (that goes beyond the cointegrating space) $\mathbf{f}_2 = \mathbf{h}_2 - \sum_{s=1}^{S-1}\mathbf{\Gamma}_s\mathbf{h}_2 = \mathbf{\Psi}\mathbf{h}_2 = -\mathbf{\Gamma}\mathbf{h}_2$ measuring the influence of the intercept (not the time variable) on the formation of first increments. From $\mathbf{f}_2 = \mathbf{h}_2 - \sum_{s=1}^{S-1}\mathbf{\Gamma}_s\mathbf{h}_2 = = \mathbf{\Psi}\mathbf{h}_2 = -\mathbf{\Gamma}\mathbf{h}_2$, the constant term in model (32) can generate a linear trend, but only in the levels not in first differences. A closer analysis shows that in the case under consideration restriction $\mathbf{g}_1 = -\mathbf{B}^T\mathbf{h}_1$ was dominated by condition $\mathbf{g}_2 = -\mathbf{B}^T\mathbf{h}_2$. Therefore, $\mathbf{g}_1 = -\mathbf{B}^T\mathbf{h}_1$ still prevents a constant term from generating a linear trend in the cointegration space but it does not preclude the presence of such a trend in the model in general due to the occurrence of this trend in the DGP. At the same time, the dominant restriction $\mathbf{g}_2 = -\mathbf{B}^T\mathbf{h}_2$ excludes nonlinear trends from the system.

Restriction $\mathbf{g}_3 = -\mathbf{B}^T\mathbf{h}_3$ ensures the meeting of a similar condition for changes in the expected value of the intercept, with its interpretation in terms of cobreaking remaining the same.

In model (34), the constant term being the sum of two components of $\mathbf{f}_2^* = \mathbf{A}\mathbf{g}_1 + \mathbf{f}_2$ is taken out of the cointegrating space.

If $\mathbf{h}_3 = 0$, the model simplifies to the traditional VECM with a deterministic component containing a constant term and a trend ($\mathbf{v}_t = \mathbf{y}_t + \mathbf{h}_1 + \mathbf{h}_2 t$):

$$\Delta \mathbf{v}_t = \mathbf{A}\mathbf{B}^{*T}\mathbf{v}_{t-1}^{*} + \sum_{s=1}^{S-1} \boldsymbol{\Gamma}_s \Delta \mathbf{v}_{t-s} + \mathbf{f}_2^{*} + \boldsymbol{\xi}_t, \tag{35}$$

where

$$\mathbf{v}_{t-1}^{*} = \begin{bmatrix} \mathbf{v}_{t-1} \\ t-1 \end{bmatrix}, \mathbf{B}^{*T} = \begin{bmatrix} \mathbf{B}^T & \mathbf{g}_2 \end{bmatrix}, \mathbf{g}_2 = -\mathbf{B}^T\mathbf{h}_2, \mathbf{f}_2^{*} = \mathbf{A}\mathbf{g}_1 + \boldsymbol{\Psi}\mathbf{h}_2 = \mathbf{A}\mathbf{g}_1 + \mathbf{f}_2.$$

In the I(2) domain, formula (32) remains valid, transformations from previous case are still correct. A more convenient (economically) interpretation can be obtained by using the following I(2) representation:

$$\begin{aligned} \Delta^2 \mathbf{v}_t \quad = \quad & \mathbf{A}\left[\mathbf{B}^T\mathbf{v}_{t-1} + \mathbf{g}_1 + \mathbf{g}_2(t-1) + \mathbf{g}_3 u_{t-1}\right] + \boldsymbol{\Gamma}\Delta\mathbf{v}_{t-1} - \boldsymbol{\Gamma}\mathbf{h}_2 + \\ & -\boldsymbol{\Gamma}\mathbf{h}_3 \Delta u_{1,t-1} + \sum_{s=1}^{S-2} \boldsymbol{\Psi}_s \Delta^2 \mathbf{v}_{t-s} + \mathbf{f}_2 \Delta^2 t + \sum_{s=0}^{S-2} \mathbf{f}_3^{s,I(2)} \Delta^2 u_{t-s} + \boldsymbol{\xi}_t \end{aligned} \tag{36}$$

for $t = S+1, S+2, ...,$ where

$$\mathbf{g}_1 = -\mathbf{B}^T\mathbf{h}_1, \quad \mathbf{g}_2 = -\mathbf{B}^T\mathbf{h}_2, \quad \mathbf{g}_3 = -\mathbf{B}^T\mathbf{h}_3,$$

$$\mathbf{f}_3^{s,I(2)} = \begin{cases} \mathbf{h}_3 & \text{for } s = 0 \\ -\boldsymbol{\Psi}_s \mathbf{h}_3 & \text{for } s = 1, 2, ..., S-2 \end{cases}.$$

Due to no acceleration of the linear trend $\mathbf{f}_2\Delta^2 t = \mathbf{f}_2 0 = \mathbf{0}$. Therefore model (36) can be simplified to

$$\begin{aligned} \Delta^2 \mathbf{v}_t \quad = \quad & \mathbf{A}\left[\mathbf{B}^T\mathbf{v}_{t-1} + \mathbf{g}_1 + \mathbf{g}_2(t-1) + \mathbf{g}_3 u_{t-1}\right] + \boldsymbol{\Gamma}\Delta\mathbf{v}_{t-1} - \boldsymbol{\Gamma}\mathbf{h}_2 + \\ & -\boldsymbol{\Gamma}\mathbf{h}_3 \Delta u_{t-1} + \sum_{s=1}^{S-2} \boldsymbol{\Psi}_s \Delta^2 \mathbf{v}_{t-s} + \sum_{s=0}^{S-2} \mathbf{f}_3^{s,I(2)} \Delta^2 u_{t-s} + \boldsymbol{\xi}_t. \end{aligned} \tag{37}$$

In a more general third case, the structural break changes the constant term and the parameter associated with the trend at the same time; then $\mathbf{d}_t = \begin{bmatrix} 1 & t & \Delta u_{1t} & \Delta u_{2t} \end{bmatrix}^T$ and the data generating process is (Gosińska 2009):

$$\mathbf{v}_t = \mathbf{y}_t + \mathbf{h}_1 + \mathbf{h}_2 t + \mathbf{h}_3 u_{1t} + \mathbf{h}_4 u_{2t}, \tag{38}$$

where

$$u_{1t} \quad = \quad \begin{cases} 0 & \text{for } t < t_0 \\ 1 & \text{for } t \geq t_0 \end{cases} \quad \text{and} \quad u_{2t} = \begin{cases} 0 & \text{for } t < t_0 \\ t-(t_0-1) & \text{for } t \geq t_0 \end{cases}, \tag{39}$$

$$u_{1t} \quad = \quad \Delta u_{2t}. \tag{40}$$

The above case should be interpreted as a discrete change in the expected value of DGP and in the development tendency inherent in the processes (it still remains linear). Let us note that (39) has one more significant assumption that is accepted implicitly. It is assumed that, the change in the expected value of DGP and the change in the deterministic trend take place in the same period, which may not always be true. The following VECM should be considered (alternative derivation of the VECM representation for this case is given by Gosińska 2015):

$$\begin{aligned}\Delta \mathbf{v}_t &= \mathbf{\Pi}\mathbf{y}_{t-1} + \sum_{s=1}^{S-1} \mathbf{\Gamma}_s \Delta \mathbf{y}_{t-s} + \mathbf{H}\Delta \mathbf{d}_t + \boldsymbol{\xi}_t = \\ &= \mathbf{A}\mathbf{B}^T \mathbf{v}_{t-1} - \mathbf{A}\mathbf{B}^T \mathbf{h}_1 - \mathbf{A}\mathbf{B}^T \mathbf{h}_2 (t-1) + \mathbf{A}\mathbf{B}^T \mathbf{h}_3 u_{1,t-1} + \\ &\quad -\mathbf{A}\mathbf{B}^T \mathbf{h}_4 u_{2,1-t} + \sum_{s=1}^{S-1} \mathbf{\Gamma}_s \Delta \mathbf{y}_{t-s} + \mathbf{H}\Delta \mathbf{d}_t + \boldsymbol{\xi}_t.\end{aligned} \tag{41}$$

Appropriate transformations of formula (41) leads to the VECM representation with a constant term, a trend, a change in the intercept and a structural change in the trend (containing only the observable variables $\mathbf{v}_t$):

$$\begin{aligned}\Delta \mathbf{v}_t &= \mathbf{A}\left[\mathbf{B}^T \mathbf{v}_{t-1} + \mathbf{g}_1 + \mathbf{g}_2 (t-1) + \mathbf{g}_3 u_{1,t-1} + \mathbf{g}_4 u_{2,t-1}\right] + \\ &\quad + \sum_{s=1}^{S-1} \mathbf{\Gamma}_s \Delta \mathbf{v}_{t-s} + \mathbf{f}_2 + \sum_{s=0}^{S-1} \mathbf{f}_3^s \Delta u_{1,t-s} + \mathbf{\Psi}\mathbf{h}_4 u_{1,t} + \\ &\quad + \sum_{s=0}^{S-2} \mathbf{f}_4^s \Delta u_{1,t-s} + \boldsymbol{\xi}_t,\end{aligned} \tag{42}$$

for $t = S+1, S+2, \ldots$, where

$$\begin{aligned}\mathbf{g}_1 &= -\mathbf{B}^T \mathbf{h}_1, \quad \mathbf{g}_2 = -\mathbf{B}^T \mathbf{h}_2, \quad \mathbf{g}_3 = -\mathbf{B}^T \mathbf{h}_3, \quad \mathbf{g}_4 = -\mathbf{B}^T \mathbf{h}_4, \\ \mathbf{f}_2 &= \mathbf{h}_2 - \sum_{s=1}^{S-1} \mathbf{\Gamma}_s \mathbf{h}_2 = \mathbf{\Psi}\mathbf{h}_2, \\ \mathbf{f}_3^s &= \begin{cases} \mathbf{h}_3 & \text{for } s = 0 \\ -\mathbf{\Gamma}_s \mathbf{h}_3 & \text{for } s = 1, 2, \ldots, S-1 \end{cases}, \quad \mathbf{f}_4^s = \sum_{j=s+1}^{S-1} \mathbf{\Gamma}_j \mathbf{h}_4.\end{aligned}$$

In relation to the aforementioned case of model with deterministic and stable development tendency, additional components appeared. The term $\left(\sum_{s=0}^{S-2} \mathbf{f}_4^s \Delta u_{1,t-s}\right)$ is an analogue of $\sum_{s=0}^{S-1} \mathbf{f}_3^s \Delta u_{1,t-s}$. Consequently, $\left(\sum_{s=0}^{S-2} \mathbf{f}_4^s \Delta u_{1,t-s}\right)$ is an additional vector of $-\sum_{s=1}^{S-1} \mathbf{\Gamma}_s \mathbf{H}\Delta \mathbf{d}_{t-s}$, where $\Delta \mathbf{d}_t = \begin{bmatrix} 0 & 1 & \Delta u_{1t} & \Delta u_{2t} \end{bmatrix}^T$. In the I(1) domain, this term can be interpreted

in deceleration categories – decreasing differential rent due to faster growth after structural change. The term $\mathbf{\Psi h}_4 u_{1,t} = \mathbf{f}_4 u_{1,t}$ can be interpreted as the correction of the $\mathbf{f}_2$ slope after a structural break.
It should be noted that $\mathbf{g}_4 = -\mathbf{B}^T\mathbf{h}_4$ means that changes in the slope of the linear trend relate to levels, not first increments. It is a logical consequence of the nature of data generation process and the earlier assumption that $\mathbf{g}_2 = -\mathbf{B}^T\mathbf{h}_2$, which shows that the nonlinear trend does not exist outside the cointegration space.
Model (42) is observationally equivalent to (a slightly different parameterization has been proposed by Trenkler, Saikkonen, Lütkepohl 2008):

$$\begin{aligned} \Delta\mathbf{v}_t &= \mathbf{A}\left[\mathbf{B}^T\mathbf{v}_{t-1} + \mathbf{g}_2(t-1) + \mathbf{g}_4 u_{2,t-1}\right] + \\ &+ \sum_{s=1}^{S-1}\mathbf{\Gamma}_s\Delta\mathbf{v}_{t-s} + \mathbf{f}_2^* + \sum_{s=0}^{S-1}\mathbf{f}_3^{s^*}\Delta u_{1,t-s} + \mathbf{f}_4^* u_{1,t} + \boldsymbol{\xi}_t \end{aligned} \tag{43}$$

for $t = S+1, S+2, ...,$ where

$$\begin{aligned} \mathbf{g}_1 &= -\mathbf{B}^T\mathbf{h}_1, \quad \mathbf{g}_2 = -\mathbf{B}^T\mathbf{h}_2, \quad \mathbf{g}_3 = -\mathbf{B}^T\mathbf{h}_3, \quad \mathbf{g}_4 = -\mathbf{B}^T\mathbf{h}_4, \\ \mathbf{f}_2^* &= \mathbf{\Psi h}_2 + \mathbf{A g}_1, \quad \mathbf{f}_4^* = \mathbf{\Psi h}_4 + \mathbf{A g}_3, \\ \mathbf{f}_3^{s^*} &= \begin{cases} \mathbf{f}_3^s - \mathbf{A g}_3 + \mathbf{f}_4^s & \text{for } s = 0 \\ \mathbf{f}_3^s + \mathbf{f}_4^s & \text{for } s = 1, 2, ..., S-2 \\ \mathbf{f}_3^s & \text{for } s = S-1 \end{cases} \\ &= \begin{cases} \mathbf{h}_3 - \mathbf{A g}_3 + \sum_{j=1}^{S-1}\mathbf{\Gamma}_j\mathbf{h}_4 & \text{for } s = 0 \\ -\mathbf{\Gamma}_s\mathbf{h}_3 + \sum_{j=s+1}^{S-1}\mathbf{\Gamma}_j\mathbf{h}_4 & \text{for } s = 1, 2, ..., S-2 \\ -\mathbf{\Gamma}_{S-1}\mathbf{h}_3 & \text{for } s = S-1 \end{cases} . \end{aligned}$$

Under $\mathbf{h}_3 = \mathbf{0}$ and $\mathbf{h}_4 = \mathbf{0}$, model (43) simplifies to the traditional VECM with a deterministic component containing a constant term and a trend ($\mathbf{v}_t = \mathbf{y}_t + \mathbf{h}_1 + \mathbf{h}_2 t$). In the I(2) domain, it is more convenient to consider the following representation (resulting from (6)):

$$\begin{aligned} \Delta^2\mathbf{v}_t &= \mathbf{A}\left[\mathbf{B}^T\mathbf{v}_{t-1} + \mathbf{g}_1 + \mathbf{g}_2(t-1) + \mathbf{g}_3 u_{1,t-1} + \mathbf{g}_4 u_{2,t-1}\right] + \\ &+ \mathbf{\Gamma}\Delta\mathbf{v}_{t-1} - \mathbf{\Gamma h}_2 - \mathbf{\Gamma h}_3\Delta u_{1,t-1} - \mathbf{\Gamma h}_4\Delta u_{2,t-1} + \\ &+ \sum_{s=1}^{S-2}\mathbf{\Psi}_s\Delta^2\mathbf{v}_{t-s} + \sum_{s=0}^{S-2}\mathbf{f}_3^{s,I(2)}\Delta^2 u_{1,t-s} + \sum_{s=0}^{S-2}\mathbf{f}_4^{s,I(2)}\Delta^2 u_{2,t-s} + \boldsymbol{\xi}_t \end{aligned} \tag{44}$$

for $t = S+1, S+2, ...$, where

$$\begin{aligned}
\mathbf{g}_1 &= -\mathbf{B}^T\mathbf{h}_1, \quad \mathbf{g}_2 = -\mathbf{B}^T\mathbf{h}_2, \quad \mathbf{g}_3 = -\mathbf{B}^T\mathbf{h}_3, \quad \mathbf{g}_4 = -\mathbf{B}^T\mathbf{h}_4, \\
\mathbf{f}_3^{s,I(2)} &= \begin{cases} \mathbf{h}_3 & \text{for } s = 0 \\ -\boldsymbol{\Psi}_s\mathbf{h}_3 & \text{for } s = 1, 2, ..., S-2 \end{cases}, \\
\mathbf{f}_4^{s,I(2)} &= \begin{cases} \mathbf{h}_4 & \text{for } s = 0 \\ -\boldsymbol{\Psi}_s\mathbf{h}_4 & \text{for } s = 1, 2, ..., S-2 \end{cases}
\end{aligned}$$

because the slope of the linear trend is constant in the stability period.
Let us recall that $\Delta u_{2t} = u_{1t}$ and, consequently, $\Delta^2 u_{2t} = \Delta u_{1t}$, which implies that

$$\left(\mathbf{I} - \sum_{s=1}^{S-2}\boldsymbol{\Psi}_s\right)\mathbf{h}_4\Delta u_{1,t} = \boldsymbol{\Psi}\mathbf{h}_4\Delta u_{1,t} = \boldsymbol{\Psi}\mathbf{h}_4\Delta^2 u_{2,t} = \left(\mathbf{I} - \sum_{s=1}^{S-2}\boldsymbol{\Psi}_s\right)\mathbf{h}_4\Delta^2 u_{2,t}.$$

Table 1 presents the comparison of the base model forms and the interpretation of basic matrices and restrictions imposed on them for VECM with a structural change as the complexity of its deterministic component gets complicated.
The key role in all considered models is played by conditions $\mathbf{g}_s = -\mathbf{B}^T\mathbf{h}_s$ $(s = 1, 2, 3, 4)$. The idea of all such restrictions can be understood on the basis of the concept of co-dominant components formulated by Granger, Terasvirta and Patton (2006), which attributes a special role to the matrix which is usually identified with the matrix of baseline cointegrating relations (the role of the cointegrating matrix and the matrix of weights in dependencies other than cointegration is discussed more in detail in Wróblewska 2015). It has been proved by Granger et alia (2006) that cointegration is only the case of mutual annihilation of dominant factors in a properly defined space (a system of variables). The stochastic trends can be recognized as one of these factors. However, co-cyclical (not considered in this paper), cotrending discussed above ($\mathbf{g}_s = -\mathbf{B}^T\mathbf{h}_s$, $s = 1, 2$) or cobreaking, which are implied by restrictions $\mathbf{g}_s = -\mathbf{B}^T\mathbf{h}_s$ $(s = 3, 4)$ can be treated as the co-dominant factors. There is some analogy between cotrending and cointegration. In the cointegration analysis involving the I(1) domain, the stochastic process I(1) is interpreted in terms of a stochastic trend and the I(0) shocks are transitory. In the I(2) domain, the I(1) shock ceases to be the dominant component, as this role is taken over by the I(2) trend which generates a stochastic trend and the shock I(1) is only a stochastic cyclical. In the I(3) domain, which is quite hypothetical from the economic point of view (Majsterek 2008) stochastic trends are I(3) shocks, while I(2) and I(1) shocks are stochastic cycles with respectively longer and shorter fluctuations. The cotrending restriction cannot be reduced in the VECM to $\mathbf{g}_2 = -\mathbf{B}^T\mathbf{h}_2$, because the interpretation of all such restrictions changes with the complexity of the deterministic structure in the VECM. In particular, the simplest type of cotrending can be identified even when the time variable is absent from the DGP. The constant term can then be treated as a deterministic trend of zero order. However, the restriction $\mathbf{g}_1 = -\mathbf{B}^T\mathbf{h}_1$ still plays an annihilation role.

Table 1: Comparison of the deterministic structure for the case of the change in a constant term (case 1), change in intercept and constant linear trend (case 2), the break in both a constant term and the linear trend (case 3)

Case 1: $\mathbf{v}_t = \mathbf{y}_t + \mathbf{h}_1 + \mathbf{h}_3 u_t$	Case 2: $\mathbf{v}_t = \mathbf{y}_t + \mathbf{h}_1 + \mathbf{h}_2 t + \mathbf{h}_3 u_t$	Case 3: $\mathbf{v}_t = \mathbf{y}_t + \mathbf{h}_1 + \mathbf{h}_2 t + \mathbf{h}_3 u_{1t} + \mathbf{h}_4 u_{2t}$
$\Delta\mathbf{v}_t = \mathbf{A}[\mathbf{B}^T\mathbf{v}_{t-1} + \mathbf{g}_1 + \mathbf{g}_3 u_{t-1}] + \\ + \sum_{s=1}^{S-1}\mathbf{\Gamma}_s\Delta\mathbf{v}_{t-s} + \sum_{s=0}^{S-1}\mathbf{f}_3^s\Delta u_{t-s} + \boldsymbol{\xi}_t$	$\Delta\mathbf{v}_t = \mathbf{A}[\mathbf{B}^T\mathbf{v}_{t-1} + \mathbf{g}_1 + \mathbf{g}_2(t-1) + \\ + \mathbf{g}_3 u_{t-1}] + \sum_{s=1}^{S-1}\mathbf{\Gamma}_s\Delta\mathbf{v}_{t-s} + \mathbf{f}_2 + \\ + \sum_{s=0}^{S-1}\mathbf{f}_3^s\Delta u_{t-s} + \boldsymbol{\xi}_t$	$\Delta\mathbf{v}_t = \mathbf{A}[\mathbf{B}^T\mathbf{v}_{t-1} + \mathbf{g}_1 + \mathbf{g}_2(t-1) + \\ + \mathbf{g}_3 u_{1,t-1} + \mathbf{g}_4 u_{2,t-1}] + \\ + \sum_{s=1}^{S-1}\mathbf{\Gamma}_s\Delta\mathbf{v}_{t-s} + \mathbf{f}_2 + \sum_{s=0}^{S-1}\mathbf{f}_3^s\Delta u_{1,t-s} + \\ + \mathbf{\Psi}\mathbf{h}_4 u_{1t} + \sum_{s=1}^{S-1}\mathbf{f}_4^s\Delta u_{1,t-s} + \boldsymbol{\xi}_t$
$\mathbf{g}_1 = -\mathbf{B}^T\mathbf{h}_1$ cotrending of order zero – a constant term (also intercept in cointegrating space) does not generate a trend	$\mathbf{g}_1 = -\mathbf{B}^T\mathbf{h}_1$ cotrending of order zero, this restriction is not dominant (admittedly a constant term does not generate a trend, but there exist another sources of a linear trend)	$\mathbf{g}_1 = -\mathbf{B}^T\mathbf{h}_1$ cotrending of order zero, this restriction is not dominant (admittedly a constant term does not generate a trend, but there exist another sources of a linear trend)
not applicable	$\mathbf{g}_2 = -\mathbf{B}^T\mathbf{h}_2$ cotrending of order one – linear trend (also tendency in cointegrating space) does not generate a quadratic trend	$\mathbf{g}_2 = -\mathbf{B}^T\mathbf{h}_2$ cotrending of order one – linear trend (also tendency in cointegrating space) does not generate a quadratic trend
$\mathbf{g}_3 = -\mathbf{B}^T\mathbf{h}_3$ cobreaking of order zero – changes in DGP mean takes place outside cointegration space, relate to levels, not first increments, this means that structural change leads to a shift, but not to a change in the nature of the trend	$\mathbf{g}_3 = -\mathbf{B}^T\mathbf{h}_3$ cobreaking of order zero – changes in DGP mean take place outside cointegration space, relate to levels, not first increments, this means that structural change leads to a shift, but not to a change in the nature of the trend	$\mathbf{g}_3 = -\mathbf{B}^T\mathbf{h}_3$ cobreaking of order zero, this restriction is not dominant (admittedly changes in a constant term does not generate changes in the nature of trend, but there exist another sources of slope change of a linear trend)

Table 1 (cont.): Comparison of the deterministic structure for the case of the change in a constant term (case 1), change in intercept and constant linear trend (case 2), the break in both a constant term and the linear trend (case 3)

Case 1: $\mathbf{v}_t = \mathbf{y}_t + \mathbf{h}_1 + \mathbf{h}_3 u_t$	Case 2: $\mathbf{v}_t = \mathbf{y}_t + \mathbf{h}_1 + \mathbf{h}_2 t + \mathbf{h}_3 u_t$	Case 3: $\mathbf{v}_t = \mathbf{y}_t + \mathbf{h}_1 + \mathbf{h}_2 t + \mathbf{h}_3 u_{1t} + \mathbf{h}_4 u_{2t}$
not applicable	not applicable	$\mathbf{g}_4 = -\mathbf{B}^T \mathbf{h}_4$ cobreaking of order one (slope change of a linear trends outside cointegration space, relate to levels, not first increments)
$\mathbf{B}^{*T} = \begin{bmatrix} \mathbf{B}^T & \mathbf{g}_1 & \mathbf{g}_3 \end{bmatrix}$	$\mathbf{B}^{*T} = \begin{bmatrix} \mathbf{B}^T & \mathbf{g}_1 & \mathbf{g}_2 & \mathbf{g}_3 \end{bmatrix}$	$\mathbf{B}^{*T} = \begin{bmatrix} \mathbf{B}^T & \mathbf{g}_1 & \mathbf{g}_2 & \mathbf{g}_3 & \mathbf{g}_4 \end{bmatrix}$
$\mathbf{f}_1 = \mathbf{A}\mathbf{g}_1 = -\mathbf{A}\mathbf{B}^T \mathbf{h}_1$ purely technical transformation relating to the constant term	$\mathbf{f}_1 = \mathbf{A}\mathbf{g}_1 = -\mathbf{A}\mathbf{B}^T \mathbf{h}_1$ purely technical transformation relating to the constant term	$\mathbf{f}_1 = \mathbf{A}\mathbf{g}_1 = -\mathbf{A}\mathbf{B}^T \mathbf{h}_1$ purely technical transformation relating to the constant term
not applicable	$\mathbf{f}_2 = \mathbf{h}_2 - \sum_{s=1}^{S-1} \mathbf{\Gamma}_s \mathbf{h}_2 = \mathbf{\Psi} \mathbf{h}_2$ removing the linear trend from short-term relations (at the first increments), thus preventing the square trend in levels, in I(1) domain matrices act as decelerators of the impact of structural change	$\mathbf{f}_2 = \mathbf{h}_2 - \sum_{s=1}^{S-1} \mathbf{\Gamma}_s \mathbf{h}_2 = \mathbf{\Psi} \mathbf{h}_2$ removing the linear trend from short-term relations (at the first increments), thus preventing the square trend in levels, in I(1) domain matrices act as decelerators of the impact of structural change

Table 1 (cont.): Comparison of the deterministic structure for the case of the change in a constant term (case 1), change in intercept and constant linear trend (case 2), the break in both a constant term and the linear trend (case 3)

Case 1: $\mathbf{v}_t = \mathbf{y}_t + \mathbf{h}_1 + \mathbf{h}_3 u_t$	Case 2: $\mathbf{v}_t = \mathbf{y}_t + \mathbf{h}_1 + \mathbf{h}_2 t + \mathbf{h}_3 u_t$	Case 3: $\mathbf{v}_t = \mathbf{y}_t + \mathbf{h}_1 + \mathbf{h}_2 t + \mathbf{h}_3 u_{1t} + \mathbf{h}_4 u_{2t}$
not applicable	$\mathbf{f}_2 \Delta^2 t = \mathbf{f}_2 0 = \mathbf{0}$ no linear trend acceleration (expected value of such acceleration is zero)	$\mathbf{f}_2 \Delta^2 t = \mathbf{f}_2 0 = \mathbf{0}$ no linear trend acceleration (expected value of such acceleration is zero)
not applicable	$\mathbf{f}_2^* = \mathbf{A}\mathbf{g}_1 + \mathbf{f}_2$ technical transformation aimed at removing a constant term outside cointegration space, also removes nonidentification of the model‘s parameters	$\mathbf{f}_2^* = \mathbf{A}\mathbf{g}_1 + \mathbf{f}_2$ technical transformation aimed at removing a constant term outside cointegration space, also removes nonidentification of the model‘s parameters
not applicable	not applicable	$\mathbf{f}_4^s = \sum_{j=s+1}^{S-1} \mathbf{\Gamma}_j \mathbf{h}_4$ removing the change in a linear trend slope from short-term relations (at the first increments), a consequence of the restriction captured on $\mathbf{f}_2$(see above), in I(1) domain matrices act as decelerators of the impact of structural change
not applicable	not applicable	$\mathbf{f}_4^{s,I(2)} = \begin{cases} \mathbf{h}_4 & \text{for } s = 0 \\ -\mathbf{\Psi}_s \mathbf{h}_4 & \text{for } s = 1, 2, ..., S-2 \end{cases}$ removing the change in a linear trend slope from short-term relations (at the first increments), a consequence of the restriction captured on $\mathbf{f}_2$ (see above), in I(2) domain matrices does not act as decelerators of the impact of structural change (only LHS second increments are stationary)

Table 1 (cont.): Comparison of the deterministic structure for the case of the change in a constant term (case 1), change in intercept and constant linear trend (case 2), the break in both a constant term and the linear trend (case 3)

Case 1: $\mathbf{v}_t = \mathbf{y}_t + \mathbf{h}_1 + \mathbf{h}_3 u_t$	Case 2: $\mathbf{v}_t = \mathbf{y}_t + \mathbf{h}_1 + \mathbf{h}_2 t + \mathbf{h}_3 u_t$	Case 3: $\mathbf{v}_t = \mathbf{y}_t + \mathbf{h}_1 + \mathbf{h}_2 t + \mathbf{h}_3 u_{1t} + \mathbf{h}_4 u_{2t}$
not applicable	not applicable	$\mathbf{f}_4^* = \mathbf{\Psi}\mathbf{h}_4 + \mathbf{A}\mathbf{g}_3$ technical transformation aimed at removing changes in a constant term outside cointegration space
$\mathbf{f}_3^s = \begin{cases} \mathbf{h}_3 & \text{for } s = 0 \\ -\mathbf{\Gamma}_s \mathbf{h}_3 & \text{for } s = 1, 2, ..., S-1 \end{cases}$ in I(1) domain matrices act as decelerators of the impact of structural change	$\mathbf{f}_3^s = \begin{cases} \mathbf{h}_3 & \text{for } s = 0 \\ -\mathbf{\Gamma}_s \mathbf{h}_3 & \text{for } s = 1, 2, ..., S-1 \end{cases}$ in I(1) domain matrices act as decelerators of the impact of structural change	$\mathbf{f}_3^s = \begin{cases} \mathbf{h}_3 & \text{for } s = 0 \\ -\mathbf{\Gamma}_s \mathbf{h}_3 & \text{for } s = 1, 2, ..., S-1 \end{cases}$ in I(1) domain matrices act as decelerators of the impact of structural change
$\mathbf{f}_3^{s,I(2)} = \begin{cases} \mathbf{h}_3 & \text{for } s = 0 \\ -\mathbf{\Psi}_s \mathbf{h}_3 & \text{for } s = 1, 2, ..., S-2 \end{cases}$ in I(2) domain matrices does not act as decelerators of the impact of structural change (only LHS second increments are stationary)	$\mathbf{f}_3^{s,I(2)} = \begin{cases} \mathbf{h}_3 & \text{for } s = 0 \\ -\mathbf{\Psi}_s \mathbf{h}_3 & \text{for } s = 1, 2, ..., S-2 \end{cases}$ in I(2) domain matrices does not act as decelerators of the impact of structural change (only LHS second increments are stationary)	$\mathbf{f}_3^{s,I(2)} = \begin{cases} \mathbf{h}_3 & \text{for } s = 0 \\ -\mathbf{\Psi}_s \mathbf{h}_3 & \text{for } s = 1, 2, ..., S-2 \end{cases}$ in I(2) domain matrices does not act as decelerators of the impact of structural change (only LHS second increments are stationary)

In turn, generalizing the discussion of cotrending for the case of polynomial trends of higher order (such as a quadratic trend), the restriction $\mathbf{g}_2 = -\mathbf{B}^T\mathbf{h}_2$ is neither a necessary condition nor, more importantly, a sufficient one for cotrending to occur, because such a role is played by $\mathbf{g}_5 = -\mathbf{B}^T\mathbf{h}_5$ (provided that the deterministic component consists of a constant term, a linear trend, a change of intercept, change of a linear trend and a quadratic trend, i.e. $\mathbf{H} = [\mathbf{h}_1 \vdots \mathbf{h}_2 \vdots \mathbf{h}_3 \vdots \mathbf{h}_4 \vdots \mathbf{h}_5]$, $\mathbf{d}_t = \begin{bmatrix} 1 & t & u_{1t} & u_{2t} & t^2 \end{bmatrix}$). An interesting issue is that the interpretation of some aspects of cotrending changes depending on the complexity of the deterministic structure of the model. In the simplest case, $\mathbf{g}_1 = -\mathbf{B}^T\mathbf{h}_1$ prevents the constant term from generating a linear trend (cotrending of order zero). In the case of a linear trend, $\mathbf{g}_2 = -\mathbf{B}^T\mathbf{h}_2$ (cotrending of order one) prevents the linear trend included in cointegration regression from generating a quadratic trend in the data. The simultaneous fulfilment of $\mathbf{g}_1 = -\mathbf{B}^T\mathbf{h}_2$ (it becomes a dominated restriction) indicates the absence of any additional sources of the linear trend. Conditions $\mathbf{g}_3 = -\mathbf{B}^T\mathbf{h}_3$ and $\mathbf{g}_4 = -\mathbf{B}^T\mathbf{h}_4$ that result directly from $\mathbf{g}_1 = -\mathbf{B}^T\mathbf{h}_1$ and $\mathbf{g}_2 = -\mathbf{B}^T\mathbf{h}_2$, respectively, should be interpreted in the same way, taking account that instead of cotrending the cobreaking occurs. Also in this case, it can be seen that condition $\mathbf{g}_3 = -\mathbf{B}^T\mathbf{h}_3$ (interpreted in terms of cobreaking when only the intercept is changed) is not a dominant restriction when the change also takes place in the deterministic trend inherent in the system. Condition $\mathbf{g}_3 = -\mathbf{B}^T\mathbf{h}_3$ means that the change in the constant term does not affect the deterministic trend (cobreaking of order zero). If this restriction is dominated by $\mathbf{g}_4 = -\mathbf{B}^T\mathbf{h}_4$ (cobreaking of order one), the change in the constant term does not directly affect the deterministic trend, but at the same time the linear tendency changes (it does not generate nor change the nonlinear trends, because of the absence of non-linear trends not only in the cointegrating space, but also in the data).

4 Vector Error Correction Model with the structural change in stochastic component

Let us assume that a change only takes place in the stochastic part of the DGP. Additionally, a structural change occurs only in one period t_0. If the deterministic part consists of the constant term and the trend, then the data generating process is given by $\mathbf{v}_t = \mathbf{y}_t + \mathbf{h}_1 + \mathbf{h}_2 t$, where $\mathbf{y}_t$ has a VAR representation with time varying parameters:

$$\mathbf{y}_t = (1 - u_t) \sum_{s=1}^{S} \mathbf{\Pi}_{s,1} \mathbf{y}_{t-s} + u_t \sum_{s=1}^{S} \mathbf{\Pi}_{s,2} \mathbf{y}_{t-s} + \boldsymbol{\xi}_t, \tag{45}$$

where u_t is defined as:

$$u_t = \begin{cases} 0 & \text{for } t < t_0 \\ 1 & \text{for } t \geq t_0 \end{cases}.$$

Given that:

$$\begin{aligned} \Delta \mathbf{v}_t &= (1-u_t)\left[\mathbf{\Pi}_1 \mathbf{y}_{t-1} + \sum_{s=1}^{S-1} \Gamma_{s,1} \Delta \mathbf{y}_{t-s}\right] + \\ &\quad + u_t \left[\mathbf{\Pi}_2 \mathbf{y}_{t-1} + \sum_{s=1}^{S-1} \Gamma_{s,2} \Delta \mathbf{y}_{t-s}\right] + (1-u_t)\mathbf{H}\Delta \mathbf{d}_t + u_t \mathbf{H} \Delta \mathbf{d}_t + \boldsymbol{\xi}_t, \end{aligned} \tag{46}$$

VECM for observable variables is as follows:

$$\begin{aligned} \Delta \mathbf{v}_t &= (1-u_t)\left[\mathbf{\Pi}_1 \mathbf{v}_{t-1} - \mathbf{\Pi}_1 \mathbf{H} \mathbf{d}_{t-1} + \sum_{s=1}^{S-1} \Gamma_{s,1} \Delta \mathbf{v}_{t-s} - \sum_{s=1}^{S-1} \Gamma_{s,1} \mathbf{H} \Delta \mathbf{d}_{t-s}\right] + \\ &\quad + u_t \left[\mathbf{\Pi}_2 \mathbf{v}_{t-1} - \mathbf{\Pi}_2 \mathbf{H} \mathbf{d}_{t-1} + \sum_{s=1}^{S-1} \Gamma_{s,2} \Delta \mathbf{v}_{t-s} - \sum_{s=1}^{S-1} \Gamma_{s,2} \mathbf{H} \Delta \mathbf{d}_{t-s}\right] + \\ &\quad + (1-u_t)\mathbf{H}\Delta \mathbf{d}_t + u_t \mathbf{H}\Delta \mathbf{d}_t + \boldsymbol{\xi}_t \end{aligned} \tag{47}$$

because $\mathbf{\Pi}_i \mathbf{y}_{t-1} = \mathbf{\Pi}_i \mathbf{v}_{t-1} - \mathbf{\Pi}_i \mathbf{H} \mathbf{d}_{t-1}$, $i = 1, 2$ and $\Delta \mathbf{y}_{t-s} = \Delta \mathbf{v}_{t-s} - \mathbf{H} \Delta \mathbf{d}_{t-s}$. For each regime it can be transformed in the same way as in the cases of changes in the deterministic part (cf. 27):

$$\begin{aligned} \Delta \mathbf{v}_t &= (1-u_t)\left\{\mathbf{A}_1 \mathbf{B}_1^T \mathbf{v}_{t-1} - \mathbf{A}_1 \mathbf{B}_1^T \mathbf{h}_1 - \mathbf{A}_1 \mathbf{B}_1^T \mathbf{h}_2 (t-1) + \sum_{s=1}^{S-1} \mathbf{\Gamma}_{s,1} \Delta \mathbf{v}_{t-s}\right\} + \\ &\quad + u_t \left\{\mathbf{A}_2 \mathbf{B}_2^T \mathbf{v}_{t-1} - \mathbf{A}_2 \mathbf{B}_2^T \mathbf{h}_1 - \mathbf{A}_2 \mathbf{B}_2^T \mathbf{h}_2 (t-1) + \sum_{s=1}^{S-1} \mathbf{\Gamma}_{s,2} \Delta \mathbf{v}_{t-s}\right\} + \\ &\quad - u_t \sum_{s=1}^{S-1} \mathbf{\Gamma}_{s,2} \mathbf{H} \Delta \mathbf{d}_{t-s} - (1-u_t) \sum_{s=1}^{S-1} \mathbf{\Gamma}_{s,1} \mathbf{H} \Delta \mathbf{d}_{t-s} + \\ &\quad + (1-u_t)\mathbf{H}\Delta \mathbf{d}_t + u_t \mathbf{H}\Delta \mathbf{d}_t + \boldsymbol{\xi}_t \\ &= (1-u_t)\mathbf{A}_1 \left[\mathbf{B}_1^T \mathbf{v}_{t-1} + \mathbf{g}_{1,1} + \mathbf{g}_{2,1}(t-1)\right] + u_t \mathbf{A}_2 [\mathbf{B}_2^T \mathbf{v}_{t-1} + \mathbf{g}_{1,2} + \\ &\quad + \mathbf{g}_{2,2}(t-1)] + (1-u_t) \sum_{s=1}^{S-1} \mathbf{\Gamma}_{s,1} \Delta \mathbf{v}_{t-s} + u_t \sum_{s=1}^{S-1} \mathbf{\Gamma}_{s,2} \Delta \mathbf{v}_{t-s} + \\ &\quad + (1-u_t)\mathbf{f}_{2,1} + u_t \mathbf{f}_{2,2} + \boldsymbol{\xi}_t \end{aligned} \tag{48}$$

for $t = S+1, S+2, ...$, where

$$\begin{aligned}\mathbf{g}_{1,1} &= -\mathbf{B}_1^T\mathbf{h}_1, \quad \mathbf{g}_{1,2} = -\mathbf{B}_2^T\mathbf{h}_1, \quad \mathbf{g}_{2,1} = -\mathbf{B}_1^T\mathbf{h}_2, \quad \mathbf{g}_{2,2} = -\mathbf{B}_2^T\mathbf{h}_2,\\ \mathbf{f}_{2,1} &= \mathbf{\Psi}_{(1)}\mathbf{h}_2, \quad \mathbf{f}_{2,2} = \mathbf{\Psi}_{(2)}\mathbf{h}_2,\\ \mathbf{\Psi}_{(1)} &= -\mathbf{\Gamma}_{(1)} = -\left(\sum_{s=1}^{S-1}\mathbf{\Gamma}_{s,1} - \mathbf{I}\right), \quad \mathbf{\Psi}_{(2)} = -\mathbf{\Gamma}_{(2)} = -\left(\sum_{s=1}^{S-1}\mathbf{\Gamma}_{s,2} - \mathbf{I}\right)\end{aligned}$$

which is equivalent to (an alternative representation has been proposed in Gosińska 2015):

$$\begin{aligned}\Delta\mathbf{v}_t &= (1-u_t)\mathbf{A}_1\left[\mathbf{B}_1^T\mathbf{v}_{t-1} + \mathbf{g}_{2,1}(t-1)\right] + u_t\mathbf{A}_2\left[\mathbf{B}_2^T\mathbf{v}_{t-1} + \mathbf{g}_{2,2}(t-1)\right] + \\ &\quad +(1-u_t)\sum_{s=1}^{S-1}\mathbf{\Gamma}_{s,1}\Delta\mathbf{v}_{t-s} + u_t\sum_{s=1}^{S-1}\mathbf{\Gamma}_{s,2}\Delta\mathbf{v}_{t-s} + (1-u_t)\mathbf{f}_{2,1}^* + u_t\mathbf{f}_{2,2}^* + \boldsymbol{\xi}_t,\end{aligned} \tag{49}$$

where $\mathbf{f}_{1,2}^* = \mathbf{A}_1\mathbf{g}_{1,1} + \mathbf{f}_{2,1}$, $\mathbf{f}_{2,2}^* = \mathbf{A}_2\mathbf{g}_{1,2} + \mathbf{f}_{2,2}$.
The key role of the matrix $\mathbf{\Psi}$ is visible (this is the same matrix which in I(2) domain becomes interpretation in terms medium-run CI(1,1) cointegration matrix). The matrix $\mathbf{\Psi}$ describes the transmission of structural changes. A structural change in the stochastic part of the data generating process is also transferred into changes in the parameters associated with the deterministic variables, which makes this case clearly different from that considered previously. A change in the cointegration mechanism implies a change in the cotrending mechanism. Obviously, the cobreaking mechanisms should not be considered if a change only affects the stochastic processes.
In the presence of the stochastic process I(2), VECM can be written as:

$$\begin{aligned}\Delta^2\mathbf{v}_t &= (1-u_t)\left[\mathbf{A}_1\mathbf{B}_1^T\mathbf{y}_{t-1} + \mathbf{\Gamma}_{(1)}\Delta\mathbf{y}_{t-1} + \sum_{s=1}^{S-2}\mathbf{\Psi}_{s,1}\Delta^2\mathbf{y}_{t-s}\right] + \\ &\quad +u_t\left[\mathbf{A}_2\mathbf{B}_2^T\mathbf{y}_{t-1} + \mathbf{\Gamma}_{(2)}\Delta\mathbf{y}_{t-1} + \sum_{s=1}^{S-2}\mathbf{\Psi}_{s,2}\Delta^2\mathbf{y}_{t-s}\right] + \mathbf{H}\Delta^2\mathbf{d}_t + \boldsymbol{\xi}_t\end{aligned} \tag{50}$$

where, again, $\mathbf{v}_t = \mathbf{y}_t + \mathbf{h}_1 + \mathbf{h}_2 t$ (so $\Delta^2\mathbf{v}_t = \Delta^2\mathbf{y}_t + \Delta^2\mathbf{h}_1 + \Delta^2\mathbf{h}_2 = \Delta^2\mathbf{y}_t$ and $\mathbf{H}\Delta^2\mathbf{d}_t$ can be omitted).
The representation (50) can be transformed in the same way as in the cases involving

changes in the deterministic part:

$$\begin{aligned}\Delta^2\mathbf{v}_t &= (1-u_t)\mathbf{A}_1\left[\mathbf{B}_1^T\mathbf{v}_{t-1}+\mathbf{g}_{1,1}+\mathbf{g}_{2,1}(t-1)\right]+ \\ &+u_t\mathbf{A}_2\left[\mathbf{B}_2^T\mathbf{v}_{t-1}+\mathbf{g}_{1,2}+\mathbf{g}_{2,2}(t-1)\right]+ \\ &+u_t\mathbf{\Gamma}_{(2)}(\Delta\mathbf{v}_{t-1}-\Delta\mathbf{h}_1-\mathbf{h}_2\Delta(t-1))+ \\ &+(1-u_t)\mathbf{\Gamma}_{(1)}(\Delta\mathbf{v}_{t-1}-\Delta\mathbf{h}_1-\mathbf{h}_2\Delta(t-1))+ \\ &+(1-u_t)\sum_{s=1}^{S-2}\mathbf{\Psi}_{s,1}\Delta^2\mathbf{v}_{t-s}+u_t\sum_{s=1}^{S-2}\mathbf{\Psi}_{s,2}\Delta^2\mathbf{v}_{t-s}+ \\ &+(1-u_t)\mathbf{f}_{2,1}^{I(2)}\Delta t^2+u_t\mathbf{f}_{2,2}^{I(2)}\Delta t^2+\boldsymbol{\xi}_t\end{aligned} \tag{51}$$

for $t = S+1, S+2, ...$, where $\mathbf{g}_{1,1} = -\mathbf{B}_1^T\mathbf{h}_1$, $\mathbf{g}_{1,2} = -\mathbf{B}_2^T\mathbf{h}_1$, $\mathbf{g}_{2,1} = -\mathbf{B}_1^T\mathbf{h}_2$, $\mathbf{g}_{2,2} = -\mathbf{B}_2^T\mathbf{h}_2$.
Due to the linearity of a trend, $\mathbf{f}_{2,1}^{I(2)}\Delta t^2 = \mathbf{f}_{2,2}^{I(2)}\Delta t^2 = \mathbf{0}$ and $\Delta\mathbf{h}_1 = \mathbf{h}_1 - \mathbf{h}_1 = \mathbf{0}$ because of no acceleration of a linear trend.
Then (51) simplifies to:

$$\begin{aligned}\Delta^2\mathbf{v}_t &= (1-u_t)\mathbf{A}_1\left[\mathbf{B}_1^T\mathbf{v}_{t-1}+\mathbf{g}_{1,1}+\mathbf{g}_{2,1}(t-1)\right]+ \\ &+u_t\mathbf{A}_2\left[\mathbf{B}_2^T\mathbf{v}_{t-1}+\mathbf{g}_{1,2}+\mathbf{g}_{2,2}(t-1)\right]+ \\ &+u_t\mathbf{\Gamma}_{(2)}(\Delta\mathbf{v}_{t-1}-\mathbf{h}_2)+(1-u_t)\mathbf{\Gamma}_{(1)}(\Delta\mathbf{v}_{t-1}-\mathbf{h}_2)+ \\ &+(1-u_t)\sum_{s=1}^{S-2}\mathbf{\Psi}_{s,1}\Delta^2\mathbf{v}_{t-s}+u_t\sum_{s=1}^{S-2}\mathbf{\Psi}_{s,2}\Delta^2\mathbf{v}_{t-s}+\boldsymbol{\xi}_t.\end{aligned}$$

The analysis of Table 2 allows some general conclusions on the DGP and makes it possible to compare the effects of changes in the stochastic and deterministic parts of the model. For simplicity, the table is limited to the model with a change in the constant term and in the linear development tendency. Firstly, in contrast to the changes considered in the second section, the assumption about the integration order of the DGP plays a key role in the case of changes in stochastic part of DGP. This conclusion is clear because the change in stochastic mechanisms that generate data is crucial in the analysed case.
The second conclusion is less obvious. A change in the stochastic part of the DGP affects both the stochastic part of all VECM and the deterministic part (structure), which must be appropriately changed. This seemingly unexpected result is due to the fact that $\mathbf{v}_t = \mathbf{y}_t + \mathbf{h}_1 + \mathbf{h}_2 t$. It is not possible to transform VECM for $\Delta\mathbf{v}_t$ in a manner that the changes in the deterministic part of this model after changing DGP for $\mathbf{y}_t$ are avoided. The reason for this is formula (4a), which is the straight consequence of the additive nature of the considered process and consequently is the base explanation for the observed changes in the deterministic part caused by the changes in the stochastic part.

Table 2: Degree of complexity comparison of the deterministic structure for the case of the change of a constant term and a trend and the stochastic process I(1) and I(2)

Case 1: $\mathbf{v}_t = \mathbf{y}_t + \mathbf{h}_1 + \mathbf{h}_2 t + \mathbf{h}_3 u_{1t} + \mathbf{h}_4 u_{2t}$ change in deterministic part	Case 2: $\mathbf{v}_t = \mathbf{y}_t + \mathbf{h}_1 + \mathbf{h}_2 t$ change in stochastic I(1) process	Case 3: $\mathbf{v}_t = \mathbf{y}_t + \mathbf{h}_1 + \mathbf{h}_2 t$ change in stochastic I(2) process
$\Delta\mathbf{v}_t = \mathbf{A}[\mathbf{B}^T\mathbf{v}_{t-1} + \mathbf{g}_1 + \mathbf{g}_2(t-1) + $ $+\mathbf{g}_3 u_{1,t-1} + \mathbf{g}_4 u_{2,t-1}] + \sum_{s=1}^{S-1}\mathbf{\Gamma}_s\Delta\mathbf{v}_{t-s} +$ $+\mathbf{f}_2 + \sum_{s=0}^{S-1}\mathbf{f}_3^s\Delta u_{1,t-s} + \mathbf{\Psi}\mathbf{h}_4 u_{1t} +$ $+\sum_{s=0}^{S-1}\mathbf{f}_4^s\Delta u_{1,t-s} + \boldsymbol{\xi}_t$	$\Delta\mathbf{v}_t = (1-u_t)\mathbf{A}_1[\mathbf{B}_1^T\mathbf{v}_{t-1} +$ $+\mathbf{g}_{2,1}(t-1)] + u_t\mathbf{A}_2[\mathbf{B}_2^T\mathbf{v}_{t-1} +$ $+\mathbf{g}_{2,2}(t-1)] + (1-u_t)\sum_{s=1}^{S-1}\mathbf{\Gamma}_{s,1}\Delta\mathbf{v}_{t-s} +$ $+u_t\sum_{s=1}^{S-1}\mathbf{\Gamma}_{s,2}\Delta\mathbf{v}_{t-s} + (1-u_t)\mathbf{f}_{2,1}^* +$ $+u_t\mathbf{f}_{2,2}^* + \boldsymbol{\xi}_t$	$\Delta^2\mathbf{v}_t = (1-u_t)\mathbf{A}_1[\mathbf{B}_1^T\mathbf{v}_{t-1} + \mathbf{g}_{1,1} +$ $+\mathbf{g}_{2,1}(t-1)] + u_t\mathbf{A}_2[\mathbf{B}_2^T\mathbf{v}_{t-1} + \mathbf{g}_{1,2} +$ $+\mathbf{g}_{2,2}(t-1)] + u_t\mathbf{\Gamma}_{(2)}(\Delta\mathbf{v}_{t-1} - \mathbf{h}_2) +$ $+(1-u_t)\mathbf{\Gamma}_{(1)}(\Delta\mathbf{v}_{t-1} - \mathbf{h}_2) +$ $+(1-u_t)\sum_{s=1}^{S-2}\mathbf{\Psi}_{s,1}\Delta^2\mathbf{v}_{t-s} +$ $+u_t\sum_{s=1}^{S-2}\mathbf{\Psi}_{s,2}\Delta^2\mathbf{v}_{t-s} + \boldsymbol{\xi}_t$
$\mathbf{g}_1 = -\mathbf{B}^T\mathbf{h}_1$ cotrending of order zero, this restriction is not dominant (admittedly a constant term does not generate trend, but there exist another sources of a linear trend)	$\mathbf{g}_{1,1} = -\mathbf{B}_1^T\mathbf{h}_1$ $\mathbf{g}_{1,2} = -\mathbf{B}_2^T\mathbf{h}_1$ these constraints inform us about the rate of cotrending processes of order zero	$\mathbf{g}_{1,1} = -\mathbf{B}_1^T\mathbf{h}_1$ $\mathbf{g}_{1,2} = -\mathbf{B}_2^T\mathbf{h}_1$ these constraints inform us about the rate of cotrending processes of order zero
$\mathbf{g}_2 = -\mathbf{B}^T\mathbf{h}_2$ cotrending of order one – linear trend (also tendency in cointegrating space) does not generate a quadratic trend	$\mathbf{g}_{2,1} = -\mathbf{B}_1^T\mathbf{h}_2$ $\mathbf{g}_{2,2} = -\mathbf{B}_2^T\mathbf{h}_2$ these cotrending restrictions are not nested in co-dominant components, these constraints inform us about the rate of cotrending processes of order one	$\mathbf{g}_{2,1} = -\mathbf{B}_1^T\mathbf{h}_2$ $\mathbf{g}_{2,2} = -\mathbf{B}_2^T\mathbf{h}_2$ these cotrending restrictions are not nested in co-dominant components, these constraints inform us about the rate of cotrending processes of order one

Table 2 (cont.): Degree of complexity comparison of the deterministic structure for the case of the change of a constant term and a trend and the stochastic process I(1) and I(2)

Case 1: $\mathbf{v}_t = \mathbf{y}_t + \mathbf{h}_1 + \mathbf{h}_2 t + \mathbf{h}_3 u_{1t} + \mathbf{h}_4 u_{2t}$ change in deterministic part	Case 2: $\mathbf{v}_t = \mathbf{y}_t + \mathbf{h}_1 + \mathbf{h}_2 t$ change in stochastic I(1) process	Case 3: $\mathbf{v}_t = \mathbf{y}_t + \mathbf{h}_1 + \mathbf{h}_2 t$ change in stochastic I(2) process
$\mathbf{g}_3 = -\mathbf{B}^T \mathbf{h}_3$ cobreaking of order zero, this restriction is not dominant (admittedly changes in a constant term does not generate in the nature of a trend, but there exist another sources of slope change of a linear trend)	not applicable	not applicable
$\mathbf{g}_4 = -\mathbf{B}^T \mathbf{h}_4$ cobreaking of order one (slope change of a linear trends outside cointegration space, relate to levels, not first increments)	not applicable	not applicable
$\mathbf{B}^{*T} = \begin{bmatrix} \mathbf{B}^T & \mathbf{g}_1 & \mathbf{g}_2 & \mathbf{g}_3 & \mathbf{g}_4 \end{bmatrix}$	$\mathbf{B}_1^{*T} = \begin{bmatrix} \mathbf{B}_1^T & \mathbf{g}_{1,1} & \mathbf{g}_{2,1} \end{bmatrix}$ $\mathbf{B}_2^{*T} = \begin{bmatrix} \mathbf{B}_2^T & \mathbf{g}_{1,2} & \mathbf{g}_{2,2} \end{bmatrix}$	$\mathbf{B}_1^{*T} = \begin{bmatrix} \mathbf{B}_1^T & \mathbf{g}_{1,1} & \mathbf{g}_{2,1} \end{bmatrix}$ $\mathbf{B}_2^{*T} = \begin{bmatrix} \mathbf{B}_2^T & \mathbf{g}_{1,2} & \mathbf{g}_{2,2} \end{bmatrix}$
$\mathbf{f}_2 = \mathbf{h}_2 - \sum_{s=1}^{S-1} \boldsymbol{\Gamma}_s \mathbf{h}_2 = \boldsymbol{\Psi} \mathbf{h}_2$ removing the linear trend from short-term relations (at the first increments), thus preventing the square trend in levels, in I(1) domain matrices act as decelerators of the impact of structural change	$\mathbf{f}_{2,1} = \boldsymbol{\Psi}_{(1)} \mathbf{h}_2$ $\mathbf{f}_{2,2} = \boldsymbol{\Psi}_{(2)} \mathbf{h}_2$ removing the linear trend from short-term relations (at the first increments) in both regimes, thus preventing the square trend in levels	$\mathbf{f}_{2,1}^{I(2)} \Delta t^2 = \mathbf{0}$ $\mathbf{f}_{2,2}^{I(2)} \Delta t^2 = \mathbf{0}$ expected value of increments is zero both before and after structural change

Table 2 (cont.): Degree of complexity comparison of the deterministic structure for the case of the change of a constant term and a trend and the stochastic process I(1) and I(2)

Case 1: $\mathbf{v}_t = \mathbf{y}_t + \mathbf{h}_1 + \mathbf{h}_2 t + \mathbf{h}_3 u_{1t} + \mathbf{h}_4 u_{2t}$ change in deterministic part	Case 2: $\mathbf{v}_t = \mathbf{y}_t + \mathbf{h}_1 + \mathbf{h}_2 t$ change in stochastic I(1) process	Case 3: $\mathbf{v}_t = \mathbf{y}_t + \mathbf{h}_1 + \mathbf{h}_2 t$ change in stochastic I(2) process
$\mathbf{f}_2^* = \mathbf{A}\mathbf{g}_1 + \mathbf{f}_2 = \mathbf{\Psi}\mathbf{h}_2 + \mathbf{A}\mathbf{g}_1$ technical transformation aimed at removing a constant term outside cointegration space	$\mathbf{f}_{2,1}^* = \mathbf{A}_1\mathbf{g}_{1,1} + \mathbf{f}_{2,1}$ $\mathbf{f}_{2,2}^* = \mathbf{A}_2\mathbf{g}_{1,2} + \mathbf{f}_{2,2}$ technical transformations aimed at removing a constant term outside cointegration spaces in both regime	$\mathbf{f}_{2,1}^{I(2)}\Delta t^2 = \mathbf{0}$ $\mathbf{f}_{2,2}^{I(2)}\Delta t^2 = \mathbf{0}$
$\mathbf{f}_4^s = \sum_{j=s+1}^{S-1} \mathbf{\Gamma}_j \mathbf{h}_4$ removing the change in a linear trend slope from short-term relations (at the first increments), a consequence of the restriction captured on $\mathbf{f}_2$(see above), in I(1) domain matrices act as decelerators of the impact of structural change	not applicable	not applicable
$\mathbf{f}_4^* = \mathbf{\Psi}\mathbf{h}_4 + \mathbf{A}\mathbf{g}_3$ technical transformation aimed at removing changes in a constant term outside cointegration space	not applicable	not applicable
$\mathbf{f}_3^s = \begin{cases} \mathbf{h}_3 & \text{for } s = 0 \\ -\mathbf{\Gamma}_s\mathbf{h}_3 & \text{for } s = 1, 2, ..., S-1 \end{cases}$ in I(1) domain matrices act as decelerators of the impact of structural change	not applicable	not applicable

In the second part of this paper, formula (4a) had only a limited interpretation of the total change of process decomposition into change caused by the stochastic and deterministic factors. With regard to the structural change in the stochastic part, it also reveals changes in the process generating first increments in variables. The changes can be taken outside the cointegration space in both the I(1) the I(2) domain. In the first case, it is due to the fact that first increments are stationary and so they do not cointegrate, so they can be considered exogenous from the cointegration analysis point of view. In the I(2) domain, first increments may participate in CI(1,1) cointegration relations, but they take place in the space of medium-run relationships, so moving them outside the space of long-run equilibrium compounds is fully justified. It is also notable that the order of DGP integration does not significantly modify the change in the deterministic part of the model (the differences are of slight modification).

If additionally $\mathbf{\Gamma}_{(k)}\mathbf{h}_2 = -\mathbf{A}_k\mathbf{g}_{1,k}$ $(k = 1, 2)$ then it guarantees no quadratic trend in data, meaning that the changes in the stochastic part may not be free. The proportions between adjustment processes with respect to cotrending mechanisms in the long- and medium-run equilibrium cointegration subspace must be preserved if the structural change is in the stochastic process only.

The above considerations can be generalized for a larger number of structural changes, including changes discussed in the second section of the deterministic structure and changes in the stochastic part of the DGP. However, it should definitely be expected that the number of such changes (especially with reference to the system containing I(2) processes) should not be too large. Particular cases of structural changes under consideration in this part are not only quantitative but also qualitative changes in the sense that the integration order of the stochastic process changes. One should only assume that these changes are more frequent in nature, but it is very difficult to consider such changes of a stepwise nature. A discussion of the problems in determining the optimal moment of structural change can be found in Gosińska (2015).

5 Summary

A very important conclusion from the study is that the integration order of stochastic component of the data generating process does not significantly affect VECM when the structural change only takes place in the deterministic part (the differences between VECM are caused by the optimal VECM transformation different in the I(2) case). This may be explained by the fact that the structural change took place only in the deterministic part. On the other hand, the cotrending and cobreaking restrictions should be interpreted with some caution. It is clear that there is a hierarchy of restrictions $\mathbf{g}_n = -\mathbf{B}^T\mathbf{h}_n$ $(n = 1, ..., N)$, with the coexistence of several types of such restrictions, and those for the highest n are dominant.

The structural change in the stochastic part of the data generating process is completely different approach. In Section 4 it has been demonstrated that this kind of change affects not only the stochastic structure but also the deterministic part of all relevant VECM representations.

The paper presents interconnections between the stochastic and deterministic components of the DGP. The conclusions from the third and fourth sections seem to suggest that the stochastic process takes precedence in relation to the deterministic tendency (the latter must be modified because of changes in the DGP, not the reverse). However, it would be wrong to hypothesise that the stochastic process is exogenous because it is only independent of assumptions concerning the deterministic tendency in DGP. The mechanism that generates this process can be influenced by other economic processes. It is also difficult to decide which trends (stochastic or deterministic) will dominate and whether this domination will be permanent and for how long.

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References

[1] Beveridge S., Nelson Ch., (1981), A new approach to decomposition of economic time series into permanent and transitory components with particular attention to measurement of the business cycle, *Journal of Monetary Economics* 7(2),151–174.

[2] Gosińska E., (2009), Analiza kointegracyjna modelu z zaburzeniami struktury na przykładzie modelu handlu zagranicznego Polski, *Bank i Kredyt* 40(6), 41–58.

[3] Gosińska E., (2015), Modelowanie procesów ekonomicznych generowanych przez niestacjonarne procesy stochastyczne ze zmianą strukturalną, Ph. D. dissertation under supervision of A. Welfe, unpublished manuscript, University of Lodz, Łódź.

[4] Granger C. W. J., Terasvirta T., Patton A., (2006), Common factors in conditional distributions for bivariate time series, *Journal of Econometrics* 132(1), 43–57.

[5] Haldrup N., (1999), An Econometric Analysis of I(2) Variables, [in:] Practical Issues in Cointegration Analysis, [eds.:] M. McAleer, L. Oxley, Blackwell Publishers, Oxford.

[6] Johansen S., (1988), Statistical Analysis of Cointegration Vectors, *Journal of Economic Dynamics and Control* 12, 231–254.

[7] Johansen S., (1995a), *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*, Oxford University Press, Oxford.

[8] Johansen S., (1995b), A Statistical Analysis for I(2) Variables, *Econometric Theory* 11, 25–29.

[9] Juselius K., (2006), *Cointegrated VAR Model. Methodology and Applications*, Oxford University Press, Oxford.

[10] Lütkepohl H., (2005), *New Introduction to Multiple Time Series Analysis*, Springer Verlag.

[11] Majsterek M., (2008), *Wielowymiarowa analiza kointegracyjna w ekonomii*, Wydawnictwo Uniwersytetu Łódzkiego, Łódź.

[12] Paruolo P., (1996), On the Determination of Integration Indices in I(2) Systems, *Journal of Econometrics* 72(1/2), 313–356.

[13] Paruolo P., (2000), A Characterization of I(2) VAR(1) Process, *Econometric Theory* 17, 235–258.

[14] Pesaran H., Shin Y., Smith R., (2000), Structural Analysis of Vector Error Correction Models with Exogenous I(1) Variables, *Journal of Econometrics* 97, 293–343.

[15] Saikkonen P., Lütkepohl H., (2000), Testing for the cointegrating rank of a VAR process with structural shifts, *Journal of Business and Economic Statistics* 18, 451–464.

[16] Trenkler C., Saikkonen P., Lütkepohl H., (2008), Testing for the cointegrating rank of a VAR process with level shift and trend break, *Journal of Time Series Analysis* 29, 331–358.

[17] Wróblewska J., (2015), Common Trends and Common Cycles - Bayesian Approach, *Central European Journal of Economic Modelling and Econometrics* 7(2), 91–110.