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Diversity and Innovation in Economic Evolution

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Abstract

The basic idea of the paper is to apply a multi-attribute notion of diversity proposed by Nehring and Puppe to technological changes appearing as a consequence of innovations in Schumpeter's sense of the term in the production sphere of the economy modelled by the use of the Arrow and Debreu topological apparatus. The paper is inspired by the work of Malawski and Woerter who used Stirling diversity concept to prove that innovative processes are the source of growing diversity in the Schumpeterian vision of economic development. We show that, under certain conditions, nondecreasing multi-attribute diversity in the production sphere of the private ownership economy is a necessary and sufficient condition for the occurrence of innovation in the economy under study.

Keywords: diversity, innovation, technological change

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1 Introduction

Diversity is inseparably connected with evolution. Indeed Witt (1996) writes: "On an abstract level, evolution may be taken to mean the self-transformation of a system through the generation and dissemination of novelty.[...] To talk about evolution in a specific field without knowing the variety, the phenomenological richness which it has produced there would be pointless." Thus, economic development could also be studied in terms of diversity.

One of the founders of an evolutionary approach to economic analysis is the Austrian economist Joseph A. Schumpeter, who considered an economic development as a historical process in which changes are "brought about by innovation, together with all their effects, and the responses to them by economic system" (Schumpeter 1939). Innovations, like "the new consumers' goods, the new methods of production or transportation, the new markets, the new forms of industrial organization that capitalist enterprise creates" (Schumpeter 1950) "illustrate the same process of industrial mutation – if I may use that biological term – that incessantly revolutionizes the economic structure from within, incessantly destroying the old one, creating a new one" (ibid). In Schumpeter's view, economic development is initiated by producers who become innovators and entrepreneurs seeking extraordinary profits which arise from a temporary monopoly position occupied by each innovator until successful imitators are able to enter the market. Development, however, must have a starting point which Schumpeter called a circular flow (Schumpeter 1982). In this case "the same products are produced every year in the same way", consumers' needs and preferences rules and the market is governed by the Walrasian tâtonnement process (ibid). The process of setting of equilibrium at a considered time interval can be regarded as well as one of the stages of Schumpeterian economic evolution (Shionoya 2007), so a circular flow together with economic development should be seen as parts of an evolutionary process as a whole (Lipieta, Malawski 2016b).

Following this line, Andrzej Malawski (1991) initiated the research program for studying Schumpeterian evolution in the Arrow-Debreu set-up by modelling economic development as a specific extension of the Debreu economy with private ownership (Debreu 1959). The set-theoretical and topological apparatus borrowed from general equilibrium theory is what makes this approach essentially different from the mainstream of modern modelling of Schumpeterian evolution (i.e. the neo-Schumpeterian research program and Schumpeterian endogenous growth theory). The Malawski's motivation for using a general equilibrium framework was Schumpeter's vision on economic evolution strongly inspired by Walrasian thinking. Although one can be surprised by the use of methods belonging to a static general equilibrium theory to model economic development, there are ways to consider time in a private ownership economy. It was done for example by Lipieta (2018a, 2018b), through the adjustment processes defined by Hurwicz (see for example Hurwicz 1986). The Malawski's program has been continuously developed in many directions (see for instance: Malawski, Woerter 2006; Malawski 2013; Ciałowicz 2015; Lipieta, Malawski

2016b; Ćwięczek, Lipieta 2018; Lipieta, Malawski 2018).
Diversity occurs in various forms in economic models (Safarzyńska, van den Bergh 2010). Being sometimes a synonym of variety it "denotes the existence of a large number of possibilities or variations. As applied, in particular, to an ecosystem or an economic system, it connotes the existence of many species, many niches, a wide variety of jobs, business opportunities, available products models, landscapes, and so forth" (Ayres 1994). Metcalf (1994) saw variety as a range of actual innovations introduced into the economy. For Saviotti and Pyka (2009) variety was a measure of the extent of differentiation of the economic system. Acemoglu (2011) showed that greater diversity in the competences of researchers increases research directed at substitute varieties and the equilibrium rate of economic growth. Pattanaik and Xu (2000) observed that the extent of diversity among the alternatives in the opportunity set is an important consideration in judging an agent's freedom of choice. Bartkowski (2017) concerned the economic value of biodiversity in ecosystem service provision.
There are many different approaches to measure diversity of an abstract set of objects. If diversity is understood as a pure heterogeneity (i.e. quantitative measure that reflects how many different types there are in a dataset), entropy indices (Rényi 1961) like Shanon-Wiener index (Shannon 1948, Wiener 1948), Berger-Parker index (Berger, Parker 1970) or Simpson index (Simpson 1949), known in the economics as Herfindahl-Hirschman index (Hirschman 1964) are used. However, in most studies on diversity, it is assumed that some information about the similarity and dissimilarity between different alternatives in the set is available and based on it, appropriate diversity indicators are created. This information may take the form of binary similarity relation (Pattanik, Xu 2000), quaternary similarity relation (Bervoets, Gravel 2007) and sometimes numerical values of dissimilarities between objects of the given set are available (Weitzman 1992, 1998). Stirling (1998), (2007) suggested defining diversity through its three properties named variety, balance and disparity. Klaus Nehring and Clemens Puppe proposed in their seminal paper (Nehring, Puppe 2002) a natural and intuitive multi-attribute notion of diversity based on the concept of the feature. Developed by the authors in a series of articles (Nehring, Puppe 2003, 2004a, 2004b) it generalizes Weitzman's approach and has a strong theoretic foundation (the proposed diversity function is a von Neumann-Morgenstern utility function satisfying an additional condition). An overview of alternative approaches to measuring diversity of a set of objects can be found for example in work of Gravel (2008), Nehring and Puppe (2009) or Baumgärtner (2004), where one can also find well-described differences in perspective on diversity between ecologists and economists.
The only contribution to consider diversity and economic development given by innovations in the evolutionary model of Malawski is the work of Malawski and Woerter (2006), where the authors modified Stirling (1998) diversity concept by imposing a hierarchical structure on its three subcategories (called variety, balance and disparity) and suggested to use it to a comparative analysis of changes in

production systems. Although Malawski and Woerter proved that in the Schumpeter's evolutionary economics the diversity is not changing in the circular flow and it is growing in the economic development, they noted, however, that "the causality between innovation and diversity could not be addressed explicitly within this framework"(Malawski, Woerter 2006). Furthermore, a construction of a hierarchical structure of diversity used by the authors seems rather artificial and reveals the most important problem of finding the right measure of diversity in a given context.

We continue Malawski and Woerter's (2006) research on changes in diversity occurring as a result of introducing innovations to the economic system, however, we consider a different approach to diversity, namely the one proposed by Nehring and Puppe in their inspiring paper (Nehring, Puppe 2002). This so-called "multi-attribute approach" to diversity was used by its authors among others to consider diversity as a metric of consumers opportunity (Nehring, Puppe 2008) as well as to model cost complementarities in terms of joint production (Nehring, Puppe 2004a). Its basic idea is to think about the diversity of a given set as derived from the number and weight of the different attributes possessed by its elements (Nehring, Puppe 2009). It refers to the concept of Lancaster (1966), who suggested (in the context of consumer theory) that properties or characteristics of goods are what determines their utility. The idea was then extended by Rosen (1974) on producers and market equilibrium and developed by Mass-Collel in his seminal contribution (Mas-Colell 1975). According to the approach made by Nehring and Puppe one can construct a proper diversity function for a given set of objects (i.e. a set of species living in some area or a set of products available to buy in some supermarket) by considering objects' attributes, which are interesting in a given situation (i.e. being a mammal or being a water animal in case of species; being appropriate for vegetarian or being edible in case of products). The diversity function is constructed by summing weights of attributes possessed by the elements of the given set. These weights reflect a relative importance of considered attributes and the established function can be used to compare a diversity value of ecosystems or a diversity value of given commodity bundles.

In the paper, we adapt the measure of diversity, proposed for a finite-dimensional space by Nehring and Puppe (2002), to the l-dimensional space of commodities and prices. This allows us to measure and compare the diversity of subsets of $\mathbb{R}^l$. We are able therefore to measure diversity of consumption sets as well as diversity of producers' feasible technologies sets. But the main idea of the paper is, considering economic development, to answer the question: whether and how the occurrence of innovation affects the diversity of the production system of the evolving economy. Innovation is here a new technology for the production of a certain (known) good or a technology for the production of a good that was not previously present on the market. Concerning an evolving economy in two points of time, we define innovation as a production plan (a point in $\mathbb{R}^l$) absent in the production system in the first point of time (for example because of lack of appropriate technology or physical resources in the system) but realized by some producer in the second one. To analyze and

compare innovative changes and their "sizes" in the given economy we use a notion of an innovative transformation of a production system (Lipieta, Malawski 2018) together with an innovation index of such a transformation. By diversity function, we gain a useful description of the innovation and changes it introduces to the economic model. We are also able to explain the phenomenon of creative destruction coined by Schumpeter (1982), i.e. the elimination of outdated and useless technologies as a result of technological progress. Indeed, even though useless products and technologies are eliminated from the economic system, the attributes corresponding to their essential features and valuable functions are preserved in the economy. They are so called "relevant attributes" and are taken over by innovations.
Finally, we reach the following conclusions:

1. The introduction of innovation is a necessary condition for the growing diversity of the economy (Proposition 3).

2. The creative destruction does not reduce the diversity of the economic system (Theorem 5).

Diversity is a very intuitive, although precisely formalized concept. When applied to the production system of the economy, it turns out to be a marker of its innovative transformation and can serve as a research tool for economic development. We hope that our results achieved in a general and rigorous model of economy with private ownership can be used as a kind of an equilibrium "microfundations" for some growth theory models, e.g. with increasing technological complexity or growing product variety (Acemoglu 2009).
The paper consists of four sections. In the second one, the notion of diversity proposed by Nehring and Puppe (2009) is introduced. A brief exposition of Debreu economy and its transformation (Lipieta 2018a) is given in Section 3. In Section 4 our main results are stated and proved. The paper is ended with a conclusion.

2 Multi-attribute notion of diversity

We introduce the notion of diversity following Nehring and Puppe (2009). The basic idea underlying this approach is to view the diversity of a set as determined by different features possessed by its elements.
Let X be a finite set and denote by $2^X := \{D : D \subset X\}$ its power set. Let us consider some, important from ones point of view, features of the objects of X. We call a set $A \subset X$ "*an attribute*", if there is a family of features possessed by exactly the objects in A. If $x \in A$, then we say that *an element* $x \in X$ *possesses* the attribute A. When an attribute is possessed by at least one element of a set $S \subset X$, i.e. $A \cap S \neq \emptyset$, then we say that *an attribute* A *is realized by the set* S.

Example 1. *Denote by X a finite set of commodities possible to buy in a given supermarket and let*

$$S = \{vege\ hot\text{-}dog,\ hot\text{-}dog,\ vege\ soap,\ soap\} \subset X.$$

Natural attributes in this case are "being soap" connected with the set $\{vege\ soap,\ soap\}$, "being edible" corresponding to $\{vege\ hot\text{-}dog,\ hot\ dog\}$, "being vege product" that is $\{vege\ soap,\ vege\ hot\text{-}dog\}$, "being gluten free", etc.

Let us assign to every attribute $A \subset X$ a number of its relative importance $\lambda_A \geq 0$, called *the weight of the attribute A*. We say that $A \neq \emptyset$ is *a relevant attribute* if $\lambda_A > 0$. Since X is finite, there are as many as $2^{\#X} - 1$ potentially relevant attributes.

Definition 1 (Nehring, Puppe 2002). *A function $v : 2^X \to \mathbb{R}$ is called a diversity function, if there exists a measure $\lambda : 2^{2^X} \to \mathbb{R}_+$ such that for all $S \subset X$*

$$v(S) = \lambda(\{A \subset X : A \cap S \neq \emptyset\}) = \sum_{A \subset X : A \cap S \neq \emptyset} \lambda_A, \tag{1}$$

where $\lambda_A = \lambda(\{A\})$ and $v(\emptyset) := 0$.

The function $\lambda : A \mapsto \lambda_A$ is referred to as the *attribute weighting function*. The support $\Lambda := \{A \subset X : \lambda_A \neq 0\}$ is called the *family of relevant attributes*.
The diversity value $v(S)$ is the total weight of all attributes realized by S. One can observe that each attribute occurs at most once in the sum (1). Each element $x \in X$ contributes to the diversity of a set S weights of all attributes not possessed by any already existing objects in S (Nehring, Puppe 2002).

Example 2. *Consider X and S as in the previous example. Assume that "being vege product" and "being edible" are the only relevant attributes and both of them have weights equal to 1. Let us calculate for example: $v(\{vege\ soap\}) = v(\{vege\ soap,\ soap\} = \lambda(\{vege\ soap,\ vege\ hot\ dog\}) = 1$; $v(\{hot\ dog\}) = \lambda(\{vege\ hot\text{-}dog,\ hot\ dog\}) = 1$; $v(\{vege\ hot\text{-}dog\}) = v(\{hot\ dog,\ vege\ hot\text{-}dog\}) = \lambda(\{vege\ hot\text{-}dog,\ hot\ dog\}) + \lambda(\{vege\ soap,\ vege\ hot\ dog\}) = v(S) = 2$.*

A diversity function has the following interesting properties (Nehring, Puppe 2002).

Remark 1. *A diversity function is monotonic, i.e. if $S \subset T$, then $v(S) \leq v(T)$, for $S, T \subset X$.*

Remark 2. *A diversity function is submodular, i.e. for $S, T \subset X$, $S \subset T$ for $x \in X$ there is $v(S \cup \{x\}) - v(S) \geq v(T \cup \{x\}) - v(T)$.*

The property above, catching the fact that the marginal diversity decreases in the size of the set, is an interpretation of the intuition that it becomes the harder for an object to add to the diversity of a set the larger that set already is.
A diversity function determines uniquely an underlying attribute weighing function as well as the family of relevant attributes Λ.

Proposition 1 (Nehring, Puppe 2002). *For any function $v : 2^X \to \mathbb{R}$ with $v(\emptyset) = 0$ there exists a unique function $\lambda : 2^X \to \mathbb{R}$, the conjugate Möbius inverse, such that $\lambda_\emptyset = 0$ and, for all $S \subset X$ the value $v(S)$ is given by (1). Furthermore, the conjugate Möbius inverse λ is given by the following formula. For all $A \neq \emptyset$*

$$\lambda_A = \sum_{S \subset A} (-1)^{\#(A \setminus S)+1} \cdot v(X \setminus S).$$

Let a diversity function v be given.
The *distinctiveness of the object $x \in X$ from the set $S \subset X$* is denoted (Nehring, Puppe 2002) by

$$d(x, S) := v(S \cup \{x\}) - v(S) = \sum_{A \subset X : x \in A,\, A \cap S = \emptyset} \lambda_A. \tag{2}$$

For two objects $x, y \in X$ the *dissimilarity* from x to y is defined (Nehring, Puppe 2002) by

$$d(x, y) := d(x, \{y\}) = v(\{x, y\}) - v(\{y\}) = \sum_{A \subset X : x \in A,\, y \notin A} \lambda_A.$$

The dissimilarity from x to y is thus equal to the weight of all attributes possessed by x but not by y. The function $d : X \times X \to \mathbb{R}$ satisfies the triangle-inequality: $d(x, y) + d(y, z) \geq d(x, z)$ for all $x, y, z, \in X$ and is symmetric if and only if $v(\{x\}) = v(\{y\})$ for every $x, y \in X$. It is called (Nehring, Puppe 2002) the *dissimilarity pseudo-metric* associated with a diversity function.
The decision-theoretic foundation of the concept of diversity is proposed by Nehring and Puppe (2002), who suggested looking at the diversity function as von Neumann-Morgernstein utility function (Morgenstern, von Neumann 1953).

3 A model of Debreu economy

We give a brief exposition of Debreu economy following Lipieta and Malawski (2018a). The *l - dimensional space of commodities and prices* is, by definition, a l - dimensional Euclidean space $\mathbb{R}^l = \{(x_1, x_2, ...) : x_k \in \mathbb{R}, \forall_{k>l}\, x_k = 0\}$ with the standard scalar product:

$$x \cdot y = (x_1, x_2, ...) \cdot (y_1, y_2, ...) = \sum_{k=1}^{\infty} x_k y_k, \quad x, y \in \mathbb{R}^l.$$

The prices of commodities are real numbers (positive for rare commodities, equal to zero for free goods and negative for harmful ones).
Two kinds of economic agents (producers and consumers) are operating on the market.

Denote by

$H = \{a_1, ..., a_m\}$ a finite set of consumers, $a_i \neq a_j$ if $i \neq j$, $m \in \mathbb{N}$,

$B = \{b_1, ..., b_n\}$ a finite set of producers, $b_i \neq b_j$ if $i \neq j$, $n \in \mathbb{N}$,

$Pref \subset \mathbb{R}^l \times \mathbb{R}^l$ a family of all preference relations (i.e. a set of reflexive, transitive and complete relations in $\mathbb{R}^l$).

The producers are characterized by a correspondence of production sets:

$$y : B \ni b_j \mapsto Y_{b_j} \subset \mathbb{R}^l,$$

which assigns to every producer $b \in B$ ($b = b_j$, $j = 1, ..., n$) a non empty production set Y_b of the producers feasible production plans. A vector $y_b \in Y_b$ describes technologies used in a production process of a producer $b \in B$ (realized at some period of time t), where the amounts of inputs are given by negative coordinates of the vector y_b, while the amounts of outputs are given by its add coordinates.
The consumers are characterized by

$\chi : H \ni a_i \mapsto \chi(a_i) = X_{a_i} \subset \mathbb{R}^l$ (a correspondence of consumption sets),

$\epsilon : H \ni a_i \mapsto \epsilon(a_i) \in X_{a_i}$ (an initial endowment mapping),

$\varepsilon : H \ni a_i \mapsto \preceq_{a_i} \subset X_{a_i} \times X_{a_i}$ (a correspondence which assigns to every consumer his/her preference relation).

Let $p \in \mathbb{R}^l$ be a price vector.

Definition 2. *A two-range relational system $P_q = (B, \mathbb{R}^l; y, p)$ is called a quasi-production system. If, additionally, for every $b \in B$*

$$\eta_b(p) := \{y_b^* \in Y_b : p \cdot y_b^* = \max_{y_b \in Y_b} p \cdot y_b\} \neq \emptyset,$$

then P_q is called a production system.

In a production system, every producer maximizes his/her profit at given prices and technologies by choosing his/her optimal production plan $y_b^* \in \eta_b(p)$.
In a quasi-production system, instead of aiming at the profit maximization, a producer may undertake some research activities and choose a production plan which offers the possibility of future profits. Such a "quasi" system can serve as a modelling tool for the supply side of an economy to function under bounded rationality (Lipieta, Malawski 2016b).

Definition 3. *A three-range relational system $C_q = (H, \mathbb{R}^l, Pref; \chi, \epsilon, \varepsilon, p)$ is called a quasi-consumption system. If, additionally, for every $a \in H$ ($a = a_i$, $i = 1, ..., m$):*

$$\beta_a(p) := \{x \in X_a : p \cdot x \leq p \cdot \epsilon(a)\} \neq \emptyset,$$

$$\varphi_a(p) = \{x_a^* \in \beta_a(p) : \forall_{x_a \in \beta_a(p)}\ x_a \preceq_a x_a^*,\ \preceq_a \in Pref\} \neq \emptyset$$

then C_q is called a consumption system.

In a consumption system every consumer maximizes his/her preferences on the budget set $\beta_a(p)$. The element $x_a^* \in \beta_a(p)$ is called the optimal plan of a consumer $a \in H$.
The "quasi-type" of consumption system allows a situation where there is no upper bound on the budget set for the preference relation of a consumer. However, if there exists a consumption plan maximizing the preference relation of a given consumer on his/her budget set, then the consumer realizes it (one of his/her best plans) (Lipieta, Malawski 2018).
It is assumed that consumers are the owners of the firms and that their shares in the firms' profits are given by a mapping $\theta : H \times B \to [0,1]$ satisfying

$$\sum_{i=1}^{m} \theta(a_i, b_j) = 1, \ \ j = 1, ..., n.$$

Definition 4. *A relational system $E_q = (\mathbb{R}^l, P_q, C_q, \theta, \omega)$, where*

$P_q = (B, \mathbb{R}^l; y, p)$ is a quasi-production system,

$C_q = (H, \mathbb{R}^l, Pref; \chi, \epsilon, \varepsilon, p)$ is a quasi-consumption system in which the budget set of every consumer $a_i \in H$ is modified to the set

$$\beta_{a_i}(p) = \{x \in X_{a_i} : p \cdot x \leq p \cdot \epsilon(a_i) + \sum_{j=1}^{n} \theta(a_i, b_j)(p \cdot y_{b_j})\},$$

$\omega = \sum_{i=1}^{m} \epsilon(a_i)$

is called a private ownership economy, in short an economy. If additionally P_q is a production system and C_q is a consumption system, then E_q is called Debreu economy and denoted by E.

The private ownership economy operates as follows (Lipieta, Malawski 2018). Let a price vector $p \in \mathbb{R}^l$ be given. Every producer $b \in B$ realizes a production plan $y_b \in Y_b$ and his/her profit by realization of a plan $y_b \in Y_b$ is divided among all consumers according to the function θ. For every consumer $a \in H$:

if $\beta_a(p) \neq \emptyset$ and $\varphi_a(p) \neq \emptyset$, then consumer a chooses a consumption plan $x_a = x_a^* \in \beta_a(p)$ which maximizes his/her preference on the budget set $\beta_a(p)$;

if $\beta_a(p) \neq \emptyset$ and $\varphi_a(p) = \emptyset$, then consumer a chooses a consumption plan $x_a \in \beta_a(p)$ due to his/her own criterion;

if $\beta_a(p) = \emptyset$, then it is assumed that $x_a = 0 \in \mathbb{R}^l$.

An allocation $((x_a)_{a\in H},(y_b)_{b\in B})$ is called *feasible* if

$$\sum_{a\in H} x_a - \sum_{b\in B} y_b - \omega = 0.$$

Let $E = (\mathbb{R}^l, P, C, \theta, \omega)$ be Debreu economy. Then every agent realizes his/her optimal plan, namely every consumer $a \in H$ a plan $x_a^* \in \varphi_a(p)$ and every producer $b \in B$ a plan $y_b^* \in \eta_b(p)$. If allocation $((x_a^*)_{a\in H},(y_b^*)_{b\in B})$ is feasible, then the sequence $((x_a^*)_{a\in H},(y_b^*)_{b\in B},p)$ is called *the state of the Warlas equilibrium of economy E.*

3.1 Innovations in the private ownership economy

Let us consider an economy $E_q = (\mathbb{R}^l, P_q, C_q, \theta, \omega)$, with its quasi-production system $P_q = (B, \mathbb{R}^l; y, p)$. We follow the Schumpeter's vision of economic development and focus on the production sector of an economy as a source of possible innovation.

For comparing changes and analyzing their results in a private ownership economy, we use an idea of a transformation of an economic system (Lipieta, Malawski 2018).

Let $t_0 = 0$ be an initial point of time for observing an evolving economy E_q and let $t, t' \in \{1, 2, ...\}$, $t < t'$.

We call the quasi-production system $P_q = (B, \mathbb{R}^l, y, p)$ *a transformation* of the quasi-production system $P_q' = (B', \mathbb{R}^l, y', p')$ if components of the system P_q at time t are transformed into the components of the system P_q' at time t'. We denote the relationship between P_q and P_q' by $P_q \subset P_q'$.

Let $P_q \subset P_q'$, $B = B'$ and let Y_b be closed for every $b \in B$. From now on we denote $Y := \bigcup_{b\in B} Y_b$, $Y' := \bigcup_{b\in B} Y_b'$.

The following definition is only a slight modification of one presented by Lipieta (2018a).

Definition 5. *A quasi-production system P_q' is called an innovative transformation of the quasi-production system P_q, shortly $P_q \subset_{in} P_q'$, if at time t' there exists at least one innovative production plan with respect to time t, i.e.*

$$P_q \subset_{in} P_q' \Leftrightarrow \exists b \in B\ \exists y_b' \in Y_b' : y_b' \notin Y. \tag{3}$$

In case of an innovative transformation, a new commodity is introduced or new technologies are used by a producer $b \in B$ who become *an innovator*, namely such a producer $b \in B$ who chooses a production plan $y_b' \in Y_b' \backslash Y$. If the innovative changes are observable the economic development has already been started (Lipieta, Malawski 2018).

Let us provide an exposition of a way an economy may evolve. Consider a quasi-production system $P_q = (B, \mathbb{R}^l, y, p)$ of an economy E_q and points of time $t_1 \leq t_2 \leq t_3 \leq t_4$; $t_k \in \mathbb{N}$, $k = 1, 2, 3, 4$. At t_1, an economy operates in the standard way. At t_2, some producer in $b \in B$ chooses his/her optimal production plan $y_b^* \in Y_b$

and makes a profit equal to $p \cdot y_b^* = \max_{y_b \in Y_b} p \cdot y_b$, but he/she decides to allocate a part of it (for example $\frac{1}{r}(p \cdot y_b^*)$, $r > 1$) to undertake some research activities (hoping for future profits). Thus the balance of the economic system is disturbed. At t_3, the same producer $b \in B$ introduces an innovative production plan $y_b' \in Y_b' \backslash Y$ to the market and sets its price according to his/her own criterion (we assume that every innovation is introduced to the market by exactly one producer at the same time). Still, there is no market equilibrium. At t_4, the price of y_b' is corrected by the economic system and, if no innovation is planed or introduced, the state of the market equilibrium can be achieved. For a direct construction of an equilibrium at unchanged prices after a mild evolution of the production sector we refer the reader to Lipieta (2015, 2018a, 2018b).
We will use the following definitions (Lipieta 2018a).

Definition 6. *A quasi-production system P_q' is called an imitative transformation of the quasi-system P_q, shortly $P_q \subset_{im} P_q'$, if P_q' is a transformation of P_q but not an innovative transformation of it.*

The definition of an imitative transformation is equivalent to the condition $Y' \subset Y$. In this case, no new commodity is produced nor new technology is used in the production process.

Definition 7. *A quasi-production system P_q' is called a destruction of the quasi-system P_q, shortly $P_q \subset_{dt} P_q'$, if $Y \backslash Y' \neq \emptyset$ or for some $b \in B$ such that $Y_b \neq \{0\}$ there is $Y_b' = \{0\}$.*

The destruction of a quasi-production system means that some commodity disappears from the market, some technology is no longer used or an activity of some producer ceases.
To measure *whether* and *how much* innovative the given transformation is, we make the following definitions.
Let $P_q \subset P_q'$. Denote for any $b \in B$ and $y_b' \in Y_b'$

$$c_{b,y_b'} := \operatorname{dist}(y_b', Y) := \inf\{\tilde{d}(y_b', \tilde{y}) : \tilde{y} \in Y\}, \tag{4}$$

where $\tilde{d} : \mathbb{R}^l \times \mathbb{R}^l \to \mathbb{R}_+$ is an Euclidean metric. It can be noticed that $c_{b,y_b'} \geq 0$ and $c_{b,y_b'} = \min\{\tilde{d}(y_b', \tilde{y}) : \tilde{y} \in Y\}$ (since Y_b is closed and B is finite).
By a *size of an innovation* introduced by the producer $b \in B$ within the transformation $P_q \subset P_q'$ we mean a non-negative number

$$c_b := \sup\{c_{b,y_b'} : y_b' \in Y_b'\}. \tag{5}$$

If a size of an innovation for $b \in B$ is positive, then $b \in B$ is an innovator. There is no innovator and no innovation in the production system P_q', if $c_b = 0$ for all $b \in B$.

Remark 3. *$P_q \subset_{in} P'_q$ if, and only if, there exist $b \in B$ such that $c_b > 0$.*

Definition 8. *An innovation index of a transformation $P_q \subset P'_q$ is a number*

$$c := \sum_{b \in B} c_b. \tag{6}$$

An innovation index takes values from the set $[0, +\infty)$. There is a following proposition as a consequence of Remark 3 and Definition 8.

Proposition 2. *Let c be an innovation index of a transformation $P_q \subset P'_q$. Then*

$$P_q \subset_{in} P'_q \Leftrightarrow c > 0$$

and

$$P_q \subset_{im} P'_q \Leftrightarrow c = 0.$$

One can extend the notion of an innovative transformation to the entire economic system. Let $E_q = (\mathbb{R}^l, P_q, C_q, \theta, \omega)$, $E_q = (\mathbb{R}^l, P'_q, C'_q, \theta', \omega')$ be economies adequately at points of time t and t', so components of the economy E_q are transformed into the components of the economy E'_q. Then E'_q is called *a transformation of the economy E_q*, and the relationship between E_q and E'_q is denoted by $E_q \subset E'_q$.

Definition 9. *Let $E_q \subset E'_q$. An economy E'_q is said to be an innovative transformation of the economy E_q, shortly $E_q \subset_{in} E'_q$, if $P_q \subset_{in} P'_q$.*

4 Technological diversity in a private ownership economy

Let us consider a private ownership economy $E_q = (\mathbb{R}^l, P_q, C_q, \theta, \omega)$ with its quasi-production system $P_q = (B, \mathbb{R}^l; y, p)$ and a quasi-consumption system $C_q = (H, \mathbb{R}^l, Pref; \chi, \epsilon, \varepsilon, p)$. In the original approach to diversity presented in Section 2 of the paper, diversity function given by formula (1) is defined on the subsets of a finite set X. However, in the economy E_q, objects of the interest are subsets of l-dimensional space of commodities, namely $\mathbb{R}^l$. An attribute A (i.e. a set of commodities defined by the family of features which are possessed by the elements of A and exactly by them) also takes the form of a subset of $\mathbb{R}^l$.
Let us reformulate Definition 1 to fit our case.

Definition 10. *A function $v : 2^{\mathbb{R}^l} \to \mathbb{R}$ is called a diversity function if there exists a measure $\lambda : 2^{2^{\mathbb{R}^l}} \to \mathbb{R}_+$ such that for all $S \subset \mathbb{R}^l$*

$$v(S) = \lambda(\{A \subset \mathbb{R}^l : A \cap S \neq \emptyset\}) = \sum_{A \subset \mathbb{R}^l : A \cap S \neq \emptyset} \lambda_A, \tag{7}$$

where $\lambda_A = \lambda(\{A\})$ and $v(\emptyset) := 0$.

To guarantee that $v(S)$ is well defined for every $S \subset \mathbb{R}^l$ we will assume that a family of relevant attributes $\Lambda = \{A \subset \mathbb{R}^l : \lambda_A \neq 0\}$ is finite.
Let $v : 2^{\mathbb{R}^l} \to \mathbb{R}$ be a diversity function as in Definition 10. Let $P_q \subset P'_q$. The following proposition states that the occurrence of innovation is a necessary condition for increasing diversity of the evolving production system.

Proposition 3. *If $v(Y) < v(Y')$, then $P_q \subset_{in} P'_q$.*

Proof. Let $v(Y) < v(Y')$ and assume to the contrary that $\neg(P_q \subset_{in} P'_q)$. From Definition 5 we have $Y' \subset Y$ and thus $v(Y) \geq v(Y')$, contradiction. □

Let us consider a family of successive transformations of the system $P_q \subset P'_q \subset P''_q$ and denote $Y'' = \bigcup_{b \in B} Y''_b$.

Remark 4. *If $v(Y) < v(Y'')$, then $P_q \subset_{in} P'_q$ or $P'_q \subset_{in} P''_q$.*

There is an immediate conclusion from the Proposition 3 and Definition 6.

Remark 5. *If $P_q \subset_{im} P'_q$, then $v(Y) \geq v(Y')$.*

One of the main paradigms of Schumpeter's thought is that technological progress is made through the mechanism of creative destruction. This process is a kind of an innovative transformation in which an innovative change results in the elimination of old or less competitive technologies, products or even producers from the market.
One can notice that as long as the evolution of the production system is only innovative and not destructive, there is no decrease in the value of diversity of the production sphere of the economy. Namely, from the monotonicity of diversity function and the Proposition 3 we have the following.

Lemma 4. *If $[P_q \subset_{in} P'_q$ and $\neg(P_q \subset_{dt} P'_q)]$, then $v(Y) \leq v(Y')$.*

In the case of the process of creative destruction, we are dealing with certain destructive changes related to the innovative transformation of the production system. Some of the products or technologies previously used turn out to be useless, i.e. the attributes connected with them are irrelevant in P'_q. But the essential features and valuable functions of the disappearing products or technologies are taken over by innovations. To approximate Schumpeter's idea, we will assume that $\Lambda \subset 2^{Y'}$. This condition ensures that the relevant attributes are preserved in the evolution of the economic system and that the essential features and valuable functions of the disappearing products or technologies are transferred to the new ones in P'_q.

Theorem 5. *Let $P_q \subset P'_q$, and let $v : 2^{\mathbb{R}^l} \to \mathbb{R}$ be a diversity function as in Definition 10 and such that $\Lambda \subset 2^{Y'}$. Then*

$$P_q \subset_{in} P'_q \Rightarrow v(Y) \leq v(Y').$$

Proof. Let $P_q \subset_{in} P'_q$ and assume to the contrary that $v(Y) > v(Y')$. Then from Definition 10 there is such an attribute $A \subset \mathbb{R}^l$, $\lambda_A > 0$ that $A \cap Y \neq \emptyset$ and $A \cap Y' = \emptyset$. But $\Lambda \subset 2^{Y'}$, what means $A \subset Y'$, contradiction. □

Theorem 5 provides a necessary condition for an innovative transformation of the production system, namely, that a diversity does not decrease in the set of existing technologies. It may be true that $P_q \subset_{in} P'_q$ and $v(Y') < v(Y)$ if more weight is assigned to products and technologies that are definitely eliminated from the market within the transformation of the system, than it is assigned to the new attributes connected with innovations, i.e. if

$$\lambda(\{A \subset \mathbb{R}^l : A \cap Y \neq \emptyset,\, A \cap Y' = \emptyset\}) > \lambda(\{A \subset \mathbb{R}^l : A \cap Y = \emptyset,\, A \cap Y' \neq \emptyset\}),$$

although in such case we are dealing with destruction which can hardly be called creative.

In Example 2, we adopted a point of view of a consumer, who recognizes some attributes as relevant ones by assigning his/her own subjective weights to them, so resulting diversity function corresponds to his/her own preferences. There are many ways to link the family of consumer's relevant attributes Λ^i to his/her preference relationship. For $x \in X_{a_i}$ an attribute may take a form of a single commodity bundle $\{x\}$ or an indifference set $\{\tilde{x} \in X_{a_i} : x \preceq_{a_i} \tilde{x} \text{ and } \tilde{x} \preceq_{a_i} x\}$ of a given preference relation $\preceq_{a_i} \subset X_{a_i} \times X_{a_i}$. In a much simpler case of X being a finite set, Nehring and Puppe (2008) considered for every consumer $i = 1, ..., m$ and any $x \in X_{a_i}$ the associated *level set* $A_i(x) = \{\tilde{x} \in X_{a_i} : x \preceq_{a_i} \tilde{x}\}$ and observed that family $\Lambda^i = \{A_i(x)\}_{x \in X_{a_i}}$ represents a structure of a line model (Nehring, Puppe 2002). The result stays true in our case under assumption $\lambda(X_{a_i}) < +\infty$ for every $i = 1, ..., m$.

A diversity function may correlate with consumer's preferences through the attribute weighting function. Assume that every consumer $a_i \in H$ $(i = 1, ..., m)$ recognizes a finite number of attributes as important ones from his/her point of view and that he/she assigns weights to them according to his/her own preferences. (For example every consumer has a certain *ideal* consumption plan which possesses attributes of some finite family.) Denote the consumer's $a_i \in H$ attribute weighting function by $\lambda^i : 2^{2^{\mathbb{R}^l}} \to \mathbb{R}_+$. Then for a given consumer $a_i \in H$ and $S \subset \mathbb{R}^l$ we define consumer's a_i diversity function by

$$v^i(S) = \lambda^i(\{A \subset \mathbb{R}^l : A \cap S\} \neq \emptyset\}) = \sum_{A \subset \mathbb{R}^l : A \cap S \neq \emptyset} \lambda^i_A.$$

In particular, for a commodity bundle $x = (x_1, x_2, ..., x_l) \in X_{a_i}$

$$v^i(\{x\}) = v^i((x_1, x_2, ..., x_l)) = \lambda^i(\{A \subset \mathbb{R}^l : A \cap \{x\} \neq \emptyset\}) = \sum_{x \in A} \lambda^i_A.$$

By choosing $\Lambda := \Lambda^1 \cup ... \cup \Lambda^m$ and $\lambda_A := \max\{\lambda_A^1, ..., \lambda_A^m\}$ we get a diversity function given by formula (7).
Although the discussion on the relationship between preferences and the relevant attributes is very interesting (as well as reflecting on the problem of aggregating individual preferences), it is not the focus of the paper. Our research is general in nature and does not depend on the way we connect consumer preferences to a family of relevant attributes.
Let v be a diversity function given by Definition 10. Let $P_q \subset P'_q$. According to (2) the distinctiveness of the production plan $y_b \in Y'_b$ of a producer $b \in B$ from the set Y is given by

$$d(y'_b, Y) = v(Y \cup \{y'_b\}) - v(Y) = \sum_{A \subset \mathbb{R}^l:\, y'_b \in A,\; A \cap Y \neq \emptyset} \lambda_A.$$

If $d(y'_b, Y) > 0$, i.e. if there is a relevant attribute possessed by $y'_b \in Y'_b$ and not realized by Y, we call y'_b a *relevant innovation*. Let us refer to the number $d_b := \sup\{d(y'_b, Y) : y'_b \in Y'_b\}$ as a *size of the relevant innovation* of a producer $b \in B$. Let $b \in B$. The relationship between the size of the innovation (c_b) and the size of the relevant innovation (d_b) is as follows.

Remark 6. *If $d_b > 0$, then $c_b > 0$.*

Due to Theorem 5 there is a connection between the increase in diversity and the sign of an innovation index c of the transformation $P_q \subset P'_q$.

Corollary 6. *Under the hypotheses of Theorem 5*

$$v(Y') > v(Y) \Leftrightarrow c > 0.$$

The following reformulation of Theorem 5 using Definition 9 provides the criterion of innovativeness of a given economy.

Theorem 7. *Let $E_q \subset E'_q$ and let $v : 2^{\mathbb{R}^l} \to \mathbb{R}$ be a diversity function such that $\Lambda \subset 2^{Y'}$. Then*

$$E_q \subset_{in} E'_q \Leftrightarrow v(Y) \leq v(Y').$$

4.1 Technological diversity and the wealth of economy with successful innovators

Let us look more closely at the relation between technological diversity and the wealth of a transforming economy. Considering an innovative transformation of a production system in Section 3.1, we assumed that the price of a new product is given by the innovator in a purely monopolistic way "according to his/her own criterion" and (implicitly) covers the cost of the innovation. Then the given price is corrected by

the market and, if no innovation is planed or introduced, the state of the market equilibrium can be achieved. But until successful imitators are able to enter the market, there arises an extraordinary profit from a temporary monopoly position occupied by the innovator.
Let $P_q \subset P'_q$.

Definition 11 (Lipieta, Malawski 2016b). *A producer $b^\star \in B$ is called a successful innovator if $Y'_{b^\star} \backslash Y \neq \emptyset$ and $\exists y'_{b^\star} \in Y'_{b^\star}$ such that $p \cdot y_b < p' \cdot y'_b$ for all $y_b \in Y$, $b \in B$.*

From the definition above and Theorem 5 we have the following conclusion.

Proposition 8. *Let $P_q \subset P'_q$, and let $v : 2^{\mathbb{R}^l} \to \mathbb{R}$ be a diversity function as in Definition 10 and such that $\Lambda \subset 2^{Y'}$. If there exists a successful innovator $b^\star \in B$, then $v(Y) \leq v(Y')$.*

Let us consider an economy $E_q = (\mathbb{R}^l, P_q, C_q, \theta, \omega)$. Let $x_a \in X_a$, $a \in H$ be consumption plans realized by the consumers and $y_b \in Y_b$, $b \in B$ be production plans chosen by the producers at a given prices $p \in \mathbb{R}^l$.
The *total wealth of economy* E_q at state $((x_a), (y_b), p)$ is, by definition (Lipieta, Malawski 2018), a number

$$w := \sum_{a \in H} p \cdot x_a. \tag{8}$$

Denote the *index of the total wealth of the economy* E_q at state $((x_a), (y_b), p)$ by

$$W := w - p \cdot \omega,$$

where $\omega = \sum_{a \in A} \epsilon(a)$. Keeping in mind that every consumer $a \in A$ chooses his/her consumption plan x_a as the best from the set

$$\beta_a(p) = \{x \in X_a : p \cdot x \leq p \cdot \epsilon(a) + \sum_{a \in H,\, b \in B} \theta(a, b)(p \cdot y_b)\},$$

it can be shown that

$$W = \sum_{b \in B} p \cdot y_b.$$

Let E'_q be an innovative transformation of the economy E_q and let us denote by W and W' the total wealth indices of E_q and E'_q, respectively. Assume that there exists a successful innovator $b^\star \in B$. The wealth of the economy within the transformation $E_q \subset_{in} E'_q$ may rise, namely $W' - W > 0$, if the profit of the successful innovator compensates for the possible losses of other producers within such transformation. If the profits from innovation are not big enough, the activity of innovators on the market may not result in the increase in the total wealth of the economy E_q. There is no direct relationship between the change in diversity of the production system of the economy and the increase of its wealth resulting from Proposition 8.

5 Conclusions

The paper extends the research program of modelling Schumpeterian evolution in the Arrow-Debreu set up. The measure of diversity proposed by Nehring and Puppe (2002) is adapted to the space of commodities and prices $\mathbb{R}^l$. It is shown that the occurrence of innovation affects the diversity of the production system of the evolving economy, thus an innovative transformation of an economy is associated with the growth of its technological diversity. A detailed description of a phenomenon of creative destruction in terms of diversity is proposed. The answer to the question of why creative destruction does not reduce the diversity of an economic system under evolution is suggested.

One may consider an impact of the demand sphere of economy on innovative processes. In the Arrow-Debreu set-up Ciałowicz (2015) refers an innovativeness of consumers to the predisposition to adopt innovative commodities in the consumption plans. Since some of the consumers are characterized by a pro-innovative preference relation, the weights of attributes connected with innovation may grow. It would be interesting to observe how innovative changes in such a case affect the diversity of the economy.

There is some contribution connecting attributes and product prices. Rosen (1974) analysed model of product differentiation based on the hedonic hypothesis that goods are valued for their utility-bearing characteristics (called also attributes). He considerd a market for a class of differentiated commodities that are completely described by a vector of n objectively measured characteristics $z = (z_l, z_2, ..., z_n)$, where a coordinate z_i measures the amount of the i-th characteristic contained in some good. The value of the good $p = w \cdot z$ is then given by the prices of characteristics $w = (w_1, ..., w_n)$. Färe, Grosskopf and Margaritis (2019) use Shepard dual lemma (Shephard 2012) to derive shadow prices of property characteristics, from observed data p and z. The idea of using the distance functions to price innovation is promising and we leave it to be explored in the future.

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